RURAL ECONOMY

Symmetry Problem in the Linear AIDS Model: Further Results

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Staff Paper 96-16

Staff Paper

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The author is Assistant Professor, Department of Rural Economy, University of Alberta, Edmonton. I would like to acknowledge helpful comments from Adolf Buse, Yanning Peng, and Janaki Alavalapati on the earlier draft.

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Abstract

This paper derives a set of linear and nonlinear restrictions to make a *n*-goods linear AIDS symmetric when all prices are allowed to vary. When prices are scaled by their means, the conventional restrictions are sufficient to make the linear AIDS symmetric at mean. This indicates an additional advantage of scaling prices at their means before estimation. Nevertheless, price-scaling does not produce a globally-symmetric linear AIDS. When prices are measured in natural units, additional nonlinear restrictions are needed to make the linear AIDS globally symmetric. These restrictions can be imposed with relative ease but convert the linear AIDS into the nonlinear one. The significance of the problem was illustrated using both the US and Canada meat consumption data sets. In both cases, there is a marked difference between the nonlinear and linear AIDS. The bias in the implied values of demand elasticities is more serious in the US data set. Also the mean-symmetric linear AIDS does not improve much on the asymmetric and symmetric linear AIDS.

Symmetry Problem in the Linear AIDS Model: Further Results

To avoid the inherent nonlinearities of the almost ideal demand system (AIDS) and the potential collinear movement of prices, many empirical applications have estimated the linear AIDS (Alston and Chalfant, Buse). This practice has recently been questioned on two accounts. First, the adequacy of Stone index as an approximation to an exact price index and its effect on the estimated demand elasticities (elasticity problem). Second, the underlining theoretical properties of the linear AIDS (symmetry problem). While elasticity problem has attracted some research efforts (Pashardes, Alston *et. al.*, Buse, Moschini), symmetry problem has so far received little analytical attention. Though the linear AIDS was suspected not to possess plausible theoretical properties before (Thomas, Green and Alston), only Hahn tackled the problem how to make the linear AIDS symmetric. He derived a set of nonlinear restrictions for a case when only one price is allowed to be different from the others. The fact that all prices changes in reality excludes the possibility of imposing these restrictions in practice. To apply Hahn's insight, the restrictions for a case when all prices allowed to change are in order.

The main purpose of this paper is to derive these restrictions and examine the effect of these restrictions on the estimation of the linear AIDS model. Several new findings emerged. First, the conventional linear restrictions, which make a two-goods nonlinear AIDS symmetric, would also make a two-goods linear AIDS symmetric. Second, a set of nonlinear restrictions to make a *n*-goods linear AIDS symmetric is derived when all prices are allowed to vary. Third, when prices are scaled by their mean before estimation, the underlying linear AIDS is symmetric at mean.

The paper is organized as follows. The next section sets out the structure of the AIDS and its linear approximation. Section III derives a set of conditions for symmetric linear AIDS. Comparisons to the nonlinear AIDS, symmetric linear AIDS, mean-symmetric linear AIDS, and asymmetric linear AIDS are made in Section IV, using the United States and Canada's annual meat consumption data. In Section V, concluding comments are drawn.

The AIDS and Its Linear Approximation

A *n*-goods AIDS in share form is given

$$w_i = \quad {}_i + \sum_{j=1}^{n} \quad {}_{ij} \log\left(p_j\right) + \quad {}_i \log\left(\frac{M}{P}\right) + \quad {}_i \tag{1}$$

where $w_i = \frac{p_i q_i}{M}$ is the budget share of good *i*, p_i is the price of good *i*, q_i is the quantity consumed of good *i*, *M* is the total expenditures on *n*-goods, ${}_i$, ${}_{ij}$, and ${}_i$ are parameters, ${}_i$ is the error term, and *P* is the translog price index defined by

$$\log P = \quad {}_0 + \sum_{i=1}^{n} \quad {}_i \log\left(p_i\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \quad {}_{ij} \log\left(p_i\right)\log\left(p_j\right) \tag{2}$$

To meet adding-up, homogeneity, and Slutsky symmetry restrictions of a standard consumer utility maximization problem subject to a linear budget constraint, the following restrictions on the parameters are required

Adding-up: $\sum_{i}^{n} \quad {}_i = 1,\ \sum_{i}^{n} \quad {}_i = 0, \text{ and } \sum_{i}^{n} \quad {}_{ij} = 0,\ \forall\, i$

Homogeneity: $\sum_{j}^{n} \quad {}_{ij} = 0,\ \forall\, j$

Symmetry: $\quad {}_{ij} = \; {}_{ji}, \ \forall\ i$ and j

Three empirical problems are associated with the use of (2). First, the specification of the price index as in (2) makes the AIDS a nonlinear econometric model and thus complicates the task of estimating AIDS model. Second, high correlation among prices in (2) can introduce serious multicollinearity problem in estimation. Third, estimating ${}_{0}$ is troublesome as ${}_{i}$ cannot be identified separately from ${}_{0}$ in conventional estimation. Hence, in empirical applications, (2) is often approximated by Stone's price index

$$\log P^s = \sum_{k}^{n} w_k \log\left(p_k\right) \tag{3}$$

Equation (3) can be calculated directly before estimation of (1). The use of (3) hence transforms a nonlinear demand system into a linear one. The linear AIDS can be written as

$$w_i = \; {}_{i}^{l} + \sum_{j=1}^{n} \; {}_{ij}^{l} \log\left(p_j\right) + \; {}_{i}^{l} \log\left(\frac{M}{P^s}\right) + \; {}_{i}^{l} \tag{4}$$

where superscript l stands for the linear AIDS.

The Symmetric Linear AIDS Model

Following Green and Alston, the linear AIDS is treated as the demand system. As a result, Stone's price index is no longer exogenous. Substituting (3) into (4) yields

$$w_i = \; {}_{i}^{l} + \sum_{j=1}^{n} \; {}_{ij}^{l} \log\left(p_j\right) - \; {}_{i}^{l} \sum_{k}^{n} w_k \log\left(p_k\right) + \; {}_{i}^{l} \log\left(M\right) + \; {}_{i}^{l} \tag{5}$$

It is clear that a simultaneity problem is inherent in equation (4) as dependent variable also appears in the right hand side of equation (5). This simultaneity problem was recognized by Eales and Unnevehr, Buse, and Alston *et. al.*. Eales and Unnevehr suggested the use of lagged share of good i, while Alston *et. al.* used three-stage least square (3SLS) to account for the potential simultaneous bias. Buse, however, showed that neither seemingly unrelated regression (SUR) nor three-stage least square (3SLS), produce consistent estimates under this classical errors-in-variables case.

To meet adding-up, homogeneity, and Slutsky symmetry conditions, the following restrictions on the parameters of the linear AIDS, similar to that on the parameters of the nonlinear AIDS, are usually imposed

Adding-up: $\sum_{i}^{n} {}^{l}_{i} = 1, \ \sum_{i}^{n} {}^{l}_{i} = 0, \text{ and } \sum_{i}^{n} {}^{l}_{ij} = 0, \ \forall\, i$

Homogeneity: $\sum_{j}^{n} {}^{l}_{ij} = 0, \ \forall\, j$

Symmetry: ${}^{l}_{ij} = {}^{l}_{ji}, \ \forall\, i \text{ and } j$.

The question is "are the above restrictions implies theoretically-consistent demand system?" While the above restrictions guarantee the linear AIDS meet adding-up and homogenous conditions, they don't guarantee the linear AIDS symmetric. To prove this more formally, taking the total differential of (5) with respective to $\log(p_i)$ and $\log(M)$ and solving for the price and income derivatives yield

$$\frac{w_j}{\log(p_i)} = {}^{l}_{ji} \frac{1 + \sum_{s \neq j}^{n} {}^{l}_{s} \log(p_s)}{1 + \sum_{i}^{n} {}^{l}_{i} \log(p_i)} - {}^{l}_{j} \frac{w_i + \sum_{s \neq j}^{n} {}^{l}_{si} \log(p_s)}{1 + \sum_{i}^{n} {}^{l}_{i} \log(p_i)} \qquad (6)$$

$$\frac{w_i}{\log(M)} = w_i + \frac{{}^{l}_{i}}{\left[1+\sum_{i=1}^{n} {}^{l}_{i} \log(p_i)\right]} \qquad (7)$$

Symmetric condition in share form can be written

$$\frac{w_i}{\log(p_j)} + w_j \frac{w_i}{\log(M)} = \frac{w_j}{\log(p_i)} + w_i \frac{w_j}{\log(M)},\ \forall i \neq j \qquad (8)$$

Substituting (6) and (7) into (8) yields

$$\begin{aligned} & {}^{l}_{ij}\left(1+\sum_{s\neq i}^{n} {}^{l}_{s}\log(p_s)\right) - {}^{l}_{i}\sum_{s\neq i}^{n} {}^{l}_{sj}\log(p_s) \\ = & {}^{l}_{ji}\left(1+\sum_{s\neq j}^{n} {}^{l}_{s}\log(p_s)\right) - {}^{l}_{j}\sum_{s\neq j}^{n} {}^{l}_{si}\log(p_s) \end{aligned} \qquad (9)$$

It is evident that, in general, conventional symmetric restrictions (${}^{l}_{ij} = {}^{l}_{ji},\ \forall\ i$ and j) would not make a n-goods linear AIDS symmetric. The linear AIDS under the conventional restrictions (${}^{l}_{ij} = {}^{l}_{ji},\ \forall\ i$ and j) is symmetric for every possible realization of prices only under limited conditions. First, the symmetry holds globally if ${}^{l}_{i} = 0\ \forall$ i. These restrictions, however, imply homothetic preference and force the income elasticities of goods equal to unity, which is usually undesirable.

Second, the symmetry holds globally if the following conditions are met

$$\frac{{}^{l}_{i}}{{}^{l}_{j}} = \frac{{}^{l}_{ik}}{{}^{l}_{jk}},\ \forall\ i, j \text{ and } k\ . \qquad (10)$$

There will be $\frac{n!\ \times n}{2\times(n-2)!}$ extra restrictions, in addition to $\frac{n!}{2\times(n-2)!}$ conventional restrictions, to make n-goods linear AIDS symmetric. However, with conventional restrictions in place, n of $\frac{n!\ \times n}{2\times(n-2)!}$ extra restrictions will be redundant (can be implied by the combination of the rest restrictions). To make 3-goods linear AIDS symmetric, for example, total restrictions are $\frac{n!}{2\times(n-2)!}(n+1)-n=4\frac{3!}{2\times 1!}-3=9$, while, to make 5-goods linear AIDS symmetric, total restrictions are increased to $\frac{n!}{2\times(n-2)!}(n+1)-n=6\frac{5!}{2\times(5-2)!}-5=55$. Above restrictions clearly limit the flexibility of linear AIDS model. While it is not impossible to impose these restrictions econometrically, it destroys the linearity of the linear AIDS.[1]

With adding-up and homogeneity imposed, the number of extra restrictions can be further reduced to *n-1*. To make 3-goods linear AIDS symmetric, for example, 9 extra restrictions is reduced to 2

$$\frac{{}^{l}_{1}}{{}^{l}_{2}}=\frac{{}^{l}_{11}}{{}^{l}_{12}} \tag{11}$$

$${}^{l}_{11}\;{}^{l}_{22}=\left({}^{l}_{12}\right)^2 \tag{12}$$

It is also worth noting that the conventional symmetric restrictions, along with adding-up, would make a 2-goods linear AIDS symmetric. To see this, recall that adding-up implies that ${}^{l}_{1}+{}^{l}_{2}={}^{l}_{12}+{}^{l}_{22}={}^{l}_{11}+{}^{l}_{21}=0$. Adding-up with the conventional symmetric restrictions satisfies equations (11) and (12) automatically.

Finally, if individual prices are scaled such that $\log(p_i)=0\ \forall\ i$ at means, then symmetry is satisfied by the conventional symmetric restrictions at a reference point. Given the fact that above two conditions either limit the flexibility of the linear AIDS model or

[1] However, the nature of this nonlinearity differs from the nonlinear AIDS as the nonlinear AIDS are nonlinear in the right-hand-side variables.

complicate the estimation of the linear AIDS, it may be of some interest to consider the symmetric restrictions only at a reference point. This result gives an additional rationale to scaling price at means before estimation, as advocated recently by Moschini.[2]

An Application

In order to illustrate potential consequences that may arise with non-symmetric linear AIDS and with imposition of extra symmetric restrictions, two sets of data on meat demand are used. One is Moschini's data for the United States and the other is for Canada from Chen.[3] In both data sets, annual prices and per capita consumption of beef, pork, and chicken are available. The United States data runs from 1958 to 1985, while Canada's data is from 1960 to 1987. In the United States, beef, pork and chicken quantity are expressed in pounds per capita (retail-weight-equivalent for beef and pork, and ready-to-cook weight for chicken) and beef, pork, and chicken prices are in $/lb. In Canada beef, pork and chicken quantity are expressed in kilograms per capita (retail-weight-equivalent for beef and pork, and eviscerated weight for chicken) and beef, pork, and chicken prices are in $/kg. For both data sets, expenditures on these meats are obtained as the product of the aforementioned prices and quantities and used as the income variable. This raises the possibility of correlation between the income variable and the error term (LaFrance). Moreover, the use of Stone index may result in simultaneity. Hence, mean-symmetric and non-symmetric linear AIDS are estimated by using a three stage least square (3SLS) to account for a simultaneity problem, while the symmetric linear and

[2] Moschini actually considered a number of scale-invariant indexes such as Paasche, Laspeyre, and Torquist indexes.

[3] The author is grateful to both Moschini and Chen for permissions to use their data.

nonlinear AIDS are estimated by using a nonlinear three stage least square (N3SLS).[4] The three-stage estimators are robust against the possibility that expenditure is endogenous.

Four models were estimated: the conventionally--restricted linear AIDS without price-scaling at means, the conventionally-restricted linear AIDS with price-scaling at means, the symmetric-linear AIDS, and the symmetric-nonlinear AIDS. Using the same US data set, Moschini concluded that the nonlinear AIDS model is quite successful in estimating the 'true' elasticity parameters. The nonlinear AIDS model is thus used as a benchmark to compare with other models. As noted in Deaton and Muellbauer, estimating $_0$ is problematic in the nonlinear AIDS model. Deaton and Muellbauer suggest a two-stage estimation procedure by first setting the value of $_0$ through a welfare argument and then estimating the rest of the parameter. As a result, the estimation of the nonlinear AIDS becomes a kind of constrained estimation. As $_0$ was not well defined by the data sets used, three different values of $_0$ are tried ($_0 = 0$, $_0 = 1$, and $_0 = l$og of the average expenditure on meat). There is, however, little effect of different values of $_0$ on the estimates. The nonlinear AIDS estimates with $_0 = 0$ was reported.

Estimated parameters for the US are reported in Table 1 and for Canada in Table 2. The statistical performance of the models are quite good, considering very simple models used. Most of the price and income elasticity coefficients are statistically significant. From the results, the following three observations can be made: 1) the nonlinear AIDS estimates are similar using both the US and Canada's data, while the linear

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[4] As noted before, Buse showed that neither seemingly unrelated regression (SUR) nor three-stage least square (3SLS) produce consistent estimates for the linear AIDS. He argued that 3SLS had smaller finite sample biases than SUR but 3SLS also had much larger standard error. No attempt is made in this paper to circumstance this concern.

AIDS estimates differ dramatically between two data sets, 2) the nonlinear AIDS and other three linear AIDS differs dramatically and more so in the US data set; 3) unlike Moschini's finding, the results from this study do not show significant improvement of the mean-symmetric linear AIDS over the asymmetric-linear AIDS; and 4) unexpectedly, there is no clear evidence suggesting that the mean-symmetric linear AIDS is statistically superior to the symmetric-linear AIDS performances. This last point suggests that imposing extra symmetric conditions on the linear AIDS is a mild restriction for the data sets used here.

Another issue of interest is the implied values of demand elasticities. Elasticity estimates for the United States are presented in Table 3 and Canada in Table 4. As noted in Green and Alston, Alston *et. al.* and Buse, alternative formulas to compute elasticities with the linear AIDS give different values. Regarding the most appropriate formula for the linear AIDS, they are divided. Green and Alston, and Alston *et. al.* suggest that elasticities constructed specifically for the linear AIDS are superior to the conventional formula derived from the nonlinear AIDS, while Buse suggests the opposite. Our elasticities for the linear AIDS are computed at means and on the basis of equations (6) and (7). The formulas in the equations (6) and (7) give the desired elasticities for the linear AIDS when dividing by the corresponding share. It is also worthy noting that all terms within summation disappear after price-scaling. As a result, the elasticity formulas for the nonlinear AIDS and mean-symmetric linear AIDS are identical such that

$$\bar{\ }_{ij} = -{}_{ij} + \frac{{}_{ij}}{\bar{w}_i} - {}_i \frac{\bar{w}_j}{\bar{w}_i} \tag{13}$$

$$\bar{\ }_{i} = 1 + \frac{{}_i}{\bar{w}_i} \tag{14}$$

while the elasticity formulas for the symmetric and asymmetric linear AIDS are

$$\bar{\ }_{ij} = -{}_{ji} + \frac{{}_{ij}}{\bar{w}_i} \frac{1+\sum_{s\neq i}^{n} {}_s \log(p_s)}{1+\sum_{j}^{n} {}^{l}_{j} \log(p_j)} - \frac{{}_i}{\bar{w}_i} \frac{\bar{w}_j + \sum_{s\neq j}^{n} {}_{sj} \log(p_s)}{1+\sum_{j}^{n} {}_j \log(p_j)} \tag{15}$$

$$\bar{\ }_{i} = 1 + \frac{{}_i}{\bar{w}_i\left[1+\sum_{i=1}^{n} {}_i \log(p_i)\right]} \tag{16}$$

where ${}_{ij}$ is the Kronecker delta (${}_{ij} = 1$ for $i = j$; ${}_{ij} = 0$ for $i \neq j$).

All of the own-price elasticities of demand and income elasticity are plausible, noting that the expenditure on meat is hold constant, rather than total income, so it is expected that the elasticites are generally larger in magnitude than if total income were held constant. Most of the cross-price elasticites of demand change signs across models. While the implied values of demand elasticities from the three linear AIDS are similar, they differ markedly from the implied values of demand elasticities from the nonlinear AIDS. Unlike Pashardes' finding, Stone index is found to result in either understated or overstated demand elasticities (in absolute value). However, the bias in elasticity estimates is more evident in the US when the linear AIDS is used.

Concluding Comments

This paper derived a set of linear and nonlinear restrictions to make a *n*-goods linear AIDS symmetric when all prices are allowed to vary. When prices are scaled by their means, the conventional restrictions are sufficient to make the linear AIDS symmetric at mean. This indicates an additional advantage of scaling prices at their means before estimation. Nevertheless, price-scaling does not produce a globally-symmetric linear AIDS. When prices are measured in natural units, additional nonlinear restrictions are needed to make the linear AIDS symmetric globally. These restrictions can be imposed with relative ease but convert the linear AIDS into the nonlinear one. The significance of the problem was illustrated using both the US and Canada's meat consumption data sets. In both cases, there is a marked difference between the nonlinear and linear AIDS. The bias in the implied values of demand elasticities is more serious in the US case when the linear AIDS is used. Also the mean-symmetric linear AIDS does not improve much on the asymmetric and symmetric linear AIDS. It is, however, worth noting that, in comparison, the nonlinear AIDS is considered to be the "true" model. A relevant question to ask is whether estimating nonlinear AIDS under highly-correlated prices are capable of producing the 'true' elasticity values. What is needed is a Monte Carlo evaluation of the finite sample properties of the nonlinear estimator. Claim "given the high correlation to be expected between price indexes, the selection of the Stone price approximate is likely to be unimportant" (Chalfant) should be examined more carefully.

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Table 1. Estimates of Four AIDS Models, US Meat Data, 1958-85

Parameters	The Nonlinear AIDS	The Symmetric Linear AIDS	The Mean Symmetric Linear AIDS	The Asymmetric Linear AIDS
1	.4825 (.0380)	-.1890 (.1999)	-.6297 (.1330)	-.6633 (.1527)
2	.4578 (.0297)	.5656 (.1345)	1.0740 (.1310)	1.1613 (.1376)
3	-.9402 (.0376)	-.3766 (.1903)	-.4443 (.1443)	-.4981 (.1552)
11	0.0391 (.0249)	.0056 (.0071)	.0238 (.0133)	.0278 (.0149)
12	.0031 (.0167)	-.0187 (..0130)	-.0231 (.0105)	-.0239 (.0109)
13	-.0422 (.0170)	-.0132 (.0061)	-.0007 (.0010)	-.0039 (.0110)
22	-.0168 (.0175)	-.0630 (.0147)	-.0033 (.0147)	-.0013 (.0148)
23	.0137 (.0139)	.0442 (.0130)	.0263 (.0114)	.0252 (.0115)
33	.0285 (.0189)	-.0311 (.0155)	-.0256 (.0129)	-.0213 (.0134)
1	.0240 (.0115)	.1442 (.0388)	.2267 (.0253)	.2358 (.0310)
2	-.0416 (.0029)	-.0429 (.0275)	-.1434 (.0249)	-.1589 (.0279)
3	.0176 (.0137)	-.1011 (.0386)	-.0833 (.0275)	-.0770 (.0318)

Table 2. Estimates of Four AIDS Models, Canadian Meat Data, 1960-1987

Parameters	The Nonlinear AIDS	The Symmetric Linear AIDS	The Mean Symmetric Linear AIDS	The Asymmetric Linear AIDS
1	.4482 (.0250)	.0688 (.1334)	-.1335 (.2128)	-.0705 (.1590)
2	.4707 (.0190)	.2646 (.0980)	1.5395 (.1386)	1.2006 (.1007)
3	-.9189 (.0154)	-1.3334 (.14358)	-1.4010 (.1950)	-1.2711 (.1347)
11	.0792 (.0291)	.0573 (.0269)	.052 (.0258)	.0577 (.0265)
12	-.0251 (.0206)	-.0235 (.0106)	-.0164 (.0158)	-.0196 (.0156)
13	-.0540 (.0178)	-.0338 (.0213)	-.0360 (.0231)	-.0381 (.0221)
22	.0259 (.0260)	.0096 (.0066)	-.0832 (.0225)	-.0757 (.0213)
23	-.0008 (.0204)	.0139 (.0060)	-.0669 (.0222)	-.0561 (.0206)
33	.0548 (.0233)	.0199 (.0167)	.1029 (.0316)	.0942 (.0287)
1	.0013 (.0074)	.0863 (.0333)	.1083 (.0393)	.0853 (.0381)
2	-.0325 (.0055)	-.2102 (.0232)	-.2176 (.0256)	-.2046 (.0236)
3	.0312 (.0046)	.1239 (.0352)	.1093 (.0360)	.1193 (.0322)

Table 3. Marshallian Demand Elasticities at the Mean of 1958-85, USA

	Beef	Pork	Chicken	Expenditure Elasticity
The Symmetric Nonlinear AIDS				
Beef	-0.951	-.014	-.035	1.043
Pork	.071	-.993	-.078	.872
Chicken	-.428	.048	-.620	1.148
The Symmetric Linear AIDS				
Beef	-1.173	-.047	-.067	1.259
Pork	.152	-1.192	.185	.868
Chicken	.396	.739	-.999	.144
The Mean Symmetric Linear AIDS				
Beef	-1.184	-.173	-.050	1.407
Pork	.175	-.867	.134	.558
Chicken	.385	.450	-1.133	.297
The Asymmetric Linear AIDS				
Beef	-1.205	-.192	.070	1.423
Pork	.214	-.836	.168	.510
Chicken	.370	.483	-.980	.350

Table 4. Marshallian Demand Elasticities at the Mean of 1960-87, Canada

	Beef	Pork	Chicken	Expenditure Elasticity
The Symmetric Nonlinear AIDS				
Beef	-0.825	-.057	-.118	1.003
Pork	-.029	-.888	-.084	.911
Chicken	-.368	.084	-.547	1.169
The Symmetric Linear AIDS				
Beef	-.974	-.094	-.095	1.192
Pork	.193	-.789	.123	.426
Chicken	-.442	-.159	-1.130	1.672
The Mean Symmetric Linear AIDS				
Beef	-.992	-.125	-.124	1.241
Pork	.222	-.555	-.073	.406
Chicken	-.461	-.580	-.551	1.593
The Asymmetric Linear AIDS				
Beef	-.972	-.096	-.096	1.190
Pork	.197	-.619	-.064	.442
Chicken	-.454	-.499	-.753	1.647