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OPTIMIZING MATERIALS-HANDLING SYSTEMS BY MATHEMATICAL PROGRAMMING

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OPTIMIZING MATERIALS-HANDLING SYSTEMS BY MATHEMATICAL PROGRAMMING*

The problem of selecting equipment and methods and organizing them into an efficient materials-handling system is important in agriculture. The current trend toward specialization, mechanization, and automation is speeding changes in farm materials-handling systems. Many research workers have noted the need to consider the entire system and its interrelationships rather than to select equipment and methods on the basis of only one process.

Sammet (22) outlined the "systems engineering" concept and its applicability to agricultural problems. He emphasized the division of systems into stages, with a number of alternatives at each stage. He noted the large number of possibilities for different systems and mentioned new methods of mathematical programming and use of electronic computers to perform the calculations. In a similar vein, Pinches (16) wrote, "Systems engineering in agriculture should start with analysis of farm operations or processes, and proceed through work flow, or process layout, to implementation and farm layout." He stressed the interaction of processes in the total operation of a farm. Hall (9) outlined several theoretical methods applicable to the design of materials-handling systems and suggested the use of a flow chart. He gave two examples of possible uses of linear programming for materials-handling problems.

Ross (21) applied industrial engineering techniques to the analysis of materials handling on an Indiana hog farm. He used the process chart and flow chart to present methods and their relationships to the entire system. The number of different systems made up of different combinations of the alternative methods totaled 8,160. Ross suggested the development of machine methods of analysis in order to better evaluate the large number of alternative systems. MacHardy (13) used industrial engineering methods similar to those used by Ross. He used the flow chart to show relationships between alternative methods and compared the various methods in each process separately, using the "pay-off period" as a cost criterion.

Winter (28) summarized industrial engineering methods applicable to the design of agricultural materials-handling systems. Using similar techniques, Todd (24) analyzed a sugarcane-harvesting processing system and designed a revised system.

DeForest (5) stressed the need for system design as opposed to piecemeal improvements. He and Forth (6) stressed the system-planning approach by developing a colorful, illustrated flow chart that showed alternative methods (paths on the chart) and their interrelationship in the movement of all materials. McKenzie (14) developed system-design fundamentals, including the recommendation of multiple use of equipment in a complete materials-handling system. Kleis (11), in a study of materials handling on 320 Michigan farms, stated as one of his conclusions the evident need to develop systems that would make the separate methods "compatible, complementary, coordinated and integrated."

The purpose of the research reported here was to construct mathematical models that described the interrelationships between equipment and methods in the entire system, and to apply and develop mathematical programming methods for optimizing the systems. The work was in the field of operations research, especially

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*Research reported herein was conducted at the Purdue Agricultural Experiment Station as a contributing project to the NC-48 Regional Materials Handling Research Project. A more detailed report of this research may be found in a doctoral thesis by the senior author (15).

linear programming. Known methods were applied and new methods were developed for solving three types of systems-selection problems.

Linear Programming

Linear programming problems require the solution of a set of linear algebraic equations, fewer in number than the number of unknowns, so as to optimize a given linear function of these unknowns. This set of linear equations is usually derived from a set of inequalities, and these equations are called constraint equations. The function to be optimized is termed the cost or profit equation, the criterion function, the objective function, or the functional equation.

Three methods of solution are available for different types of problems. The simplex method solves all general linear programming problems. It was developed by Dantzig (4) and was explained and further developed by Charnes, Cooper, and Henderson (2). Randolph (19) presented a good explanation based on the solution of linear algebraic equations. Boles (1) wrote a clear exposition of the method from the standpoint of selecting farm enterprises for maximum profit. Hall (9) and Tribus and Rowe (25) suggested agricultural engineering applications of linear programming and explained the solution techniques.

The transportation method solves problems that may be formulated in a manner analogous to that of finding the minimum-cost method of shipping products from a number of sources to a number of destinations. A good description of this technique is given by Snodgrass and French (23).

The assignment method solves a further specialization of the transportation problem. As the name implies, a certain number of machines or men can be assigned to the same number of jobs at maximum total effectiveness or minimum total cost. Flood's assignment method of solution was presented by Churchman, Ackoff, and Arnoff (3).

Flow Charts

The flow chart as used by Ross (21), MacHardy (13), and others was the fundamental method of presenting the problem in this research. For simplicity of mathematical analysis, it was essential that a complete materials-handling system be represented by a single path. The following definition was developed:

Flow chart - A flow chart is a network of directed links that represent alternate methods of performing specific operations such that any complete single path through the network represents a complete system.

Definitions of "process" and "method" given by Vaughan and Hardin (27) were used:

Process - Any regular course of action adhered to in performing work--a plan followed regularly.

Method - A definite system of procedure as to how a work process or any parts thereof are done. It involves order of work, hand and body motions, arrangement of work place, and kind of equipment used.

As used in this research, a process may be performed by one of several alternative methods. Each method is represented on the flow chart by one directed link. The points of connection between links are called nodes, and the terms "equipment" and "machine," as used, refer to buildings and building modifications as well as to machinery.

Figure 1 shows a sample flow chart for moving feed into storage; removing, grinding, and mixing it; and delivering it to the livestock. Note that the chart shows the infeasibility of using certain methods together. In Figure 1, Method 2 represents movement of ear corn, while Method 3 involves grinding with a small grinder that will not handle ear corn. Since it is not feasible to use these two methods together, the flow chart is constructed so that no single path can include both. The flow chart permits the combining of materials, since they may change from one process to the next. For example, processes involved in handling bedding could be attached to the end of Process III in Figure 1 to provide a complete farm flow chart.

The flow chart was constructed so that each system was practical and well-engineered. This point was important because cost alone was to be the criterion. For example, a less expensive, unsafe machine would not be preferable to a more expensive safe machine, so safety was considered in the design of each method and any possible combination of methods formed a safe, complete system.

Man and Machine Process Charts

A special process chart was developed to aid in describing the methods represented on the flow chart and in making detailed cost estimates. A set of shorthand symbols was developed to shorten the amount of writing necessary to describe the operation. The underlying principle was to include only enough information to show how the operation was performed. The equipment was designated by letter; and after the first mention, only the letter was used to identify the item.

Operating costs of all equipment, including labor, and fixed costs of all machines that were not used in other processes were listed. Fixed costs of multiple-use machines were handled separately, as is shown later. Table I lists the shorthand symbols used, and Table II gives an example of the man and machine process chart for a particular method.

Table I. Shorthand Symbols Used in Man and Machine Process Charts

→	Transported to
	Hitched or connected to
	Unhitched or disconnected from
V	Opened, including dismounting and remounting power unit when necessary
Λ	Closed, including dismounting and remounting when necessary
	Adjusted or positioned

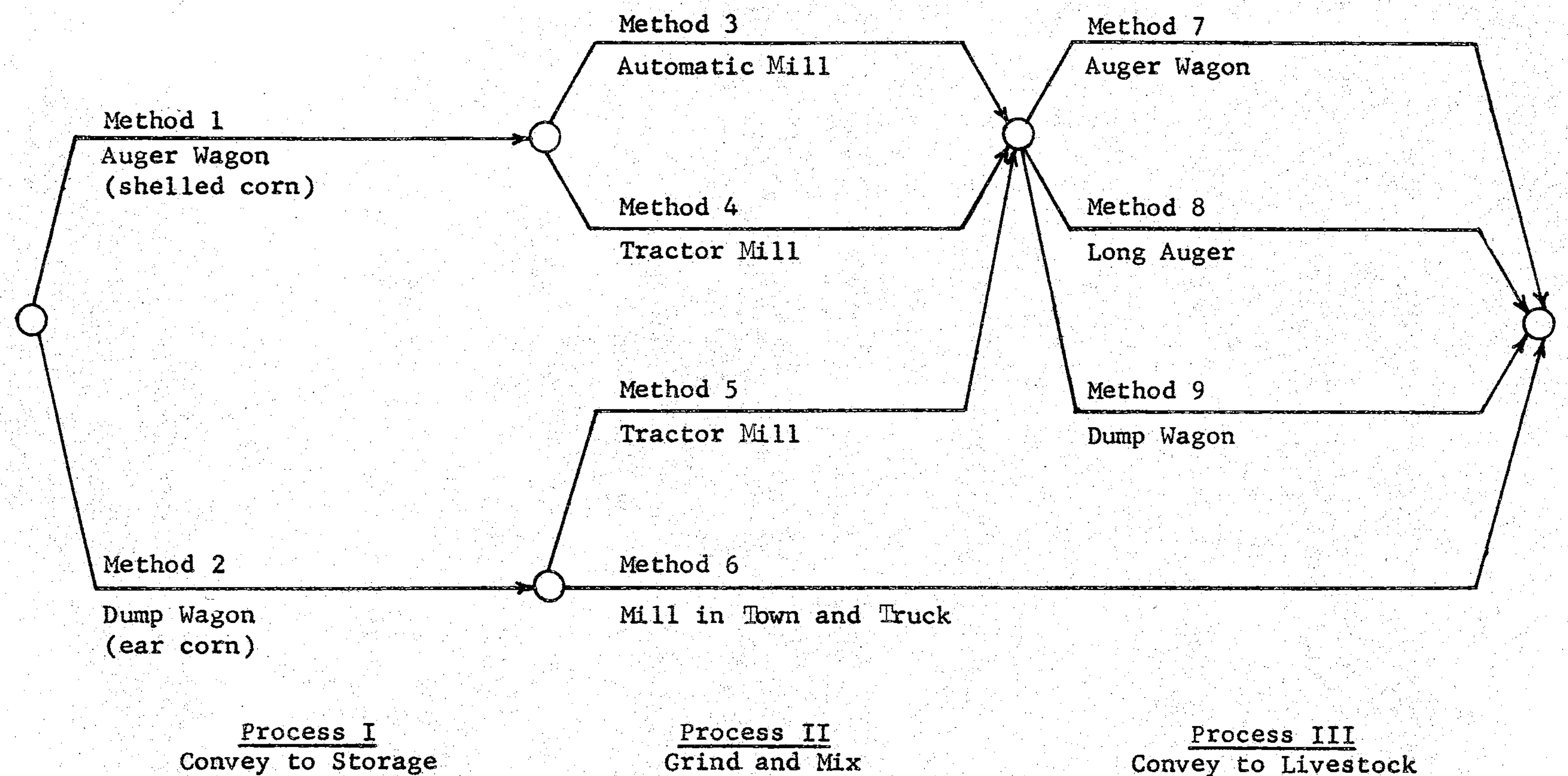


Figure 1. Simple Materials-Handling Flow Chart

Table II. Man and Machine Process Chart for Feeding Hogs on Pasture

Description	Remarks	Time: Man	Dump Truck, T	Auger, A	Feeder, F
1. Loaded truck (T) portable auger → field.	2 T. load, 2,000 ft.	3 min.	3 min.		
2. Feeder (F) V .		-	-		
3. T A to fill F.	1 T. cap.	2	2		No Op. Cost
4. F filled & Λ .	15 T./hr.	4	4	4 min.	
5. A T → second feeder (F2).		2	2		
6. F2 V , T , A to fill F2.		2	2		
7. F2 filled & Λ .		4	4	4	
8. A T → building H.		3	3		
		20 min.	20 min.	8 min.	
Annual Totals	48 T.	8.0 hr.	8.0 hr.	3.2 hr.	
Operating Cost	Rate	$1.50	$0.80	$0.05	
	Total	12.00	6.40	.16	
Fixed Cost		-	m.u.	m.u.	$32.40

Total Cost for Table 2 - $50.96

m.u. - Multiple-use

Cost Functions

The form of the cost functions for the various methods is important, since the total cost is to be minimized in the optimal system. In linear programming, costs for each variable must be linear with the amount of use. However, methods have been developed for handling a non-linear cost in linear programming when the cost function is convex to the use axis, as shown in Figure 2. The convex function may be approximated by a piecewise linear function, and the single variable is replaced by one variable for each linear portion. In a cost minimization problem, the simplex solution method for linear programming naturally selects the lowest-cost variable first (use from 0 to a, at minimum cost, c_1) so that the solution will contain the variables in their proper order according to the amount of use. Puterbaugh (18) applied this separable convex function theory to agricultural production functions.

Because of the fixed cost, equipment costs are frequently discontinuous at zero use. If the equipment is not used, it need not be purchased, and the cost is zero. If it is used even a small amount, the cost is the fixed cost plus the operating cost. If the cost above zero use is assumed to be linear, an approximation that is continuous is a concave function, as shown by OAB in Figure 3. If these two linear segments were used to represent the variable in a cost-minimization problem, the segment with the least slope (AB) could appear in the solution without the segment representing the fixed cost, and the solution would not be correct.

The cost problem is simpler if the equipment is to be used a specified amount or not at all. Then the cost may correctly be assumed to be linear through zero and through the specified amount of use as shown by OC in Figure 3. On a flow chart with given amounts of material to be handled, a method using equipment that is not used in any other process may be given such a cost function. If all methods on the flow chart are of this type, the problem is simply one of finding the minimum-cost path through a network with constant-cost branches. Several methods, mentioned later under "minimum-path methods," are available for solving these relatively simple problems.

In farm materials-handling systems, "multiple-use" equipment is common, that is, equipment which may be used in several processes. For example, a self-unloading forage wagon may be used for many different processes in a materials-handling system. Such multiple-use equipment poses a problem of determining the actual cost of a method using such a machine, since the fixed-cost allocation depends upon how many other uses of the machine are in the system. The number of uses of the machine determines the cost of the method, but it is difficult to determine whether to include a given method if its cost is in question.

Hirsch and Dantzig (10) stated this problem in linear programming terms, except that the criterion function to be minimized was not the usual

$$z = \sum_{i=1}^{n} c_i x_i, \qquad (1)$$

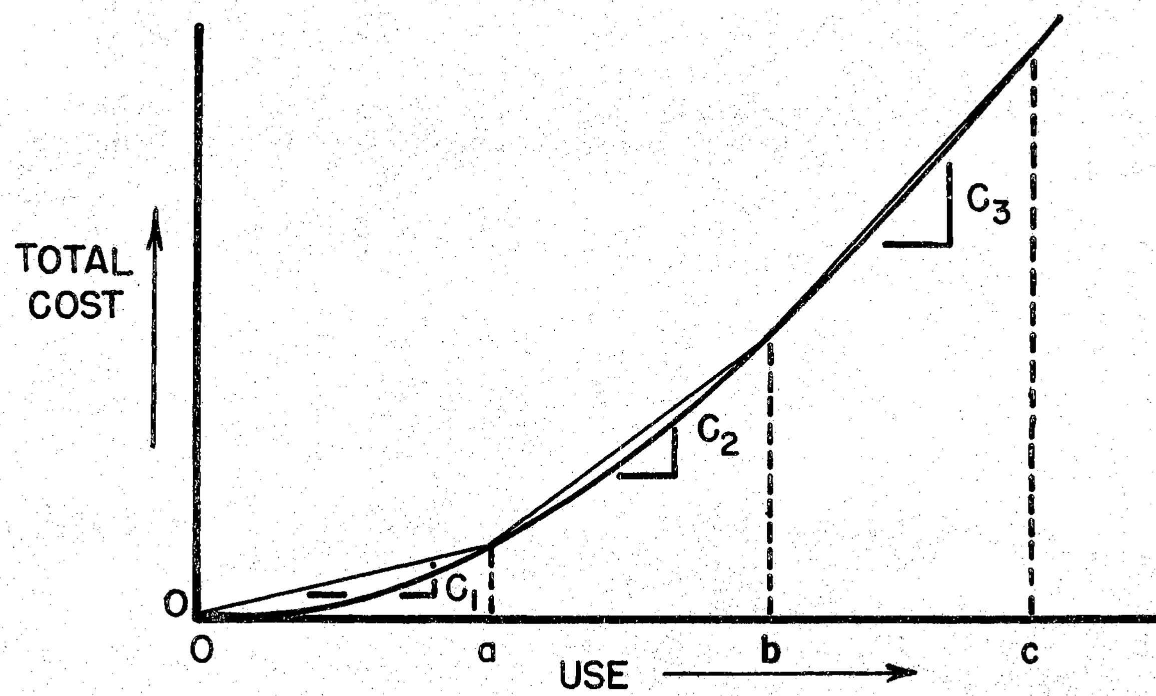


Figure 2. Approximation of a Convex Cost Function by Linear Segments

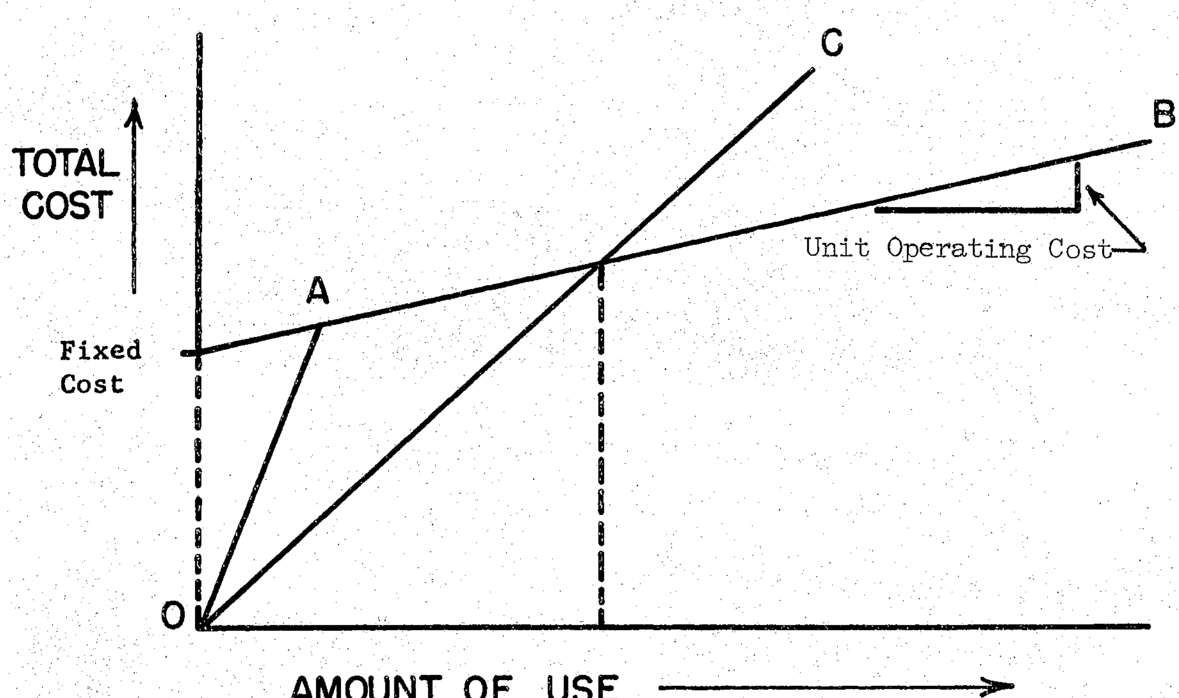


Figure 3. Equipment Cost Functions

but was

$$z = \sum_{i=1}^{n} c_i x_i + b_i (f(x_i)), \qquad (2)$$

where

$$f(u) = \begin{cases} 1 \text{ for } u > 0 \\ 0 \text{ for } u = 0, \end{cases}$$

b_i = the fixed cost associated with x_i, and

c_i = the unit operating cost associated with x_i.

Equation 2 gives the total cost for the system, with the fixed cost of each variable x_i added only once for any value of x_i greater than 0. They pointed out that no computational algorithm had developed for solving such a problem. This paper presents a method that does solve this problem in most cases, but that does not guarantee the desired type of answer in all cases, as is explained later.

Three general types of materials-handling problems were considered. They are called minimum-path problems, fixed-size problems with multiple-use machines, and variable-size problems. Available minimum-path methods solve the first type. The unit-flow model, which is the major topic of this paper, was developed to solve the second type. Further restrictions on the fixed-size problem and problems involving variable sizes of enterprises require methods for integer solutions to linear programs.

Minimum-Path Methods

As previously mentioned, one type of minimum-path problem involves a fixed amount of material to be handled by each method, and no multiple use of equipment between processes. If the fixed costs of multiple-use equipment are negligible, multiple usage may be included; and if the fixed costs of all equipment are negligible, the material to be handled may be variable within the problem. However, the negligible fixed-cost assumption is seldom realistic. Solutions for minimum labor, energy, or capital are readily found with minimum-path methods.

Many algorithms are available for solving minimum-path problems. The smaller problems can be solved manually, and the larger problems with a digital computer. Pollack and Wiebenson (17) presented a good review of several of these methods; and Randolph, Bartlett, and Peart (20) described others.

The Unit-Flow Model

In this research the unit-flow model was developed to handle fixed-size problems with multiple-use equipment. The unit-flow model is a linear programming model so constructed that the solution, except in rare cases, is the single-path system with the minimum total of operating costs and fixed costs.

Classification of Equipment According to Use. The first step in constructing this model is to list on the flow chart the equipment and buildings to be added or modified. A good deal of engineering effort must precede this step, because all reasonable methods of performing the various processes must be considered and their interrelationships must be specified in constructing the flow chart. On the flow chart, each method is represented by a directed link; and the method number, equipment designation (in parentheses), and brief description are listed on the directed link for each method. The flow chart of Figure 4 is used for illustration purposes.

Equipment is classified according to use as (1) single-use, (2) must-use, (3) multiple-ordered-use, or (4) multiple-use. To aid in describing these terms precisely, the following definitions of "parallel" and "series" are used.

Parallel - A group of methods or a group of uses of a single machine is a parallel group if no single-path system can contain more than one of the methods or machine uses.

Series - A group of methods or uses of a single machine is in series if more than one method or use of the machine can be a part of one single-path system.

A machine is a single-use machine if all its uses are parallel. In Figure 4, E, K, M, H, and D are examples of single-use equipment. The fixed cost of a single-use machine is added to the operation costs of each method in which it is specified.

A must-use machine is used in every possible system represented on the flow chart. Machine A is a must-use machine in Figure 4. Fixed cost of a must-use machine has no bearing on which system is the least-cost, and it may be neglected, added to the optimal system, or added to all methods in one process, such as Methods 1 and 2 for Machine A.

A machine has multiple-ordered-use if its uses are in series,and the use nearest the origin node must be in any path including subsequent uses. Machine G has multiple-ordered-use in Figure 4, because it cannot appear in a system unless Method 2 is in the system. This method is called the "initial" method for multiple-ordered-use Machine G. The fixed cost of a multiple-ordered-use item is added to the operating cost of its initial method.

A multiple-use machine has two or more series uses that may each be in a system without the other. Machines B, C, and L, in Figure 4, are multiple-use machines. Methods 1 and 7, which use Machine B, may both be in a system, or either may be in a system without the other. It follows that there must be at least one system using neither method.

There is no correct place on the flow chart to assign the fixed cost of a multiple-use item, and this is the crux of the optimizing problem. The solution of this problem lies in constructing separate "purchase variables" for the multiple-use machines and assuring an integer solution for these variables.

Costs of Individual Methods. Once the equipment is classified according to use, the fixed costs of all except multiple-use equipment are added to the operating costs of the appropriate method as specified above. These combined costs are referred to as method costs. The man and machine process charts constructed for the alternative methods help to determine detailed costs. It is helpful at this point to designate all multiple-use equipment on the flow chart, as has been done by underlining on Figure 4.

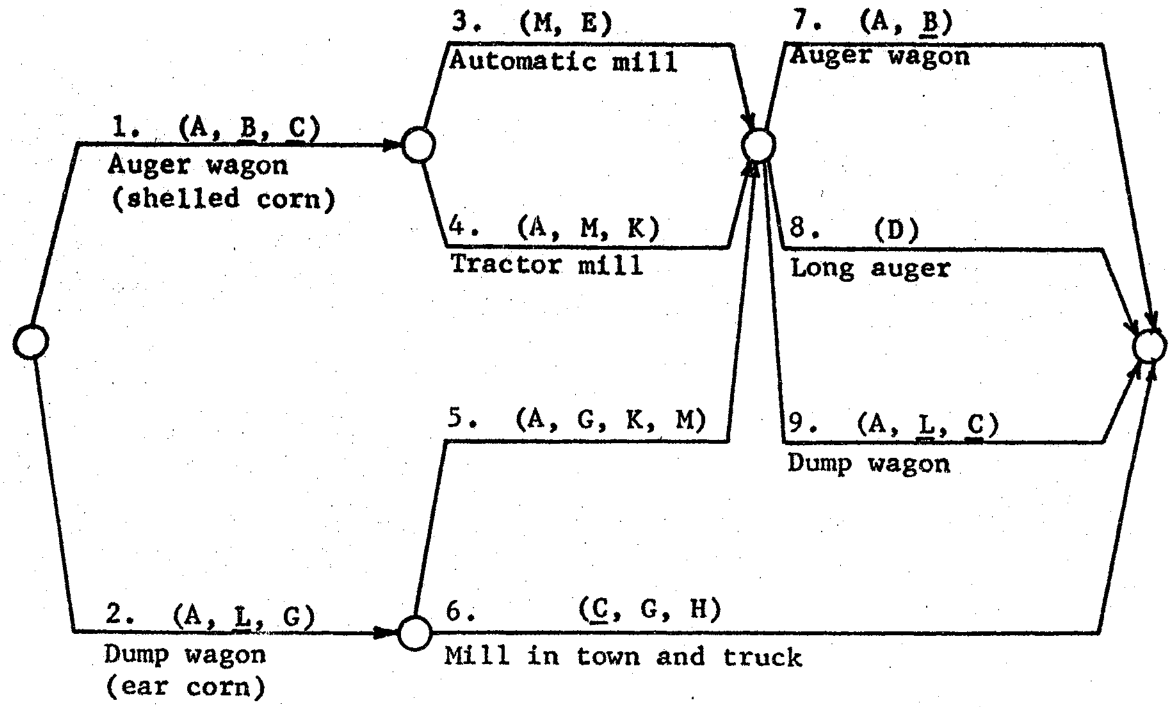


Figure 4. Example Flow Chart With Equipment Designations

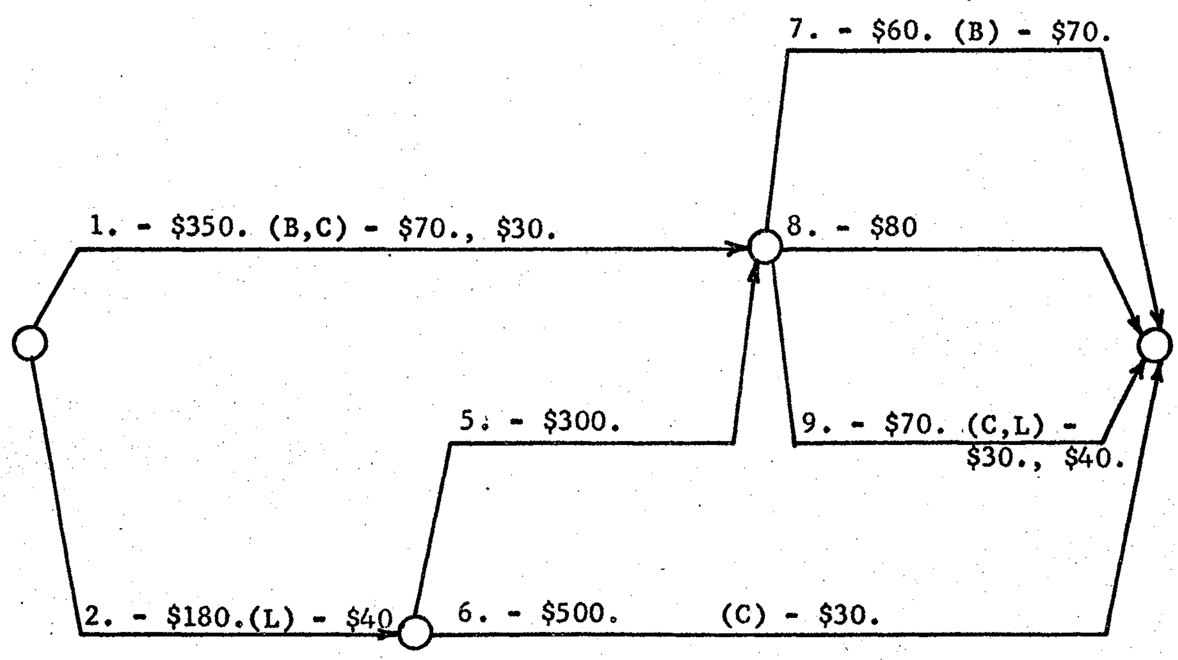


Figure 5. Revised Flow Chart With Multiple-Use Machines and Costs

After these method costs are computed, it may be possible immediately to eliminate some of the higher cost methods from the problem. In a group of methods connecting two nodes, any method may be eliminated that has a higher cost and uses all multiple-use equipment used by a lower-cost method in the group. In Figure 4, Methods 3 and 4 have no multiple-use equipment and Method 4 costs more, so it may be eliminated from further consideration. Methods 1 and 3 are then combined and called Method 1.

Figure 5 shows the revised flow chart after the methods have been eliminated or combined and only multiple-use machines are shown. The comparison of methods between common nodes is simple, even in large flow charts, and it may reduce the size of the problem.

Variables. Each method is represented by a variable, x_i, which equals the number of units of material handled by Method *i*. These variables are called "method variables."

Each multiple-use machine is represented by a variable, x_J, which equals the number of units of Machine *J* that are to be used or purchased. These variables are called "purchase variables."

The mathematical model. After the alternative methods have been designed, the flow chart constructed, the equipment classified according to use, the fixed and operating costs allocated, and the variables named, construction of the mathematical model is relatively simple. The mathematical model consists of a set of linear equations and inequalities. They are of two types, network equations and purchase inequalities.

The network equations are written according to Kirchoff's law of electrical currents; that is, the flow into a node must equal the flow out of the node. A critical point in the construction of the unit-flow model is the definition of a flow of one unit through a method as the amount of material used in determining the cost of that method. Thus the flow chart is viewed as a network carrying one unit of flow even though the amounts and types of actual material that are handled may change from one process to the next. This definition does not limit or approximate the model, as costs are based on the actual amounts and types of material handled in each process.

In a network of *n* nodes, *n-1* independent equations of flow through the nodes must be written. The following network equations apply to the flow chart of Figure 5. In this case an equation is written for each node except the final one:

$$x_1 + x_2 = 1. \tag{3}$$

$$x_2 - x_5 - x_6 = 0. \tag{4}$$

$$x_1 + x_5 - x_7 - x_8 - x_9 = 0. \tag{5}$$

The purchase inequalities are written so that the purchase variable must take on a value at least as great as any method variable associated with it. In terms of the example, if either x_1 or x_7 equals one, x_B should also equal one, since

Machine B is needed for either method. The following two inequalities accomplish this:

$$x_1 \qquad -x_B \leq 0. \tag{6}$$

$$x_7 \quad -x_B \leq 0. \tag{7}$$

Similarly, for machine L, two inequalities are formed:

$$x_2 \qquad -x_L \leq 0. \tag{8}$$

$$x_9 \quad -x_L \leq 0. \tag{9}$$

<u>One inequality is required for each possible parallel group of uses for each multiple-use machine</u>. For example, the following inequalities are necessary for Machine C, since (x_1, x_6) and (x_6, x_9) are all the possible parallel groups of methods using Machine C:

$$x_1 \quad + x_6 \qquad -x_C \leq 0. \tag{10}$$

$$x_6 \quad + x_9 \quad -x_C \leq 0. \tag{11}$$

The inequality sign is used rather than equality so that the purchase variable may equal one while the method variables in one inequality are zero.

In the cost equation, the annual method costs, as previously computed, are the coefficients of the method variables, and the annual fixed costs are the coefficients of the purchase variables. For Problem 2, the cost equation is as follows:

$$350x_1 + 180x_2 + 300x_5 + 500x_6 + 60x_7 + 80x_8 + 70x_9 + 70x_B + 30x_C$$
$$+ 40x_L = z = \text{cost (to be minimized)}.$$

<u>Solutions</u>. A materials-handling problem formulated in this way and submitted to the simplex algorithm of solving a linear programming problem yields an optimal solution with each variable usually equal to either zero or one. Non-integer values for the solutions are not desirable because they mean that part of the material should be handled by one method and part by another method. Even worse, a fractional solution for a purchase variable means the purchase of part of a machine, which is an unrealistic solution for the usual materials-handling equipment.

Although non-integer solutions are mathematically possible, a study of the problem shows that they are unlikely. First, in its simplest form, the requirement for a possible non-integer solution is a situation in which two multiple-use machines are used together in one method of one process; while in another process, each is part of one of two parallel or "competing" methods. This situation in its simplest form is shown in Figure 6, Second, in addition to the first requirement, the following cost relationship must hold to produce a non-integer solution:

$$(c_2 - c_1) < (c_3 + c_A - (c_4 + c_B)) < (c_1 - c_2) > 0,$$
$$\text{or } (c_1 - c_2) > |c_3 + c_A - (c_4 + c_B)|.$$

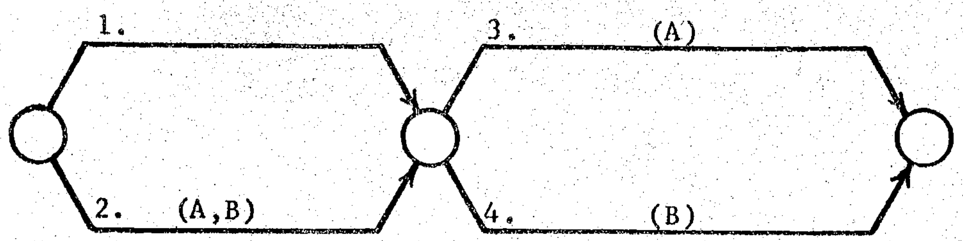


Figure 6. Simplest Flow Chart Required for Possible Non-Integer Solution

Because of these cost relationships and the unlikely nature of this type of multiple-use machine placement on a flow chart, the possibility for non-integer solutions is not believed to be a significant disadvantage of the unit-flow method.

Information in addition to designation of the optimum system may be obtained with this method. Cost of one specific method or of one multiple-use machine may be varied automatically with some available linear programming computer routines. The maximum cost at which a particular method or machine is included in the optimal solution is valuable information.

During the research work on the unit-flow model, several problems were set up and solved. They ranged in size from small problems that were computed manually to two larger problems handled on the Datatron 220. The largest problem was constructed with data from an actual 289-acre grain-hog farm in Indiana. The flow chart contained 82,944 different complete systems. Different methods of conveying materials accounted for many of the alternatives. It was generally not possible to choose one best conveying method because of the variety of materials handled. Even more alternative conveying methods would have been feasible on a farm with the feeding area closer to the feed storage area. On this farm the two were about half a mile apart.

Once the alternative materials-handling methods were designed, the flow chart constructed and the costs computed, writing of the equations for the unit-flow model took only a short time--less than three hours. Of course, the initial design and cost work was considerable and important, but it would have been necessary for any type of analysis. Computer operating time on a Datatron 220 totaled less than four hours, and this is considered a medium-speed machine. The size of the problem required data to be stored on a magnetic tape unit, and that greatly reduced access speed.

The flow chart for the full-size problem is shown in Figure 7, and the methods in the optimal system are starred. Table III lists the multiple-use machines by index letters used on the flow chart and gives their annual fixed costs. Costs for the feed grinding alternatives were $3.00 per ton for on-the-farm custom grinding, and $2.00 per ton at a custom mill in town five miles away. A penalty of one percent of the feed cost (for wastage) was charged to the no-grinding method.

Note that the programming solution method makes all these choices, in effect, simultaneously and considers the interrelationships among all the choices.

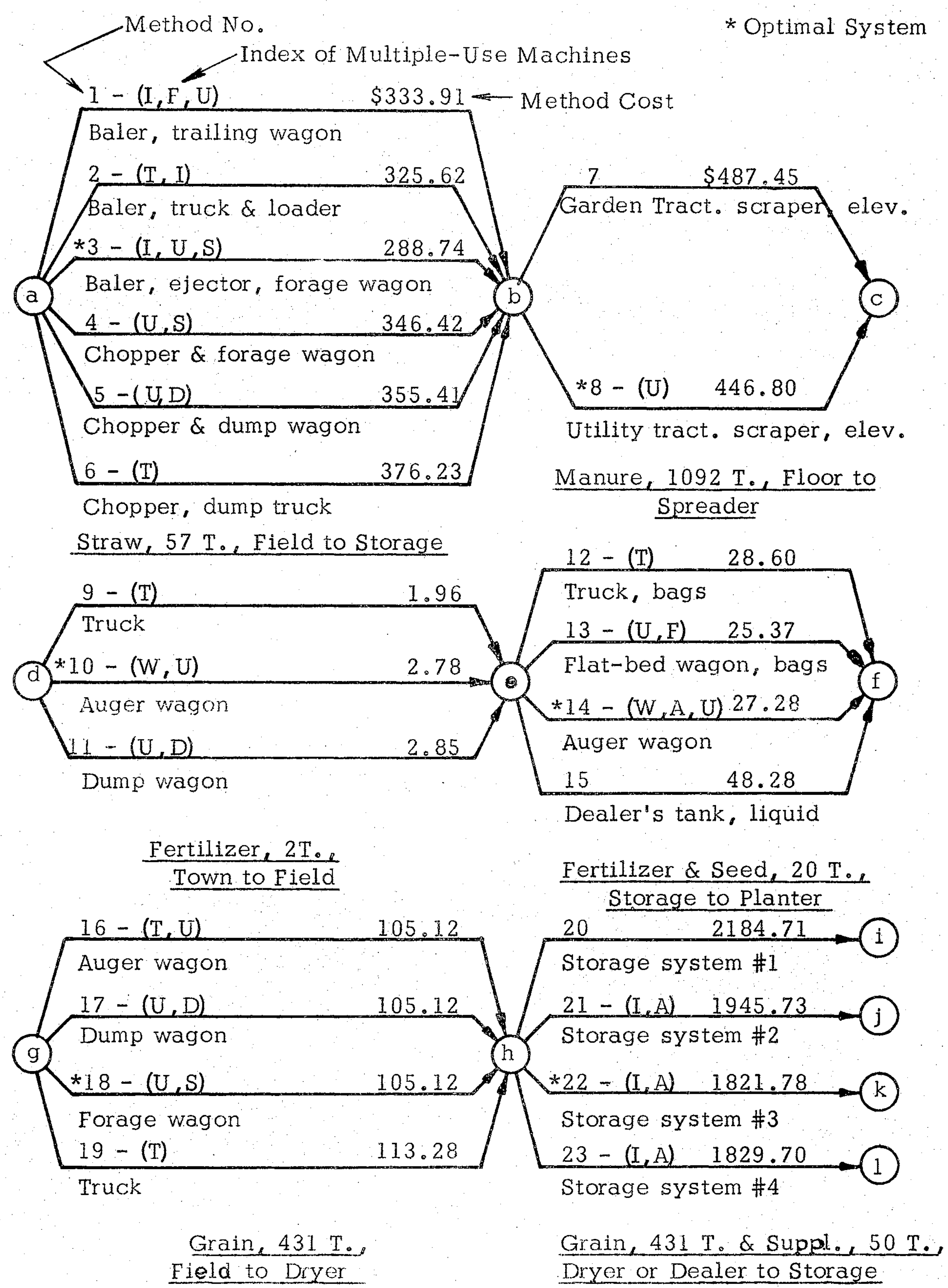


Figure 7. Flow Chart for Corn-Hog Farm

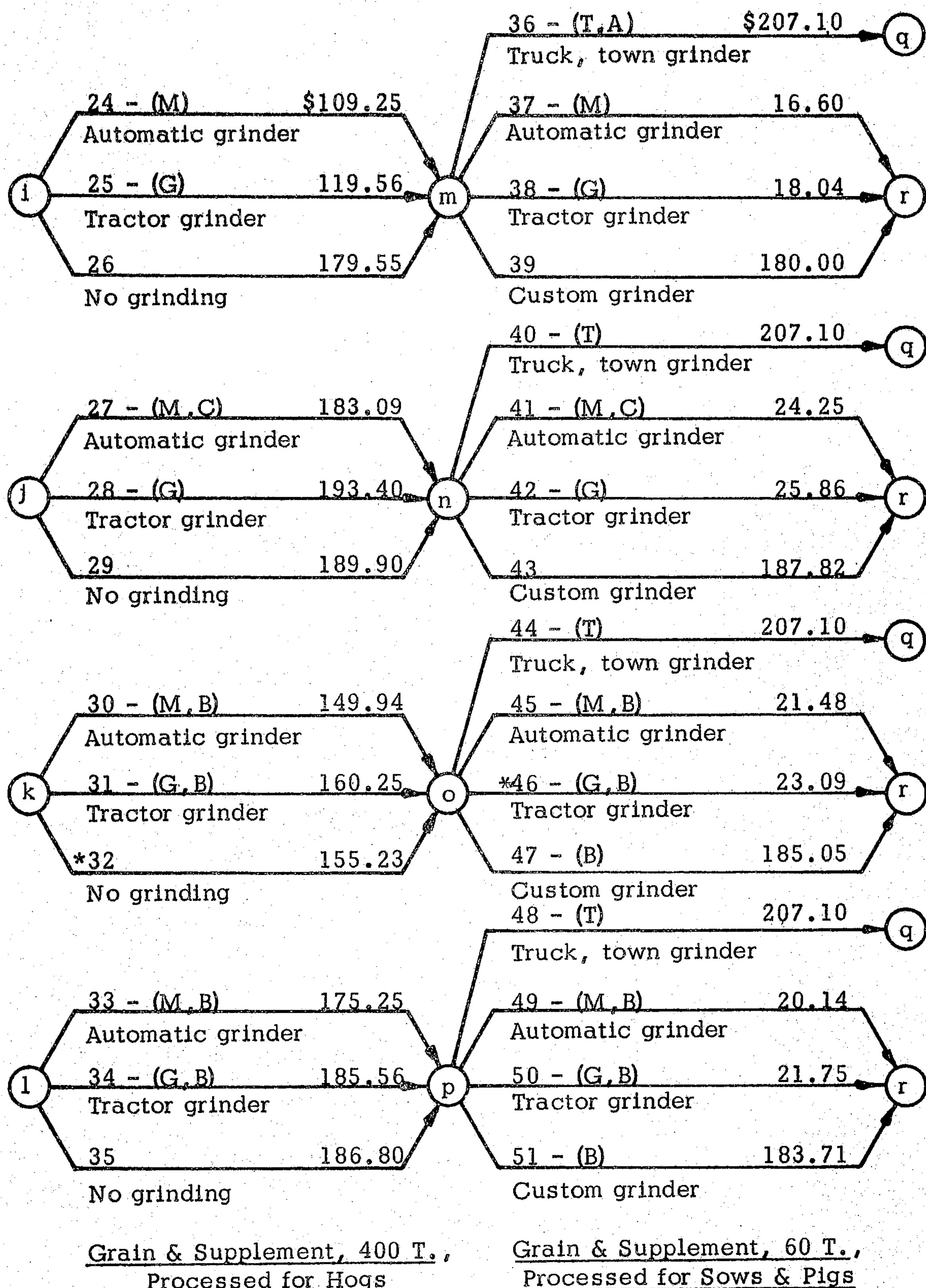
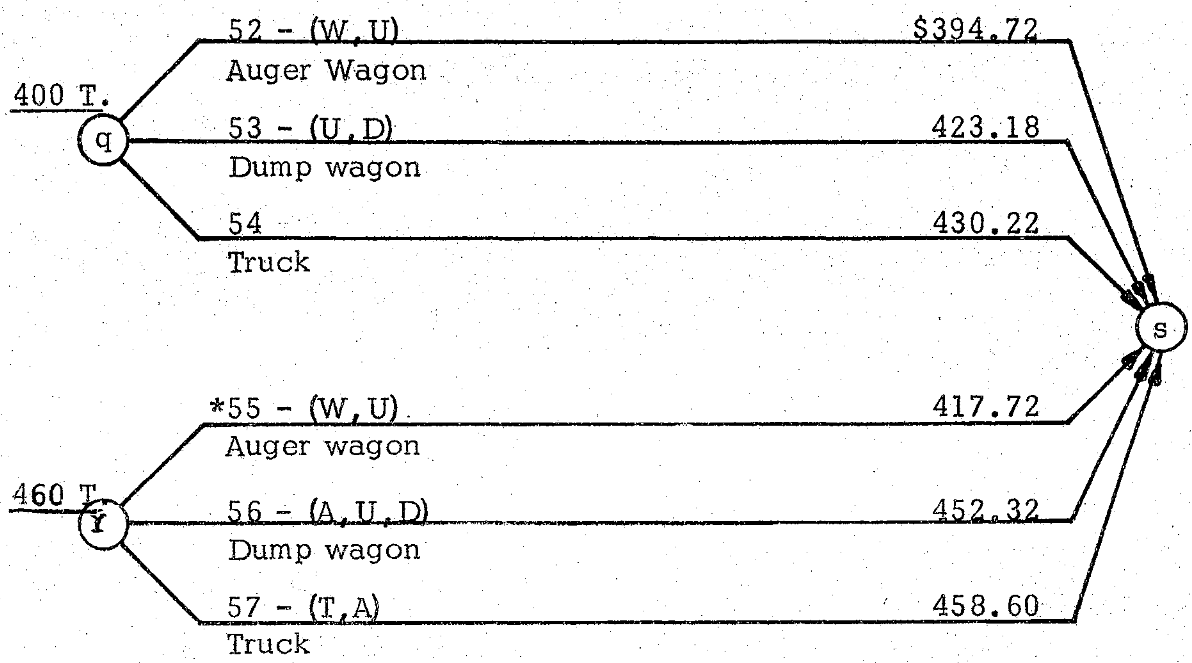


Figure 7. (continued) Flow Chart for Corn-Hog Farm



Feed, Processing Area to Feeders

Figure 7. (continued) Flow Chart for Corn-Hog Farm

Table III. Multiple-Use Machines Listed on Flow Chart of Figure 7

Index	Machine	Annual Fixed Cost	Index	Machine	Annual Fixed Cost
T	- Dump truck	$489.50	F	- Flat-bed wagon	$36.50
W	- Auger wagon	71.00	U	- Utility tractor	96.80
I	- Inclined chain elevator	48.30	D	- Dump-bed wagon	48.90
A	- Portable auger	23.30	S	- Self-unloading forage wagon	76.07
M	- Automatic mixer-grinder & auger	88.77	G	- Tractor-powered grinder & feed meters	69.00
C	-- Horiz. chain conveyor	30.75	B	- Hoppered overhead bin	36.80

<u>Summary of procedure for the unit-flow model.</u> The unit-flow model developed in this research is believed to be a useful tool for selecting minimum-cost materials-handling systems for fixed-size operations with possibilities for multiple use of machines. The procedure for setting up and solving such problems is summarized here:

1. Necessary processes are established for the materials-handling problem under study.

2. Practical, efficient alternative methods of performing these processes are designed, using up-to-date engineering knowledge and practice. These methods are specified on man and machine process charts, and cost data are obtained and calculated.

3. The flow chart is constructed, and necessary machines are listed for each method.

4. Machines are classified as single-use, must-use, multiple-ordered-use, or multiple-use.

5. Method costs, including operating costs and fixed costs of the appropriate machines, are computed.

6. Methods connecting the same pair of nodes are compared; and when any method uses all the multiple-use machines specified in a lower-cost method, the higher-cost one is eliminated.

7. Network equations, purchase inequalities, and the cost equation are written to form a linear programming problem.

8. The problem is solved by using the simplex algorithm, which is available for practically all digital computers.

9. For selected multiple-use machines or for certain methods of particular interest, cost figures may be varied and the problem re-run until the maximum cost is obtained for which that item remains in the optimal solution. Computer routines are available or may be written for performing this operation automatically (26).

Integer-Solution Methods

Further restrictions on the fixed-size problem. In a fixed-size materials-handling problem, it may be desirable to place a limitation on the amount of labor, seasonal or total, and/or on the amount of capital available. These types of restrictions may be readily added to the linear programming model. A linear inequality is written which states the desired limitation. For example, in the problem shown in Figure 5, suppose the initial capital expenditure for Machines B, C, and L are \$700, \$300, and \$400, respectively, and the maximum permissible capital outlay is specified as \$1,000. The appropriate inequality would be added to the original set of equations and inequalities as follows:

$$700x_B + 300x_C + 400x_L \leq 1000.$$

Labor restrictions would be handled similarly.

The linear programming solutions to such problems will rarely give the desired zero and unity values for the variables. A recently developed algorithm by Gomory (7, 8) or one by Land and Doig (12) may be applied to the problem of finding integer-valued solutions to linear programming problems. With formulation of the problem in the unit-flow model, integer-valued solutions will be zero and unity solutions. Actually, computer programs for the integer-solution method are still in the process of development, but they hold promise of greatly widening the scope of problems capable of solution by linear programming methods.

Selection of optimal enterprises and equipment--variable-size problems. The use of linear programming for selecting farm enterprise levels for an optimal management program is a well-established part of agricultural economics research and practice. In such models the variables are amounts of activities, such as hog production under a given management system, corn production, corn selling or buying, fertilizer use, and any other farm management activity desired in the model. The constraints, or equations and inequalities describing the problem, are usually based on limiting factors of production, such as land, labor, housing facilities, and capital. A constant profit or cost factor per unit of the variable is used in the functional equation. As explained previously, decreasing-returns functions or increasing-cost functions may be handled by approximation with several linear functions.

The use of integer solution methods for linear programs makes it possible to handle decreasing-cost functions, such as equipment costs composed of both fixed and operating costs.

As is shown by one of the authors (15), the flow chart, equations similar to the network equations of the unit-flow model, and a set of purchase inequalities may be used to specify a materials-handling problem when the amount of various farm enterprises is unknown and is actually selected by the linear programming method. Again the integer-solution method must be used to obtain zero or unity answers for the machine purchase variables. This technique shows possibilities of being effective not only for determining optimal levels of farm enterprises, but at the same time for determining the optimal materials-handling system for that combination of enterprises. To the authors' knowledge, no application of this method has yet been made, but the future availability of a computer routine for the integer solution method of linear programming will make such applications feasible. The integer solution method is an important tool for dealing with a large group of problems, of which these applications to farm materials-handling systems are but a part.

Value of a Mathematical Method

A mathematical method of selecting an optimal materials-handling system has several advantages as an aid in system synthesis. A mathematical model can describe the entire system so that selection need not be made on a process-by-process basis. This allows recognition of interrelationships between all parts of the system, such as multiple use of a machine in different processes. By being able to describe and select systems as a whole, rather than piecewise, mathematical programming can make a major contribution in optimizing materials-handling systems.

A mathematical model encourages the concise and complete statement of the problem and the variables. It promotes objective decision-making. Subjective decisions will continue to be made in the original design of alternative systems, but a model capable of handling many variables allows many more factors to be quantified.

A mathematical method that is efficiently programmed for solution on a digital computer saves computation time and allows larger problems to be studied. The increasing availability of computers at colleges and universities and in business firms makes possible the general use of such a method. A mathematical method may permit analysis of the sensitivity of the solution to changes in costs or other coefficients of the problem. Such an analysis would be as valuable as the optimal solution itself.

The usefulness and ease of application of the models studied and developed in this research justify further research in their application and in the theory of various models. The unit-flow model could be used effectively to determine recommended systems for typical sizes and types of farms. For example, the optimum system for a 200-head cattle-feeding operation on a 240-acre farm with CCSOCl rotation could be determined. A typical building and equipment situation could be used, or no buildings could be assumed, in which case the minimum-cost system of new buildings and equipment would be determined. Accurate data on labor requirements and costs are needed for the various methods.

It is possible that in the future the method could be economically applied to specific farmsteads by farm feed and equipment distributors or by farm managers, using computers available privately or through universities. Actual cost of proposed new equipment or buildings would be available, and the farmer could make a good estimate of his own labor requirements. Linear programming is currently being used on individual farms by farm management consultants to select enterprises, and its extension to equipment selection is logical.

Further research should be conducted in developing methods of applying integer-solution techniques to problems of determining optimum enterprises and materials-handling systems on actual-size problems. This is a large problem and could benefit from research by many individuals in various departments.

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