PRICE RISK MANAGEMENT STRATEGIES

FOR GRAIN IMPORTERS

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ABSTRACT

The presence of multiple sources of uncertainty complicates hedging decisions. One of these is the output price and its correlation with input prices. The other is how far forward a firm covers its positions. This study analyzes hedging strategies for grain importers as processors. The analytical model addresses questions of the hedge horizon and accounts for the correlation between input and output prices and exchange rate risk. A theoretical model was developed explicitly modeling the operations of a grain importing firm. The concept of strategic demand for hedging was developed in the context of an analytical model, representing the adjustment in hedge ratios in relation to the hedge horizon and input-output price correlations. Results indicated that the hedging demand diminishes as the time horizon increases, that input-output price correlations have an important impact on hedging, as does the exchange rate. The model also illustrated the impact of price controls and/or output contracting on risk exposure and the demand for hedging.

Keywords: price risk uncertainty, hedging, grain, importers

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William W. Wilson and Robert Wagner[1]

INTRODUCTION

Hedge ratio calculations have been the subject of a substantial amount of research and the theory of hedging and price risk management is well-documented and adapted in practice. For end-users and importers, the presence of multiple sources of uncertainty complicates hedging decisions. One of these is the output price and its correlation with input prices. The second is the hedge horizon which is the time period for which the strategy is implemented. Simply, it is *how far forward* a firm covers its positions. The hedge horizon implicitly refers to the length of time the hedging strategy covers.

Effective procurement strategies can give end-users a competitive advantage over rivals and can increase profitability. However, the conduct of an industry affects risk management strategies by influencing how end-users react to input price changes. In the case where an importer performs a trading function, the firm essentially buys and sells the same commodity leading to high correlations of input and output prices over short periods of time. In contrast, when an importer purchases the commodity as an input to some value-added processing activity (i.e., flour milling or bread baking), correlations of input and output prices may be lower, depending on the importance of the particular commodity's price relative to other input prices, price regulations, and competitive pressure. For companies involved in international transactions, additional risks may include fluctuations in ocean freight rates (or other transportation costs) and foreign currency exchange rates. Procurement strategies must consider each of these sources of risk.

This study analyzes hedging strategies for grain importers as processors. The analytical model addresses questions of the hedge horizon and to account for the correlation between input and output prices. Stochastic procedures were applied to empirically analyze hedging strategies in the Mexican bread flour market.

OPTIMAL HEDGE RATIOS

Hedgers are firms with positions in the cash market, including producers, merchandisers, and end-users, who use commodity futures markets to transfer a portion of their risk to speculators. Traditionally hedging involves a position in the futures market, which is equal and opposite of the position in the underlying cash market. This approach has intuitive appeal and there has been considerable research in the literature focused on improving the estimation of hedge ratios.

Utility-maximizing models are used by Sakong et al., 1993; Lapan et al., 1991; Collins, 1997; Garcia et al., 1994; Rolfo, 1980; Haigh and Holt, 1995; among others, which make assumptions about the utility function of the decision maker. These models include a risk

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aversion parameter and expectations of movements in futures prices. Derivation of the representative model is described in numerous articles (Blank et. al, 1991; Sakong et al., 1993; Lapan et al., 1991; Collins, 1997; among others). The mean-variance model with a single source of uncertainty yields the following optimal hedge ratio:

$$H^* = \frac{E(f_1) - f_0}{2\lambda\sigma_f^2} - \frac{\sigma_{sf}}{\sigma_f^2}$$

where $E(f_1) - f_0$ is the futures market bias, λ is the risk aversion coefficient, σ_f^2 and σ_{sf} are variance of futures and covariance of cash and futures price, respectively. The demand for futures is comprised of the speculative and hedging components which are represented on the right-hand side, respectively.

Hedging strategies for international transactions are complicated by foreign exchange risk. Nayak and Turvey (2000) developed a risk-minimizing hedging model to address the effectiveness of minimum-variance hedges by the amount of risk reduction gained from a particular strategy. Haigh and Holt (1995) extended traditional hedge ratio models to include decisions of an international trading entity facing multiple sources of risk. They included currency exchange rates and transportation costs as sources of uncertainty. Due to large volumes of grain in international transactions, even small increases in transportation costs can potentially diminish profits.

Our model extends this framework. Some important differences relate to including both revenues from selling output and the costs of inputs and production. Inclusion of both input and output prices allows explicit modeling of the relationship between those markets. Cash and futures prices for inputs and output prices are stochastic variables. The firm's expected payoff is:

$$E(\Pi) = Q_{O,t\to t+n} E(\widetilde{P}_{O,t+n}) - Q_{I,t\to t+n} E(\widetilde{P}_{I,C,t}) + F_I \left[E(\widetilde{P}_{I,F,t}) - P_{I,F,t-m} \right] - (Q_{O,t\to t+n})(C_{\$/O}) \quad (1)$$

where $Q_{O,t\to t+n}$ is the number of units of output planned to be produced from t to $t+n$, and $Q_{I,t\to t+n}$ is the quantity of inputs needed to produce the desired output from t to $t+n$, F_I is the quantity of inputs purchased in the futures markets on day $t-m$, $\widetilde{P}_{O,t+n}$ is the price of processed products (output) sold at time $t+n$, $\widetilde{P}_{I,C,t}$ is the price of inputs in the spot market at time t, $\widetilde{P}_{I,F,t}$ is the futures price at time t, and $P_{I,F,t-m}$ is the futures price at time $t-m$. The relationship between the quantity of inputs and outputs is fixed (constant conversion ratio). Non-ingredient costs of production are assumed non-stochastic and represented by $C_{\$/O}$ (costs in dollars per unit of output). Stochastic variables are represented by tildas.

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The first term represents the revenue from selling output at time *t+n*. A production decision is made at *t-m* which will result in sale of output at time *t+n*. The second term is the price paid for cash inputs at time *t* in the cash markets when the firm acquires the physical inputs. The third term represents the payoffs from hedging activities in the futures markets. Here, payoffs from the positions in futures markets offset the price fluctuations in cash markets. Payoff functions typically do not include both costs and revenues. Models of producers tend to focus on hedging output prices, while models of processors are concerned primarily with input costs. An important distinction in this model is that input and output prices are considered simultaneously.

All decisions are made at time *t-m* for the production period from *t* to *t+n*, where the sequence of days is given by *t-m < t < t+n*. Inputs are purchased at time *t* in the spot market, and if a hedge was placed in the futures markets at time *t-m*, it is also lifted at *t*. Risk can be described as the change in input prices from time *t-m* to time *t*, when actual spot inputs are purchased, as well as changes in output prices from day *t-m* till day *t+n*.

There are several possible combinations of *m* and *n* time periods and as a result both inputs and outputs may be "priced" at different times. The length of periods *m* and *n* depend on industry practices. In the formulation below it is assumed that output prices are random until the sale at *t+n*. The timeline of the decision making process is illustrated by Figure 1.
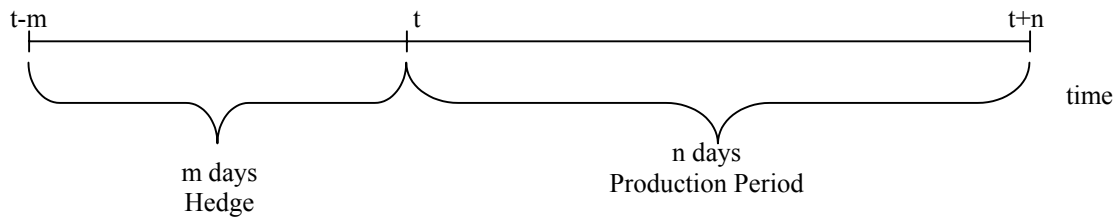
Figure 1. Timeline of Hedging and Processing Periods

The firm could also sell their output forward at *t* and in this case $\widetilde{P}_{O,t+n}$ would no longer be random. Output may be contracted at a specific price and, therefore, part of the price risk could be eliminated. Alternatively, a processor can contract inputs, which makes $\widetilde{P}_{I,C,t}$ a non-random variable. These issues are considered in the empirical analysis and simulations.

The variance of payoffs is:

$$Var(\Pi) = \left(Q_{O,t\to t+n}\right)^2 Var\left(P_{O,t+n}\right) + \left(Q_{I,t\to t+n}\right)^2 Var\left(P_{I,C,t}\right) + F_I^2 Var\left(P_{I,F,t}\right)$$
$$- \left[2Q_{O,t\to t+n}Q_{I,t\to t+n}Cov\left(P_{O,t+n}, P_{I,C,t}\right)\right] + \left[2Q_{O,t\to t+n}F_I Cov\left(P_{O,t+n}, P_{I,F,t}\right)\right]$$
$$- \left[2Q_{I,t\to t+n}F_I Cov\left(P_{I,C,t}, P_{I,F,t}\right)\right] \tag{2}$$

Covariances among input cash, futures, and output prices are in the variance equation which allows explicit modeling of the interaction of input and output prices. This also incorporates how price changes in input markets translate into price changes in output markets. Measuring this relationship is an empirical issue and depends on the competitive conduct in the industry, business practices, or the length of the time lag.

The optimal futures position is found by maximizing the mean-variance utility function of the firm by choosing the appropriate level of futures position. The problem is represented as:

$$Max\ J = E(\Pi) - \frac{\lambda}{2}Var(\Pi) \tag{3}$$

The first order condition is:

$$\frac{\partial J}{\partial F_I} = E\left[P_{I,F,t} - P_{I,F,t-m}\right] - \frac{\lambda}{2}\left[2F_I Var\left(P_{I,F,t}\right) + 2Q_{O,t\to t+n}Cov\left(P_{O,t+n}, P_{I,F,t}\right) - \right.$$
$$\left. -2Q_{I,t\to t+n}Cov\left(P_{I,C,t}, P_{I,F,t}\right)\right] = 0 \tag{4}$$

Rearranging and solving for F_I, the optimal futures position is:

$$F_I = \frac{Q_{I,t\to t+n}Cov\left(P_{I,C,t}, P_{I,F,t}\right)}{Var\left(P_{I,F,t}\right)} + \frac{E\left[P_{I,F,t}\right] - P_{I,F,t-m}}{\lambda Var\left(P_{I,F,t}\right)} - \frac{Q_{O,t\to t+n}Cov\left(P_{O,t+n}, P_{I,F,t}\right)}{Var\left(P_{I,F,t}\right)} \tag{5}$$

where F_I is the number of units purchased in futures markets and the hedge ratio is:

$$\frac{F_I}{Q_{I,t\to t+n}} = \frac{Cov\left(P_{I,C,t}, P_{I,F,t}\right)}{Var\left(P_{I,F,t}\right)} + \frac{E\left[P_{I,F,t}\right] - P_{I,F,t-m}}{Q_{I,t\to t+n}\lambda Var\left(P_{I,F,t}\right)} - \frac{Q_{O,t\to t+n}Cov\left(P_{O,t+n}, P_{I,F,t}\right)}{Q_{I,t\to t+n}Var\left(P_{I,F,t}\right)} \tag{6}$$

Important differences are the inclusion of the relationship between input and output prices, the time lag between purchasing, processing, and selling the commodity, and explicitly incorporating the quantity of inputs and outputs.

The first term is the hedging demand for futures. An increase (decrease) in $Cov\left(P_{I,C,t}, P_{I,F,t}\right)$ leads to a proportionate increase (decrease) in the hedge ratio when the covariance term is zero. An increase (decrease) in $Var\left(P_{I,F,t}\right)$ results in a decline (rise) in the optimal hedge ratio. The second term is the speculative demand for futures. The numerator is the bias in the futures markets, and the denominator is the product of the risk-aversion coefficient and the variance of futures prices.

The last term is what we refer to as the *strategic demand* for futures and includes the covariance (which in concept is related to the correlation) of input futures and output product prices. In durations over which the covariance (or correlation) between input and output markets is very low, this term converges to zero. Greater correlations result in a reduction in the optimal hedge ratio. In this case, profit margins are protected by similar fluctuations in input and output prices.

For a firm with a short input position, the hedge ratio at *t-m* would presumably be positive as inputs are purchased in the futures markets. However, the company may also be exposed to price risk after purchasing inputs at time *t* in the spot market. In that case they have a long spot ingredient position at t $\left(Q_{I,t} > 0\right)$, which may pose additional risks. At that point, the firm is already protected against rising input prices. However, a decline in input costs may give an advantage to its competitors who will be relatively lower cost producers.

Hedging With Multiple Outputs

The model developed can be extended for multiple outputs (e.g., by-products, mill-feeds, etc.). Using the same notation as above, with subscripts *1* and *2* designating outputs 1 and 2, the payoff function is given by:

$$\Pi = \left[\left(Q_{O1,t \to t+n}\tilde{P}_{O1,t+n}\right) + \left(Q_{O2,t \to t+n}\tilde{P}_{O2,t+n}\right)\right] - \left(Q_{I,t \to t+n}\right)\left(\tilde{P}_{I,C,t}\right) + F_I\left(\tilde{P}_{I,F,t} - P_{I,F,t-m}\right)$$
$$-\left(Q_{I,t \to t+n}\right)\left(C_{P/I}\right) \tag{7}$$

The variance of the payoff function is:

$$Var(\Pi) = \left(Q_{O1,t \to t+n}\right)^2 Var\left(P_{O1,t+n}\right) + \left(Q_{O2,t \to t+n}\right)^2 Var\left(P_{O2,t+n}\right) + \left(Q_{I,t \to t+n}\right)^2 Var\left(P_{I,C,t}\right) \tag{8}$$
$$+F_I^2 Var\left(P_{I,F,t}\right) + \left[2Q_{O1,t \to t+n}Q_{O2,t \to t+n}Cov\left(P_{O1,t+n}, P_{O2,t+n}\right)\right] - \left[2Q_{O1,t \to t+n}Q_{I,t \to t+n}Cov\left(P_{O1,t+n}, P_{I,C,t}\right)\right]$$
$$+\left[2Q_{O1,t \to t+n}F_I Cov\left(P_{O1,t+n}, P_{I,F,t}\right)\right] - \left[2Q_{O2,t \to t+n}Q_{I,t \to t+n}Cov\left(P_{O2,t+n}, P_{I,C,t}\right)\right]$$
$$+\left[2Q_{O2,t \to t+n}F_I Cov\left(P_{O2,t+n}, P_{I,F,t}\right)\right] - \left[2Q_{I,t \to t+n}F_I Cov\left(P_{I,C,t}, P_{I,F,t}\right)\right]$$

The first order conditions are:

$$\frac{\partial J}{\partial F_I} = E\left[P_{I,F,t} - P_{I,F,t-m}\right] - \frac{\lambda}{2}\left[2F_I Var\left(P_{I,F,t}\right) + 2Q_{O1,t\rightarrow t+n}Cov\left(P_{O1,t+n},P_{I,F,t}\right)\right. \qquad (9)$$

$$\left. +2Q_{O2,t\rightarrow t+n}Cov\left(P_{O2,t+n},P_{I,F,t}\right) - 2Q_{I,t\rightarrow t+n}Cov\left(P_{I,C,t},P_{I,F,t}\right)\right] = 0$$

Solving for the futures position F_I gives:

$$F_I = \frac{Q_{I,t\rightarrow t+n}Cov\left(P_{I,C,t},P_{I,F,t}\right)}{Var\left(P_{I,F,t}\right)} + \frac{E\left[P_{I,F,t}\right] - P_{I,F,t-m}}{\lambda Var\left(P_{I,F,t}\right)} - \frac{Q_{O1,t\rightarrow t+n}Cov\left(P_{O1,t+n},P_{I,F,t}\right)}{Var\left(P_{I,F,t}\right)} \qquad (10)$$

$$-\frac{Q_{O2,t\rightarrow t+n}Cov\left(P_{O2,t+n},P_{I,F,t}\right)}{Var\left(P_{I,F,t}\right)}$$

There is a further adjustment in the quantity of inputs purchased in the futures markets as a result of correlations between both output product prices and input futures prices. Extending the model to *n* outputs would result in *n* strategic demand terms.

For importers involved in international transactions, an additional source of uncertainty stems from fluctuation in the exchange rate between their home currency and that to which their accounts have to be settled. To accommodate these impacts the payoff function is:

$$\Pi = \left[\left(Q_{O1,t\rightarrow t+n}\tilde{P}_{O1,t+n}\right) + \left(Q_{O2,t\rightarrow t+n}\tilde{P}_{O2,t+n}\right)\right] - \left(Q_{I,t\rightarrow t+n}\right)\left(\tilde{P}_{I,C,t}\right)\left(\tilde{P}_{FX,S,t}\right) \qquad (11)$$

$$+F_I\left[\left(\tilde{P}_{I,F,t}\right)\left(\tilde{P}_{FX,S,t}\right) - \left(P_{I,F,t-m}\right)\left(P_{FX,S,t-m}\right)\right] + F_{FX}\left[\left(\tilde{P}_{FX,F,t}\right) - \left(P_{FX,F,t-m}\right)\right]$$

$$-\left(Q_{I,t\rightarrow t+n}\right)\left(C_{P/I}\right)$$

where the notation is the same as earlier, with the addition of subscripts *1* and *2* representing quantity and price of multiple outputs, $P_{FX,S,t-m}$ and $\tilde{P}_{FX,S,t}$ are exchange rates at times t-m and t, and $P_{FX,F,t-m}$ and $\tilde{P}_{FX,F,t}$ are futures rates for exchange at times *t-m* and *t*. Equation 11 allows for two outputs and for the hedging of the exchange rate. Exchange rate risk also enters the formulation as part of the wheat futures transactions, since the wheat futures are priced in U.S. dollars.

EMPIRICAL PROCEDURES

An empirical analysis was conducted using data from a Mexican flour milling firm importing wheat from the United States. In addition to the risk of changing input prices, exchange rates and multiple outputs were added to the model. Results were obtained by stochastic simulation of the firm's payoff function using *@Risk*™ and *Risk Optimizer*™ to find optimal positions in both peso and wheat futures contracts.

Statistical Behavior of Key Variables

Monthly prices for Mexican flour and mill-feed prices were collected from 1996 to 2000, along with exchange rates and U.S. Gulf FOB import-port cash wheat prices. All variables except U.S. Gulf cash wheat prices are measured in Mexican pesos. Sample statistics were calculated for various combinations of $m = 1, 2, 4$ month and $n = 0, 1, 2, 4$ month periods. The distribution of percentage price changes and correlations were measured from each price series. This was done on a time series representing percentage returns in all of the variables over each particular combination of m and n periods. The standard deviation of returns and correlation between the returns in each variable were then calculated to estimate probability distributions.

Sample statistics for a 2-month hedge ($m = 2$) are reported in Table 1. There is a strong correlation (0.9092) between Kansas City wheat futures $\left(P_{I,F,t}\right)$ and U.S. Gulf cash wheat prices $\left(P_{I,C,t}\right)$. As the time lag n increases, there is a weakening relationship between input (cash wheat) and output (Mexican flour) prices. The correlation between $P_{I,C,t}$ and $P_{O1,t+n}$ declined from 0.4539 for $n = 0$ to 0.2172 for $n = 4$. The contrary is true for the by-product (Mexican mill-feed), where the correlation between $P_{I,C,t}$ and $P_{O2,t+n}$ is first negative, then slightly positive with a longer time lag. There is a weak but significant relationship between spot or futures exchange rates and wheat prices. There is a weak relationship between Mexican flour and by-product prices, but this may be explained by the effect of alternative feeds (such as corn) on mill-feed prices.

Simulation Procedures

The model determined the minimum-variance hedge ratios (assuming no bias in futures) over various hedge horizons and production periods. Base case values at $t\text{-}m$, the time hedging decisions are made and shown in Table 2. The probability distribution and correlations of the random variables were estimated from historical data. *@Risk*™ was used to simulate the firm's payoff function and find the combination of hedge ratios in *KCBT* futures and *CME* Mexican peso futures contracts that results in a minimum-variance hedge portfolio (optimal positions). *Risk Optimizer*™ was then used to verify the results from the *@Risk*™ simulations.

Table 1. Two Month Hedge Sample Statistics

Variables		n=0	n=1	n=2	n=4
	Mean		Std. (σ)		
Cash wheat input*	-0.01920	0.074964			
KCBT wheat futures*	-0.01799	0.070196			
Mexican flour	-0.03021	0.048589	0.059441	0.067715	0.08242
Mexican by-product	0.02542	0.102614	0.135297	0.161154	0.18117
Spot peso/$ exch. Rate	0.01008	0.036756			
CME peso futures	0.00864	0.047552			
			Correl.		
Corr($P_{I,C,t}$, $P_{I,F,t}$)		0.9092			
Corr($P_{I,C,t}$, $P_{O1,t+n}$)		0.4539	0.3338	0.2784	0.2172
Corr($P_{I,C,t}$, $P_{O2,t+n}$)		-0.1369	-0.0619	0.0864	0.2518
Corr($P_{I,C,t}$, $P_{FX,S,t}$)		-0.0009			
Corr($P_{I,C,t}$, $P_{FX,F,t}$)		-0.1724			
Corr($P_{I,F,t}$, $P_{O1,t+n}$)		0.4082	0.3189	0.2524	0.1912
Corr($P_{I,F,t}$, $P_{O2,t+n}$)		-0.0348	-0.0297	0.0760	0.2485
Corr($P_{I,F,t}$, $P_{FX,S,t}$)		0.0996			
Corr($P_{I,F,t}$, $P_{FX,F,t}$)		-0.0896			
Corr($P_{O1,t+n}$, $P_{O2,t+n}$)		-0.1024	-0.0603	-0.0340	0.0173
Corr($P_{O1,t+n}$, $P_{FX,S,t}$)		0.3640	0.4623	0.3782	0.3739
Corr($P_{O1,t+n}$, $P_{FX,F,t}$)		0.0724	0.3192	0.3283	0.2654
Corr($P_{O2,t+n}$, $P_{FX,S,t}$)		-0.1601	-0.1363	-0.0464	0.1817
Corr($P_{O2,t+n}$, $P_{FX,F,t}$)		-0.1781	-0.1776	-0.1808	0.0653
Corr($P_{FX,S,t}$, $P_{FX,F,t}$)		0.7111			

* Values are in U.S. dollars. (All other variables are measured in terms of Mexican Pesos.)

Table 2. Base Case Values

Variable	Price
U.S. Gulf FOB import port	
Cash wheat	3.39 $/bushel
KC wheat futures	3.90 $/bushel
Mexican flour	2.16 MP/kg
Mexican by-product	1.11 MP/kg
Spot exchange rate	9.45 MP/$
CME MP futures	9.48 MP/$

EMPIRICAL RESULTS

Base Case Results

Table 3 shows the results for combinations of *m* and *n* time periods. Results indicate that the firm should optimally take a long position in *KCBT* wheat futures, and either a long or a short position in *CME* peso futures depending on the time horizon of the decision and the correlation of the random variables. The futures positions for wheat are given as a percentage of the wheat-equivalent cash flour inputs. The size of the long wheat futures position is different for each combination of *m* and *n* periods, but is approximately 40 to 50 percent of the underlying cash position. As the time horizon increases the optimal hedge ratio declines. These findings are consistent with the notion that over a longer time period, output prices generally adjust to changes in input prices. It also supports some observed hedging strategies where buyers hedge a small portion of their position for deferred cash needs. Then, as they approach the processing period, they increase their hedge.

The size of the position in peso futures is less stable, ranging from small long positions to relatively large short positions. The positions are given as a percentage of the underlying spot currency position that is purchased in *CME* peso futures. These results depend on the levels of correlation, and probability distributions of each random variable, and do not reflect any expectations about future price changes.

Table 3. Base Case Results

m (months)	n=0	n=1	n=2	n=4
Min Risk hedge ratio in KC wheat futures (F_I):				
m=1	0.3645	0.3739	0.3178	0.1892
m=2	0.4816	0.5046	0.4771	0.3560
m=4	0.4533	0.4458	0.4594	0.4895
Min Risk hedge ratio in CME peso futures (F_{FX}):				
m=1	0.2304	-1.5563	-2.2507	-1.9533
m=2	0.0777	-1.3934	-1.6089	-2.0857
m=4	0.4490	-0.3797	-1.1033	-1.9321

The expected return and variance for selected scenarios are summarized in Figure 2 which shows the variance for various (possible optimal) combinations of peso and wheat hedge ratios for $m = n = 2$. The edges of the graph are highlighted to show the top left and front. The arrow indicates the relative position of the surface to the floor of the graph. The choice of hedge ratios has a large impact on the variance of payoffs. The minimum-variance of payoff is 8.0185, which is achieved by taking a long position in wheat futures $\left(F_I = 0.4771\right)$ and a short position in peso futures $\left(F_{FX} = -1.6089\right)$. Deviating from either of these hedge ratios results in an increased variance.

The impact of the peso hedge ratio is much smaller. The results suggest that risk can be reduced from 10.3877 without hedging to 8.0185 with the minimum-variance hedge ratio.[2]
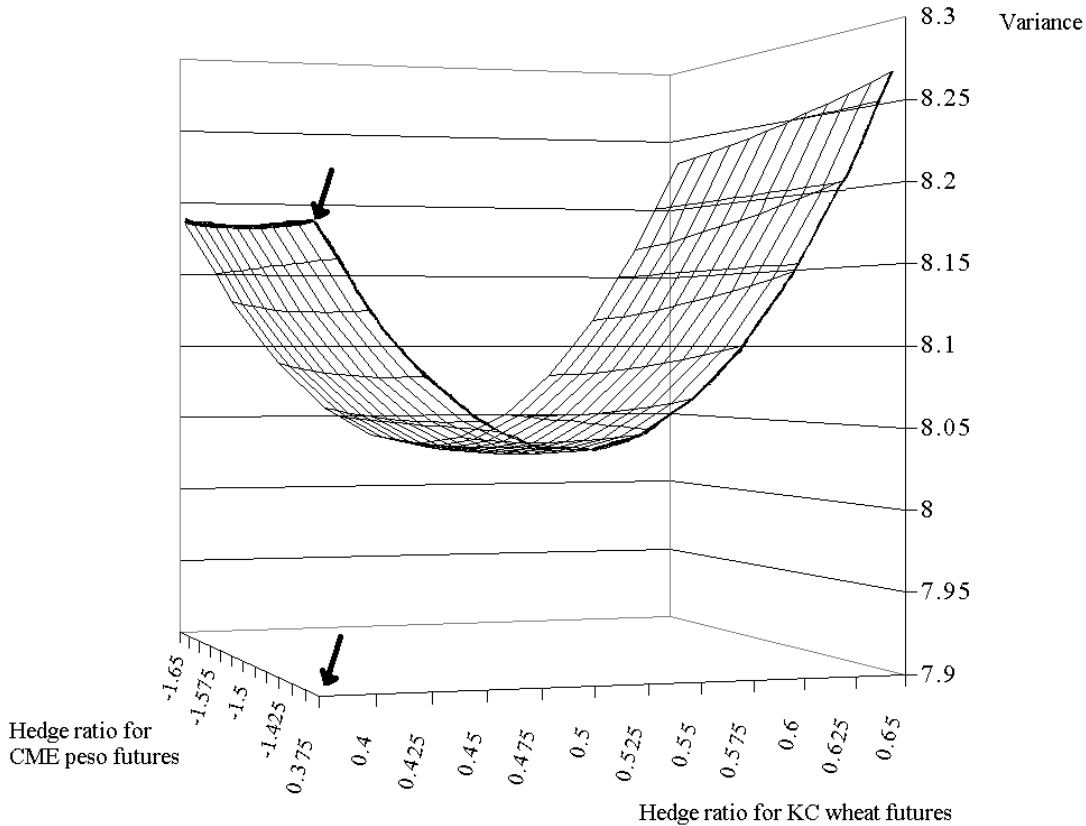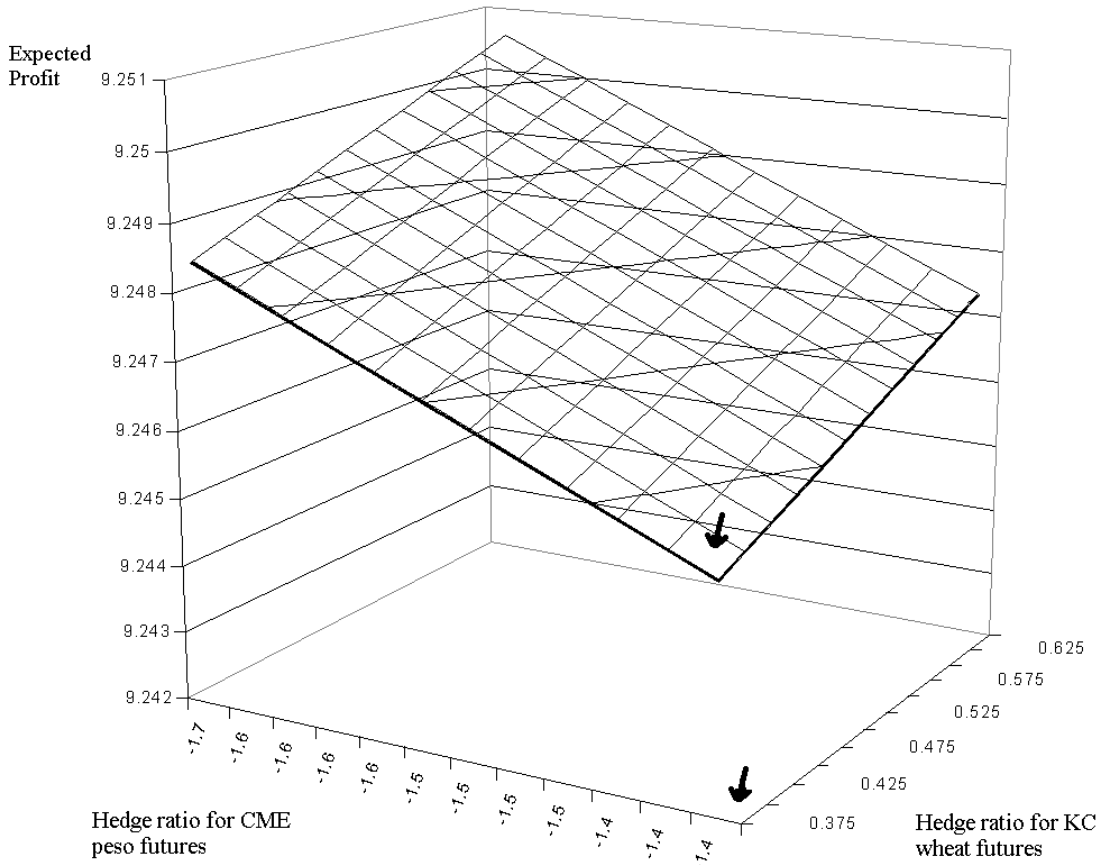

Figure 2. Variance of Profits as a Function of Peso and Wheat Hedge Ratios with Zero Bias

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[2] It is not possible to decompose the relative size of hedging, speculative, and strategic demand when using stochastic simulation.

The relationship between expected profits and the hedge ratios differs compared to a single hedge ratio model (Figure 3). In this case, where the expectation of futures prices $E\left[P_{I,F,t}\right]$ is multiplied by the expectation of spot exchange rates $E\left[P_{FX,S,t}\right]$, the covariance (or correlation) between the random variables results in a slightly different expected profit for each combination of optimal hedge ratios. This result can be derived by taking the expectation of the firm's profit function (Π) which results in:

$$
\begin{aligned}
E(\Pi) = \; & Q_{O1,t\rightarrow t+n}E(P_{O1,t+n}) + Q_{O2,t\rightarrow t+n}E(P_{O2,t+n}) \\
& - Q_{I,t\rightarrow t+n}E(P_{I,C,t})E(P_{FX,S,t}) - Q_{I,t\rightarrow t+n}Cov(P_{I,C,t},P_{FX,S,t}) \\
& + F_I E(P_{I,F,t})E(P_{FX,S,t}) + F_I Cov(P_{I,F,t},P_{FX,S,t}) - F_I(P_{I,F,t-m})(P_{FX,S,t-m}) \\
& + F_{FX}E(P_{FX,F,t}) - F_{FX}(P_{FX,F,t-m}) - (Q_{I,t\rightarrow t+n})(C_{P/I}) \; .
\end{aligned}
$$

Figure 3. Expected Profits as a Function of Peso and Wheat Hedge Ratios with Zero Bias (for $\lambda = 0.5$ and m=n=2)

When bias is zero $E[P_{I,F,t}] = P_{I,F,t-m}$ and $E[P_{FX,S,t}] = P_{FX,S,t-m}$, the covariance term remains. Since the wheat hedge ratio (F_I) is multiplied by the covariance term, the expected value differs as the hedge ratio changes. The values of expected profits are shown for combinations of wheat and peso hedge ratios, again for the case where $m = n = 2$. If there is no exchange rate risk, the plane would be reduced to a single point, since the hedge ratios would have no effect on the level of expected profits.

While the expected profit changes with different hedge ratios, these changes are comparatively small. The corresponding changes in variance, however, are comparatively large, as shown by Figure 2 for the same scenario. These relationships and the effects of hedge ratios on risk reduction are illustrated in Figure 4, where expected profit and variance of profit are plotted for each combination of wheat and peso hedge ratios. Each point is the result of a stochastic simulation, with a corresponding portfolio of cash and futures positions. As illustrated, changes in wheat hedge ratios (F_I) have a larger effect on the variance than changes in peso hedge ratios (F_{FX}).[3] The minimum-variance hedge position is indicated by the arrow on the left side. It corresponds to a hedge ratio of 0.4771 for KC wheat and 1.6089 for CME peso futures. Values of the peso hedge ratio are indicated on the right side of the graph and show that most of the effect on expected profit and the variance is due to the size of the wheat futures position.

The E-V frontier is found by tracing out portfolios where variance is minimized for each possible level of expected profit. The combinations of wheat and peso hedge futures positions in Figure 4 only show an interval close to the "optimum" portfolio under the base case assumptions. In this case, the E-V frontier is not visible, since relatively large peso futures positions would have to be taken to be on the frontier.

The amount of risk reduction for the optimal strategy was compared to a "no-hedge" scenario ($F_1 = 0$ and $F_{FX} = 0$). The no-hedge and the minimum-variance ($F_1 = 0.4771$ and $F_{FX} = 1.6089$) hedge strategies were simulated and compared. The resulting cumulative probability density functions are reported by Figure 5. This is an approximation of the probability density function of payoffs without hedging, and with minimum-variance hedge ratios. The minimum-variance hedge ratio reduces the dispersion (the variance) in the distribution of profits. The extent of risk reduction is related to the level of correlation between the cash commodity (inputs) and the hedging instrument.

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[3] The graph could be 'filled-in' completely within the E-V frontier running simulations with very small incremental changes in both hedge ratios. 'Jumps' between clusters of observations are due to the intervals selected for the hedge ratios from simulation to simulation.
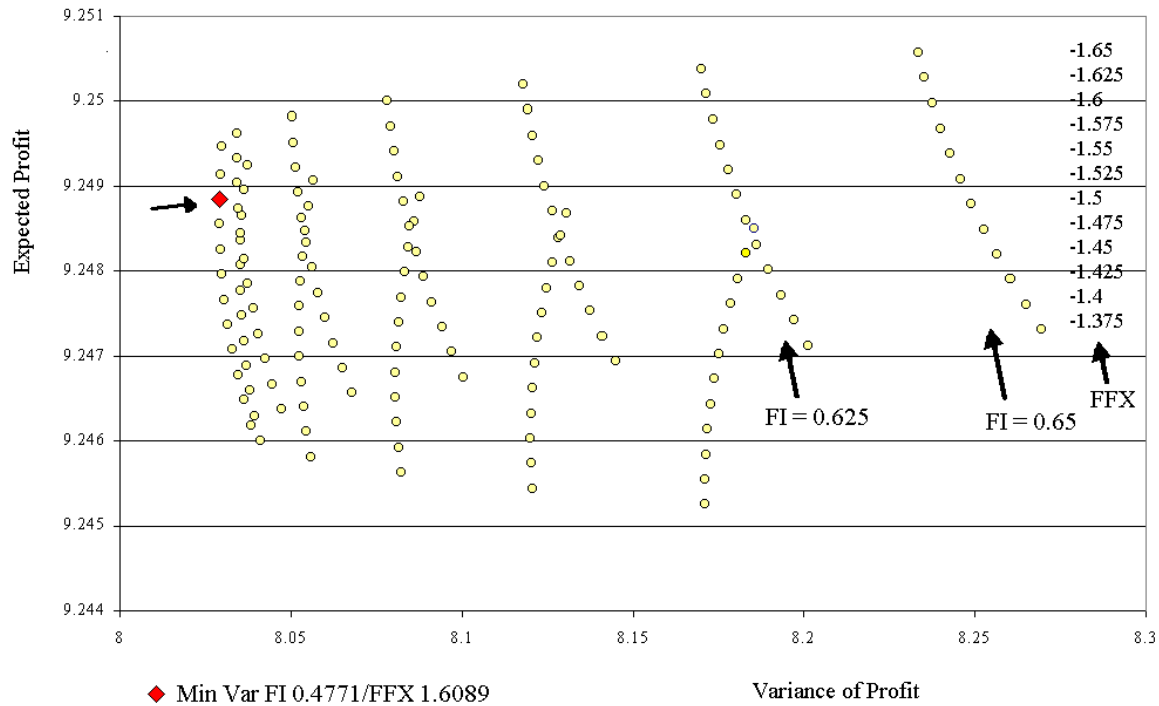
Figure 4. Expected Profits and Variance for Mexican Flour, Base Case with Zero Bias

Table 4 compares the outcomes of alternative hedging strategies with the minimum-variance hedge ratios. If no hedges were placed, the expected profit would be slightly lower, and the variance higher than the minimum risk hedge ratios. These results reflect the correlations between not only the input spot and futures prices, but also the correlation between wheat and peso futures.

Table 4. Comparison of Hedging Strategies

FI	FFX	E(P)	VaR(P)
0	0	9.21	10.39
1	0	12.91	11.83
0	-1	9.22	10.24
0.48*	-1.61*	11.01	8.02

*Minimum-variance hedge ratios

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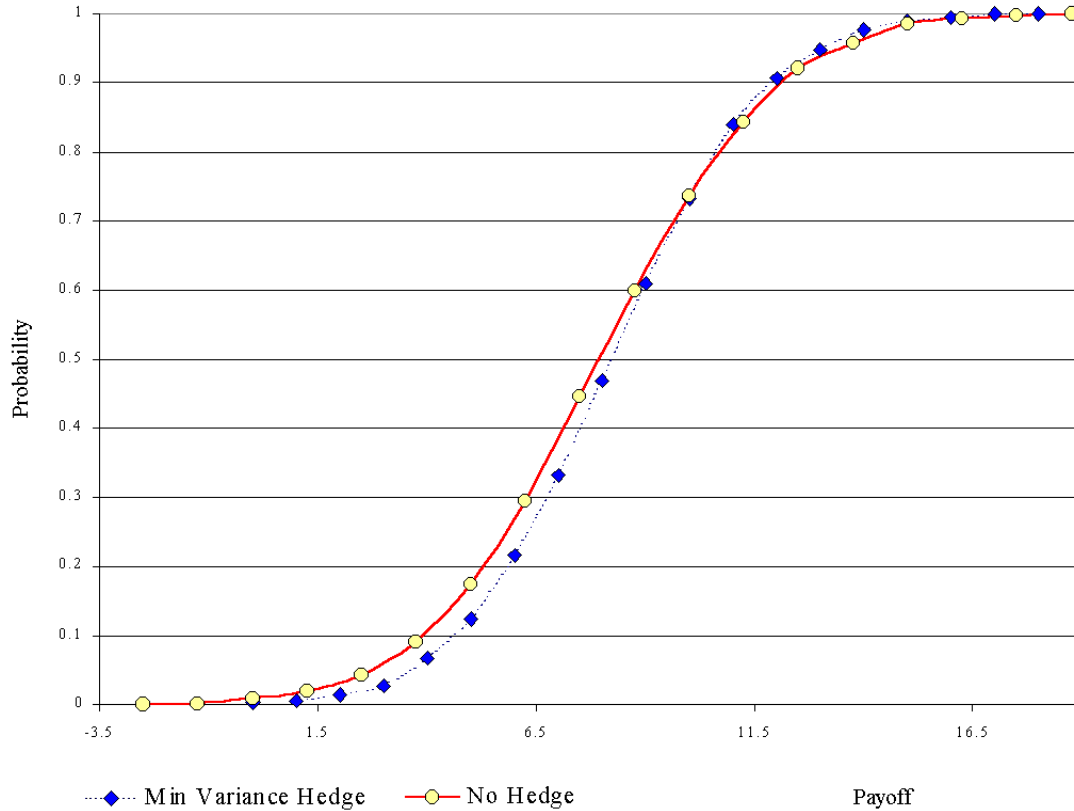
Figure 5. Base Case Cumulative Probability Density Functions

Sensitivity Analysis: Bias and λ

The analysis was expanded to include the case where the firm believes wheat futures prices are biased during m, the hedging period $\left(P_{I,F,t-m} < E\left[P_{I,F,t}\right]\right)$. The model was changed to include expectations of a 10 percent increase in Kansas City (KC) wheat futures prices over the hedging period. In this case, the firm may take these expectations into account in its hedge ratio formulation.

For the base case, $\lambda = 0.5$ and the bias was 10 percent. Expected profits and variances were derived for combinations of hedge ratios in KC wheat and CME peso futures. These numbers were then inserted into the mean-variance utility function for a given level of risk-aversion to find utility-maximizing hedge positions. Figure 6 shows the level of utility from each hedge ratio combination for the firm with increasing wheat futures price expectations. The firm would maximize its utility by taking a long position in wheat futures $\left(F_I = 1.25\right)$ and a short position in peso futures $\left(F_{FX} = -2\right)$. By incorporating the expectation of increasing prices, the firm takes a larger long position than when its goal was risk minimizing $\left(F_I = 0.4771\right)$. The

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larger long position in wheat futures benefits the firm if wheat futures increase. Figure 6 also shows that the firm's utility is not as sensitive to the size of the peso futures position. Even relatively large changes in the position in peso futures (the hedge ratio) lead to small changes in utility.
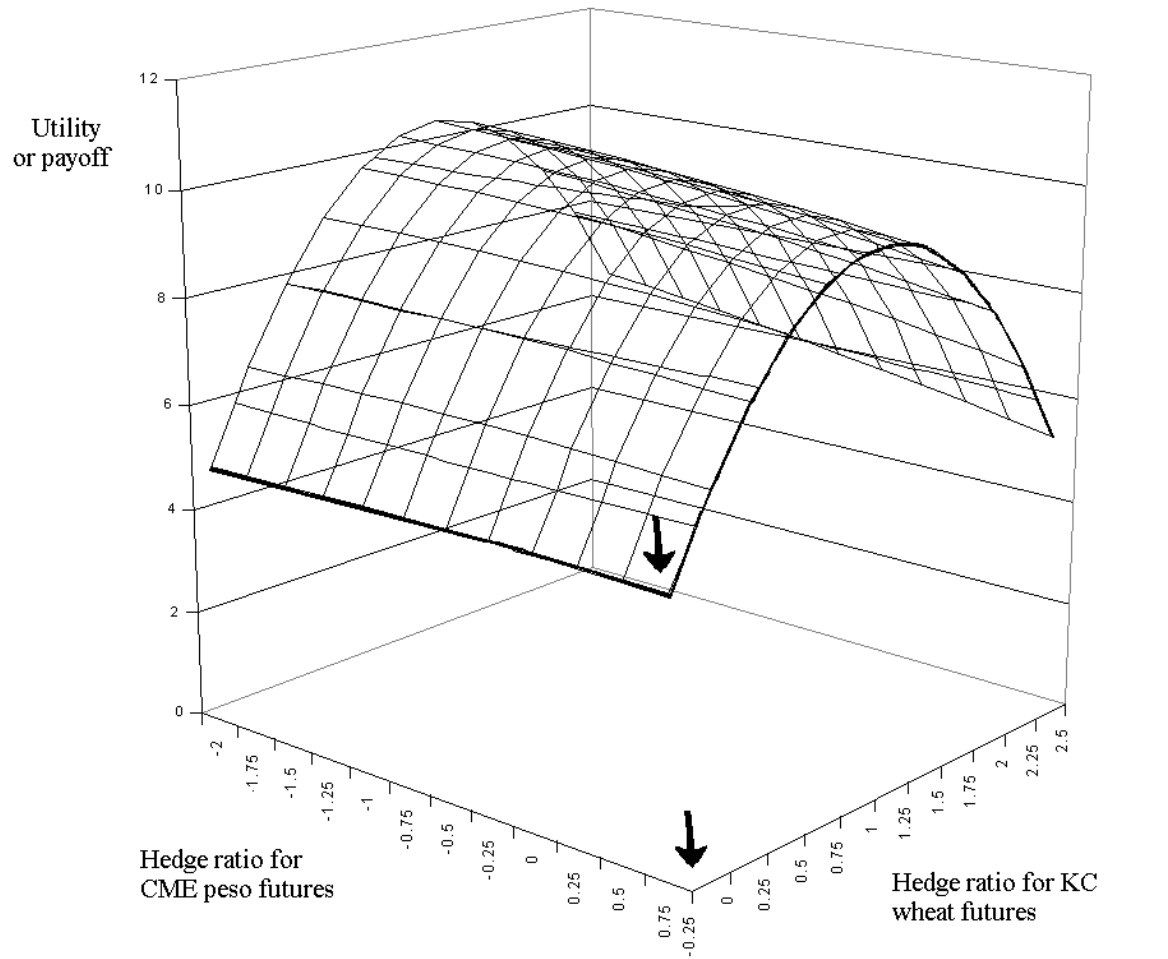
Figure 6. Utility Maximizing Hedge Ratios for $\lambda = 0.5$

The expectation of rising wheat futures prices (*ceteris paribus*) leads the hedger to take positions in both wheat and peso futures in anticipation of an expected return which would be higher than under the 'no-bias' scenario. The degree to which the firm takes their expectations (market outlook) into consideration in the hedging decision again depends on the risk aversion parameter.

Firms may have differing attitudes toward risk aversion, λ. Sensitivity analysis was conducted to examine the adjustment in the size of hedge ratios as a result of changing the risk-

aversion coefficient. Table 5 shows the results for the sensitivity analysis. As the hedger becomes more risk averse (λ increases), both the hedge ratios in KC wheat and CME peso futures converge toward the risk minimizing hedge positions. As λ nears infinity, the utility maximizing hedge ratios converge toward the risk-minimizing hedge ratios. When the firm is less risk-averse (λ decreases), it takes a larger long position in wheat futures and a larger short position in peso futures.

Table 5. Sensitivities to Risk Aversion Parameter

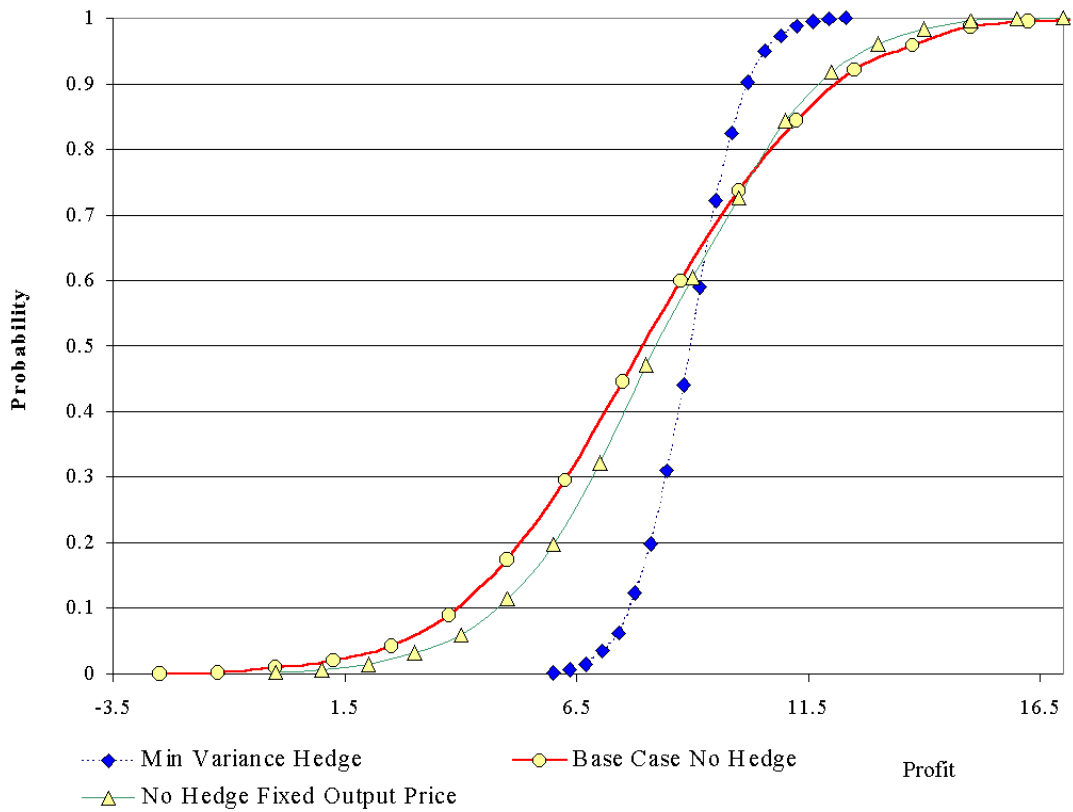| λ (risk aversion)	F_I (KC wheat futures)	F_{FX} (CME peso futures)	$E(\Pi)$	$VaR(\Pi)$
0.07	6	*		
0.25	2	-4.5	16.61	29.41
0.5	1.25	-3	13.74	13.32
1	0.75	-2	12.01	8.68
3	0.5	-1.75	10.17	8.05
∞	0.47	-1.6	9.24	8.02

*The futures position becomes very large with respect to the underlying exposure.

Strategic Decisions on Prices and Contracting Output

The relationship between input and output prices is an important factor in the formulation of hedging strategies. Two additional cases were considered: output contracting and output price regulation, both of which are common in some import markets. The miller could contract his output (flour or by-products) before it is produced. Similarly, in some countries, government price regulations fix the output price which has a similar effect on price correlations. The impact in both cases is equivalent to setting the standard deviation of output prices and the correlation between input and output prices to zero.

Cumulative probability functions are compared in Figure 7 for "no-hedge" scenarios for the base case and fixed output prices, as well as minimum-variance hedges for the fixed output price case. When the firm chooses not to hedge, overall risk is as if output prices are fixed (or contracted), without much reduction in the level of expected profits. However, when input prices are hedged in wheat and peso futures, overall risk is substantially reduced. These results illustrate that with forward contracting for outputs, hedging input costs would be a beneficial and a very important risk management strategy. The potential negative effect of hedging is the reduction of not only the "down-side" but also the "up-side" potential for profits.

Risk Optimizer™ was used to find the minimum-variance hedge ratios under the above restrictions. While changing the hedging period *m*, resulted in a different hedge ratio for both wheat and peso futures, changing the processing period, *n*, did not affect the results (Table 6). Comparing the results from the base case shows the increase in the long wheat futures position for each hedging period, *m*. This is due to the reduced risk in output prices, as the firm now takes futures positions strictly to manage their input costs (i.e., strategic demand is zero).

Figure 7. Cumulative Probability Density Functions for Fixed Output Prices

Table 6. Price Regulation and Output Contracting

m (months)	F_I (KC wheat futures)	F_{FX} (CME peso futures)
1	0.709	-0.084
2	0.837	-0.368
4	0.923	0.008

SUMMARY

The model developed in this paper and the analytical results provide an understanding of some of the fundamental differences in the use of hedging instruments by importers as processors versus other market participants. Results illustrate that their demand for futures is impacted by traditional hedging and speculative demand and also by strategic demand. Due to the latter, the correlation among input and output prices, as well as exchange rate, has an important impact on hedging. In some cases, this may result in more or less hedging than conventionally would be prescribed. Finally, forward contract on products and/or price regulations on products affects hedging strategies in a very important way.

Importers confront multiple risk sources of price risk, including input, output, and foreign exchange rates. Hedging demand is influenced by the relationship between input and output prices and also depends on the time period or lag between procurement decisions and selling processed outputs. Hedging demand is influenced by the ability to fix prices of outputs using forward contracts. In order to find optimal hedge positions under multiple risks, a model was developed and applied to a case representing a Mexican flour milling firm importing wheat. Simulation procedures were used to examine the effects of multiple risks and outputs on the firm's hedging strategy.

The results show the hedge ratio is affected partly by the strategic demand for futures, which is determined by the relationship between input futures and output prices. The model is more complex, since inputs and outputs are denominated in different currencies, and the firm is faced with the additional risk of foreign exchange rate fluctuations. Peso futures were included but their effect on risk reduction was limited compared to wheat futures.

The impact of the hedge horizon on hedging strategies is measured by the size of the strategic demand for futures. This factor captures the effect of input-output price correlations, which tend to reduce the demand for hedging. The sample statistics suggest that the need to cover input positions declines as the time horizon increases (e.g., typically more important to cover the next few week's exposure than 12 months out).

In some markets, output prices could be fixed either through contracting or government regulation. In that case, the correlation between inputs and outputs is reduced to zero, and the firm is exposed to the risk of rising input costs. The results show that fixed output prices substantially reduced the variance of expected profits, but at the same time the probability of earning profits also decreases. The effects of contracting output prices and government price regulation were also illustrated. Hedging demand changes with the length of the time period, as well as with the correlation and volatility of markets.

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