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Strategic Hedging for Grain Processors

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ABSTRACT

Price risk management problems confronting grain processors differ somewhat from conventional motives for hedging. There are two components of this problem that are addressed in this study. One is the competitive characteristics of the processing sector, the structure and conduct of which ultimately determines the relationship between input and output prices. In some cases, these are highly correlated and in others they are not. The second refers to the hedge horizon, or, how far forward a firm should cover its inevitable short cash positions. This study incorporates these two components of hedging into a mean-variance framework to evaluate how they impact price risk management decisions for processors. A theoretical model is developed which is then solved numerically to illustrate the relationships between optimal hedge ratios, the correlation between input and output prices, and the hedge horizon. The model is applied to the case of the United States bread baking industry to further illustrate how these impact hedging in a particular industry.

Key Words: hedging, optimal hedge ratios, food processors, risk management

STRATEGIC HEDGING FOR GRAIN PROCESSORS

William W. Wilson, Robert Wagner, and William Nganje[1]

INTRODUCTION

Unique sources of uncertainty affect risk management strategies in the grain processing industry. One is the hedge horizon which is the time period for which a firm covers its inevitable short cash position. The hedge horizon refers to the period of time the hedging strategy covers.[2] The second is the correlation between input and output prices which differs by industry. Although several studies have shown that price risk is significantly reduced when hedge ratios explicitly account for despondencies between markets (Haigh and Holt, 1995; Garcia, Roh, and Leuthold, 1995; and Manfredo, Garcia, and Leuthold, 2000), correlation between input and output markets are yet to be explicitly modeled. In the case of grain merchandising and some processing sectors, input and output prices are typically highly correlated and equal and opposite positions are generally adopted. Hedging strategies can be substantially different for processors. When input and output prices are highly correlated, the processing margin can be hedged effectively. However, if the correlation is weak and/or when the hedge horizon is longer, hedging strategies can be affected.

The structure and conduct of an industry determines how input cost changes are translated into output price changes. Higher added value in a product tends to reduce the correlation between the price of a raw commodity and that of processed goods (Blank et al., 1991). Depending on the structure and competitiveness of a particular industry, rivals may react differently to actions of competitors. The reaction of the baking industry to rising input-costs during the summer of 2000 is illustrated by the following quote: "leading wholesalers indicated that price increases (bread prices) had been initiated to compensate for rising costs" (*Milling & Baking News*, September 2000). Depending on the structure of the industry, output prices may or may not be closely related to input prices, and this relationship can have an important effect on risk management strategies.

The objective of this study is to incorporate the hedge horizon and the input-output price relationship into a hedging model. These are commercial and risk factors typically confronting hedging decisions by grain processing firms. An analytical model is developed to derive hedge ratios that reflect the demands for futures by end-users, with a focus on hedge horizon, and input-output price relationships. The model is illustrated using a numeric example and data from the U.S. bread baking industry. A contribution of this study is that price risks are significantly reduced for processors when these dependencies (hedge horizon and input-output relationships) are accounted for in hedging models.

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[1] Wilson is Professor, Wagner is a former Graduate Research Assistant, and Nganje is Assistant Professor in the Department of Agribusiness and Applied Economics, North Dakota State University, Fargo, ND.

[2] The hedging horizon has a subtle difference with the concept of time-varying hedge ratios used commonly in the literature. Although both concepts incorporate the impact of covariance between cash and futures markets as time varies, the hedging horizon emphasizes the risk between price and quantity simultaneously, by normalizing prices in cash and futures markets to represent unit returns per unit of input in these markets.

BACKGROUND AND PREVIOUS STUDIES

Risk management involves the identification, evaluation, and implementation of strategies to reduce uncertainty in revenue flows (Baker and Gloy, 1999). Implementing risk-reducing strategies comes at a cost, and these have to be weighed against potential benefits. While hedging generally results in reduced risk, it also reduces profits, which reflects a tradeoff between risk and return. Hedging models typically fail to explain observed behavior of processors and hedgers. Specifically, "processors do routinely hedge, but the observed hedge ratios vary considerably from year to year and in general are inconsistent with any existing model of hedging behavior" (Collins, 1997).

A majority of the hedging literature focuses on agents with long cash positions, typically which are known and have a fixed time horizon and which are covered by short futures position. Typically, these would be farmers, elevators, and traders. Only a few studies of hedging have considered agricultural processors, who are normally short cash and long futures. End-users are concerned with price risk on ingredients (i.e., inputs), but also how changes in input prices affect output prices, as well as the hedge horizon. Risk reduction strategies confronting processing industries is typically more complex than the simple prescriptions from traditional hedging models.

If inputs and outputs are highly correlated, any change in the input price is largely offset by a change in the output price. In this case, changes in output prices provide a hedge against changes in input prices. Indeed, this is the case of a grain merchandiser: output prices and input prices are highly if not perfectly correlated, with instantaneous concurrent changes in both, and typically there are no lags between pricing in the input and output markets. In contrast, end-users confront more complex relationships between input and output prices, and important time lags between pricing inputs and outputs. Lags can allow for significant differences between input prices at the time of purchase, and output price at the time of sale. This, has the effect of reducing the correlation and suggests that holding the input price hedge, until the actual pricing of the finished product, may be necessary for desired risk reduction (Jackson, 1980).

Although processor hedging strategies are not common in the literature, those that have focused in this area address several motivating points. Johnson (1960) discussed the use of futures in consumer goods industries and addressed several of these issues. A firm's pricing strategy and lags inherent in the production process are two crucial factors which determine the firm's hedge horizon. This strategic aspect of hedging is impacted by a firm's size and market share. Smaller firms may be able to extract larger benefits from hedging when competitors deal only in the cash market. However, larger firms may be vulnerable to competitors if they do not conform to industry hedging practices. A discussion of the differences between traders and end-users also outlines how processors differentiate their products, which tends to reduce correlation between inputs and outputs.[3]

Jackson (1980) described how intermediate processors benefit from risk reducing measures that are not as practical for most in consumer goods industries. Included are contracts

[3] An example illustrated how M&M/Mars was able to anchor marketing strategies around futures market activities which also shows the important implications of hedging for end-users.

for output prices which vary with input price changes, and other formulas that attempt to divide price risk between producer and processor cannot be employed as easily in end-user situations. She also discusses the importance of the hedge horizon.

Finally, Hull (1998) discussed competition and hedging strategies. He noted that hedging decisions for processors are risky and are affected by hedging decisions of competitors. He illustrated that if a majority of the firms in an industry act similarly with respect to hedging and procurement, it can be very risky to adopt a contrasting strategy. This is especially the case if the processor controls a large share of the market, which would look attractive to competitors, should they have a cost advantage and the ability to reduce price below this processor's acceptable level.

Optimal Hedge Ratios

Traditionally, hedging strategies involve holding a position in the futures market, equal and opposite the position in the underlying cash market. More recent approaches have sought to derive hedge ratios which depend on the correlation implied in cash and futures markets. Hedge ratio derivation follows the same procedure where the profit function is maximized in terms of the choice variables (Lapan et al., 1991; Martinez and Zering, 1992; Rolfo, 1980; Lence and Hayes, 1994; Moschini and Lapan, 1995; Blank et al., 1991; Vukina et al., 1996; and Nayak and Turvey, 2000; among others). As the number of choice variables and possible risk management instruments increases, the variance of profit function becomes complex because of the correlation effects between these variables.

The risk minimizing hedge ratio is $H = -\left(\sigma_{sf} / \sigma_f^2\right)$ where H is the risk minimizing hedge ratio, σ_{sf} and σ_f^2 are covariance between cash and futures markets and variance of futures, respectively. These vary as more instruments are added. This hedge ratio is calculated with the objective of minimizing the variance of income, once the position in the cash market has been determined (Johnson, 1960; Blank et al., 1991; Rolfo, 1980; Lence and Hayes, 1994; among others).

Utility maximizing models [Sakong et al. (1993), Lapan et al. (1991), Collins (1997), Garcia et al. (1994), Rolfo (1980), and Haigh and Holt (1995), among others] include risk aversion, as well as expectations of movements in futures prices. The mean-variance approximation of utility maximizing models yields the following optimal hedge ratio:

$$H^* = \frac{E(f_1) - f_0}{2\lambda\sigma_f^2} - \frac{\sigma_{sf}}{\sigma_f^2} \quad 1$$

Demand for futures is comprised of the speculative and hedging in demand, the two right-hand side components, respectively. The speculative component (Blank et al., 1991; Vukina et al., 1996) is determined by the bias in futures prices $E(f_1) - f_0$ and the risk aversion (λ). An advantage of the utility or profit maximization model is that hedge ratios are derived by maximizing returns and accounting for bias and risk aversions.

Many of the models fail to explain the observed behavior of a wide range of agents. Collins constructs his positive model of hedging behavior as a financial decision where the firm's objective is to maximize equity by making choices about current operations. The model explains hedging as avoidance of financial failure, as opposed to an approach to reduce price uncertainty. It explains why some agents such as most farmers choose not to hedge at all while others such as arbitragers typically hedge most of their positions (Collins, 1997). Collins notes that the risk minimizing hedge ratio is inappropriate for processors, as it "does not match the behavior of processors and farmers who frequently hedge only part or none of their commitments" (Collins, 1997). He concludes that the risk minimizing model tends to be best at predicting the behavior of arbitrage traders who take close to equal and opposite positions in futures markets.

Recent literature indicates that the expected utility model, as used in this study, is superior to the minimum variance model on theoretical grounds. The decision to minimize variance of expected returns is essentially arbitrary (Frechette and Tuthill, 2000). A more appropriate objective, consistent with economic theory, is the maximization of expected utility. However, the reliance of the expected utility model on the amount of risk and the risk premium makes it agent dependent. Two approaches have been used to overcome the limitations of the expected utility model. The first is to use a weighted expected utility hedge ratio (WEUHR) suggested by Frechette and Tuthill. They showed that although the WEUHR overcomes Allais paradox type limitations or linearity in probability its superiority, in terms of causing agents to over or under hedge, over the expected utility hedge ratio (EUHR) are areas for further research. This is particularly true when two agents in the same risk and return environment adopt different hedging strategies, while the WEUHR may suggest otherwise. A second and frequently used method is to use numerical simulations. The latter may be appealing in markets where participants' risk preferences are important.

In this study, we adopt a numerical simulation approach to account for risk preference. Our approach does not detract from the main focus of incorporating the relation between input and output markets. Unlike the WEUHR, the properties and behavior of the EUHR have been well documented in the literature, especially for a multi-risk environment.

THEORETICAL MODEL OF HEDGING FOR GRAIN PROCESSORS

Our analysis builds on utility maximization models and incorporates two features important to end-users. One is the relationship between input and output prices. The second is the *hedge horizon*, which refers to the length of coverage whereby the firm determines how far forward to purchase inputs in futures markets. "In the real world, there is a delay between the decision to produce, the time of production, and the time of sale. The price of the product at the time of sale may differ from that which was expected when the production decision was made," (Markowitz, 1991). Taking this into consideration, the effects of hedge horizon and input-output price correlations are incorporated into the model. First, the hedging model is developed, then results from a numerical analysis are presented to explain effects of key parameters on the hedging decision.

The Model

Inclusion of both input and output prices allows the explicit modeling of the relationship between those markets. Input cash and futures prices, as well as output prices, are stochastic variables. For simplicity, the expected payoff to processing is:[4]

$$E(\Pi) = Q_{O,t\to t+n}E(\widetilde{P}_{O,t+n}) - Q_{I,t\to t+n}E(\widetilde{P}_{I,C,t}) + F_I\left[E(\widetilde{P}_{I,F,t}) - P_{I,F,t-m}\right] - (Q_{O,t\to t+n})(C_{\$/O}) \tag{1}$$

where $Q_{O,t\to t+n}$ is the number of units of output planned to be produced from day *t* to day *t+n*, and $Q_{I,t\to t+n}$ is the quantity of inputs needed to produce the desired output from *t* to *t+n*. The relationship between the quantity of inputs and outputs is fixed. F_I is the quantity of inputs offset by a position in the futures markets on day *t-m*, $\widetilde{P}_{O,t+n}$ is the price of processed products (output) sold at time *t+n*, $\widetilde{P}_{I,C,t}$ is the price of inputs in the cash market at time *t*, $\widetilde{P}_{I,F,t}$ is the futures price at time *t*, and $P_{I,F,t-m}$ is the futures price at time *t-m*. The *non-ingredient* cost of production is assumed constant and represented by $C_{\$/O}$ (costs in dollars per unit of output). Stochastic variables are represented by tildas. While many of the variables included in this model are similar to those in traditional models, there are also fundamental differences.

The first term represents the revenue from selling output at time *t+n*. A production decision is made at *t-m* which will result in sale of output at time *t+n*. The second term is the price paid for inputs at time *t* in the cash markets when the firm acquires the physical inputs. The third term represents the payoffs from hedging activities in the futures markets. Here, payoffs from futures positions offset price fluctuations in cash markets. The higher the correlation between cash and futures markets, the more effective hedging will be at reducing the fluctuation in profits.

The timeline of the decision making process is illustrated by Figure 1 All decisions are made at time *t-m* for the production period from *t* to *t+n*, where the sequence of days is given by *t-m* < *t* < *t+n*. Inputs are purchased at time *t* in the spot market, hedges placed in the futures markets at *t-m* are exercised at *t*. Risk can be described as the change in input prices from time *t-m* until time *t*, when actual spot inputs are purchased, as well as changes in output prices from day *t-m* until day *t+n*. Several combinations of *m* and *n* time periods exist and, as a result, typically inputs and outputs may be "priced" at different times. The length of periods *m* and *n* depend on industry practices and commercial decisions. In this formulation, output prices are random until output is sold at *t+n*. .

The firm could also sell their output forward at *t,* in which case $\widetilde{P}_{O,t+n}$ would no longer be a random variable. Output may be contracted at a specific price and, therefore, part of the price risk could be eliminated. Alternatively, a processor can contract inputs, which makes $\widetilde{P}_{I,C,t}$ a non-random variable. These issues are considered in the empirical analysis and simulations.

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[4] Details of mathematical derivations and some extensions to the model are in Appendix A.

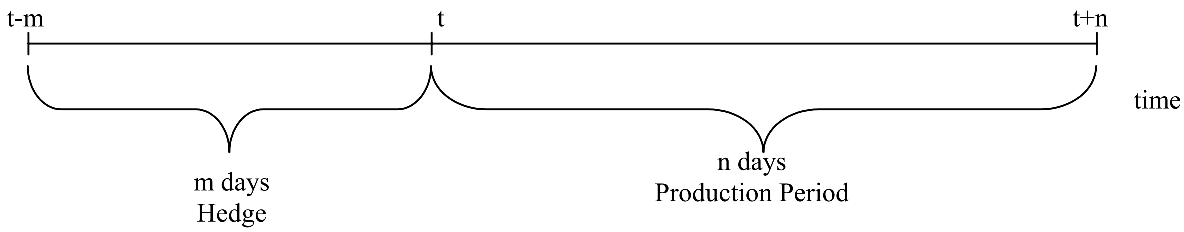

Figure 1. Timeline of Hedging and Processing Periods

The variance of output prices, $Var\left(P_{O,t+n}\right)$, and the covariance between input and output prices, $Cov\left(P_{O,t+n}, P_{I,F,t}\right)$, are non-zero. The variance is:

$$\begin{aligned} Var(\Pi) &= \left(Q_{O,t\to t+n}\right)^2 Var\left(P_{O,t+n}\right) + \left(Q_{I,t\to t+n}\right)^2 Var\left(P_{I,C,t}\right) + F_I^2 Var\left(P_{I,F,t}\right) \\ &-\left[2Q_{O,t\to t+n}Q_{I,t\to t+n}Cov\left(P_{O,t+n}, P_{I,C,t}\right)\right] + \left[2Q_{O,t\to t+n}F_I Cov\left(P_{O,t+n}, P_{I,F,t}\right)\right] \\ &-\left[2Q_{I,t\to t+n}F_I Cov\left(P_{I,C,t}, P_{I,F,t}\right)\right] \end{aligned} \tag{2}$$

Covariances among input, futures, and output prices allows the explicit modeling of the interaction of input and output prices and incorporates how price changes in input markets translate into price changes in output markets.

By substituting Equation 1 and 2 in a mean variance formulation, we obtain Equation 3, the proxy for the expected utility model.

$$Max\ J = E(\Pi) - \frac{\lambda}{2} Var(\Pi) \tag{3}$$

The hedger maximizes the above function to select the optimal size of the futures position. The first order condition is:

$$\begin{aligned} \frac{\partial J}{\partial F_I} &= E\left[P_{I,F,t} - P_{I,F,t-m}\right] - \frac{\lambda}{2}\left[2F_I Var\left(P_{I,F,t}\right) + 2Q_{O,t\to t+n}Cov\left(P_{O,t+n}, P_{I,F,t}\right) - \right. \\ &\left. -2Q_{I,t\to t+n}Cov\left(P_{I,C,t}, P_{I,F,t}\right)\right] = 0 \end{aligned} \tag{4}$$

Rearranging the optimal futures position is:

$$F_I = \frac{Q_{I,t\to t+n} Cov\left(P_{I,C,t}, P_{I,F,t}\right)}{Var\left(P_{I,F,t}\right)} + \frac{E\left[P_{I,F,t}\right] - P_{I,F,t-m}}{\lambda Var\left(P_{I,F,t}\right)} - \frac{Q_{O,t\to t+n} Cov\left(P_{O,t+n}, P_{I,F,t}\right)}{Var\left(P_{I,F,t}\right)} \tag{5}$$

and the processor's hedge ratio is:

$$\frac{F_I}{Q_{I,t\to t+n}} = \frac{Cov\left(P_{I,C,t}, P_{I,F,t}\right)}{Var\left(P_{I,F,t}\right)} + \frac{E\left[P_{I,F,t}\right] - P_{I,F,t-m}}{Q_{I,t\to t+n}\lambda Var\left(P_{I,F,t}\right)} - \frac{Q_{O,t\to t+n} Cov\left(P_{O,t+n}, P_{I,F,t}\right)}{Q_{I,t\to t+n} Var\left(P_{I,F,t}\right)} \tag{6}$$

The differences from traditional hedging models are the inclusion of the relationship between input and output prices, and the time lag between purchasing, processing, and selling the commodity.

The first term of equation (6) is the hedging demand for futures (Blank et al., 1991). An increase (decrease) in $Cov\left(P_{I,C,t}, P_{I,F,t}\right)$ leads to a proportionate increase (decrease) in the hedge ratio. When the covariance term is zero, hedging demand is zero, since there is no benefit gained from hedging with zero correlation. An increase (decrease) in $Var\left(P_{I,F,t}\right)$ results in a decline (rise) in the optimal hedge ratio. The second term is the speculative demand for futures which is determined by the bias in the futures markets, the risk aversion coefficient, and the variance of futures prices.

We refer to the last term as the *strategic demand* for futures, which is determined by the covariance between input futures and output prices. Output quantity is multiplied by the ratio of the covariance between input futures and output prices, where the correlation is determined over *n*, the length of the hedge horizon, and the variance of futures prices. In durations over which the covariance (or correlation) between input and output markets is very low, this term converges to zero. Greater correlations result in a reduction in the optimal hedge ratio. In this case, profit margins are protected by similar fluctuations in input and output prices.

For a firm with a short input position, the hedge ratio at *t-m* would typically be positive as inputs are purchased in the futures markets. They may also be exposed to price risk after purchasing inputs at time *t* in the spot market. In that case, they have a long spot ingredient position at *t* $\left(Q_{I,t} > 0\right)$, which may pose additional risks. At that point, the firm is already protected against rising input prices. However, a decline in input costs may give an advantage to its competitors who may lower output prices.

Numerical Analysis

A numerical analysis is conducted of a processor's long hedge strategy (short cash and long futures position) to illustrate the response of the model to key variables and parameters, and highlight the relationship between these variables, using hypothetical parameter values. Base values were assumed for each of the parameters. A unit of input is processed into a single unit of output (conversion ratio = 1). Initial values are: $Cov\left(P_{I,C,t}, P_{I,F,t}\right) = 230$, $Cov\left(P_{O,t+n}, P_{I,F,t}\right) = 50$, $Var\left(P_{I,F,t}\right) = 250$, $E\left[P_{I,F,t}\right] = 5.30$, $P_{I,F,t-m}$ = \$5.25, and λ = 0.5.
Results were derived numerically.

Hedging demand is 0.92, the speculative demand is 0.0004, and the strategic demand is negative 0.2, the sum of which is the optimal hedge ratio, which is 0.72. The effects of correlation between input and output markets on the optimal hedge ratio are illustrated in Figure 2. There is a substantial adjustment in the optimal hedge ratio as a result of including the input-output price relationship. When $Cov\left(P_{O,t+n}, P_{I,F,t}\right)$ is zero, the strategic demand is zero and the optimal hedge is 0.92. Increases in the covariance between input and output prices has the effect of reducing the optimal hedge ratio.

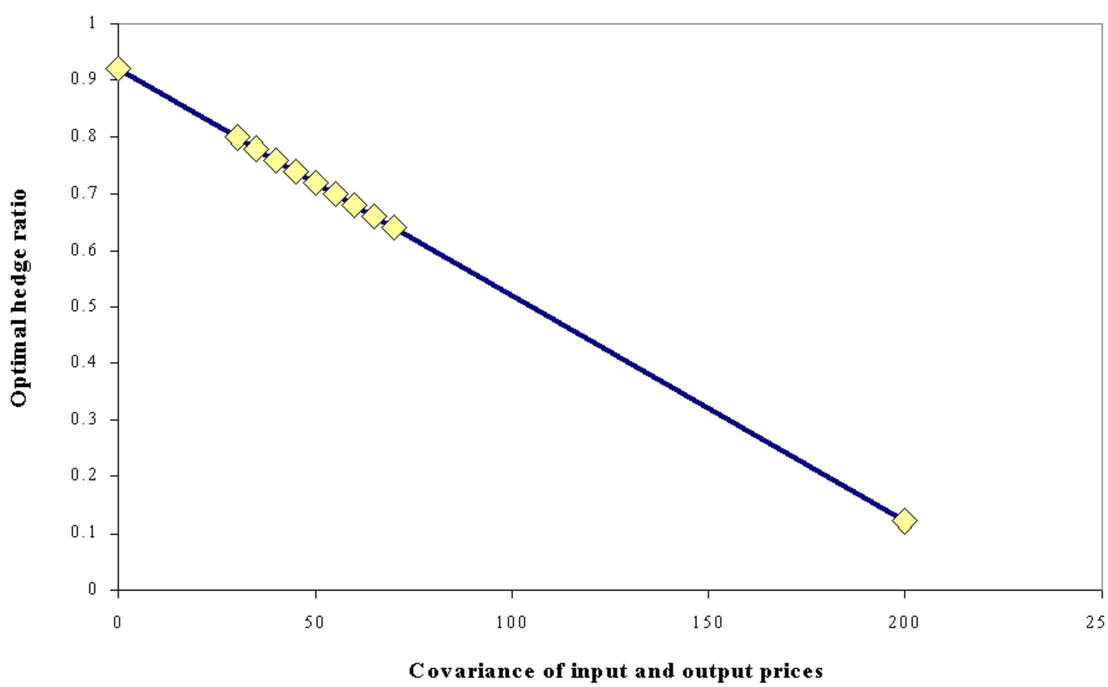

Figure 2. The Impact of Input-Output Covariance on Hedge Ratios

Hedging Strategies in U.S. Bread Baking

Bakers are a good example to illustrate grain processors hedging decisions. Outputs are bread, largely sold and priced at a period following purchase of the inputs. Inputs are flour. Neither flour nor bread have futures, but futures in wheat provide a viable hedging mechanism that is used routinely in this industry.

Data Sources and Statistics

Monthly average white pan bread prices were taken from the Bureau of Labor Statistics. Bread prices represent average bread prices from across the United States. Weekly data were collected and converted to monthly averages for Kansas City Bakers Standard Patent Bulk Flour for the same time period. The input price series (flour) uses Kansas City flour which is the most often purchased by bakery operations. Daily data were collected for Kansas City Board of Trade (KCBT) nearby HRW futures, which were converted into a monthly average for the period January 1990 to August 2000.

The production capacity of the bakery was derived using the following assumptions: 9 ounces of flour were used to produce a one pound loaf of white pan bread; a large bakery used approximately 8,000 hundredweight of flour per week; a medium-size bakery used approximately 3,000 hundredweight of flour per week. Conversions between hundredweight of flour and bushels of wheat were calculated using the same ratios as in the case of the flourmill.

Figure 3 shows monthly average prices for Kansas City Bakers Standard Patent Bulk Flour, *KCBT* HRW wheat futures, and white pan bread (U.S. average, dollars per one pound loaf). Flour (dollars per 0.005625 cwt) and wheat (dollars per 0.0129375 bushel) prices are shown for the approximate equivalent of inputs for baking a one pound white pan bread. Wheat and flour prices are measured against the left vertical axis, and bread prices are measured against the right vertical axis. The relationship between wheat and flour prices is apparent, however, neither of the two series appear to be strongly related to bread prices for the entire sample period.

As the time lags m and n are changed, the observations on which the analysis was run also changed. This leads to some of the time lags including more volatile periods than others, and in some cases, given the peculiarities of the sample data, volatility declined as the time lag increased. The variance of output prices $Var\left(P_{O,t+n}\right)$ at time $t+n$ changed with the time lag since the observations varied with each scenario. Sample statistics for a three-month hedge (m = 3) are given in Table 1. The correlation between input and output prices is weaker. While the relationship between bread prices (the output) and wheat futures (the hedge instrument) are weaker, wheat prices are highly correlated with flour prices. The variance of cash and futures markets, $Var\left(P_{I,C,t}\right)$ and $Var\left(P_{I,F,t}\right)$, and the covariance $Cov\left(P_{I,C,t}, P_{I,F,t}\right)$ remain constant since n, the time lag, does not affect these parameters.

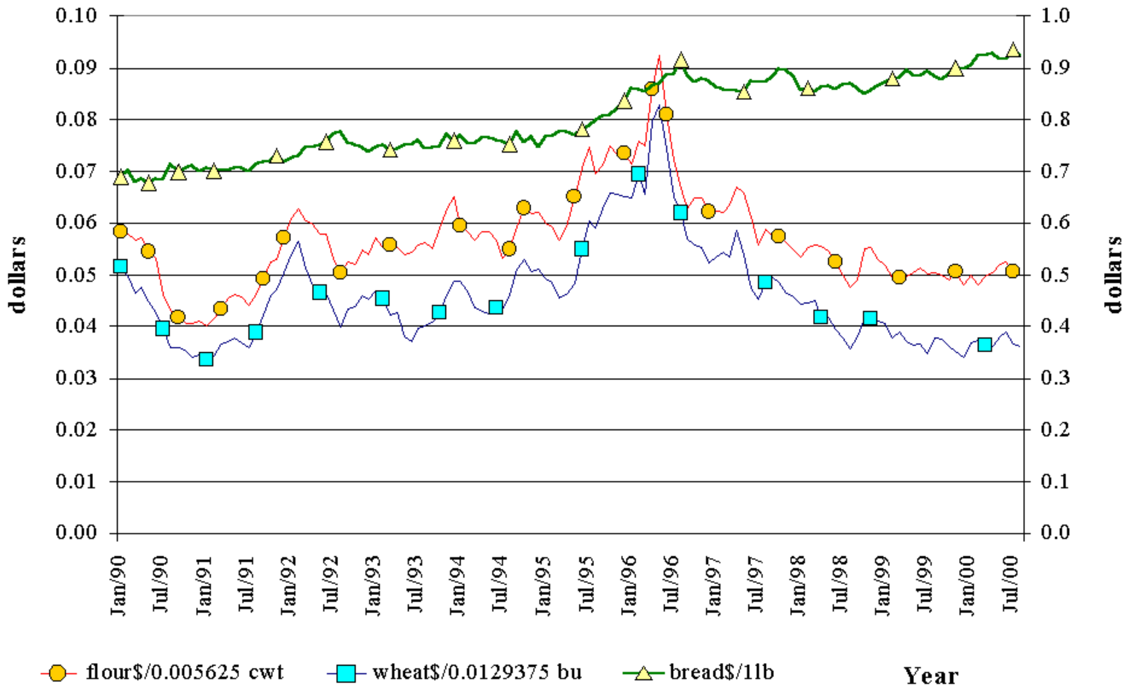

Figure 3. KC Flour, HRW Wheat Futures, and Bread Prices

Base Case Results

The base case scenario assumes zero bias. The expected futures price at the time the hedge is lifted, $E\left\lfloor P_{I,F,t} \right\rfloor$, equals the futures price when the hedge is entered, $P_{I,F,t-m}$, and the speculative demand of the hedge strategy is zero. Hedging decisions are made at time *t-m*. At this time, the processor *anticipates* selling bread at a future date, *t*+*n*. Inputs (flour and other inputs) will be purchased at *t*. The hedge is placed in wheat futures from *t-m* until *t*.

Results were derived for m = 1, 2, 3, 6 month hedging periods and for n = 0, 1, 2, 3, 6 month processing periods. In this case, the firm purchases flour at time *t* and processes it into bread during *m*. The output (bread) is priced at *t*+*n*. Inputs, represented by $Q_{I,t\rightarrow t+n}$, are not the same commodity as outputs, $Q_{O,t\rightarrow t+n}$, and there is additional value between wheat, flour, and bread in the vertical value chain.

Table 1. Three Month Hedge Sample Statistics for Bread Baking

m = 3 (months)	n=0	n=1	n=2	n=3	n=6
Variances:					
$Var(P_{I,C,t})$	2.8023				
$Var(P_{I,F,t})$	0.5831				
$Var(P_{O,t+n})$	0.004751	0.00473	0.0047	0.00471	0.00476
Covariances:					
$Cov(P_{I,C,t}, P_{I,F,t})$	1.2064				
$Cov(P_{I,C,t}, P_{O,t+n})$	0.0409	0.0443	0.0471	0.0488	0.0492
$Cov(P_{I,F,t}, P_{O,t+n})$	0.0154	0.0168	0.0180	0.0185	0.0185
Correlations:					
$Correl(P_{I,C,t}, P_{I,F,t})$	0.9519				
$Correl(P_{I,C,t}, P_{O,t+n})$	0.3581	0.3883	0.4140	0.4284	0.4298
$Correl(P_{I,F,t}, P_{O,t+n})$	0.2969	0.3232	0.3470	0.3570	0.3540

The parameter values for the base case were: $Q_{I,t\rightarrow t+n} = 0.005625$, $Q_{O,t\rightarrow t+n} = 1$, $E\lfloor P_{I,F,t}\rfloor = P_{I,F,t-m} = \$2.80/bu$, $E\lfloor P_{I,C,t}\rfloor = \$8.5/cwt$, $E\lfloor P_{O,t+n}\rfloor = 0.60/lb\ bread$, and $\lambda = 0.5$. The input price of \$8.50 per hundredweight of flour translates into 4.7 cents of flour related expenses per loaf of bread. In addition, 45 cents are assumed to represent all other processing related costs, including that for other ingredients. The output price, 60 cents, represents typical wholesale prices per loaf of bread.

Table 2 shows the results for F_I, the optimal futures position for a combination of *m* and *n* hedging and processing periods. The hedge ratios are interpreted as the percentage of wheat equivalents of inputs (flour) to produce a one pound loaf bread that is purchased in futures markets. In other words, for one loaf of bread 0.005625 hundredweight of flour is used, which is equivalent to around 0.0129375 bushels of wheat. The value of $F_I / 0.0129375$ gives the optimal hedge ratio in percentage form.

The optimal hedge ratio, F_1, is comprised of hedging, speculative, and strategic demands for futures. In this case, the results indicate that the optimal strategy would be a net short position in wheat futures. The size of the correlation between wheat futures and bread prices is sufficiently high that the strategic demand exceeds hedging demand, i.e., the firm takes a net short wheat futures position. It is important that the sign of the strategic demand depends on the correlation between bread and wheat futures prices. When the correlation is positive, the strategic demand causes a negative adjustment of the futures position, but causes a positive adjustment when correlations are negative. While these findings seem counter-intuitive, further investigation of the hedging and strategic demand factors illuminates the situation. Since the correlation between wheat futures and bread prices has been positive over the past decade, the strategic demand term has a negative sign, or in other words it represents a downward adjustment to the futures position.

The hedging strategy is highly dependent on the correlation between inputs and outputs. To contrast with the base case, the same analysis was repeated on a subset of the observation from January 1996 to August 2000. During this period, bread prices exhibited a general upward trend, while wheat and flour prices were gradually declining. These relationships resulted in a negative correlation between input and output prices, and had the effect of suggesting a larger long position (hedge ratio greater than 1). These results were mainly due to the negative sign of strategic demand which represented an upward adjustment of the futures position.

Table 2. Base Case Simulation Results for Bread Baking

m (months)	n=0	n=1	n=2	n=3	n=6
Hedging Demand (+)					
1	0.9003	0.9003	0.9003	0.9003	0.9003
2	0.8995	0.8995	0.8995	0.8995	0.8995
3	0.8954	0.8954	0.8954	0.8954	0.8954
6	0.8922	0.8922	0.8922	0.8922	0.8922
Speculative Demand (+)					
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000	0.0000
Strategic Demand (-)					
1	2.2777	2.4587	2.6251	2.7184	2.7159
2	2.1881	2.3652	2.5308	2.6197	2.5966
3	2.0536	2.2306	2.3876	2.4604	2.4529
6	1.6488	1.8037	1.9489	2.0312	2.0318
F_I					
1	-1.3774	-1.5584	-1.7249	-1.8181	-1.8156
2	-1.2887	-1.4657	-1.6313	-1.7203	-1.6971
3	-1.1581	-1.3351	-1.4922	-1.5649	-1.5574
6	-0.7566	-0.9116	-1.0567	-1.1390	-1.1397

Sensitivity Analysis

To examine the effects of changing correlation on the hedge position, the results for a three week hedge and two week processing period (m = 3 and n = 2) are presented, assuming that the base case variances do not change. For illustration purposes, the correlations and covariances, as well as the results for the optimal hedge position, are presented in Table 3.

Table 3. Sensitivity Analysis of Correlations for Bread Baking

	ρ = -0.4	ρ = 0	ρ = 0.4	ρ = 0.8
$Var(P_{I,F,t})$	0.5832	0.5832	0.5832	0.5832
$Var(P_{O,t+n})$	0.0047	0.0047	0.0047	0.0047
$Cov(P_{I,F,t}, P_{O,t+n})$	-0.02094	0.0000	0.02094	0.04188
Hedging Demand:	0.8954	0.8954	0.8954	0.8954
Speculative Demand:	0.0000	0.0000	0.0000	0.0000
Strategic Demand:	-2.7754	0.0000	2.7754	2.5507
Optimal Hedge Ratio:	3.6707	0.8954	-1.8799	-4.6553

With negative or nil correlations, the optimal hedge ratio is positive. However, for correlations greater than about .4, the optimal hedge ratio becomes negative. In these cases, the strategic demand more than offsets the hedging demand for futures. These results suggest that firms may over hedge when the correlation between input and ouput prices is positive and vice versa when the correlation is negative, if input and ouput relationships are not considered in optimum hedge ratio estimation.

In the special case that output price (bread) is set by a forward contract, the price of output would be known with certainty at the time the hedging decision is made (at t-m). In this case, there would be no output price risk. The variance of output price and the covariance converge to zero, which leads to a zero strategic demand. In this case, the hedge ratio is the sum of hedging and speculative demands (which in this case, the latter is nil).

Summary

Hedge ratio calculations have been the object of extensive research. The literature includes numerous derivations and formulations for finding optimal positions under both risk minimization and profit maximization models. Past studies focused on finding optimal hedge positions for producers and traders for managing the risk of price fluctuations in input *or* output markets separately. These models were extended to include multiple inputs or outputs and several sources of risk. However, the relationship between prices in input and output markets is influenced by competitive factors, the amount of value added to the output product, and the period of coverage, or *hedge horizon*.

Hedging in food processing was examined and explained from the perspective of hedging, speculative, and strategic demand for futures. The correlation between input and output markets, as well as the time horizon of the decision making process, was shown to be crucial parameters. The results indicate that hedging strategies depend on the correlation between input and output price changes over time. Depending on the industry, outlooks or opinions (the bias) about the direction of movements in futures prices lead to varying degrees of speculative trading. The strategic demand represents a negative adjustment to the conventional hedge position, as the 'natural protection' of margins through the parallel fluctuation of input and output prices, taken into account.

As firms determine their hedge horizon, they implicitly determine how much of the future input needs they wish to purchase at a locked-in price. Costs are locked-in by hedging in futures markets, where the effectiveness of risk management strategies is improved by a close correlation between the cash (or physical) market and the futures contract price.

The amount of value added influences the degree of correlation between the price of the base commodity and the processed product. The more processing a product is subject to, typically the lower the correlation between the source commodity and the good. This is the case when comparing the correlations between wheat prices and flour prices (high correlation) with the correlation between wheat prices and bread prices (lower correlation).

The *hedge horizon* has implications not only in terms of correlations, but also in terms of the amount of inputs (and the locked-in price) the firm decides to purchase at a given time. In terms of correlations, the period of time between purchasing inputs and selling processed outputs is important. In some industries, changes in input prices translate into similar changes in output prices almost instantaneously (naturally protecting profit margins), while in other industries this adjustment may take longer. The hedge horizon also determines the length of time for which the firm seeks coverage by hedging. These two issues are closely related and were important aspects of this analysis.

Derivation for an optimal hedge ratio (or futures position) includes a *strategic demand* as well as hedging and speculative demand for futures. The strategic demand is impacted by the relationship between price changes in input and output markets. The sign of the strategic demand is negative and a higher covariance leads to a reduced overall hedge ratio as shown by the numerical analysis. The relationship between input and output prices also depends on the time period or lag between procurement decisions and selling processed outputs. The empirical analysis considered these effects by measuring sample statistics on a variety of potential hedge horizons.

Analytical solutions were derived for the U.S. bread baking cases. Under the base case scenarios, when the bias was assumed to be zero speculative demand was zero. The resulting hedge ratios were the combination of the hedging and strategic demand. Typically, the strategic demand had a negative sign, resulting in a hedge position smaller than the traditional minimum variance hedge. These results are consistent with those derived by Frechette and Tuthill which suggest that when agents use the minimum variance hedge ratio (MVHR), they hedge too much because the MVHR ignores the benefits of higher returns.

Hedging and speculative demands are not affected by the hedge horizon, since parameters that determine the size of hedging demand do not include the time lag. The strategic demand, however, changes with the time lag, since the relationship between input and output prices (as measured by the covariance or correlation) changes with the time lag. In most instances, the longer the time lag, the smaller the strategic demand and, therefore, the closer the optimal position is to a traditional hedge ratios. Incorporating input and output price relationships in optimum hedge ratio estimation for grain processors will prevent them from under/over hedging.

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APPENDIX A

MODEL DERIVATIONS AND OPTIMALITY CONDITION

The mean-variance specification of a processor, is:

$$E(\Pi) = Q_{O,t\to t+n} E\left(\widetilde{P}_{O,t+n}\right) - Q_{I,t\to t+n} E\left(\widetilde{P}_{I,C,t}\right) + F_I\left[E\left(\widetilde{P}_{I,F,t}\right) - P_{I,F,t-m}\right] - \left(Q_{O,t\to t+n}\right)\left(C_{\$/O}\right) + \frac{\lambda}{2}$$

$$\begin{aligned}&\left(Q_{O,t\to t+n}\right)^2 Var\left(P_{O,t+n}\right) + \left(Q_{I,t\to t+n}\right)^2 Var\left(P_{I,C,t}\right) + F_I^2 Var\left(P_{I,F,t}\right) - \\ &- \left[2Q_{O,t\to t+n} Q_{I,t\to t+n} Cov\left(P_{O,t+n}, P_{I,C,t}\right)\right] + \left[2Q_{O,t\to t+n} F_I Cov\left(P_{O,t+n}, P_{I,F,t}\right)\right] - \\ &- \left[2Q_{I,t\to t+n} F_I Cov\left(P_{I,C,t}, P_{I,F,t}\right)\right]\end{aligned} \tag{1A}$$

Taking first and second order conditions we obtain:

$$\begin{aligned}\frac{\partial J}{\partial F_I} &= E\left(\widetilde{P}_{I,F,t} - P_{I,F,t-m}\right) - \lambda F_I Var\left(P_{I,F,t}\right) - \lambda Q_{O,t\to t+n} Cov\left(P_{O,t+n}, P_{I,F,t}\right) \\ &+ \lambda Q_{I,t\to t+n} Cov\left(P_{I,s,t}, P_{I,F,t}\right) = 0\end{aligned} \tag{2A}$$

$$\begin{aligned}\frac{\partial J}{\partial Q_{O,t\to t+n}} &= E\left(\widetilde{P}_{O,t+n} - C_{\$/O}\right) - \lambda Q_{O,t\to t+n} Var\left(P_{O,t+n}\right) \\ &+ \lambda Q_{I,t\to t+n} Cov\left(P_{O,t+n}, P_{I,s,t}\right) - \lambda F_I Cov\left(P_{O,t+n}, P_{I,F,t}\right) = 0\end{aligned} \tag{3A}$$

$$\begin{aligned}\frac{\partial J}{\partial Q_{I,t\to t+n}} &= E\left(-P_{I,C,t}\right) - \lambda Q_{I,t\to t+n} Var\left(P_{I,C,t}\right) \\ &+ \lambda Q_{O,t\to t+n} Cov\left(P_{O,t+n}, P_{I,C,t}\right) + \lambda F_I Cov\left(P_{I,C,t}, P_{I,F,t}\right) = 0\end{aligned} \tag{4A}$$

The system of three equations can be solved for F_I, $Q_{O,t\to t+n}$, and $Q_{I,t\to t+n}$. Since the resulting equations are considerably more complex, the variables were redefined in terms of:

$Q_{O,t\to t+n}$ = Q, $Q_{I,t\to t+n}$ = R, F_I = F, $E\left[\widetilde{P}_{O,t+n}\right] = A$, $E\left[\widetilde{P}_{I,C,t}\right] = S$, $E\left[\widetilde{P}_{I,F,t}\right] = G$, $P_{I,F,t-m}$ = H,

$C_{\$/O}$ = C, $Var\left(P_{O,t+n}\right)$ = W, $Var\left(P_{I,s,t}\right)$ = T, $Var\left(P_{I,F,t}\right)$ = K, $Cov\left(P_{O,t+n}, P_{I,s,t}\right)$ = V,

$Cov\left(P_{O,t+n}, P_{I,F,t}\right)$ = N, $Cov\left(P_{I,s,t}, P_{I,F,t}\right)$ = L.

The closed form solutions to the system of three equations are given by:

$$Q = \frac{\left(\text{-L}^2\text{A+L}^2\text{C-LVH+LVG+LNS-NTG+NTH-KTC+KTA-VKS}\right)}{\text{g}\left(-\text{L}^2\text{W} + 2\text{LNV} + \text{KTW} - \text{V}^2\text{K} - \text{N}^2\text{T}\right)} \tag{5A}$$

$$R = \frac{\left(\text{-KSW-KVC+KVA+LNC+SN}^2\text{-LHW-LNA-VNG+VNH+LGW}\right)}{\left(-\text{L}^2\text{W} + 2\text{LNV} + \text{KTW} - \text{V}^2\text{K} - \text{N}^2\text{T}\right)} \tag{6A}$$

$$F = \frac{\left(\text{-SLW-TNA+TGW+V}^2\text{H-THW-V}^2\text{G+VLA-VLC+TNC+SNV}\right)}{\left(-\text{L}^2\text{W} + 2\text{LNV} + \text{KTW} - \text{V}^2\text{K} - \text{N}^2\text{T}\right)} \tag{7A}$$

An interesting observation can be made about g, the risk aversion parameter. In contrast with previous models of hedging where an infinitely risk averse agent's hedge was the same as the minimum-risk hedge, in this formulation, an infinitely risk averse agent would not even be involved with any production activities.

Second order conditions were taken and the Hessian matrix was constructed to show that a maximum to the optimization problem exists.

$$H = \begin{bmatrix} -\lambda Var\left(P_{O,t+n}\right) & \lambda Cov\left(P_{O,t+n}, P_{I,C,t}\right) & -\lambda Cov\left(P_{O,t+n}, P_{I,F,t}\right) \\ \lambda Cov\left(P_{O,t+n}, P_{I,C,t}\right) & -\lambda Var\left(P_{I,C,t}\right) & \lambda Cov\left(P_{I,C,t}, P_{I,F,t}\right) \\ -\lambda Cov\left(P_{O,t+n}, P_{I,F,t}\right) & \lambda Cov\left(P_{I,C,t}, P_{I,F,t}\right) & -\lambda Var\left(P_{I,F,t}\right) \end{bmatrix} \tag{8A}$$

The conditions for maximum are:

$$\left|H_1\right| = -\lambda Var\left(P_{O,t+n}\right) < 0 \tag{9A}$$

$$\left|H_2\right| = \lambda^2\left[Var\left(P_{O,t+n}\right)Var\left(P_{I,C,t}\right) - Cov\left(P_{O,t+n}, P_{I,C,t}\right)^2\right] > 0 \tag{10A}$$

$$\begin{aligned}\left|H_3\right| = &-\lambda Var\left(P_{O,t+n}\right)\left[\lambda^2 Var\left(P_{I,s,t}\right)Var\left(P_{I,F,t}\right) - \lambda^2 Cov\left(P_{I,s,t}, P_{I,F,t}\right)\right] \\ &- \lambda Cov\left(P_{O,t+n}, P_{I,s,t}\right)\left[\lambda^2 Cov\left(P_{O,t+n}, P_{I,C,t}\right)Var\left(P_{I,F,t}\right) - \lambda^2 Cov\left(P_{O,t+n}, P_{I,F,t}\right)Cov\left(P_{I,s,t}, P_{I,F,t}\right)\right] \\ &- \lambda Cov\left(P_{O,t+n}, P_{I,F,t}\right)\left[\lambda^2 Cov\left(P_{O,t+n}, P_{I,s,t}\right)Cov\left(P_{I,C,t}, P_{I,F,t}\right) - \lambda^2 Cov\left(P_{O,t+n}, P_{I,F,t}\right)Var\left(P_{I,s,t}\right)\right] < 0\end{aligned} \tag{11A}$$

Given the upper bound of covariance between two variables is: $\left|\sigma_{XY}\right| = \sigma_X\sigma_Y$, the conditions are met, with the exception of the special case when the covariance term equals the product of standard deviations between the two variables. This special case only happens when the correlation coefficient between cash and futures equals 1.