Pricing of Rainfall Insurance in India using Gaussian and *t* Copulas

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Abstract

Low income households, especially in the developing countries such as India could suffer losses due to weather related events such as drought, hurricanes, floods etc. Such losses could cast a household into a chronic poverty cycle - a poverty trap from which the household may find it difficult to re-emerge. Rainfall derivatives are the insurance contracts that compensate a household based on the weather outcome rather than the actual crop yield. Traditional methods for pricing rainfall derivatives include burn analysis, index value simulation and daily rainfall simulation. In this work, we price the rainfall derivatives using a different method that uses the Gaussian and *t* copulas to capture the dependence between the monthly rainfalls in the monsoon season in India.

We find that though the premiums calculated using burn analysis and our proposed method were equal, the standard deviation and Value at Risk "VaR" of the insurance payoffs calculated using both the methods differed. Therefore, in practice, the actuarial pricing of the rainfall insurance contract using burn analysis and our proposed method could be different. Our method could be easily applied to price rainfall derivatives for the regions that exhibit extreme rainfall patterns.

Keywords: Weather derivatives, rainfall insurance in India, pricing in incomplete markets, Gaussian and t Copulas, agriculture yields, Monte Carlo simulations

JEL code: Q14, G13, G17, G22

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1. Introduction

Low income households especially in the developing countries such as India could suffer losses due to weather related events such as drought, hurricanes, floods etc. that could cast a household into a chronic poverty cycle - a poverty trap from which the household may find it difficult to remerge (Barnett et. al. 2008, Gien et. al. 2010). In a survey conducted by Gien et. al. (2010) in the Indian state of Andhra Pradesh, 89% of surveyed farmers indicated drought as the most important risk faced by them. Unfortunately, drought risk tends to be a ***locally*** systematic risk i.e. the drought tends to affect most households in an area simultaneously. Thus an affected household is less likely to get support from the family, friends or nearby households in a drought situation. But the risk exposure of a household to the drought is actually diversifyable because a drought based weather risk is less likely to be strongly correlated with the systematic risk factors such as the stock index returns. Thus financial markets could be tapped to manage the weather risk. A traded rainfall insurance contract could help households reduce their weather risk by providing a payoff when the rainfall is deficient (Skees et. al. 2002).

India launched its first individual crop yield insurance scheme in the year 1972-73. The scheme was launched by the General Insurance Department of Life Insurance Corporation of India for the H-4 cotton (Singh 2010). After the Comprehensive Crop Insurance Scheme (CCIS) in 1985, Government of India launched National Agricultural Insurance Scheme (NAIS) in 1999. NAIS provided insurance against the crop yield losses due to variety of risks such as drought, fire, storm, pest etc. The scheme operated on a small defined area such as a village or a district as defined by the state government and the state administration assessed the crop yields and compensated the affected farmers. By 2008, the government had paid about 8% of the premium as subsidies and claims were 2.8 times the premium collected (Singh 2010). Despite the government subsidy the scheme did not achieve desired success largely due to an inefficient insurance design and the delays in the claim settlements (Clarke et. al. 2012). Crop yield based schemes such as NAIS are plagued with the problems of moral hazard where the farmer’s behaviour could influence the crop yield which in turn determines the insurance payout and adverse selection due to asymmetric information where some farmers who are better aware of their lower risk voluntarily opt out of the insurance scheme. Furthermore, the assessment of the crop yield loss could be an administratively expensive process. As the local area level losses are not accurately measurable, crop yield based insurance products may not effectively use the financial markets for risk transfer or avail of the international reinsurance (Manuamorn 2005).

A different approach to the crop insurance is to compensate the farmers based on the weather outcomes such as the extent of rainfall rather than the actual crop yields. Weather Index Insurance are contingent claims or derivatives where the payoff is a function of a weather parameter such as rainfall, temperature, humidity etc. as recorded at a specified local weather station. As the payoff of such a weather index insurance is dependent on an objective and transparent information, issues of moral hazard and adverse selection are substantially ameliorated and administrative costs are also reduced as a local crop yield assessment is no longer necessary. Hence the claim settlement is not delayed as well (Manuamorn 2005). Furthermore, such weather index based contracts could be traded in the financial markets and these contracts could thus provide risk diversification opportunities to the other market participants as well. The tacit assumption underlying the weather index insurance is that the weather parameter such as rainfall which is the underlying, is highly correlated with the crop yield. After all the farmers need to be insured against the loss in the crop yield and not the weather parameter. The risk of insufficient correlation between the weather index and the actual crop yield in a region is called "Basis Risk" (Clarke et. al. 2012). Innovative ways such as blending of weather index insurance with rural finance needs to be found to mitigate the basis risk for sustained demand for weather index insurance (Manuamorn 2005).

The first Rainfall Insurance[3] product in India was introduced in 2003 by ICICI – Lombard General Insurance Company for Groundnut and Castor farmers of BASIX's[4] association in Mahabubnagar district of the Indian state of Andhra Pradesh (Clarke et. al. 2012). Then in 2007, the Government of India introduced its own Weather Based Crop Insurance Scheme (WBCIS).

Analysis of any weather derivative requires performing following three steps (Musshoff et. al. 2007):

1. Measuring relation between the weather variable and the crop yield
2. Statistical modelling of the weather variable - in our case rainfall
3. Developing a theoretically consistent pricing model

Our paper is structured as follows: In section 2 we try to answer the question if the rainfall affects the crop yields? In Section 3 we explore some relevant literature and methodologies for pricing rainfall insurance. In section 4, we describe a typical structure for the rainfall insurance and statistically model the rainfall variable. Then we price June, July and August rainfall insurance sub-contracts analytically. In section 5 we price the rainfall insurance contract using simple Monte Carlo technique and use Gaussian and t Couplas to capture rainfall dependence between the monsoon months. In section 6 we conclude.

[3] *We use the terms "Rainfall Index Insurance" and "Rainfall derivative" interchangeably in this work*
[4] *BAXIS is an association of Hyderabad – India based companies that provide financial services and technical assistance to the rural poor.*

2. Data and the impact of rainfall on the crop yield

The rains in India are seasonal and the south west monsoon season starts every year from the month of June and continues up till September. "Kharif" crops are the crops sown and harvested during this monsoon season[5]. The extent of monsoon rains and the cultivated Kharif crops vary from state to state in India. In this work, we focus on the state of Andhra Pradesh in India and analyse the rainfall derivative for the Mahabubnagar district in Andhra Pradesh. We define "Total Kharif rainfall" as the cumulative rainfall from June to September for the purpose of this study.

Following table lists the data used in our work:

Table 1: Data table[6]

Data	Region	Period	Frequency
Kharif and Rabi crop production in tons and Area of cultivation in hecters	All districts of Andhra Pradesh - India	1998 - 2009	Seasonal
Rainfall and Temperature	All India	1901- 2007	Monthly
Rainfall	All districts of Andhra Pradesh - India	1998 - 2009	Monthly
Rainfall	Mahabubnagar district of Andhra Pradesh - India	1901 - 2010	Monthly

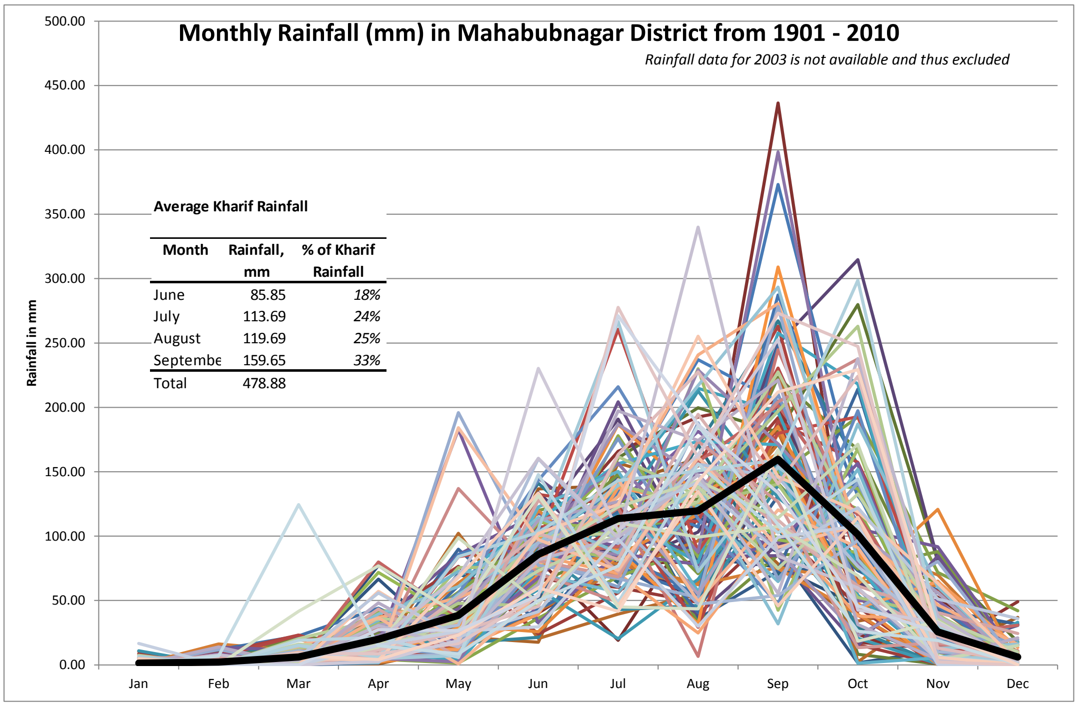
Rice (~ 32% of the cultivated area), Bajra (11%), Cotton (9%), Soyabean (7%) and Groundnut (7%) are the top five Kharif crops by the cultivated area (in hectares) in Andhra Pradesh from 1998 to 2009. The ranking of the top five Kharif crops in the district of Mahabubnagar namely Castor seed (21%), Jowar (13%), Rice (13%), Maize (13%) and Cotton (10%); differs from that in the state as a whole. Annexure 1 provides a detailed crop yield summary for both the state - Andhra Pradesh and the district - Mahabubnagar.

Figure 1 below depicts the distribution of the monthly rainfall in Mahabubnagar district from 1901 to 2010. The seasonality of the rainfall in the region is prominent. The average total rainfall in the Kharif season was ~ 479 mm and the average rainfall peaked at ~ 159 mm in September. The maximum and minimum rainfall during Kharif season was 765 mm and 256 mm respectively. The standard deviation of the rainfall during Kharif season was 121 mm and the Kurtosis was absent.

[5] *The monsoon season is also described as Kharif season.*

[6] *Data from http://www.indiawaterportal.org/; https://data.gov.in/catalog/district-wise-season-wise-crop-production-statistics; ftp://www.tropmet.res.in/pub/data/rain/iitm-regionrf.txt; "Rabi" season crops are sowed and harvested in the winter months.*

Figure 1: Monthly rainfall in Mahabubnagar district from 1901- 2010



The primary objective of rainfall derivatives is to hedge the crop yield (Crop production / Cultivated Area, kg/hectare) risk of the farmers that arise due to the deficient or excess rainfall. Studies by Prassanna (2014) and Auffhammer, Ramanathan and Vincent (2011) have found a positive impact of rainfall on the crop yields in India. We first perform a panel regression to test if the total Kharif rainfall has any explanatory power to explain the Kharif crop yields of 31 crops in 22 districts in the state of Andhra Pradesh in India from 1998 up till 2009[7].

We test the following regression model:

$$y_{ijt} = \alpha + \boldsymbol{\beta X_i} + \boldsymbol{\gamma Z_j} + \varphi t + \omega R_{Totkh,jt} + \varepsilon_{ijt}$$

Where y_{ijt} is the Kharif crop yield of crop i in the district j in the year t, $\boldsymbol{X_i}$ and $\boldsymbol{Z_j}$ are vectors of dummy variables for crop types and districts respectively, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are the corresponding parameter vectors, φ is the parameter on the annual time trend, ω is the parameter on the $R_{Totkh,jt}$

[7] *We exclude the year 2003 from our analysis as we do not have the rainfall data for this year. But for the time series analysis (table 3 and 7), June to September monthly rainfalls in the year 2003 are substituted with the corresponding June to September average monthly rainfalls estimated using rainfall data from 1901 up till 2002.*

which is the total Kharif rainfall in district *j* in the year *t* and ε_{ijt} is the error term. The *F* statistic of this panel regression is statistically significant, $F = 699$, *p*- value ~ 0 and the adjusted $R^2 = 0.96$. The parameter on the annual time trend, φ is not significant but the parameter on the total Kharif rainfall, ω is significant with the *t* value of 2.25 and a *p*-value of 0.02, indicating that the total Kharif rainfall has a significant impact on the crop yields. The detailed results are presented in the Annexure 2.

Due to the basis risk, crop insurance needs to be designed for a specific crop in a given district. Thus we now analyse if the total Kharif rainfall in the Mahabubnagar district explains the variance in the crop yields of various crops in the Mahabubnagar district. We test the following regression model for six major crops (selected based on their rank by the hecters cultivated, except Groundnut which is included in the analysis because the first rainfall index insurance was launched for Groundnut in the Mahabubnagar district):

$$y_t = \beta_o + \beta_1 R_{Totkh,t} + \varepsilon_t$$

Where y_t is the yield of a crop and $R_{Totkh,t}$ is the total Kharif rainfall in the year *t* in the Mahabubnagar district and ε_t is the error term. Table 2 below summarizes the results of the regression analysis.

Table 2: Impact of total Kharif rainfall on the crop yields of major crops in the Mahabubnagar district

Impact of Total Rainfall (Kharif) on Crop Yield in Mahabubnagar District

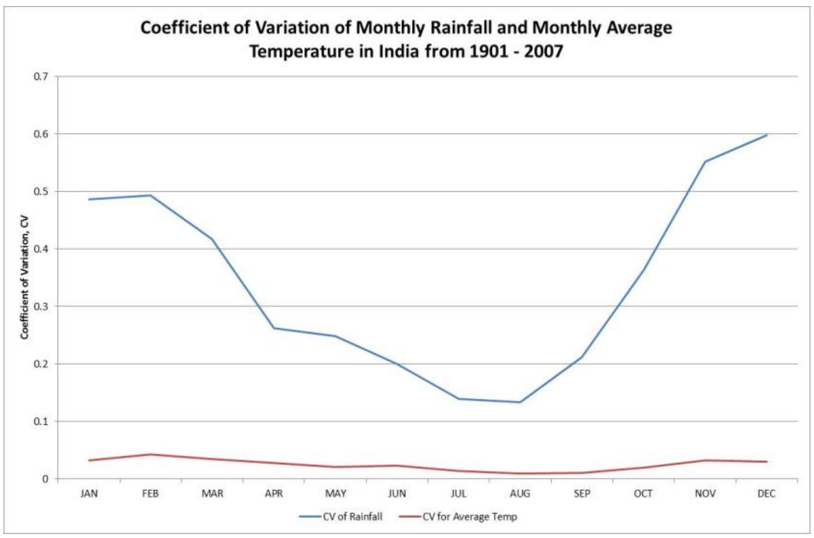
	Slope	Slope, t Value	Slope, P Value	Adjusted R2
Castor seed	0.32	1.25	0.24	5.4%
Jowar	0.61	1.99	0.08	23.0%
Rice	0.90	1.32	0.22	6.9%
Maize	**3.67**	**2.32**	**0.05**	**30.5%**
Cotton	1.05	1.68	0.13	15.4%
Groundnut	**1.22**	**2.92**	**0.02**	**43.0%**

The regression analysis indicates that the rainfall derivatives could hedge the crop yield risk of Groundnut, Maize and to some extent, that of Jowar and Cotton. This lends support to the decision by ICICI – Lombard General Insurance Company of launching the insurance product first for the Groundnut crop.

3. Methodologies for pricing rainfall derivatives and literature review

Despite its relevance to the agriculture, rainfall derivatives have been studied less often than the temperature derivatives (Jewson et. al. 2005). There are fundamental differences between the modelling of a rainfall stochastic process and a temperature stochastic process. Rainfall is more erratic and the rainfall distribution at any time is bound by zero. Figure 2 below compares the coefficient of variation (CV), a ratio of the standard deviation to the mean, of monthly rainfall and monthly temperature from the year 1901 up to 2007 in India. CV of the monthly rainfall is almost 10 times higher than that of the monthly temperature.

Figure 2: CV comparison of monthly rainfall and monthly temperature in India from 1901 - 2007



There are primarily three methods to price rainfall derivatives (Jewson et. al. 2005, Musshoff et. al., 2007):

1. **Burn analysis:** This is a non-parametric method where in the payoff of the rainfall derivative is calculated directly from the collected historical rainfall data. This method is empirical and makes no assumption about the parametric distribution of the rainfall.

2. **Index value simulation:** Under this method a parametric or a non-parametric distribution is fitted to the historical rainfall data and the fitted distribution is then used to randomly draw the values of the required rainfall index. The payoff of the rainfall derivative is then calculated from this simulated rainfall index.

3. **Daily simulation:** This method requires developing a statistical model of the underlying daily rainfall process. The rainfall index is then calculated from this daily simulated rainfall process and the payoff of the rainfall derivative is estimated from this calculated rainfall index.

Turvey (1999) estimates the prices of various rainfall derivatives under the assumption that the rainfall index is normally distributed. Stoppa and Hess (2003) develop a methodology to construct a rainfall index that correlates with the crop yield. Cao, Li and Wei (2004) estimate the rainfall derivative payoff from a daily rainfall process. They estimate the probability of the daily rainfall using a Markov chain and model the severity of the rainfall using a Gamma distribution and a mixture of two exponential distributions. Husak, Michaelsen and Funk (2006) also use Gamma distribution to estimate the severity of monthly rainfall and combine the severity with the probability of the rainfall. Musshoff, Odening and Xu (2007) use a similar methodology as that proposed by Cao et. al. (2004) for modelling the daily rainfall process. They study the hedging effectiveness of rainfall derivatives in mitigating wheat yield risk in northeast Germany and find that the daily models tend to underestimate the rainfall variability. Leobacher and Ngare (2009) propose a Markov Gamma model where in, they fit the Gamma distribution to the rainfall in every month and use a Gaussian copula to capture the correlation between two consecutive months. In their Markov model the rainfall is dependent only on the previous month. Cabrera, Odening and Ritter (2013) propose a daily rainfall model similar to that proposed by Cao et. al. (2004) and calculate the prices of rainfall derivatives using Esscher transforms.

The model that we propose extends the previous works in the following two ways:

1. We model rainfall over the Kharif season in a given year as a joint distribution of four monthly rainfall random variables, one each for four monthly rainfalls. We use copulas to capture the dependence between the four monthly rainfall marginal distributions. Each marginal distribution is assumed to be Gamma distributed and a copula couples these marginals to form a multivariate Kharif rainfall distribution. Thus in our model the rainfall in a given month may exhibit correlation with many months.
2. We model the dependence between the marginal distributions using the Gaussian and the t copulas. A t copula also captures “thick tails” i.e. the risk of extreme floods or droughts. This higher risk of extreme events may not be captured using a Gaussian copula.

Any rainfall derivative with the rainfall as an underlying would be priced in the incomplete markets because the underlying rainfall or an index based on rainfall is not a tradable asset; hence a replicating portfolio cannot be built using such an underlying. Thus there is no unique price for a rainfall derivative. But if the markets were liquid, in order to avoid arbitrage opportunities, the rainfall derivatives if traded in the market, may satisfy some internal consistency relationship between them. But currently the rainfall derivatives are not traded in India, hence the market price of rainfall risk (weather risk) is not easily known. If the rainfall risk were assumed to be not diversifyable, the insurer could price the rainfall derivative using the standard Utility Principle (Delbaen et. al. 1989, Embrechts 1996, Gerber et. al. 1998, Hamisultane 2008, Shah et. al. 2015). Under this principle the total premium for underwriting a rainfall derivative will be such that the utility of the initial wealth of the insurer is same as the expectation of the utility of the summation of the initial wealth and the premium charged less the expected payoff under the rainfall derivative.

Musshoff et. al. (2007) observe negligible correlation between the rainfall index and the stock market returns and hence they assume that the rainfall risk is not a systematic risk and the market price of rainfall risk is zero. Hull (2009 pg. 791) suggests a similar approach as well. Cao and Wei (2004) on the other hand use the investor's risk aversion level and the parameters governing the aggregate dividend process to calculate a non-zero market price of the weather risk.

Gine et. al. (2007) test the correlation between the Indian stock market returns and the rainfall insurance payouts and find that these two variables are not correlated. In this work, we also assume that the rainfall risk is idiosyncratic and diversifyable and thus the market price of the rainfall risk is zero. This assumption implies that the risk neutral distribution (***Q*** measure) of the rainfall risk coincides with the real life distribution (***P*** measure) of the rainfall risk. Thus the premium $\pi(t)$ charged by an insurer at the time *t* for underwriting a rainfall derivative is equal to the expectation of the discounted rainfall derivative payoff $X(T)$ at time T under the ***P*** measure. For a constant risk free interest rate r, $\pi(t)$ is given as (Bjork 2009):

$$\pi(t) = e^{-r(T-t)} E_t^P\big(X(T)\big)$$

This financial pricing approach is equivalent to the actuarial equivalence premium pricing principle (Bühlmann 1980, Mikosch 2009). For zero interest rate, the premium charged is given as:

$$\pi(t) = E_t^P\big(X(T)\big)$$

4. Structure of a rainfall insurance contract and modelling of the rainfall distribution

In this section we analyse the structure of a rainfall insurance contract using the rainfall data of Mahabubnagar district. We assume that the rainfall in a given month is independent of that in the previous month. This assumption implies, for example that the July monthly rainfall is independent of the June monthly rainfall. We will relax his assumption in the section 5. Now we perform a simple time series analysis of each of the twelve monthly rainfalls[8], total Kharif rainfall[9] and annual rainfall[10] time series. We use Augmented Dickey Fuller (ADF) and KPSS test to test for unit root in all the above times series. In case of an ambiguous result, we test for a unit root using Philips Parron test. Then we use Ljung Box test to check if the autocorrelations up till a specified lag are equal to zero.

The table 3 below summarizes the results of the three unit root and Ljung Box tests.

Table 3: Summary of the unit root and Ljung Box tests[11]

Rainfall time series	ADF Stat.	ADF Stat., P value	KPSS stat.	Ljung - Box, Q stat	Q stat, P value	PP Test Stat.	PP stat., P Value
January	-10.07	0.00	0.21	29.10	0.79		
February	-9.31	0.00	0.16	28.38	0.81		
March	-10.76	0.00	***0.47***	14.26	1.00	-10.76	0.00
April	-12.71	0.00	0.20	41.04	0.26		
May	-10.40	0.00	0.17	28.82	0.80		
June	-9.28	0.00	0.38	32.14	0.65		
July	-9.89	0.00	0.20	44.80	0.15		
August	-9.96	0.00	***0.83***	43.61	0.18	-10.24	0.00
September	-10.39	0.00	0.21	24.12	0.94		
October	-8.98	0.00	0.29	31.76	0.67		
November	-10.38	0.00	0.14	***62.03***	***0.00***		
December	-11.50	0.00	0.28	46.88	0.11		
Total Kharif Rainfall	-11.14	0.00	***0.49***	30.58	0.72	-11.14	0.00
Annual Rainfall	-9.32	0.00	0.60	29.73	0.76		

[8] By "monthly rainfall" we mean cumulative rainfall in a month measured at the end of the month.

[9] By "Total Kharif rainfall" we mean cumulative rainfall in the full Kharif season i.e. sum of four monthly rainfalls from June to September.

[10] By "annual rainfall" we mean cumulative rainfall in a calendar year.

[11] For the KPSS test, the null hypothesis of the absence of a unit root is rejected if the value of the test statistic is greater than 0.739 (1% significance) or 0.463 (5% significance). In the table, the bold values of the test statistics and the corresponding bold P values indicate a possible presence of a unit root.

Though the ADF test null hypothesis of the existence of a unit root was rejected in all the rainfall time series, the null hypothesis of the KPSS test of the absence of a unit root was rejected for March and the total Kharif rainfall time series at 5% significance and for August rainfall time series at 1% significance. We then also perform Philips Perron (PP) test for these three rainfall time series and find that the null hypothesis of the existence of a unit root is rejected. When we apply the Ljung Box test to test if the autocorrelations up till a specific lag are zero, we find that only in case of the November rainfall time series the autocorrelations up till 36 lags were different from zero (Gujarati et. al. 2009). But a fit of ARIMA model based on Akaike information criterion (AIC) criterion indicated that the model ARIMA(0, 0, 0) was the best fit for the November rainfall time series. The autocorrelation and partial autocorrelation plots for November rainfall time series are presented in Annexure 3.

We assume that the monthly rainfall in the month i, R_i measured at the end of the month, has a cumulative distribution function (CDF) $F_{R_i}(r_i)$ which is continuous and strictly increasing such that $F_{R_i}^{-1}$ exists and R_i is Gamma distributed. We also assume that the time series R_{it} is independent and strictly stationary[12] i.e. $R_{it-1}, R_{it}, R_{it+1}, ..$ are independent and identically distributed, for example the June monthly rainfall in the year 2009 is independent of the June monthly rainfall in the year 2008 but both the random variables are identically distributed. The CDF of the monthly rainfall R_i is given as[13]:

$$R_i \sim GAM(\theta_i, \kappa_i)$$

$$F_{R_i}(r_i) = \int_0^{r_i} \frac{1}{\theta_i^{\kappa_i}\Gamma(\kappa_i)} x^{\kappa_i - 1}\, e^{-x/\theta_i} dt\,, \quad x \geq 0,\ \ \theta_i, \kappa_i > 0$$

Where θ_i is the scale parameter and κ_i is the shape parameter (Bain et. al. 1991).

Our choice of Gamma distribution for describing the rainfall is based on the following two reasons (Husak et. al. 2006):

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[12] *Note: As the monthly rainfall time series is Gamma distributed, absence of the correlation under Ljung Box test does not guarantee independence. Only in the case of the elliptical distributions does a zero correlation implies independence. Hence we separately introduce an assumption of independence. Also in a time series, a covariance stationarity does not imply a strict stationarity, hence we introduce the assumption of the strict stationarity here.*

[13] *Because of the independence assumption we now drop the subscript t for time.*

1. Gamma distribution is bounded on the left at zero and this property is required to describe the rainfall, as the rainfall does not take negative values. Also Gamma distribution is positively skewed i.e. the distribution is long tailed on to the right and this property helps in describing the rainfall in the regions where extreme heavy rains are a possibility.
2. Gamma distribution is flexible and thus it can represent many distributions with differing shapes using its shape and scale parameters.

Because of the *iid* (independent and identically distributed) assumption for the monthly rainfall random variable, we estimate the parameters of the four monthly rainfall Gamma distributions from the rainfall data (1901 – 2010) using the maximum likelihood method. Table 4 below summarizes the shape and scale parameters of the fitted Gamma distributions. We use the Kolmogorov Smirnov (KS) test to test the null hypothesis that the empirical distribution of the monthly rainfall is Gamma distributed. Each of the four *p*-values in the table 4 indicates that we cannot reject the null hypothesis that the monthly rainfalls are Gamma distributed (Husak et. al. 2006).

Table 4: Gamma distribution parameters

Monthly Rainfall	Shape	Scale	KS Test*, P Value
June	5.272	16.283	0.351
July	5.016	22.667	0.955
August	3.285	36.430	0.236
September	4.376	36.482	0.777

The rainfall insurance structure that we price in this work is summarized below in the table 5 and the structure is very similar to that underwritten by ICICI – Lombard General Insurance Company (Gine et. al. 2007).

Table 5: Rainfall insurance contract structure (Term sheet)

Phase	**1**	**2**	**3**
Month	**June (*)**	**July**	**August**
Strike (Rainfall, mm)	$K_1 = 70$	$K_2 = 110$	$K_3 = 95$
Notional (INR/mm)	$N_1 = 10$	$N_2 = 10$	$N_3 = 10$
Policy Limit (INR)	$L_1 = 1000$	$L_2 = 1000$	
Exit Rainfall (mm)	$E_1 = 10$		

The Kharif rainfall insurance contract consists of three insurance sub-contracts – one each for the June, July and August months. The full Kharif rainfall insurance contract (for all the three months) has to be purchased before the month of June i.e. the beginning of the Kharif rainfall season. In June, if the Monthly rainfall R_1, is above 70 mm *(K_1)*, the June sub-contract expires out of money. For rainfall between 70 mm up till 10 mm *(E_1)*, INR 10 *(N_1)* is paid for every mm deficiency in the rainfall. If the recorded rainfall is below 10 mm *(E_1)*, a flat INR 1000 *(L_1)* payoff occurs. Mathematically the June payoff $X_1(T_1)$ payable at T_1 is expressed as follows (Mack 1984, Panjer 2006):

$$Payoff_1 = X_1(T_1) = \begin{cases} L_1, & R_1 < E_1 \\ N_1 \times max(K_1 - R_1, 0), & R_1 \geq E_1 \end{cases}$$

The payoff of a July insurance sub-contract, $X_2(T_2)$ is equal to the INR 10 *(N_2)* for every mm of rainfall below the strike rainfall of 110 mm *(K_2)* up till the policy limit of INR 1000 is reached *(L_2)*. Mathematically the payoff is expressed as:

$$Payoff_2 = X_2(T_2) = min\big(L_2,\ \ N_2 \times max(K_2 - R_2, 0)\big)$$

In the August sub-contract, there is no external policy limit imposed; the payoff, $X_3(T_3)$ is simply the notional *(N_3)* i.e. INR 10 for every mm of rainfall below the strike rainfall of 95 mm *(K_3)*.

$$Payoff_3 = X_3(T_3) = N_3 \times max(K_3 - R_3, 0)$$

Now we know that the premium for an insurance contract at time *t*, with the monthly rainfall for the month *i*, as the underlying is given by:

$$\pi_i(t) = E_t^P\big(X_i(T_i)\big)$$

It is trivial to derive the following analytical expressions for the premiums for the June, July and August rainfall insurance sub-contracts described above (Bain et. al. 1991):

$$\begin{aligned}\pi_1(t) = {} & L_1 F(E_1;\ \theta_1, \kappa_1) \\ & + N_1 \Big(K_1\big(F(K_1;\ \theta_1, \kappa_1) - F(E_1;\ \theta_1, \kappa_1)\big) \\ & - \theta_1\, \kappa_1\big(F(K_1;\ \theta_1, \kappa_1 + 1) - F(E_1;\ \theta_1, \kappa_1 + 1)\big)\Big)\end{aligned}$$

Where $F(K_1;\ \theta_1, \kappa_1)$ is a Gamma cumulative distribution function (CDF) with the upper limit of integration K_1 and parameters θ_1 and κ_1.

$$\pi_2(t) = L_2 F\left(K_2 - \frac{K_2}{N_2};\ \theta_2, \kappa_2\right) + N_2\left(K_2\left(F(K_2;\ \theta_2, \kappa_2) - F\left(K_2 - \frac{K_2}{N_2};\ \theta_2, \kappa_2\right)\right) - \theta_2\, \kappa_2\left(F(K_2;\ \theta_2, \kappa_2 + 1) - F\left(K_2 - \frac{K_2}{N_2};\ \theta_2, \kappa_2 + 1\right)\right)\right)$$

and

$$\pi_3(t) = N_3\big(K_3 F(K_3;\ \theta_3, \kappa_3) - \theta_3\, \kappa_3 F(K_3;\ \theta_3, \kappa_3 + 1)\big)$$

We then calculate the premiums for June, July and August rainfall insurance sub-contracts using both analytical and Monte Carlo simulation methods and the results are summarized in the table 6 below.

Table 6: Premium for the June, July and August rainfall insurance sub-contracts under the assumption of no correlation between the rainfalls in various months

Premium for the Kharif Contract (Assumption of no correlation between the rainfall in the kharif months)

Insurance Contract Month	Method	Premium, INR	Standard Error, INR	95% Confidence Interval for premium	
June	Analytical	72.32	N/A	N/A	N/A
	Monte Carlo	71.68	0.77	70.17	73.18
July	Analytical	179.06	N/A	N/A	N/A
	Monte Carlo	176.30	1.45	173.47	179.14
August	Analytical	134.36	N/A	N/A	N/A
	Monte Carlo	133.78	1.31	131.20	136.35

Note: the standard error of simulation for n number of trails is defined as $\frac{\sigma}{\sqrt{n}}$ where σ is the standard deviation of the n simulated payoffs (Hull 2009).

In the next section we assume that there is correlation between the monthly rainfalls of the Kharif season in a given year i.e. for example the monthly rainfall in August is correlated with the monthly rainfall in July in a given year; and we use a copula to capture this dependence.

5. Rainfall insurance contract pricing using Gaussian and t Copulas

We first test if the monthly rainfall in May could predict the total Kharif rainfall in the Mahabubnagar district, if yes, one could "cheaply" purchase the Kharif rainfall insurance contract given this information asymmetry. Table 3 shows that both the monthly rainfall in May and total Kharif rainfall time series do not possess a unit root thus we regress the total Kharif rainfall ($R_{Totkh,t}$) on the May rainfall ($R_{May,t}$) to test if May rainfall could predict the total Kharif rainfall. The simple regression model is given below:

$$R_{Totkh,t} = \beta_o + \beta_1 R_{May,t} + \varepsilon_t$$

We find that the regression model has an adjusted R^2 of ~ 0 and the slope coefficient $\beta_1$is not statistically significant (*t* stat = -0.87, *p*-value = 0.38). Thus we assume that the total Kharif rainfall cannot be predicted given the monthly rainfall in May.

Now we analyse the rainfall over the Kharif season as a four dimensional multivariate time series, $\boldsymbol{R_{kt}}$ with monthly rainfall in each of the four Kharif months i.e. from June, R_{1t} to September, R_{4t} as the components of that multivariate time series (Tsay 2014).

$$\boldsymbol{R_{kt}} = (R_{1t}, R_{2t}, R_{3t}, R_{4t})'$$

To measure the linear dynamic dependence of $\boldsymbol{R_{kt}}$, we define its lag l cross- covariance matrix as follows (Tsay 2014):

$$\Gamma_l = Cov(\boldsymbol{R_{kt}}, \boldsymbol{R_{kt-l}}) = E[(\boldsymbol{R_{kt}} - \boldsymbol{\mu})(\boldsymbol{R_{kt-l}} - \boldsymbol{\mu})']$$

$$\Gamma_l = \begin{bmatrix} E\left(\widetilde{R_{1t}}, \widetilde{R_{1,t-l}},\right) & \cdots & E\left(\widetilde{R_{1t}}, \widetilde{R_{4,t-l}},\right) \\ \vdots & \ddots & \vdots \\ E\left(\widetilde{R_{4t}}, \widetilde{R_{1,t-l}},\right) & \cdots & E\left(\widetilde{R_{4t}}, \widetilde{R_{4,t-l}},\right) \end{bmatrix}$$

Where $\boldsymbol{\mu} = E(\boldsymbol{R_{kt}})$ is the mean vector of $\boldsymbol{R_{kt}}$ and $\widetilde{\boldsymbol{R_{kt}}} = \boldsymbol{R_{kt}} - \boldsymbol{\mu}$ is the mean adjusted time series. The mean vector is constant and the cross covariance matrix is a function of only lag l and not time *t* under the assumption that $\boldsymbol{R_{kt}}$ is covariance stationary**.** Note the cross covariance matrix above captures not only the auto covariance between, say June monthly rainfall at time *t* and June monthly rainfall at time $t - l$ but also cross covariance between June monthly rainfall at time *t* and monthly rainfalls in July, August and September at the time $t - l$.

We test the existence of linear dynamic dependence in $\boldsymbol{R_{kt}}$ using a multivariate Ljung Box test. The test statistic is defined as (Tsay 2014):

$Q_k(m) = n^2 \sum_{l=1}^{m} \frac{1}{n-l} tr\left(\hat{\Gamma}_l' \hat{\Gamma}_0^{-1} \hat{\Gamma}_l \hat{\Gamma}_0^{-1}\right)$

Where k is the dimension= 4, in our case, n is the sample size and tr(**A**) is the trace of matrix **A**. We assume that only one year lagged rainfall i.e. lag $l = 1$, may affect the rainfall in a given year. Thus $m = 1$ and our null hypothesis is as follows:

H_0: $\Gamma_1 = 0$ and the alternative hypothesis is given by:

H_a: $\Gamma_1 \neq 0$

The value of $Q_4(1)$ in our case is 19 with a *p*-value of 0.27; thus we do not reject the null hypothesis that $\Gamma_1 = 0$.

As the lag 1 cross covariance matrix is not different from zero, we focus on the concurrent correlation matrix[14] (i.e. lag 0, cross-correlation matrix) and test if any of the correlations in the concurrent correlation matrix is significant. We first test for the significance of the Pearson correlations and find that only the correlation between June and August monthly rainfalls is significant (*p*-value = 0.03). But Pearson's correlation is an accurate measure of the association only if the random variables have an elliptical distribution. Thus we also test for the significance of the Spearman rank correlation and find that Spearman rank correlation between June and August monthly rainfalls is not significant (*p*-value = 0.1). The results are presented in the table 7 below.

[14] *Concurrent correlation matrix can be easily calculated from lag 0 cross-covariance matrix Γ_0. Lag 0 – cross correction matrix $\hat{\rho}_0 = \hat{D}^{-1}\hat{\Gamma}_0\hat{D}^{-1}$, where $\hat{D} = daig\{\hat{\gamma}_{0,11}^{1/2}, \dots \hat{\gamma}_{0,44}^{1/2}\}$ and $\hat{\gamma}_{0,ii}^{1/2}$ is the $(i,i)^{th}$ element of $\hat{\Gamma}_0$.*

Table 7: Correlation analysis

Covariance Analysis: Ordinary
Included observations: 110

Correlation t-Statistic Probability	JUN	JUL	AUG	SEP
JUN	1.000000			

JUL	-0.043018	1.000000		
	-0.447468	-----		
	0.6554	-----		
AUG	0.196250	0.088126	1.000000	
	2.079934	0.919410	-----	
	0.0399	0.3599	-----	
SEP	0.001112	0.106749	-0.077912	1.000000
	0.011553	1.115743	-0.812153	-----
	0.9908	0.2670	0.4185	-----

Covariance Analysis: Spearman rank-order
Included observations: 110

Correlation t-Statistic Probability	JUN	JUL	AUG	SEP
JUN	1.000000			

JUL	-0.032789	1.000000		
	-0.340932	-----		
	0.7338	-----		
AUG	0.155057	0.097978	1.000000	
	1.631133	1.023143	-----	
	0.1058	0.3085	-----	
SEP	-0.018723	0.142051	-0.045863	1.000000
	-0.194607	1.491359	-0.477121	-----
	0.8461	0.1388	0.6342	-----

We now summarize the assumptions:

- In a given year, total Kharif rainfall cannot be predicted using the rainfall data before the Kharif season. The June, July, August and September monthly rainfalls in a given year could exhibit some dependency between them.
- Though the monthly rainfalls during the Kharif season in a given year show some dependency, the monthly rainfalls in a given year are independent of the monthly rainfalls in the previous year. For example there could be some dependency between monthly rainfalls in June and July in a given year but the monthly rainfall in June of this year is independent of the monthly rainfalls in the previous year.
- Multivariate time series $\boldsymbol{R_{kt}}$ is strictly stationary and independent[15]**.** i.e. $\boldsymbol{R_k}$ is independent and identically distributed. This assumption implies that the rainfall over the Kharif season every year is a random draw from a stationary joint rainfall distribution of four monthly rainfall random variables.

Because of *i.i.d* assumption, dropping the subscript *t*, we consider $\boldsymbol{R_k}$ as a four dimensional vector valued random variable as below:

$$\boldsymbol{R_k} = (R_1, R_2, R_3, R_4)'$$

$$\forall (r_1, r_2 \cdots r_4) \in [0, \infty)^4$$

And assume that $\boldsymbol{R_k}$ is continuous with a joint CDF as below:

[15] *For a time series that does not have an elliptical distribution, absence of serial and cross correlation does not guarantee independence. Hence we introduce the assumption of independence separately. Similarly weak stationarity does not imply strict stationarity and hence the strict stationarity assumption.*

$$\boldsymbol{F}_{\boldsymbol{R_k}}(\boldsymbol{r_k}) = F_{R_1,R_2,R_3,R_4}(r_1,r_2,r_3,r_4) = P(R_1 \le r_1, R_2 \le r_2, R_3 \le r_3, R_4 \le r_4)$$
$$= \int_0^{r_1}\int_0^{r_2}\int_0^{r_3}\int_0^{r_4} f_{R_1,R_2,R_3,R_4}(v,w,x,y)\,dydxdwdv$$

As in the previous section, we assume that each, R_i is Gamma distributed.

$R_i \sim GAM(\theta_i, \kappa_i)$

Now every joint distribution implicitly contains both a description of the marginal distribution of the random variables and a description of the dependence structure between these random variables. We model the dependence between the monthly rainfalls in the Kharif season using the concept of a copula. Simply put, a copula is a d-dimensional distribution function on $[0,1]^d$ with standard uniform marginal distributions. A copula joins the univariate distributions to form a joint distribution. In our case a copula joins Gamma distributions of the monthly rainfalls R_i, $F_{R_i}(r_i)$ and forms a joint distribution of rainfall over the Kharif season $\boldsymbol{R_k}$, $\boldsymbol{F}_{\boldsymbol{R_k}}(\boldsymbol{r_k})$ with a defined dependence structure. Under the condition of continuity of the marginals, the famous theorem due to Sklar guarantees the uniqueness of a copula C (Schmidt 2006, Emberechts 2009 and Meucci 2011).

$$\boldsymbol{F}_{\boldsymbol{R_k}}(\boldsymbol{r_k}) = F_{R_1,R_2,R_3,R_4}(r_1,r_2,r_3,r_4) = C\left(F_{R_1}(r_1), F_{R_2}(r_2), F_{R_3}(r_3), F_{R_4}(r_4)\right)$$

Now by definition $F_{R_i}(r_i) = u_i$ where u_i is uniformly distributed i.e. $u_i \in [0,1]$ therefore we get (McNiel et. al. 2005, Alexander 2008):

$$C(u_1,u_2,u_3,u_4) = F\left(F_{R_1}^{-1}(u_1), F_{R_2}^{-1}(u_2), F_{R_3}^{-1}(u_3), F_{R_4}^{-1}(u_4)\right)$$

We use two copulas to capture the dependence between the monthly rainfalls – a Gaussian copula and a *t* copula. We use a *t* copula to capture the higher likelihood of the extreme values or "fat tails", in case, $\boldsymbol{R_k}$ were to exhibit this phenomenon.

A Gaussian copula is defined as below (McNiel et. al. 2005, Alexander 2008, Fusai 2008):

$$C_P^{Ga}(u_1,u_2,u_3,u_4) = \boldsymbol{\Phi}_P\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \Phi^{-1}(u_3), \Phi^{-1}(u_4)\right)$$

Where $\boldsymbol{\Phi_P}$ is the standard multivariate normal distribution, Φ is the standard univariate normal distribution function and *P* is the correlation matrix.

A t copula is defined as below (McNiel et. al. 2005, Alexander 2008, Fusai 2008):

$$C_{\upsilon,P}^{t}(u_1,u_2,u_3,u_4) = \boldsymbol{t}_{\upsilon,P}\left(t_{\upsilon}^{-1}(u_1), t_{\upsilon}^{-1}(u_2), t_{\upsilon}^{-1}(u_3), t_{\upsilon}^{-1}(u_4)\right)$$

Where $\boldsymbol{t}_{\upsilon,P}$ and t_{υ} are multivariate and univariate student *t* distributions respectively with υ degree of freedom and P correlation matrix. As the degree of freedom υ becomes larger a t copula approaches the corresponding Gaussian copula (Demarta et. al. 2004).

We follow the following procedure to generate the joint distribution of rainfall over the Kharif season, $\boldsymbol{R_k}$:

1. Generate cumulative density values for Kernel density i.e. u_{ji} for each data point *j* of the monthly rainfall for month *i* (in our case 109 X 4, u_{ji}values).
2. Fit both Gaussian and *t* copulas to the computed u_{ji} values in step 1. Unlike the parameter estimation for a Gaussian copula, the parameter estimation for a *t* copula is complex, as the likelihood function needs to be maximized with repect to both the degree of freedom υ and the correlation matrix *P* (Kjersti 2004). Hence we assume that the *t* copula has the same correlation matrix as that of the Gaussian copula and estimate only the degree of freedom. The fitted parameters for both the copulas are summarized in table 8.
3. Once both the copulas are fitted, simulate random samples of the cumulative densities[16] of $\boldsymbol{R_k}$ i.e. $\boldsymbol{U_o}$, using each copula.
4. Then use inverse cumulative density function of each R_i to get the joint distribution of $\boldsymbol{R_k}$.

[16] *Each* $\boldsymbol{U_o} = (u_{o1}, u_{o2}, u_{o3}, u_{o4})$ *where* u_{oi} *is the* o^{th} *simulation of the cumulative density of* R_1*and* $u_{oi} \in [0,1]$.

Table 8: Parameters of the fitted copulas

Correlation matrix for Gaussian copula

Monthly Rainfall	June	July	August	September
June	1.00	-0.03	0.18	-0.02
July	-0.03	1.00	0.11	0.12
August	0.18	0.11	1.00	-0.07
September	-0.02	0.12	-0.07	1.00

Correlation matrix for t copula

Monthly Rainfall	June	July	August	September
June	1.00	-0.03	0.18	-0.02
July	-0.03	1.00	0.11	0.12
August	0.18	0.11	1.00	-0.07
September	-0.02	0.12	-0.07	1.00
nu =	1,71,90,998			

Note: As the degree of freedom υ is very large the t copula will approach the Gaussian copula.

Once we have the simulated values of monthly rainfall in each Kharif month, we can calculate the premium for the full Kharif rainfall insurance contract. We also perform burn analysis and provide the calculated burn premium for the comparison with the premium calculated using our proposed method. We also calculate the premium under the assumption that the rainfall over the Kharif season, $\boldsymbol{R_k}$ follows a multivariate normal distribution implying that the marginal distributions of the monthly rainfalls are normal as well.

Table 9: Calculated premium for the full Kharif rainfall insurance contract using various methods

Premium for the Kharif Contract

Method	Premium, INR	Standard Error, INR	95% Confidence Interval for the premium	
Burn Analysis	386.91	N/A	N/A	N/A
Monte Carlo (Gaussian Copula)	384.77	2.231	380.40	389.15
Monte Carlo (t Copula)	385.96	2.230	381.59	390.33
Monte Carlo (Aussumption of Multi variate Normal distribution)	397.20	2.712	391.89	402.52

It is clear from the table 9 above, that the premiums calculated using burn analysis, Gaussian copula and t copula are equal but the premium calculated under the assumption of multivariate normal distribution for $\boldsymbol{R_k}$ is higher than the rest.

In this work we have assumed a zero market price of rainfall risk but in practice the actuarial pricing may be performed using a standard pricing principle such as the standard deviation principle given below (Mikosch 2009):

$$\pi(t) = E_t^P\big(X(T)\big) + \zeta\sqrt{Var\big(X(T)\big)}$$

Where $X(T)$ is derivative payoff at time T and ζ is a positive constant

This principle incorporates a risk measure such as standard deviation or variance in the pricing calculation. The second risk measure that we consider here is the Value at Risk (VaR). VaR is defined as a α^{th} quantile of a loss distribution. Formally, VaR of a variable (X) for a given quantile (α) at given time *t* is defined as (McNiel et. al. 2005, Panjer 2006):

$$VaR_\alpha(X) = F_X^{-1}(\alpha)$$

We provide a summary of risk measures for the Kharif rainfall insurance contract payoff in the table 10 below:

Table 10: Risk measures for the payoff of the Kharif rainfall insurance contract

Method	Standard Deviation	VaR, 99%
Burn Analysis	367.0	1,287.5
Monte Carlo (Gaussian Copula)	352.8	1,398.9
Monte Carlo (t Copula)	352.6	1,393.3

We note that the standard deviation of the payoff calculated using the burn analysis is greater than that obtained using our methodology (Monte Carlo analysis and Gaussian and t copulas). Thus our proposed method could result in lower premiums charged to farmers in comparison with the burn analysis, at least in the case where the pricing is performed using the standard deviation principle. The Value at Risk (VaR, 99%) of the insurance payoff computed using burn analysis is lower than the VaR of the payoff computed using our method. This shows that the burn analysis could result in an underestimation of the actuarial risk and thus could lower the regulatory capital requirement of the insurers.

6. Conclusion

In this work we propose a methodology for pricing a Kharif rainfall insurance contract using Gaussian and t copulas that capture the dependence between the June, July, August and September monthly rainfalls in the Mahabubnagar district in Andhra Pradesh – India. We find that though the premium calculated using burn analysis and our proposed method were equal, the standard deviation and Value at Risk "VaR" of the insurance payoffs using both the methods differed. Therefore, in practice, the actuarial pricing of the rainfall insurance contract using burn analysis and our proposed method could be different. Furthermore, our method could find more applicability in regions with extreme rainfalls where burn analysis may prove to be inappropriate especially because of limited data.

In this work, we did not quantify the basis risk in a district for a given crop which could be crucial to the efficient design of the rainfall insurance. Also, it is difficult to trade rainfall derivatives for each crop and each district separately in the capital markets as these contracts may not find enough liquidity. We leave this onerous task of designing at least a state level rainfall index for India to the future works.

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Annexures

Annexure 1: Crop yield data for Andhra Pradesh and Mahabubnagar

Production of top 10 Crops by Area of cultivation in the state of Andhra Pradesh from 1998 - 2009

Crop	Production in Tons	Area in Hectares	Crop Yield, Kg/Hectare	% Area
Rice	57,55,26,474	27,75,01,863	2,074.0	31.8%
Bajra	7,70,07,223	9,23,75,243	833.6	10.6%
Cotton	14,86,83,404	8,31,34,976	1,788.5	9.5%
Soyabean	6,41,12,751	6,45,58,566	993.1	7.4%
Groundnut	6,06,14,809	6,10,56,000	992.8	7.0%
Maize	11,20,93,192	6,06,26,330	1,848.9	7.0%
Jowar	3,90,65,293	3,92,62,499	995.0	4.5%
Arhar	2,16,47,668	3,19,54,190	677.5	3.7%
Moong	89,95,641	2,61,38,569	344.2	3.0%
Urad	82,36,758	2,02,89,881	406.0	2.3%

Production of top 10 Crops by Area of Cultivation in district Mahabubnagar from 1998 - 2009

Crop	Production in Tons	Area in Hectares	Crop Yield, Kg/Hectare	% Area
Castor seed	6,65,805	16,61,370	400.8	21.1%
Jowar	7,47,193	10,29,194	726.0	13.1%
Rice	23,82,170	10,00,061	2,382.0	12.7%
Maize	20,44,928	9,98,029	2,049.0	12.7%
Cotton	9,67,334	7,53,732	1,283.4	9.6%
Arhar	2,75,685	7,12,515	386.9	9.1%
Groundnut	3,78,751	6,08,126	622.8	7.7%
Moong	62,399	4,27,719	145.9	5.4%
Sunflower	1,35,026	2,15,091	627.8	2.7%
Bajra	72,691	1,36,301	533.3	1.7%

Annexure 2: Linear regression results

Linear Regression Table:

	Estimate	t Value	P Value
(Intercept)	2,613.05	1.10	0.27
Arhar	-281.85	-0.12	0.90
Bajra	1,024.70	0.45	0.66
Banana	17,627.06	7.34	0.00
Cashewnut	249.94	0.10	0.92
Castor.seed	-74.81	-0.03	0.97
Cotton.lint.	1,012.44	0.45	0.65
Dry.chillies	1,932.06	0.84	0.40
Groundnut	881.96	0.39	0.70
Horse.gram	89.55	0.04	0.97
Jowar	868.55	0.38	0.70
Korra	411.33	0.15	0.88
Maize	2,562.39	1.13	0.26
Masoor	79.74	0.03	0.97
Mesta	9,723.06	4.07	0.00
Moong	-443.99	-0.20	0.85
Niger.seed	2,925.69	1.05	0.29
Onion	17,619.48	6.79	0.00
other.oilseeds	18,881.21	5.89	0.00
Ragi	1,823.80	0.79	0.43
Rice	1,954.18	0.86	0.39
Samai	2,959.91	1.01	0.31
Sesamum	366.58	0.16	0.87
Small.millets	1,348.00	0.55	0.58
Soyabean	755.86	0.31	0.76
Sugarcane	79,784.24	34.78	0.00
Sunflower	200.95	0.09	0.93
Tapioca	9,870.95	3.34	0.00
Tobacco	26.66	0.01	0.99
Turmeric	5,423.24	2.32	0.02
Urad	-298.03	-0.13	0.90
Adilabad	-3,321.18	-4.51	0.00
Anantapur	-2,812.21	-3.59	0.00
Chittoor	-2,421.45	-3.03	0.00
Cuddapah	-2,949.15	-3.73	0.00
East Godavari	-2,276.18	-2.79	0.01
Guntur	-1,505.40	-1.98	0.05
Karimnagar	-2,850.03	-3.76	0.00
Khammam	-2,311.53	-3.06	0.00
Krishna	-1,152.96	-1.51	0.13
Kurnool	-2,808.18	-3.82	0.00
Mahabubnagar	-3,235.37	-4.47	0.00
Medak	-3,099.56	-4.23	0.00
Nalgonda	-3,198.53	-3.90	0.00
Nellore	54.29	0.06	0.95
Nizamabad	-2,904.13	-3.88	0.00
Prakasam	-2,644.72	-3.50	0.00
Rangareddi	-3,024.81	-4.10	0.00
Srikakulam	-4,980.70	-5.89	0.00
Vishakhapatnam	-6,055.93	-8.09	0.00
Vizianagaram	-5,857.75	-7.89	0.00
Warangal	-3,126.55	-4.27	0.00
Trend	36.04	1.47	0.14
Total Rainfall (Kharif)	**1.10**	**2.25**	**0.02**

Note: Crop yields of areas less than 1000 hecters excluded in the analysis

Annexure 3: ACF and PCF plots of November rainfall time series