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SOME PRODUCTION FUNCTIONS FOR THE PUNJAB

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I. INTRODUCTION

𝕿HIS paper reports some production functions of the Cobb-Douglas type estimated with the Farm Management[1] data for two districts of the Punjab, Ferozepore and Amritsar, for the years 1954-55, 1955-56 and 1956-57.

One of the main objectives of this analysis, as of any production function analysis, is to arrive at some judgment about the efficiency of the prevalent factor proportions in production and hence to suggest changes in these proportions in the optimal direction. An empirical production function enables us to do so because the marginal product of each input estimated from the function can be compared with its acquisition cost.

Secondly, the data used for estimating the production functions are also used to examine the correlations, if any, that exist between the yield per acre, the average product of labour and total average cost on the one hand, and the acre-size and the output-size of the farm on the other.

Thirdly, returns to scale are estimated.

Finally, we show how the estimated function can be used to determine not only the *direction* of desirable input changes but even the *magnitude* of the optimum land input and the optimum labour input on certain assumptions.

Thus, the familiar and important topics—the relationship between farm size and efficiency, and the determination of optimal factor combinations in general and the "optimum holding" in particular—are treated with the production function technique.

II.   THE ESTIMATED FUNCTIONS

The data relate to 200 holdings: 10 each from 20 villages selected by stratified sampling.[2]   A different sample of holdings was studied in the same villages every year.

Three sets of equations were estimated.   Their coefficients are presented in Tables I, III and V.

In these Tables L is the standard unirrigated acreage: the sum of the unirrigated acreage, and the irrigated acreage times 1.67 (the standardisation factor).[3] M stands for man-days of human labour and B for bullock labour days.   E is the expenditure on seeds, manures and fertilizers, the use of implements and interest on working capital.   I is the expenditure on irrigation: operation of wells in the case of well-irrigation, and canal dues paid to the Government in the case of canal irrigation.   E* is the sum of E and I.   And Y is the value of output.

TABLE I—COEFFICIENTS OF PUNJAB PRODUCTION FUNCTIONS

No.	Year	L	M	B	E	I	R²
55·1	1954-55	0·30 (0·08)	0·20 (0·11)	0·02 (0·10)	0·41 (0·10)	*	·64
56·1	1955-56	0·50 (0·11)	0·57 (0·15)	-0·33 (0·15)	0·03 (0·12)	0·17 (0·06)	·60
57·1	1956-57	0·27 (0·08)	0·30 (0·11)	0·10 (0·10)	0·35 (0·10)	0·04 (0·04)	·72

*   For 1954-55, irrigation expenditure was not recorded.   Therefore variable I does not appear in equation 55·1.

The coefficient of B is not significant[4] in equations 55.1 and 57.1.   In equation 56.1 it is significant but negative.   The coefficient of M is not significant in equation 55.1, that of E is not significant in equation 56.1 and that of I in equation 57.1.

Besides, as often happens in production function analysis, considerable inter-collinearity was found to exist between the input variables.

2.   Studies in Economics of Farm Management in the Punjab, 1954-55, p. 29.

3.   This factor is the weighted average of the ratio of the yields of the main crops per irrigated acre to the yield per unirrigated acre in crop-cutting experiments carried out in the Punjab over three decades.   It has been computed from data in various annual issues of the Agricultural Statistics of India and the Report on the Season and Crops of the Punjab.

4.   At the 5 per cent level.

The simple correlation coefficients between pairs of variables are shown below

TABLE II—SIMPLE CORRELATION COEFFICIENTS BETWEEN DIFFERENT VARIABLES

Year ending June		L	M	B	E	I
M	1955	·79				
	1956	·73				
	1957	·78				
B	1955	·80	·86			
	1956	·72	·88			
	1957	·78	·90			
E	1955	·81	·82	·77		
	1956	·74	·76	·79		
	1957	·82	·81	·80		
I	1955	—	—	—	—	
	1956	·64	·61	·52	·45	
	1957	·61	·62	·56	·56	
Y	1955	·75	·72	·68	·76	—
	1956	·71	·67	·57	·56	·63
	1957	·78	·79	·77	·79	·57

The Table shows that the bullock labour input is highly[5] correlated with land, manual labour and operating expenditure, and operating expenditure with land and manual labour.

In the event of high inter-collinearity between input variables a choice has to be made between them. In the second set of experiments, therefore, considering the correlations between different variables, it was decided to drop B from all the three regressions 55.1, 56.1 and 57.1 and to add up E and I. The coefficients of the set of equations so obtained (55.2, 56.2 and 57.2) are given in Table III.

TABLE III—COEFFICIENTS OF PUNJAB PRODUCTION FUNCTIONS

No.	Year	L	M	E*	R²
55·2	1954-55	0·31 (0·08)	0·21 (0·09)	0·41 (0·10)	·64
56·2	1955-56	0·55 (0·13)	0·39 (0·13)	0·03 (0·18)	·56
57·2	1956-57	0·18 (0·07)	0·40 (0·08)	0·48 (0·11)	·71

5. 'High' is taken here to mean a correlation coefficient equal to or greater than ·8.

All the coefficients are significant except the coefficient of E* in 56.2.

An examination of the simple correlation matrix of L, M and E* again revealed high inter-collinearity between E* and L and M. (Table IV).

TABLE IV—SIMPLE CORRELATION COEFFICIENTS BETWEEN E*, L AND M

Year	r between E* and	
	L	M
1954-55	·81	·82
1955-56	·84	·83
1956-57	·82	·83

A third set of equations was, therefore, estimated with only L and M as the input variables (Table V).

TABLE V—COEFFICIENTS OF PUNJAB PRODUCTION FUNCTIONS

No.	Year	L	M	R^2
55·3	1954-55	0·47 (0·24)	0·40 (0·27)	·60
56·3	1955-56	0·56 (0·09)	0·41 (0·10)	·56
57·3	1956-57	0·35 (0·06)	0·60 (0·07)	·68

The marginal products of the inputs at geometric means computed from the three sets of equations are given in Table VI.

TABLE VI—MARGINAL PRODUCT AT GEOMETRIC MEANS

Equation No.	L .	M	B	E	I	E*
55·1	27·96	0·79	0·10	2·47	—	—
56·1	39·81	2·46	—2·00	0·20	5·33	—
57·1	33·54	2·18	0·95	2·87	1·41	—
55·2	27·80	1·16	—	2·70	—	—
56·2	43·58	1·70	—	—	—	0·17
57·2	22·50	2·91	—	—	—	3·10
55·3	43·09	1·59	—	—	—	—
56·3	44·37	1·78	—	—	—	—
57·3	43·76	4·37	—	—	—	—

It will be seen that the marginal product estimates from equation sets (1) and (2) remain very unstable due, in part, to changes in function specifications. The estimates of the marginal product of "land" from the equation set (3), however, are relatively stable, while the marginal product of "labour"[6] estimated from equation (3) rises very steeply in 1956-57.

6. Since inputs other than land and human labour have been excluded the coefficients (partial elasticities) and marginal products of L and M have to be interpreted as effects of the inputs L and M *combined with* the inputs highly correlated with them.

TABLE VII—AVERAGE COST OF INPUTS

	1954-55 Rs.	1955-56 Rs.	1956-57 Rs.
Human labour per day[7]	2·49	2·59	2·84
Bullock labour per day	1·83	2·23	2·96
Rent of land per acre[7]	50·00	56·00	65·19
Estimated rent of land per standard unirrigated acre[8]	32·62	36·62	42·81

Source : Studies in Economics of Farm Management in Punjab, 1954-55, pp. 19, 53, 71; 1955-56, pp. 6, 45, 83 and 1956-57, pp. 6, 42, 76.

If we compare the marginal product estimates in Table VI with the actual wages of labour and the rent of land shown in Table VII, we find that the marginal product of land is consistently more than rent. This suggests that the land-use market is imperfect and/or land is relatively under-applied. On the other hand, in the first two years the wage rate exceeds and in the third it remains much below the marginal product of labour.[9]

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7. Labour cost is the weighted average of monthly wages of hired male adult agricultural workers, the weights being the proportion of total annual labour input employed in each month.

The rise in rent is not due to a mere rise in the proportion of irrigated to dry land in the sample because this proportion remained almost the same in the 3 years.

8. The rent per acre calculated from Farm Management Studies is the sum total of the rent of irrigated land and the rent of unirrigated land reported therein divided by the total area. This has been transformed into rent per standard unirrigated acre so that it can be compared with the marginal product per standard unirrigated acre. Let $r = a_w r_w + a_d r_d$ where r is the rent per acre, r_w the rent per irrigated acre, r_d the rent per unirrigated acre, a_w the proportion of irrigated area to total area and a_d the proportion of unirrigated area to total area. Since in estimating the input of land for the production function one irrigated acre has been considered equivalent to $1 \cdot 67$ unirrigated acres,

$$r = a_w (1 \cdot 67) (r_d) + a_d r_d$$

so that the rent per standard unirrigated acre

$$r_d = \frac{r}{a_d + (1 \cdot 67) a_w}$$

Thus for 1956-57, for example,

$$r_d = \frac{65 \cdot 19}{\cdot 22 + (1 \cdot 67) (\cdot 78)} = 42 \cdot 81.$$

9. A part of the difference between the estimated marginal products and acquisition costs must be due simply to the fact that the former should be slightly biased upwards owing to the exclusion of inputs other than land and human labour.

We also observe that while the (geometric) average land input remained roughly the same (21 to 23 acres) the average labour input declined substantially from 495 to 360 mandays during the three years(Table VIII). This decline came about inspite of the increase in the total "earner" manpower in the sample families from 464 to 532 persons. Thus the rise in the marginal product of labour was associated with the fact that the farmers economised labour. They might have done so because in the first two years the wage rate was higher than the marginal product of labour.

TABLE VIII

Year	$\bar{Y}$ (Rs.)	$\bar{L}$ (acres)	$\bar{M}$ (days)	$\bar{B}$ (days)	$\bar{E}$ (Rs.)	$\bar{I}$ (Rs.)	$\bar{E}^*$ (Rs.)
1954-55	1,963·90	21·42	494·99	333·97	325·99	—	—
1955-56	1,827·30	23·06	420·34	303·11	248·20	59·14	323·37
1956-57	2,620·40	20·96	360·08	276·75	321·66	67·39	405·23

On the whole, the orders of magnitude of the estimated marginal product of land and labour derived from equations (3) seem to be very plausible. The most significant aspect of the marginal product estimates is that they are not so wildly out of line with acquisition costs as the usual references to the irrationality of the Indian peasant imply. In fact, the two are close enough to justify the inference that the Punjab peasant is trying to use the resources accessible to him as efficiently as he can—efficiency being defined in the orthodox theoretical sense of the equality of marginal factor product and acquisition cost.[10] That the quantum of these resources (including knowledge) is limited is another matter. The small differences between marginal products and acquisition costs can be due to errors of measurement and/or exclusion of some inputs, and/or the well-known imperfections of resource mobility. Though much emphasis is usually laid on these imperfections, their measured effect on the efficiency of resource-combination seems to be only residual. Adjustments in the resource-mix do take time. But they *are* made—in the direction of optimality. Hence, for instance, the significant adjustment of the labour input mentioned above. This adjustment, *inter alia*, has clearly led to a noticeable increase in the productivity of labour—both average and marginal—in the agriculture of the two Punjab districts.

III. SIZE AND EFFICIENCY

The usual discussions of the size of the "optimum holding" are based on a comparison of the average yield per acre or the average cost in different acre-size groups. The limitations of such comparisons have been discussed elsewhere.[11] Here we have estimated from the data for 1956-57, the regression and correlation coefficients between pairs of productivity and size magnitudes.

10. Dr. W. D. Hopper reached a similar conclusion in his "Allocation Efficiency in Traditional Indian Agriculture," IADP Staff Working Paper No. 6203, based on a study of farms in eastern U.P.

11. Raj Krishna, "The Optimum Firm and the Optimum Farm," *The Economic Weekly* October 6 and 13, 1962.

TABLE IX—COEFFICIENTS OF SIMPLE CORRELATION BETWEEN AVERAGE
PRODUCTIVITY AND SIZE: 1956-57

Relation No.	Productivity or cost variable (dependent)		Size variable (independent)		Linear regression coefficient	Linear coefficient of determination	Quadratic coefficient of determination†
(1)	(2)		(3)		(4)	(5)	(6)
57·4	Output per acre	$\frac{(Y)}{(L)}$	Acreage	(L)	$-\cdot40$ $(\cdot14)$	·038	—
57·5	Output per man-day	$\frac{(Y)}{(M)}$	Acreage	(L)	$\cdot034$ $(\cdot008)$	·078	—
57·6	Average cost A	(C_a)	Acreage	(L)	$\cdot0013$ $(\cdot0004)$	·002	·002
57·7	Average cost C	(C_c)	Acreage	(L)	$-\cdot00133*$ $(\cdot00072)$	·017	—
57·8	Output per acre	$\frac{(Y)}{(L)}$	Output	(Y)	$-\cdot0056$ $(\cdot0010)$	·075	·106
57·9	Output per man-day	$\frac{(Y)}{(M)}$	Output	(Y)	$-\cdot00014$ $(\cdot00006)$	·022	·226
57·10	Average cost A	(C_a)	Output	(Y)	$-\cdot0000089$ $(\cdot0000032)$	·036	·045
57·11	Average cost C	(C_c)	Output	(Y)	$-\cdot000028$ $(\cdot000005)$	·130	·146

† Adjusted for degrees of freedom.
* Coefficient non-significant.

Cost A includes the expenditure actually incurred by the farmers and excludes rent paid or imputed, interest on fixed capital and compensation for family labour. Cost C includes these excluded items as well.

The Table shows that there is no high (linear) correlation between output per acre and the size of the holding nor between output per man-day and the size of the holding nor between average cost (A or C) and the size of holding. (See relations 57.4 to 57.7).

The linear relationship between the productivity of land, and output, the productivity of labour, and output, and average cost (A or C), and output, are also very weak. (See relations 57.8 to 57.11).

None of the 8 linear relations explains more than 13 per cent of the variance of cost or productivity.[12]

In view of the theoretical presumption that cost and productivity relations are likely to be non-linear (quadratic) rather than linear, all the 8 relations were tested for deviation from linearity. Relations (57.4), (57.5) and (57.7) were n)t

12. Even if M is specified as a variable additional to L in the productivity relations 57·4 and 57·5 the improvement in the explained variance is not substantial.

found to be significantly deviant from linearity. But the other five were all found to be significantly deviant from linearity. However, the attempt to fit quadratic curves did not yield much better results in terms of explained variance except in one case $\left(\dfrac{Y}{M} = F(Y) \right)$. (See the last column of Table IX). Three of the estimated non-linear functions are given below. The other two are not given because in these both Y and Y^2 turned out to be highly non-significant.

57·9′ $\dfrac{Y}{M}$ = Constant + ·000060 Y − ·000000052* Y^2
                           (·000014)   (·000000089)

  *$\bar{R}^2 = 0·226$

57·10′ C_a = Constant   − ·000017 Y + ·00000053* Y^2
                           (·000007)   (·00000036)

  *$\bar{R}^2 = ·045$

57·11′ C_c = Constant   − ·000047 Y + ·0000012 Y^2
                           (·000010)   (·0000006)

  *$\bar{R}^2 = ·146$

* Coefficient not significant.

*$\bar{R}^2$ is R^2 adjusted for degrees of freedom.

Thus the equations which we may consider now are the linear ones 57.4 to 57.8 and the quadratic ones 57.9′ to 57.11′.

The proportion of variance explained by all these equations is small. But, all the linear regression coefficients except that of equation 57.7 are significant. The coefficient of Y is significant in all the three quadratic equations and the coefficient of Y^2 is significant in equation 57.11′.

The *signs* of the coefficients indicate a slight tendency of the output per acre to fall as the size of the farm increases (57.4 and 57.8). Output per man-day seems to increase with size when it is measured by acreage and to increase and then decrease as size increases when size is measured by output (57.5 and 57.9′). Average cost A also seems to increase with size when size is measured by acreage and to decrease and then increase as size increases when size is measured by output (57.6 and 57.10′). Average cost C seems to fall as size increases when size is measured in acreage, but it falls and then rises as size increases when size is measured by output (57.7 and 57.11′).

Thus the behaviour of productivity and average cost (A and C) is evidently very sensitive to the measure of size that is chosen. Since, theoretically, output is a better measure of size than any single input,[13] the important relations are 57.8, 51.9′, 57.10′ and 57.11′. From these it seems that output per acre tends to fall as size increases. But it is interesting to note that the positive sign of Y and the negative sign of Y^2 in relation 57.9′ suggest the normal parabolic labour productivity curve of micro-theory, concave to the horizontal axis; and the negative sign of Y and the positive sign of Y^2 in relations 57.10′ and 57.11′ also imply normal parabolic cost curves of theory, convex to the horizontal axis.

13. See Raj Krishna, *Op. cit.*

We may conclude that the available Punjab data do not reveal the existence of any *strong* average product or average cost relationships. But if we are merely interested in the *direction* of change of productivity and cost as size increases there is some indication that the productivity of land declines with size; and the productivity of labour and average cost vary in the theoretically expected way: the former rises and then falls after a point and the latter falls and then rises after a point.

In order to examine the effect of grouping, non-linear cost curves were also fitted to *grouped* data. The average cost of farms in each of 11 output-level classes was correlated with average class output. The equations turned out to be :

57·10″ $C_a =$ Constant $- \cdot000005$ Y $- \cdot00000013$† Y^2
                                  $(\cdot000002)$ $(\cdot00000007)$
           $*\bar{R}^2 = \cdot30$

57·11″ $C_c =$ Constant $- \cdot000017$ Y $- \cdot00000045$ Y^2
                                  $(\cdot000003)$ $(\cdot00000011)$
           $*\bar{R}^2 = \cdot75$

† Coefficient not significant.
* Adjusted for degrees of freedom.

The coefficients of determination of these equations are much higher than those of the corresponding equations estimated from ungrouped data. And the signs of *both* Y and Y^2 are negative so that the curves are concave to the origin. From grouped data, therefore, a steadier and stronger tendency of the average cost to fall as output increases can be deduced. But this deduction is evidently the statistical outcome of the 'heroic' averaging of groups of observations—of a very large number of small farms and a very small number of large farms—rather than the reflection of a true economic relationship. We should, therefore, rely on inferences from the regressions based on ungrouped data instead of those based on grouped data. Incidentally, the foregoing results confirm the earlier *a priori* statement about the unreliability of conclusions usually drawn from the grouped data given in the Studies in the Economics of Farm Management.[14]

It is necessary to emphasise that even the directional tendencies indicated by the equations estimated from ungrouped data are very weak and cannot be taken very seriously because the equations fit the data very poorly.

The only definite statement that can be made on the basis of ungrouped data is that no definite size-cost or size-productivity relations are discernible in the agriculture of the two Punjab districts (and presumably other districts) for the time being.

If the estimation of productivity/cost relations from ungrouped data for other regions yields similar results it will follow that *no strong productivity or cost relations have emerged so far in Indian agriculture*. Such a finding will have the important implication that all the current beliefs about these relations (based on acre-size-class, average cost or average yield tables in Farm Management reports) have a very lean empirical foundation. The policy implication will be that there is no strong case yet, on efficiency grounds alone, for favouring or discriminating against any particular farm size in respect of the supply of inputs or the sale of outputs.

14. Raj Krishna, *Op. cit.*

IV. RETURNS TO SCALE

Adding up the elasticities of production of the two inputs "land" and "labour" in the third set of equations we get a measure of the returns to scale :

1954-55	·87
1955-56	·97
1956-57	·95

These sums of the elasticities of production of L and M were tested for deviation from unity. The sum for 1954-55 is significantly different from unity but the sums for 1955-56 and 1956-57 are not. The estimates of scale return for the last 2 years suggest that the constancy of the returns to the scale of "land" and "labour" inputs over a considerable range is not an implausible assumption about agriculture in the two Punjab districts in those years.

V. THE "OPTIMUM HOLDING" AND THE OPTIMUM LABOUR INPUT

We can use the coefficients estimated by us to compute the optimum land input ("the optimum holding") and the optimum labour input, assuming (1) that the inputs of land and labour services ought to be such that their acquisition costs per unit equal the estimated values of their marginal product at the geometric means and (2) that the average rent and the average wage rate given in Table VII are the "right" acquisition costs of these inputs. Alternatively, instead of regarding both inputs as variable we can assume that in the determination of the optimum land input, the labour input is fixed at its sample geometric mean, and in the determination of the optimum labour input, the land input is fixed at its sample geometric mean.

In the first case (both inputs variable) we get the optimum inputs L* and M* by solving the two optimality conditions : $P_l = MP_l$, $P_m = MP_m$ where P_l and P_m are the acquisition costs. In the second case (one input fixed) we fix $L = \bar{L}(GM)$ and equate P_m to MP_m to get M* and then fix $\bar{M} = \bar{M}(GM)$ and equate P_l to MP_l to get L*. L* and M* thus obtained in both cases for the 3 years are shown in Table X.

TABLE X—OPTIMUM LAND AND LABOUR INPUTS

Year	Both Inputs Variable		Other Input Fixed at GM	
	L* (acres)	M* (man-days)	L* (acres)	M* (man-days)
1954-55	19·09	212·85	36·14	233·68
1955-56	6·12	63·33	35·68	223·10
1956-57	4,401·50	1,13,734·76	21·70	1,056·60

On the assumption of complete input variability at constant input prices, the optimal land and labour input levels indicated for 1954-55 are less than the actual mean input levels, apparently because the estimated returns to scale in that year turned out to be diminishing. The sum of the coefficients of land and labour

was only .87. But for 1955-56, and 1956-57, since the estimated returns to scale are nearly constant (.97 and .95) the optimal input levels turn out to be very small and very large respectively. In other words, on the assumptions made, the least-cost scale could be indifferently very small or very large. This result is consistent with the finding from the size-productivity relations noted above that in the agriculture of the two Punjab districts no particular scale of farm operation can yet be said to have crystallised as *the* optimal scale.

In the more realistic case of one input being assumed to be fixed, we find that given the labour availability the area of the "average" farm could be profitably enlarged to some extent from a little more than 20 acres to a little more than 30 acres in the first two years. In the third year the average farm area is not much below the (labour-constrained) optimum. Given the availability of land, on the other hand, the mean labour input seems to have been excessive in the first two years (by about 200 man-days) but very deficient (nearly a third of the optimal) in the third year. It seems that, given the size distribution of holdings, in some areas of the Punjab the scarcity of farm labour is becoming serious enough in recent years to warrant major labour-saving changes.