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A MODEL OF AGRICULTURAL TRADE FOR DIFFERENTIATED PRODUCTS

by

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Abstract

of

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The model described in this paper was developed to take account of international product differentiation through imperfect substitutability. Utilizing a CRESH function to represent a demand and a supply function appropriate for differentiated products, we arrived at a model for the analysis of the effects on agricultural trade of various government policies.

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# A MODEL OF AGRICULTURAL TRADE FOR DIFFERENTIATED PRODUCTS

A number of internationally traded agricultural commodities are imperfect substitutes in the view of prospective buyers who tend to differentiate products by country or region of origin (Schuh 1981, 1982; Thompson). The importance of product differentiation in trade of food and agricultural products has been supported by several empirical studies. First, the law of one price that should prevail if products are homogeneous has been rejected for a number of world agricultural markets (Blejer and Hillman; Grennes, Johnson, and Thursby 1978a; Richardson). Second, trade statistics frequently show, even for the most narrowly defined categories, two-way trade, or exports and imports of the same good (Johnson, Grennes, and Thursby; Pagoulatos and Sorensen). For example, the United States both exports and imports such products as breakfast cereals, chewing gum, brandy, and roasted coffee. The existence of two-way trade suggests the possibility that products are differentiated internationally.

Additional evidence of product differentiation in agricultural trade has been presented by Oleson and Grennes, Johnson, and Thursby (1978b) who treated wheat as a nonhomogeneous commodity. Sarris (1980, 1982) analyzed world trade in fruit and vegetables as products differentiated according to their country of origin. Finally, Capel, Reekie and Sirhan--in three individual studies summarized by Johnson (1971)--showed that produce heterogeneity exists in the wheat, cotton, and flue-cured tobacco industries.

The implications of these studies are that product differentiation is an important factor in world agricultural trade and that traditional trade models, which assume perfect-substitutability, may not be appropriate for differentiated commodities (Sarris 1981; Schuh 1982; Thompson). A model that incorporates differentiated products has been developed by Armington (1969a,b).

Armington's theoretical framework assumes that consumers differentiate among varieties of the same good on the basis of their country of origin. A key assumption of the model is that for any one market, the elasticity of substitution between every pair of traded products is constant and equal across all of them. Under these assumptions, Armington obtained a set of product demand functions that have been utilized in the analysis of trade policy by Grennes, Johnson, and Thursby (1978a,b), De Melo and Robinson, and Sarris (1980, 1982). The constant elasticity of substitution (CES) assumption, however, is inconsistent with the basic assumption of Armington's model, that consumers differentiate among varieties of the same good on the basis of their country of origin. Recent work by Artus and Rhomberg and Deppler has relaxed the CES assumption by utilizing Hanoch's constant ratio elasticity of substitution homothetic (CRESH) function. While the CRESH function increases the flexibility of the model, its implicit nature complicates the estimation of the product-demand functions. Further research is clearly needed in this area.

It is the objective of this paper to derive an empirical trade model that is applicable to agricultural markets characterized by international product differentiation. The model obtained in this paper has its theoretical foundation in Lancaster's characteristics approach to product differentiation and the theory of demand. Utilizing Hanoch's CRESH function to represent demand and developing an appropriate framework for dealing with the supply of differentiated products, we arrive at a testable formulation of the trade model. Finally, government policy in the form of export subsidies, and exchange rate changes is introduced into the model to increase its operational usefulness.

## Main Features of the Model

The model identifies T countries (K = 1 ... T), each producing a different variety of N goods ($X_1$ ... $X_N$). Following Lancaster, we can assume that consumers derive their satisfaction from the enjoyment of the characteristics embodied in the goods they consume. Furthermore, they rank their preference orderings directly over collections of characteristics, and only indirectly over collections of goods. Identical commodities constitute a product if they possess the same characteristics in the same proportions and consumers are indifferent between them. Commodities combining the same characteristics in different proportions belong to the same commodity group if they can be produced employing the same engineering technology. They constitute a good, and a good comprises one or more differentiated products. Commodities offering characteristic combinations that can only be attained by employing different production technologies are called different goods.

According to these definitions, then, consumers differentiate among varieties of the same good on the basis of their country of origin. This implies the existence of N x T differentiated products ($X_{11}$... $X_{1T}$,

$X_{21} \ldots X_{2T}, \ldots, X_{N1} \ldots X_{NT}$). If each country is a potential consumer of its own and all others' products, we can identify, for each product, a single supply function and a set of T demand functions. The model is closed by imposing the usual identity that for each product, domestic demand plus exports must equal domestic supply plus imports.

Demand Equations

The demand equations are derived from the consumer's utility maximization problem. We assume here that the utility function is given in terms of N x M characteristics ($Z_{11} \ldots Z_{1M}, Z_{21} \ldots Z_{2M}, \ldots, Z_{N1} \ldots Z_{NM}$), M being supplied by each of the N goods. The utility function is also weakly separable: the first set of M characteristics serving consumption objective $R_1$ and being supplied by good $X_1$, the second set serving consumption objective $R_2$ and being supplied by $X_2$, etc. Furthermore, we follow Lancaster in assuming that each consumption activity produces a fixed vector of characteristics and that the relationship between the characteristics and the product-space is linear. Under these assumptions, country m's utility maximization problem can be written as:

(1) Maximize $U^m = U^m[R_1^m(Z_{11} \ldots Z_{1M}), R_2^m(Z_{21} \ldots Z_{2M}) \ldots R_N^m(Z_{N1} \ldots Z_{NM})]$

(2) Subject to: $Z_{ji}^m = \sum_{k=1}^{T} C_{jik} X_{jk}^m \qquad (i = 1 \ldots M)$

$$I^m = \sum_{j=1}^{N} \sum_{k=1}^{T} P_{jk}^m X_{jk}^m$$

where:

$C_{jik} = \frac{Z_{jik}}{X_{jk}}$ defines the amount of characteristic ji per unit of product j coming from country k,

$I^m$ = country m's expenditures on goods $(x_1 \ldots x_n)$,

$P^m_{jk}$ = price of country k's variety of good j in country m,

$X^m_{jk}$ = amount of country k's variety of good j consumed in country m.

The solution of the above optimization problem yields individual product demand functions in country m as a function of the country's total money expenditure on the good and of the prices of all products. To operationalize the resulting demand functions we follow Artus and Rhomberg and Deppler in adopting Hanoch's function and assume that a good $(X_j)$ is a linear homogeneous function of its constituent products $(X_{jk})$ and that this relationship can be implicitly defined by a CRESH function:

$$(3) \qquad F(X_j, X_{jk}) = \sum_{k=1}^{T} b_{jk}(X_{jk}/X_j)^{\rho_{jk}} - 1 \equiv 0$$

where $b_{jk}$ and $\rho_{jk}$ are the distribution and substitution parameters respectively, and

$$(4) \qquad a_{jk} = \frac{1}{1 - \rho_{jk}}$$

is the elasticity of substitution parameter.

Although the function $F(X_j, X_{jk}) = 0$ is an implicit function, Hanoch demonstrated (1971, p. 697) that it implies the existence of an explicit function $X_j = \phi(X_{jk})$ if, and only if, $b_{jk}$ is positive for at least one k and the products $b_{jk}\rho_{jk}$ have the same algebraic sign for all

k. This restricts all the products in the same group to being substitutes rather than complements, which is not an unreasonable assumption within the context of differentiated products.

Again following Deppler we further assume that the appropriate price index function associated with the CRESH function (3) is given by:

$$(5) \qquad P_j = \pi_k P_{jk}^{s_{jk} a_{jk}/A}$$

where

$P_{jk}$ is the price of product k belonging to good j,

$S_{jk} = \dfrac{P_{jk} X_{jk}}{\sum_{u=1}^{T} P_{ju} X_{ju}}$ is the market share of product k with respect to total sales of good j, and

$a_{jk} = \dfrac{1}{1-\rho_{jk}}$ is the elasticity of substitution of product k vis-a-vis all products in j taken together,

and

$$A = \sum_{k=1}^{T} S_{jk} a_{jk}.$$

Under these assumptions, the utility maximization problem presented earlier (equations 1 and 2) can be solved, after transforming the characteristics into the goods-space, by means of a two-stage procedure that takes the form (omitting country subscripts):

Stage one

(6) Maximize $U(X_1 \ldots X_N)$

(7) subject to: $\sum_{j=1}^{N} P_j X_j - I = 0$

The solution to this first stage yields demand equations of the form

(8) $\overset{*}{X}{}^{D}_{j} = X_j[P_1 \ \ldots \ P_N, \ I]$

where

$\overset{*}{X}{}^{D}_{j}$ is the amount of good j optimally demanded irrespective of its origin, and

$P_j$ is the price of composite good j in a numeraire currency.

<u>Stage two</u>

(9) Minimize $\sum_{k=1}^{T} P_{jk}X_{jk}$

(10) subject to $\sum_{k=1}^{T} b_{jk}(X_{jk}/X^{*}_{j})^{\rho_{jk}} - 1 = 0$

where the value of $X^{*}_{j}$ is given by the solution to the first stage.

Rearranging terms, forming the Lagrangean, and solving for the first-order condition, we obtain the desired product demand functions:

(11) $X^{D}_{jk} = B_{jk}X_jP_{jk}^{\varepsilon_{jk/jk}} \left(\prod_{u \neq k} P_{ju}^{\varepsilon_{jk/ju}}\right)$

where

$B_{jk} = (\rho_{jk}b_{jk})^{\frac{1}{1-\rho_{jk}}}$

$\varepsilon_{jk/jk} = S_{jk}\sigma_{jk/jk}$ is the own-price compensated (in the sense of fixed $X_j$) elasticity of demand,

$\varepsilon_{jk/ju} = S_{ju}\sigma_{jk/ju}$ is the cross-price compensated elasticity of demand

$\sigma_{jk/ju}$ is the elasticity of substitution of products k,u belonging to good j, and

$\sigma_{jk/jk}$ is the own elasticity of substitution.

Supply Equations

The supply side of the model incorporates Mayer's determination of the most profitable product specification in the Armington trade model. The assumptions utilized in the supply side are that: i) each country produces a unique variety of each good, ii) each country's output is small compared to the world's total output of that good, and iii) each country tries to maximize profits as a single firm would do.

If there are only two factors of production, capital and labor, and the product's specification is again defined in terms of characteristics, the production function for country k's variety of good j can be written in the form:

$$(12) \quad X_{jk} = X_{jk}(e_{jk}, L_{jk}, K_{jk})$$

where $e_{jk}$ is the quality control parameter representing the variety of the good and $L_{jk}$ and $K_{jk}$ represent the labor and capital requirements for the production of product jk.

Let us assume that the production function above is homogeneous of degree one in inputs alone and that the marginal rate of substitution between capital and labor is independent of the choice $e_{jk}$, so that the production function can be written in the separable form:

$$(13) \quad X^S_{jk} = h_{jk}(e_{jk})\ g_{jk}(L_{jk},k_{jk}) = h_{jk}(e_{jk})\ L_{jk}f_{jk}(k_{jk})$$

where

$$k_{jk} = \frac{K_{jk}}{L_{jk}}$$

Even if individual countries' production is small, the fact that they are able to offer products with different specifications enables them to alter the price of whatever their outputs are. Consequently, for each product

(14) $\quad P_{jk} = P_{jk}(e_{jk})$

and, under the above stated assumptions, product jk's profit function becomes:

(15) $\quad \pi_{jk} = P_{jk}(e_{jk})h_{jk}(e_{jk})L_{jk}f_{jk}(k_{jk}) - L_{jk}(w^k - r^k k_{jk}).$

Taking advantage of the duality between the production and the variable profit function, a Generalized Leontief production function can be derived from equation (15) above (Woodland 1977).

$$X^S_{jk} = a_{xx} + a_{xL}\left(\frac{w^k}{P_{jk}}\right)^{1/2} + a_{xk}\left(\frac{r^k}{P_{jk}}\right)^{1/2}$$

where the a's represent the coefficients of the function and $w^k$ $r^k$ are the input prices in country k, which are assumed to be determined under competitive condition and are therefore, exogenous to the model. The product prices will also be considered exogenous.

Government Policy

A number of government policies can be introduced in the trade model in the previous sections. As an illustration, we present the case of export subsidies and exchange rates. To do so, we assume that a country does not import and export the same product and that the product under consideration is produced domestically. Imports would then be equal to zero since, by definition, country m is the only supplier of such a product. Market equilibrium for product jm (variety m of the composite good $X_j$) in country m requires that domestic supply equal domestic demand plus exports.

$$(17) \quad D^m_{jm} + E_{jm} = S^m_{jk}$$

where

$D^m_{jm}$ = country m's demand for project jm for domestic use,

$E_{jm}$ = export demand for product jm to the rest of the world,

$S^m_{jm}$ = domestic supply of product jm in country m.

The value of $E_{jm}$ represents the sum of the quantities demanded of product jm by all countries except country m. The explicit forms of equations $D^m_{jm}$ and $E_{jm}$ can be derived from the product demand funtions obtained in the model developed in earlier pages (equation 11). These take the form:

$$(18) \quad D^m_{jm} = D^m_{jm}(X^m_j, P^m_{jm}, P^m_j) = \left[B^m_{jm}\ X^m_j\ P^{m\ \varepsilon^m_{jm/jm}}_{jm} \left(\prod_{u \neq m} P^{m\ \varepsilon^m_{ju/jm}}_{ju}\right)\right]$$

and

$$(19) \quad E_{jm} = \sum_{k \neq m} D^k_{jm}(X^k_j, P^k_{jm}, P^k_j) = \sum_{k \neq m} \left[B^k_{jm}\ X^k_j\ P^{k\ \varepsilon^k_{jm/jm}}_{jm} \left(\prod_{u \neq m} P^{k\ \varepsilon^k_{ju/jm}}_{ju}\right)\right]$$

where

$$(20) \quad B^m_{jm} = (\rho^m_{jm}\ b^m_{jm})^{\frac{1}{1-\rho^m_{jm}}}$$

and

$P^m_{jm}$ = price of product jm in m's currency

$P^m_j$ = price of good j in m's currency,

$P^k_{jm} = \dfrac{P^m_{jm}}{ER^k_m(1 + te^m_{jm})}$ = price of product jm in k's currency,

$ER^k_m$ = exchange rate between countries m and k,

$te^m_{jm}$ = rate of export subsidy in country m for product jm,

$\varepsilon^m_{jm/jm}$ = own-price elasticity of demand for product m in country m,

$\varepsilon^m_{jm/jk}$ = cross-price elasticity of demand for product m with respect to product k, in country m.

The explicit form of equation $S^m_{jm}$ can be obtained from equation (16) presented earlier:

$$(21) \quad S^m_{jm} = S^m_{jm}\,(P^m_{jm},\ w^m,\ r^m) = a_{xx} + a_{xL}\left(\frac{w^m}{P^m_{jm}}\right)^{1/2} + a_{xk}\left(\frac{r^m}{P^m_{jm}}\right)^{1/2}$$

where $w^m$ and $r^m$ are the price of labor and capital, respectively, in country m.

Totally differentiating equation (17) we obtain the expression

$$(22) \quad \hat{D}^m_{jm} D^m_{jm} + \hat{E}_{jm} E_{jm} = \hat{S}^m_{jm} S^m_{jm}$$

where the superscript "^" refers to percentage changes.

In order to obtain these expressions in terms of elasticities, we define the following

$$(23) \quad \hat{D}^m_{jm} = -\varepsilon^m_{jm/jm} \hat{P}^m_{jm}$$

and

$$(24) \quad \hat{E}_{jm} = \sum_{k \neq m} [- \varepsilon^k_{jm/jm} \hat{P}^k_{jm}]$$

Differentiation of equation (21) and substitution of (18), (19), (23), (24) and (22) yields, after some manipulations

$$(25) \quad \hat{P}^m_{jm} = \frac{-GJ}{F - BH - GH} \hat{CE}^m$$

which is the equation we want to compute, and where

$$(26) \quad B = B^m_{jm} X^m_j \left( \pi_{u \neq m} P_{ju}^{\varepsilon^m_{jm/jm}} \right)$$

$$(27) \quad G = \sum_{k \neq m} B^k_{jm} X^k_j \left( \pi_{u \neq m} P^k_{ju}{}^{\varepsilon^k_{ju/jm}} \right)$$

$$(28) \quad F = \frac{1}{2} \left[ a_{xL} \left(\frac{w^m}{P^m_{jm}}\right)^{1/2} + a_{xk} \left(\frac{r^m}{P^m_{jm}}\right)^{1/2} \right]$$

$$(29) \quad H = \varepsilon^m_{jm/jm} P^m_{jm}{}^{\varepsilon^m_{jm/jm}}$$

$$(30) \quad J = \varepsilon^k_{jm/jm} \left(\frac{P^m_{jm}}{CE^m}\right)^{\varepsilon^k_{jm/jm}}$$

$$(31) \quad CE^m = ER(1 + te^m_{jm})$$

In summary, then, the proposed empirical model to be estimated consists of the set of equations implied by

$$X^k_{jm} = B^k_{jm} X^k_j P^k_{jm}{}^{\varepsilon^k_{jm/jm}} \left( \pi_{u \neq m} P^k_{ju}{}^{\varepsilon^k_{ju/jm}} \right)$$

where $X^k_{jm}$ represents all the demand functions of product jm by all countries k (k,m = 1 ... t) and can be estimated for all the countries that consume a substantial amount of the commodity considered.

The estimation of these equations will yield results for the parameters $B^k_{jk}$ and for the cross-price elasticities of demand $\varepsilon^k_{jm/jm}$ and $\varepsilon^k_{ju/jm}$. Substitution of these in the identities given by the groups of equations (26) - (31) allows us to compute the desired effects of changes in the rate of export subsidies represented by equation (25).

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