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Comparing the Danish and the Ontario Exchanges.

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ABSTRACT

This paper analyzes the design of the Danish milk quota exchange. We focus on the restriction that each producer can only submit a single bid (a quantity and a price limit). We argue that this restriction creates inefficiencies for two reasons. First, a single bid cannot express a buyer's downward sloping demand curve (the aggregation effect). Second, the buyers minimize the risk of foregoing profitable trade by submitting their average valuation rather than their marginal valuation of quota (the uncertainty effect). We use data from the (multiple bids) Ontario milk quota exchange, to evaluate the empirical impact of a single bid restriction.

Keywords: Quota exchange, single bid, multi-unit, double auctions, efficient trade.

1 Introduction

The EC introduced milk quotas to reduce the supply of milk back in 1984. Since then Denmark has allowed the quotas to be traded in different ways. Until 1997, quotas were traded along with farm land and bought and redistributed by the Danish Dairy Board. In 1997, a milk quota exchange was introduced to facilitate efficient reallocation of milk quotas and to reduce transaction costs related to the searching and matching. This paper analyzes the design of the Danish milk quota exchange.

Quota programs play an important role in agricultural policy and have generated a large literature. A number of empirical studies have computed potential efficiency gains from establishing a free market for quotas. Using simulation, Rucker et. al. (1995) calculated the deadweight loss from restrictions in the transferability of tobacco quotas in North Carolina. Ewasechko and Horbulyk (1995) and Lambert et. al. (1995) calculated potential efficiency gains from re-allocation of milk quotas across provinces in Canada. Boots et. al. (1997) estimated the cost of quantity restrictions in Dutch milk quota trade.

The theoretical literature on auctions is large as well. A recent survey is Klemperer (1999). The focus is typically on one-sided auctions, where a monopolist chooses the auction rules to maximize expected revenue. However some of the most important markets are governed by two-sided auctions, also referred to as double auctions or exchanges. Several empirical studies and laboratory experiments have shown that the double auction institution is very stable. Test auctions with as few as 2-3 buyers and 2-3 sellers have generated almost efficient outcomes (Friedman, 1984). This suggests that – if one uses an appropriate design of the auction rules – the double auction generate an efficient allocation even with very few participants.

In the Danish milk quota exchange the producers can only submit one bid each. A similar single bid restriction is found on the German milk quota exchanges (Bundesministerium für Ernährung, Landwirtschaft und Forsten, 2000). The contribution of this paper is to analyze the impact of this restriction both theoretically and empirically.

The single bid restrictions creates distortions in two ways. Firstly, a buyer cannot express a downward sloping demand curve by submitting a single bid (the aggregation effect). Secondly, the buyers try to reduce the risk of foregoing profitable trade by submitting high bids (the uncertainty

effect). In a similar manner, distortions are introduced on the seller's side. The aggregation effect and the uncertainty effect lead to inefficient trade on the Danish milk quota exchange.

We also show how the efficiency of the Danish milk quota exchange can be improved by allowing the producers to submit multiple bids. This corresponds to the trading rules on the Canadian milk quota exchanges, where the producers can submit multiple bids. We use data from the milk quota exchange in Ontario to evaluate the effect of imposing a single bid restriction on a milk quota exchange. This allows us to quantify the likely distortion generated by the single bid restriction on the Danish exchange.

The remainder of this paper is organized as follows. Section 2 describes the Danish milk quota exchange. In Section 3, we analyze the effects of single bid restrictions and in Section 4, we show how the distortions on the Danish milk quota exchange can be avoided by allowing the producers to submit multiple bids. The empirical impact of a single bid restriction is evaluated in Section 5. We discuss our assumptions and results in Section 6. Section 7 concludes the paper.

2 The Danish Milk Quota Exchange

The producers can only give a *single bid* for buying and a *single ask* for selling at the Danish quota exchange. We refer to this as *the single bid restrictions*. The basic trading rules can be summarized as follow:

- A seller submits one ask, i.e. the quantity he wants to sell and the minimum price he requires.

- A buyer submits one bid, i.e. the quantity he wants to buy and the maximum price he is willing to pay.

Based on all bids a single clearing price is found and:

- Buyers with maximum prices above or equal to the clearing price buy their requested quantity at the clearing price.

- Sellers with minimum prices below or equal to the clearing price sell their offered quantity at the clearing price.

Figure 1: The Quota Exchange
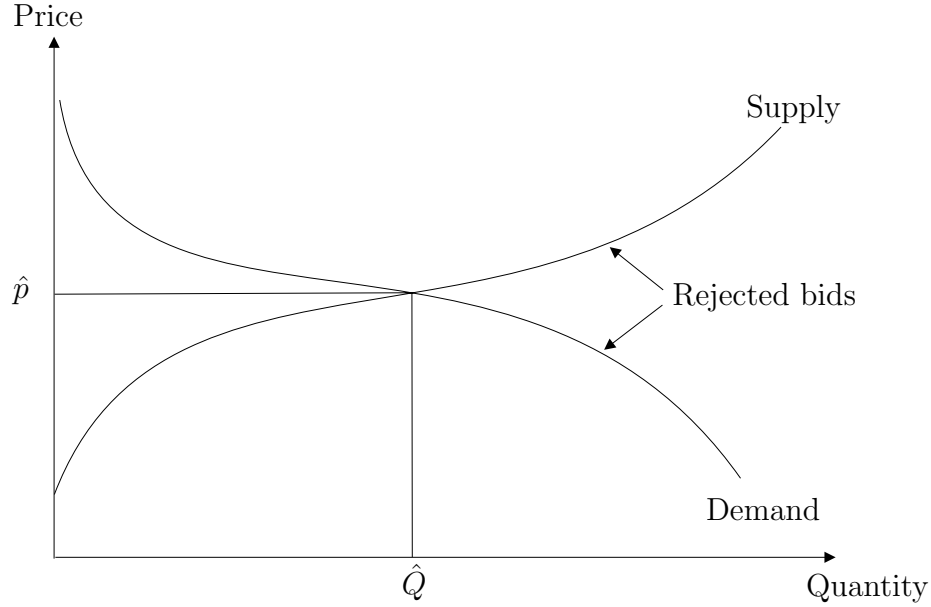
- All other bids and asks are rejected.

The Danish dairy board runs the exchange (i.e. operates the clearing house). In order to always clear the exchange using the above rules, the Danish dairy board holds a small buffer of quota. This means that buyers bidding exactly the clearing price always buy their requested quantity, even if there is excess demand for quotas at the market clearing price. Similarly, sellers asking exactly the clearing price always sell their offered quantity.

The exchange is illustrated in Figure 1 where $\hat{p}$ clears the exchange and $\hat{Q}$ is redistributed on this exchange.

The Danish milk quota exchange can be characterized as a *single price sealed bid double auction*. I.e. producers submit sealed bids and asks and all trade is settled at a single price.

In addition to the single bid restriction buyers are subject to quantitative restrictions. Although the effects of the quantitative restrictions are interesting, we restrict our analysis to the single bid restrictions.

3   Single Bid Exchange

In a perfect market all wealth enhancing trades are realized. All producers with marginal valuation of quota below the market price sell quota, and all producers with marginal valuation of quota above the market price buy quota.

Restricting the producers to submit only a single sealed bid and a single sealed ask has two effects:

The aggregation effect: The single bid restriction limits the information transmitted through the exchange. A buyer cannot express a downwards sloping demand curve and a seller cannot express an upwards sloping supply curve.

The uncertainty effect: The uncertainty about the clearing price systematically affects the producers' bids and asks.

3.1   The Aggregation Effect

Let $Z_B(q)$ be the price a buyer bids and $Z_S(q)$ be the price a seller asks for the quantity q. We show in section 3.2 how $Z_B(q)$ and $Z_S(q)$ are determined. For now, we assume that $Z'_B(q) \leq 0$ and $Z'_S(q) \geq 0$, i.e. buyers have downward sloping bid functions and sellers have an upward sloping bid functions.

Due to the single bid restriction a producer must choose a single point on $Z_B(q)$ as his bid and a single point on $Z_S(q)$ as his ask.

Once a producer has observed the market clearing price, he may want to change his bid or ask.

- A buyer who bid above the clearing price (and had his bid accepted) may wish that his bid was submitted on a larger quantity.

- A buyer who bid below the clearing price (and had his bid rejected) may wish that his bid was submitted at a lower quantity – offering a higher price so that his bid would have been accepted.

Hence, the demand curve revealed on the milk quota exchange is below the horizontal aggregation of the buyers' bid functions, $Z_B(q)$.

There are similar effects on the supply side:

- A seller who asked for a lower price than the clearing price (and had his ask accepted) may wish that his bid was submitted on a larger quantity.

- A seller who asked for a higher price than the clearing price (and had his ask rejected) may wish that his bid was submitted on a lower quantity (asking a lower price) so that his bid would have been accepted.

Hence, the supply curve revealed on the milk quota exchange is above the horizontal aggregation of the sellers bid functions, $Z_S(q)$. The aggregation effects are illustrated in Figure 2.

Figure 2: Single bid versus multiple bids auction
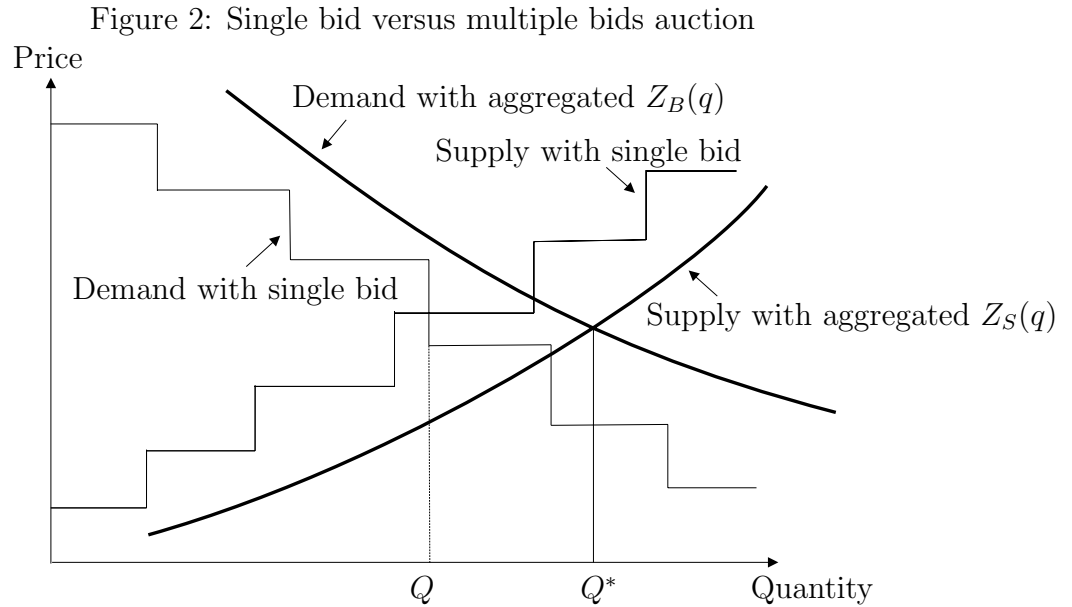
It is straightforward to see that the single bid restriction leads to less trade than a market clearing with the buyers' and sellers' horizontally aggregated bid functions. The reason is that a single bid does not transmit information about the quantity a buyer requests for prices above the bid price. Neither does a single bid transmit information about the additional quantity a buyer requests for prices below the bid price. Of course, the argument can be repeated on the sellers' side.

3.2 The Uncertainty Effect

In this section, we show how a rational producer chooses the bid and ask he submits on an exchange with a single bid restriction. That is, we determine a producer's bid functions $Z_B(q)$ and $Z_S(q)$, respectively. We also show which point on $Z_B(q)$ a buyer submits as his bid and on $Z_S(q)$ for a seller.

The uncertainty about the clearing price affects the producers' bid function. The producers try to minimize the risk of foregoing profitable trade. The producers can forego profitable trade either because their bid/ask is rejected or because they buy/sell too little on the exchange.

We make the following assumptions:

- the producers are price-takers, i.e. the individual producer cannot influence the clearing price.

- the producers are risk neutral, i.e. the individual producer maximize the expected value of his bid and ask.

- the producers have ex ante beliefs about the clearing price. These beliefs are described by a density function $f(\hat{p})$ where $\hat{p}$ is the clearing price. Our model does not require that all producers have the same beliefs, but to simplify the notation, we suppress the producer identity.

- the producers' valuation of quota are independent. This means that we do not consider the problem of winners curse[1] in our model.

In Section 6, we discuss the implications of our assumptions. we now model the milk quota exchange as a one-shot single price sealed bid double auction.

3.2.1 The Buyer's Problem

Consider a buyer with an inverse demand function $P_B(q)$. That is, $P_B(q)$ is the marginal value of quota when q units are acquired. If a bid of quantity q is accepted at a clearing price $\hat{p}$, the profit to the buyer is

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[1]The winners curse is the tendency that the producers who end up buying milk quota at the quota exchange are those who overestimate the profitability of milk production.

$$\int_0^q (P_B(x) - \hat{p})\, dx \tag{3.1}$$

The bid is accepted as long as the bid price p exceeds the clearing price $\hat{p}$. The buyers expected value, $\pi_B(p, q)$, from bidding (p, q) is therefore

$$\pi_B(p, q) = \int_0^p \left(f(\hat{p}) \int_0^q (P_B(x) - \hat{p})\, dx \right) d\hat{p} \tag{3.2}$$

The first order condition for an optimal bid price p can now be expressed as:[2]

$$\frac{\partial \pi_B}{\partial p} = f(p) \int_0^q (P_B(x) - p)\, dx = 0 \iff$$

$$\int_0^q P_B(x)dx = p \cdot q \iff$$

$$A_B(q) = p \tag{3.3}$$

where $A_B(q)$ is the average value, $\int_0^q P_B(x)dx/q$, of the quota q.

The first order condition for the quantity component q of the optimal bid is:

$$\frac{\partial \pi_B}{\partial q} = \int_0^p f(\hat{p})\, (P_B(q) - \hat{p})\, d\hat{p} = 0 \iff$$

$$\frac{\int_0^p \hat{p} \cdot f(\hat{p})d\hat{p}}{\int_0^p f(\hat{p})d\hat{p}} = P_B(q) \iff$$

$$E(\hat{p}|\hat{p} \le p) = P_B(q) \iff$$

$$D\left(E(\hat{p}|\hat{p} \le p)\right) = q \tag{3.4}$$

where $E(\hat{p}|\hat{p} \le p))$ is the expected clearing price when the bid is accepted and $D(\cdot)$ is the demand function, i.e. $D(p) = P_B^{-1}(q)$.

A buyer must solve the two first order conditions (3.3) and (3.4) simultaneously. Figure 3 illustrates this.

The first order condition for the price (3.3) states that the buyers' price strategy is to bid the average value and not the marginal value of quota.

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[2]Here and below assume that $\pi(\bullet)$ satisfy sufficient regularity conditions to ensure that the following rules apply: $\frac{\partial \int_a^b F(x,y)dx}{\partial y} = \int_a^b \frac{\partial F(x,y)}{\partial y}dx$, $\frac{d \int_a^b F(x,y)dx}{da} = -F(a)$, and $\frac{d \int_a^b F(x,y)dx}{db} = F(b)$, for the appropriate integrand F.

Figure 3: The buyer's problem
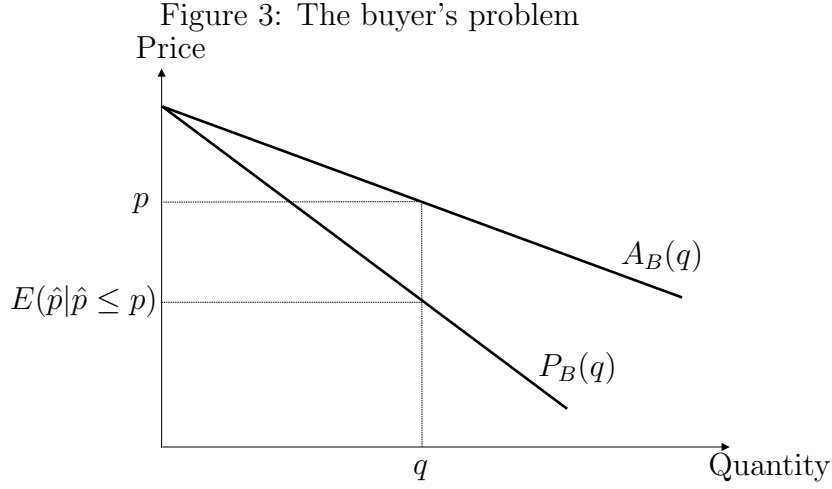
To understand this note that a producer wanting to buy quota risks ending up not buying at all on the quota exchange if his bid is too low. There is no benefit from bidding below the average value of a given quota, because the producers are price-takers and cannot affect the clearing price. On the other hand, the additional trade a producer can obtain by bidding a price above the average valuation of quota is unprofitable. Thus, the buyers deal with the uncertainty about the clearing price by bidding their average value of quota instead of their marginal value of quota.

We may summarize this in a proposition.

Proposition 3.1. *Consider a sealed bid single price exchange with a single bid restriction. A rational buyer uses his average value of quota as bid price:*

$$Z_B(q) = A_B(q) \tag{3.5}$$

and chooses the quantity, by inserting the expected price, conditional on the bid being accepted, into his demand function:

$$D\left(E(\hat{p}|\hat{p} \leq p)\right) = q \tag{3.6}$$

When the demand curve is downward sloping, the buyers price bid for a given quantity i.e. the average value of quota, is *higher* than the marginal value of quota $P_B(q)$. The uncertainty effect – therefore tends to increase the revealed demand.

3.2.2 The Seller's Problem

The solution to the seller's problem is symmetric to that of the buyer's problem. We use $P_S(q)$ as the inverse function of the seller's supply function.

If an ask of quantity q is accepted at clearing price $\hat{p}$, the profit to the seller is

$$\int_0^q (\hat{p} - P_S(x)) \, dx \tag{3.7}$$

Using a bid price of p, his bid is accepted whenever $\hat{p} \geq p$, and his resulting expected profit is therefore:

$$\pi_S(p, q) = \int_p^\infty \left(f(\hat{p}) \int_0^q (\hat{p} - P_S(x)) \, dx \right) d\hat{p} \tag{3.8}$$

The seller maximizes the expected profit.

The first order condition for an optimal ask price is:

$$\frac{\partial \pi_S}{\partial p} = -f(p) \int_0^q (p - P_S(x)) \, dx = 0 \iff$$

$$\int_0^q P_S(x) dx = p \cdot q \iff$$

$$A_S(q) = p \tag{3.9}$$

where $A_S(\cdot) = \int_0^q P_S(x) dx / q$ is the seller's average value of quota.

The first order condition for the quantity component of the bid is:

$$\frac{\partial \pi_S}{\partial q} = \int_p^\infty f(\hat{p}) (\hat{p} - P_S(q)) \, d\hat{p} = 0 \iff$$

$$\frac{\int_p^\infty \hat{p} \cdot f(\hat{p}) d\hat{p}}{\int_p^\infty f(\hat{p}) d\hat{p}} = P_S(q) \iff$$

$$E(\hat{p} | \hat{p} \geq p) = P_S(q) \iff$$

$$S\left(E(\hat{p} | \hat{p} \geq p)\right) = q \tag{3.10}$$

where $E(\hat{p} | \hat{p} \geq p)$ is the expected price given that the ask is accepted and $S(\cdot)$ is the usual supply function $S = P_S^{-1}$.

The seller must solve the two first order conditions (3.10) and (3.9) simultaneously. This is illustrated in Figure 4 below.

Figure 4: The seller's problem
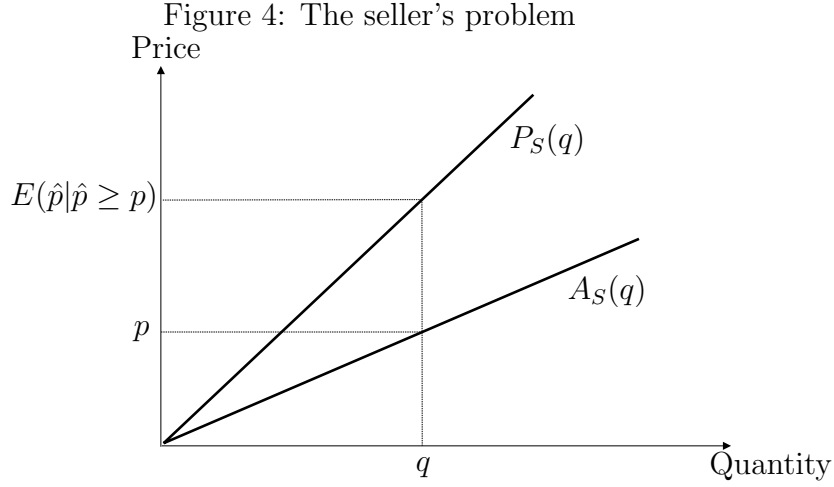
The first order condition for the price (3.9) shows that the seller's price strategy is to ask the average as opposed to the marginal value of quota. To understand this, observe that if he submit an ask price above his average value, he simply risk foregoing profitable trade. If he submits an ask below his average value, the additional trade this will generate (relative to asking the average value) is unprofitable, since he receives less for the quota than it is worth to him. Hence, it is optimal for the producer to use the average value of quota as his ask price.

We summarize the characteristics of the seller's solution in a proposition as well.

Proposition 3.2. *Consider a sealed bid single price exchange with a single bid restriction. A rational seller uses his average value of quota as ask price:*

$$Z_S(q) = A_S(q) \tag{3.11}$$

and chooses the quantity by inserting the expected price, conditional on the ask being accepted, into his supply function:

$$S\left(E(\hat{p}|\hat{p} \geq p)\right) = q \tag{3.12}$$

With an increasing supply curve, the seller's price demand for a given quantity of quota is below the marginal value of quota $P_S(q)$. The uncertainty effect therefore – as opposed to the aggregation problem – tend to increase the revealed supply.

3.3 Numerical example

We now give a simple example of the model presented above. We only illustrate the buyer's problem, since the buyer's and the seller's problems are symmetric. In the example, the producer's demand function is $P_B(q) = 1 - q$, and we assume that the producer's beliefs about the market clearing price is uniform distributed $\hat{p} \sim \text{Uni}[0; 1]$.

Using an uniform distribution simplifies the analysis, since all possible market clearing prices are weighted equally, that is: $f(\hat{p}) = 1$ for all $\hat{p} \in [0, 1]$. Using equation (3.2), the buyer's problem is given by:

$$\max_{p,q} \int_0^p \int_0^q ((1 - x) - \hat{p}) \, dx d\hat{p} \tag{3.13}$$

The two first order conditions yield:

$$1 - q = E(\hat{p}|\hat{p} \leq p) \iff$$

$$1 - q = \frac{\int_0^p \hat{p} d\hat{p}}{\int_0^p 1 d\hat{p}} = \frac{1}{2}p \iff$$

$$q = 1 - \frac{1}{2}p \tag{3.14}$$

and

$$\frac{\int_0^q (1 - x) \, dx}{q} = p \iff$$

$$1 - \frac{1}{2}q = p \tag{3.15}$$

The solution to this system of equations is: $p = \frac{2}{3}$ and $q = \frac{2}{3}$.

To emphasize the uncertainty effect, consider variations in the producer's belief. Let $\hat{p} \sim \text{Uni}[\varepsilon; 1 - \varepsilon]$ such that a large value of $\varepsilon \in [0, \frac{1}{2}[$ indicates that the buyer has good price projections. Now the density function, $f(\hat{p})$, is $\frac{1}{1-2\varepsilon}$, but the expected equilibrium price is unchanged. In our analysis we now have to use $\min\{p, 1 - \varepsilon\}$ as the upper limit for our integrals. The two first order conditions yield:

$$1 - q = E(\hat{p}|\hat{p} \leq p) \iff$$

$$1 - q = \frac{\int_{\varepsilon}^{\min\{p, 1-\varepsilon\}} \hat{p} \frac{1}{1-2\varepsilon} d\hat{p}}{\int_{\varepsilon}^{\min\{p, 1-\varepsilon\}} \frac{1}{1-2\varepsilon} d\hat{p}} = \frac{\min\{p, 1 - \varepsilon\} + \varepsilon}{2} \iff$$

$$1 - q = \frac{\min\{p + \varepsilon, 1\}}{2} \tag{3.16}$$

and

$$\frac{\int_0^q (1 - x)\, dx}{q} = p \iff$$

$$1 - \frac{1}{2}q = p \tag{3.17}$$

Solving for p and q we get the optimal bid as: $p = \min\{\frac{2+\varepsilon}{3}, \frac{3}{4}\}$, $q = \max\{\frac{2-2\varepsilon}{3}, \frac{1}{2}\}$.

Hence, with a smaller uncertainty the buyer bids a higher price and a smaller amount. In the limit $(\varepsilon \to \frac{1}{2})$ there is no uncertainty. The buyer demands $q = \frac{1}{2}$ and offers $p = \frac{3}{4}$. In fact this property holds for all $\varepsilon \geq \frac{1}{4}$ $(E(\hat{p}|\hat{p} \geq p) = E(\hat{p})$, if $p \geq 1 - \epsilon)$. Since he does not influence the equilibrium price by his bid any bid above $p = \frac{1}{2}$ is accepted and he might as well submit his average valuation.

4   Multiple Bids Exchange

The analysis in Section 3 illustrates that the single bid restriction creates distortions (the aggregation effect and the uncertainty effect). In this section we show that allowing for multiple bids removes these distortions.

In D. Nautz (1995): "Optimal bidding in multi-unit auctions with many bidders" a similar situation is analyzed. Nautz analyze a discrete auction where the seller (mechanism designer) sets a grid of prices $p_0 < p_1 < \ldots < p_{k+1}$. The seller invites bidders to submit their bids in the form of demand schedules: $B(p_0) \geq B(p_1) \geq \ldots \geq B(p_{k+1})$. $B_i = B(p_i)$ states how many units the bidder is willing to buy at p_i. That is, the bidders can submit multiple bids. Nautz proves that the optimal strategy for the bidders is to bid their true demand function, i.e. $B(p) = D(p)$.

We shall now generalize Nautz's result to a continuous[3] auction where both buyers and sellers can submit multiple bids. We assume that the pro-

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[3]That is, the clearing price can take a continuum of values.

ducers are price-takers and risk neutral and that the producers' private valuations of quota are independent. The inverse bid functions $B_B(\hat{p})$ and $B_S(\hat{p})$ on the clearing price, $\hat{p}$, state how large a quota the producer wants to buy/sell at price $\hat{p}^4$.

A buyer chooses the inverse bid function which maximizes his expected value of trading on the auction. The expected value of trading on the auction is the value of buying/selling the quantity $B(\hat{p})$ at the price $\hat{p}$ weighted by the probability of $\hat{p}$ being the clearing price. In order to maximize the expected value of trading on the auction, the producers must maximize the value of trading for each value of $\hat{p}$. This is done by bidding the demand function. The reason is that the demand function states the quantity which maximizes the value of trading at a given price. Hence, when the buyers can submit multiple bids it is optimal for them to submit their demand functions. The intuition on the sellers side is similar.

Proposition 4.1. *In a continuous single price sealed bid auction, the true demand and supply functions are the optimal inverse bid function:*

$$B_B(p) = D(p) \quad and \quad B_S(p) = S(p) \tag{4.1}$$

Proof. A buyer maximizes the expected value of trading on the auction by solving the following problem:

$$\max_{B_B(\cdot)} \pi_B(B_B(\hat{p})) = \max_{B_B(\cdot)} \int_0^\infty \left(f(\hat{p}) \int_0^{B_B(\hat{p})} (P_B(x) - \hat{p})\, dx \right) d\hat{p} \tag{4.2}$$

where $B_B(\hat{p})$ is the inverse bid function which defines the quantity the producer wants to buy for each $\hat{p}$, $\hat{p}$ is the clearing price, $f(\hat{p})$ is the density function of $\hat{p}$ and $P_B(x) = D^{-1}(x)$ is the inverse demand function.

For all $\hat{p}$ the Euler's condition for the problem is:

$$f(\hat{p})\left(P_B(B_B(\hat{p})) - \hat{p}\right) = 0 \tag{4.3}$$

Solving for $B_B(\hat{p})$ yield:

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[4] Note that $B(\hat{p})$ is the inverse of the bid function $Z_B(q)$ (used in Section 3.1), which express the price a producer bids for the quantity q.

$$B_B(\hat{p}) = P_B^{-1}(\hat{p}) = D(\hat{p}) \tag{4.4}$$

The argument runs similarly on the supply side.   □

Proposition 4.1 emphasize that a multi bid auction is a truth revealing mechanism which leads to efficient trade.

5   Empirical analysis

In Sections 3 and 4 we showed that the single bid restriction leads to inefficiency. The size of the efficiency loss depends on the producers beliefs about the clearing price and on their supply and demand functions. In this section we estimate the efficiency loss in an empirical application.

In the Danish milk quota exchange, bids and asks reflect price expectations as well as average values. One therefore needs panel-data combined with good expectations models to determine marginal values and hereby the likely behaviour in a multiple bid exchange. We do not have the necessary data therefor we cannot directly determine the expected gain from introducing multiple bids in Denmark

Several Canadian milk quota exchanges use multiple bids. Moreover the association: "Dairy Farmers of Ontario" have provided data from 11 auctions held monthly from September 1997. All producers in Ontario are allowed to buy and sell quota. There are no quantitative restrictions on the bids nor asks. The data set contains the individual bids and asks submitted on these 11 auctions[5]. On an average auction 488 buyers and 194 sellers participated. 20,3% of the buyers submitted more than one bid, and 12,2% of the sellers submitted more than one ask. We can use these data to evaluate the impact of introducing a single bid restriction.

In Section 4 we have shown that the optimal bidding strategy in a multiple bid auction is to submit the demand (supply) curve. The producers do not submit continuous demand and supply functions in practice but can provide an arbitrarily fine approximation using discrete points. Therefore,

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[5]The first 7 auctions have been used of farmers from Quebec also, but the data set contains only the bids/asks submitted in Ontario. Although the first 7 clearing prices (calculated on the data set) differ from the official clearing prices, we use all 11 auctions.

Figure 5: The buyer's optimal single bid
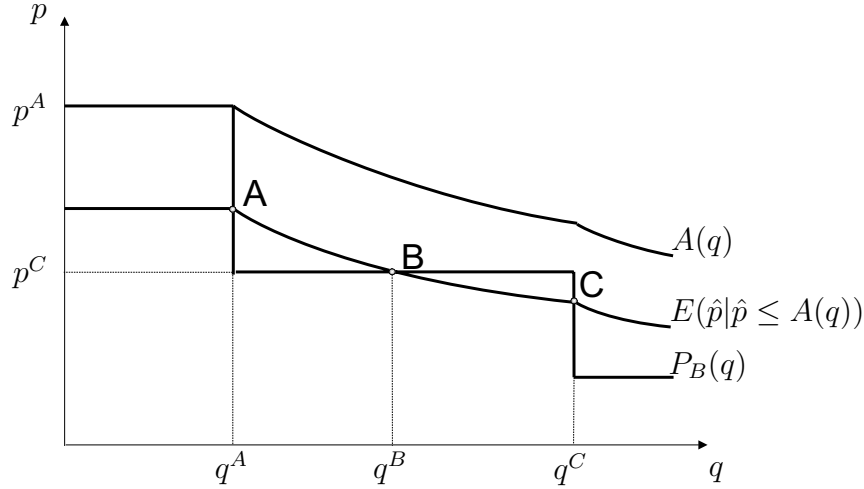
we assume that the bids submitted on the Ontario milk quota exchange reveal the producers' true marginal value of quota.

Combining the assumed behaviour with the empirically few bids suggest that the producers marginal cost of producing milk is a step function. The nature of the production can explain this. Unless a producer has free capacity, expansion of production requires investments. On the other hand, if a producer has free capacity an expansion does not affect the marginal cost. Hence changing the capacity will change the marginal cost to another constant level. This explains the shape of the demand for milk quota as illustrated in Figure 5. Direct cost models of Danish milk producers show similar characteristics[6].

We assume that the producers have the same beliefs about the future clearing prices, and that the beliefs can be modelled as a normal distribution estimated upon the 11 official clearing prices from the auctions. The distribution of the clearing price, measured in CAD/kg butterfat/day is given by $N(15484, 702)$.

In Section 3 we proved that the producers will bid their average value, $A(q)$, and choose the quantity q such that $q = D(E(\hat{p}|\hat{p} \leq A(q)))$ or equivalent $P_B(q) = E(\hat{p}|\hat{p} \leq A(q))$. Figure 5 illustrates the situation for a buyer. The figure shows the step formed marginal values $P_B(q)$, derived from bids saying that a maximum of q^A will be bought at price p^A and that further

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[6]Nielsen (1998) show that the marginal cost of producing milk is a step function.

units up to q^C will be bought at price p^C. Figure 5 also show the average values $A(q)$ and the truncated expected price $E(\hat{p}|\hat{p} \leq A(q))$.

We shall now determine how a farmer as depicted in Figure 5 would react in a single bid auction. The optimality condition from section 3 says that quantity q is set such that $P_B(q) = E(\hat{p}|\hat{p} \leq A(q))$. By the discrete nature of the problem this can happen for several levels of q. The optimal point is the one that yields the largest expected surplus to the buyer. In the figure we have 3 productions levels to consider: q^A, q^B and q^C. Bidding (q^A, p^A) gives, for example, an expected surplus of:

$$\text{Prob}(\hat{p} < p^A) \cdot (p^A - E(\hat{p}|\hat{p} \leq A(q)) \cdot q^A \tag{5.1}$$

where $\text{Prob}(\hat{p} < p^A)$ is the probability that the bid is accepted. After selecting the best single bids and asks we solve the clearinghouse. This gives us the equilibrium that would prevail if the producers were restricted to submit a single bid only. Comparing the equilibrium simulated under the single bid restriction to the equilibrium on the Ontario milk quota exchange enables us to compute the efficiency loss generated by the single bid restriction. The efficiency loss is the difference between the actual surplus in the two situations. For each buyer this can be calculated as follows:

$$\text{Surplus multiple bids} - \text{Surplus single bid}$$

$$\sum_{j=1}^{M} \max\{(p_j - \hat{p}_{\text{multi}}), 0\}\Delta q_j - \max\{(p - \hat{p}_{\text{single}}), 0\}q \tag{5.2}$$

where j indexes the multiple bids, Δq_j is the additional quantity demanded at p_j compared to p_{j-1}, and p, q is the buyer's optimal single bid.

Figure 6 shows the resulting clearing prices in the two setups, single and multiple bids/asks. The figure shows that the clearing prices under the single bid restriction lies close to the clearing prices under the multiple bids setup. This justifies the use of the same expectations about the clearing price in both situations. Moreover the model is robust to changes in the uncertainty; for instant doubling the variance hardly change our results.

Table 1 show the results for the 11 auctions, all numbers are in million CAD. The table show for each auction the surplus when allowing for multiple bids and under the single bid restriction. The net loss is the efficiency loss from introducing a single bid restriction. The last two columns show the

Figure 6: The market clearing prices
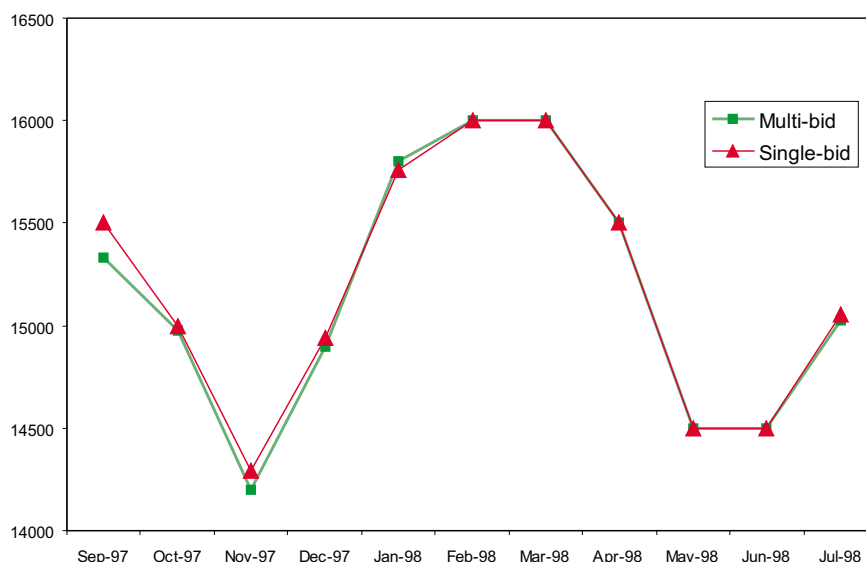
efficiency loss in percentages of total surplus and total turnover generated on the Ontario milk quota exchange.

On average the single bid restriction would reduce the total surplus on the Ontario milk quota exchange by 2.5 percent. Of course several facts can contribute to this some what limited difference.

Very important is the stepwise marginal cost curve in milk production. It makes it possible to approximate the marginal values in the relevant region using only a single or a few bids. Since the Ontario exchange are repeated frequently, possible miss allocation can be corrected without too long delay. A detailed picture of the cost condition may therefore not be submitted in each auction. This suggest that the cost curve we enter may be too simple, which in turn makes us underestimate the loss from a single bid restriction.

Both the Danish and the Ontario quota exchanges are repeated at fixed intervals. The producers can improve an inefficient allocation in the next exchange. This potential makes the inefficiencies of a temporary nature.

Table 1: Losses from single bid restrictions (in million CAD)

	Total surplus			Net loss of total:	
	Multiple	Single	Net loss	Surplus	Turnover
September 1997	2.35	2.19	0.017	7.6 %	1.19 %
October 1997	1.54	1.50	0.051	3.4 %	0.43 %
November 1997	2.88	2.86	0.019	0.7 %	0.13 %
December 1997	2.23	2.21	0.023	1.0 %	0.16 %
January 1998	1.43	1.39	0.039	2.9 %	0.26 %
February 1998	1.94	1.86	0.078	4.2 %	0.40 %
March 1998	1.67	1.64	0.029	1.8 %	0.15 %
April 1998	1.43	1.40	0.028	2.0 %	0.21 %
May 1998	4.02	3.99	0.029	0.7 %	0.15 %
June 1998	3.89	3.79	0.096	2.5 %	0.46 %
July 1998	1.99	1.93	0.065	3.4 %	0.56 %
Average	2.31	2.25	0.057	2.5 %	0.36 %

6 Discussion

In this paper we have assumed that the producers are price takers. This rules out strategic behavior. The assumption is justified by the high number of participants in the exchanges[7]. Moreover, as mentioned in the introduction, strategic behavior in double auctions does not seem important in empirical studies and laboratory experiments with even a low number of participants.

Another simplification is the assumption that the producers are risk neutral. Under the single bid restriction, introducing risk aversion make the buyer reduce the risk by bidding a lower quantity at a higher price. In a multiple bids setup, risk aversion will not change the optimal bidding strategy. The reason is that a producer cannot influence the clearing price, so the best he can do is to reveal his true marginal values. This is well known from the literature on second price auctions (Klemperer 1999). Hence, introducing risk aversion will not change the main conclusion that allowing for multiple bids is the optimal rule.

A third assumption is that of independent private values of milk quota.

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[7]On the Danish exchange from 1997 to 2000 there were approximately 5000 bids and asks per exchange.

The private values come mainly from the individual producers cost of producing milk. In reality there are common elements that can make the valuations dependent on each other, e.g. resale value, the political agenda, interest rate etc. These common effects are left out of the model.

Our analysis is static. This is important for two reasons. First a dynamic free market is difficult to approximate by a discrete set of exchanges unless the preferences are relatively stable between the exchanges. Second, if the producers have stable preferences two single bid exchanges resembles one exchange with a two bid restriction. With two or more bids both the aggregation and the uncertainty effects are reduced.

We use the static model to evaluate the impact of imposing a single bid restriction on the Ontario milk quota exchange. Most of the producers only submit one bid at each exchange. One reason for this might be that a producer can submit a new bid the following month if his bid is rejected. This reduces the impact of a single bid restriction. On the Danish quota exchange a producer has to wait for 6 months before she can submit a new bid. This makes each Danish quota exchange more similar to a one shot event than the Ontario exchange. Thus the single bid restriction probably has more impact on the efficiency of the Danish milk quota exchange than on the efficiency of the Ontario milk quota exchange.

The quantitative restrictions on the Danish exchange might reduce the inefficiency generated by the single bid restriction. This is so because the producers need less bids to express their now truncated demand. On the other hand the quantitative restrictions introduce inefficiencies of its own by restriction the trade-volume between producers with very different values. Limiting the trade introduce a new sort of inefficiency, which is not included in our model.

We have assumed all producers to be rational with sufficient analytical capacities to deduce optimal bidding strategies etc. In reality, producers may only be bounded rational. In discussing our approach with the Danish Dairy Board it has been suggested that the producers are only bounded rational. The producers find it very complicated to submit bids on the exchange. If they were to submit multiple bids the industry expect that they might get more confused. Still, we suggest that it is easier for a producer to submit multiple bids than a single bid. In order to submit multiple bids, the producer must know his marginal values, i.e. demand function $D(p)$ and supply

function $S(p)$. To submit a single bid, the producer must deduce his strategy and figure out which point to submit. This requires that he forms beliefs about the clearing price (i.e. $f(\hat{p})$). Hence, if the single bid restriction is removed a producer may have to submit more bids—but it is easier to figure out which bids to submit.

7   Conclusion

This paper has shown that the single bid restriction on the Danish milk quota exchange leads to inefficient trade. There are two effects: The aggregation effect and the uncertainty effect.

A multiple bid exchange will eliminate these inefficiencies. In other words, allowing for multiple bids will generate efficient trade. Moreover it is easier to deduce the optimal multiple bid strategy than the optimal single bid strategy.

We have also evaluated the inefficiency in an empirical context. We estimated the efficiency loss from introducing a single bid restriction on the Ontario milk quota exchange. The analysis showed that on average an efficiency loss of 2.5% of the total surplus can be expected. The somewhat limited loss in practice is due to the stepwise marginal cost curve in milk production.

In this paper we have not combined the aggregation effect and the uncertainty effect in a unified equilibrium model. As other research in double auctions has shown, the analytical work required tend to get very complex. This however is an interesting issue for further research.

References

BOOTS, M., A. LANSINK, AND J. PEELINGS (1997): "Efficiency loss due to distortions in Dutch quota trade," *European Review of Agricultural Economics*, 24(1), 31–46.

BUNDESMINISTERIUM FÜR ERNÄHRUNG, LANDWIRTSCHAFT UND FORSTEN (2000): "Agrarpolitische Mitteilungen," .

EWASECHKO, A., AND T. HORBULYK (1995): "Interprovencial Reallocation of Market Sharing Quota for Industrial Milk," *Canadian Journal of Agricultural Economics*, 43, 191–202.

FREIDMAN, D. (1984): "On the Efficincy of Experimental Double Auctions Markets," *The American Economic Review*, 74(1), 60–72.

KLEMPERER, P. (1999): "Auction Theory: A Guide to the Literature," *Journal of Economic Survey*, 13(3), 227–286.

LAMBERT, R., R. ROMAIN, B. BASILLAIS, AND R. ROY (1995): "Analyse de la libéralisation des échanges de quotas entre le Québec dans le secteur laitier," *Canadian Journal of Agricultural Economics*, 43, 209–223.

NAUTZ, D. (1995): "Optimal Bidding in Multi-Unit Auctions with Many Bidders," *Economics Letters*, 48, 301–306.

NIELSEN, K. (1998): "Analysis of a new marginal pricing scheme," Bachelor Thesis.

RUCKER, R., W. THURMAN, AND D. SUMNER (1995): "Restricting the Market for Quota: An Analysis of Tobacco Production rights with Corroboration from Congressional Testimony," *Journal of Political Economy*, 105, 142–175.

THE DANISH MILK BOARD (1999): "Milk quota in general," Aarhus, 1-5.

———— (2000): "The Danish quota exchange," Aarhus, 1-16.