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Strategic Use of Futures and Options by Commodity Processors

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ABSTRACT

In this study, the strategic impacts of input-output price relationships on end-users' demands for futures and/or options are analyzed. An analytical model is developed based on mean-variance utility and extended to account for the impact of output prices and the inclusion of both futures and/or call options in the portfolio. This study makes several contributions to the literature on risk management in agriculture. First, its focus is on end-users and captures their unique characteristics. Second, it explicitly captures the correlation between input-output prices on hedging strategies. Finally, it incorporates options into a portfolio model. The analytic model was applied to the bread baking industry, an important agribusiness processor, which is interesting because of the relation between wheat prices, the primary ingredient, and bread prices. We show the optimal portfolio of futures and options and illustrate how this varies with several critical variables.

Key Words: Futures, Options, Risk Management, Processors, Hedging

STRATEGIC USE OF FUTURES AND OPTIONS BY COMMODITY PROCESSORS

David W. Bullock, William W. Wilson, and Bruce L. Dahl*

1. Introduction

Processors make extensive use of futures and options for hedging purposes which comprise integral components of their price risk management strategies. Typically, long futures positions of varying sizes and durations are taken to offset short cash positions of ingredients. These market participants make extensive use of futures and their trading would be viewed as highly conventional. With the advent of exchange traded options, the number of trading strategies has escalated and use of these has no doubt become an integral component of risk management by agricultural processors. The extent that options are used by themselves or as a component of a broader strategy to supplement futures is unclear, but likely varies through time and across firms. Use of these instruments for risk management by first processors (e.g., crushers, flour millers) is clear. However, use of these instruments are less clear for further processors in which agricultural ingredients are only a component of the product (e.g., bread, pasta, and tortilla manufacturers, beer, etc.).

Most of the literature on use of futures and options is focused on risk management problems of farmers, traders, or importers. Some of the literature suggests that end-users have complicated risk management problems, notably with respect to the impact of input prices on output prices. Empirical models of hedging using futures are well-adopted in academic literature and in commercial practice. However, modeling of the role of options has been less obvious, despite that they have apparently become an integral component of risk management for agricultural processors.

The purpose of this study is to analyze the strategic impacts of input-output price relationships on end-users' demands for futures and/or options. An analytical model is developed based on mean-variance utility and extended to account for the impact of output prices and the inclusion of both futures and/or call (European) options in the portfolio. The former is important because it reflects the competitive environment in the product market and varies across sectors and through time. For this reason, we refer to this as a strategic motivation for hedging and stems from product pricing practices in an industry which reflects interfirm rivalry and conduct.

This study makes several contributions to the literature on risk management in agriculture. First, its focus is on end-users and captures their unique characteristics. Second, it explicitly captures the impact of correlation between input-output prices on hedging strategies. Finally, it incorporates options into a portfolio model. The analytic model is applied to the bread baking industry, an important agribusiness processor, which is interesting because of the relation

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between wheat prices, the primary ingredient, and bread prices. We show the optimal portfolio of futures and options and illustrate how this varies with several critical variables. It illustrates that options are only used in special cases and that while generally a bread manufacturer would be a long-hedger, the size of the hedge ratio would vary substantially.

2. Related Studies and Industry Practices

2.1) Mean Variance Models, Hedge Ratios, and Options. There has been extensive theoretical and empirical research on the use of futures and options as risk management tools. Most of this in the case of agriculture has focused on short hedgers. Generally, these use a form of the mean-variance model to derive portfolios consisting of futures positions to offset cash price risk and optimal hedge ratios are derived. Results show that the optimal futures position consists of speculative and hedging demands. The former depends on the market bias and traders' risk aversion coefficient. The latter depend on the size of the cash position and the covariance between cash and futures and the variance of the latter.

There is a long list of studies that have contributed to the above evolution. These date back to the portfolio choice literature starting with the seminal work by Markowitz (1952). Studies that applied the mean-variance framework to single-input futures hedging decisions include those by Johnson (1960), McKinnon (1967), Danthine (1978), Holthausen (1979), and Stein (1985). Contributions from these studies include the separation of hedging from speculative demand for holding futures positions, the role of futures prices in commodity price stabilization, the informational content of futures prices, and equilibrium futures price determination. Following these contributions, there have been numerous (too many to mention here) studies that have applied these models to varying agricultural problems and the approaches have been widely adopted in both academic and commercial research, as well as in formulating commercial risk management strategies. More recent studies have examined the application of futures hedging to processing and handling margins when there is correlation between the input and output prices. Wilson, Wagner, and Nganje (2002) applied the mean-variance model to a domestic bread baking firm. Their analytical model showed the similar result that the demand for holding futures is the sum of a speculative demand and a hedging demand. However, their hedging demand included the traditional hedge ratio plus an additional term they referred to as "strategic demand" which is determined by the covariance between the input and output prices. They used Monte Carlo simulation and genetic optimization to solve for the optimal hedge ratios for various combinations of time lags and correlations. Their results showed a very small variability in the traditional hedge ratio component while substantial variability occurred in the strategic demand component. Wilson and Wagner (2002) extended the mean-variance model to examining multiple output-input relationships using the example of a Mexican flour milling firm facing price risk in its flour, millfeed, wheat, and peso / U.S. dollar exchange rates. Their simulation results showed substantial variability in both the wheat and peso hedge ratios that depended upon the level of correlation.

Application of the mean-variance framework to hedging portfolios containing both futures and options dates to the seminal work by Wolf (1987) and Wolf and Francis (1991). Wolf derived a generic form of the variance-covariance matrix for a portfolio containing futures

and options. From this model, he was able to prove theorems illustrating that options are the preferred instrument for speculative demand while futures are the preferred instrument for hedging demand. Wolf and Francis conducted simulations of Wolf's portfolio model and found that the optimal portfolio would generally include complicated positions using combinations of futures and options.

Bullock and Hayes (1992) improved upon Wolf's model by endogenizing the variance-covariance matrix using a statistical theorem for deriving moments from a nonsmooth function of a random variable. They reaffirmed most of the theorems of Wolf and proved that the optimal option hedge ratio is not equal to the inverse of delta as has commonly been believed. Bullock and Hayes proved that the futures contract is the optimal instrument for speculating on information on the mean of the future spot price, and the option contract is the optimal instrument for speculating on information regarding the variance of the future spot price. Bullock and Hayes (1993) used this model to illustrate the value of having access to derivative securities for a risk-inverse investor.

The appropriateness of using mean-variance models with options was questioned by Lapan, Moschini, and Hanson (1991). Because options truncate the underlying price distribution, they create the possibility of introducing skewness into the overall portfolio wealth distribution. However, Hanson and Ladd (1991) showed that even when skewness is present, the mean-variance approach provides results that are consistent with mean-variance analysis. These results are later supported by simulation results of Bullock and Hayes (1993) and Garcia, Adam, and Hauser (1994).

2.2) *Processor Hedging Strategies.* Risk management strategies can give processors a competitive advantage over rivals, reduce exposure to risks, and in some cases increase the firm's profitability. Due to the importance of risk management issues for most agribusiness entities, much research effort has been dedicated to this area. This research, however, has focused on producers, traders, and handlers, often considering commodity prices as the single source of price risk. There are additional sources of uncertainty for processors which should be explicitly incorporated into models that prescribe risk management strategies. For purposes here, we envision consumer product processors including, as examples, bread, pasta, beer, tortillas, etc., for which a source of uncertainty is the product price at the time it is sold. Product price risk is that associated with the price that the firm will receive for its finished products.

There has been only scant reference to these problems in the academic literature. Johnson (1960) touched on several issues on the use of futures contracts in consumer goods industries. He emphasized that the firm's pricing strategy and lags inherent in the production process are two crucial factors that determine the optimal hedge horizon for the firm. He noted the strategic aspect of hedging and discussed how a firm's size and market share are important. Johnson states that the correlation between inputs and outputs is reduced since processors differentiate their products. Jackson (1980) elaborated on how intermediate industries can benefit from risk reducing measures that are not as practical for those in consumer goods industries. Including output price in contracts with customers as a response to price changes in

important inputs and other formulas that share price risk between producer and processor, cannot be employed as easily in end user situations.

Finally, Hull (2000, p. 83-84) discussed problems of hedging by processors and the role of competition. In industries where firms tend to re-price products infrequently, changes in input prices would have little effect on output prices and, therefore, low correlations would exist. In a competitive industry, where small cost advantages or disadvantages could strongly affect a firm's profits, hedging and procurement strategies are critical. In these situations, it is important to anticipate actions of competitors in making hedging decisions. If competitors hedge, it could be risky for a firm not to use a similar strategy. The opposite case is also true, where competitors do not use any form of hedging. This is because deviating from competitors' strategies will almost always cause either price advantages or disadvantages and make margins more risky. He concluded that if hedging is not the norm in an industry, it may in fact be highly risky to be different from competitors. In markets where ingredient and product prices are correlated, a firm that does not hedge can expect margins to be relatively constant. If a firm does hedge, it can expect its margins to be more volatile.

Another factor impacting hedging strategies is the transformation process. As the amount of value added to an input increases, the correlation between the input and output prices decreases. The less a finished product resembles the input, the lower the price correlation. The correlation of prices is affected by the relative distance of processed goods from the base commodity in a vertically integrated industry structure. In addition, the structure and conduct within an industry determines how input cost changes are translated into output price changes. Higher *value-added* in a product tends to reduce the correlation between the price of a raw commodity and that of processed goods (Blank, Carter, and Schmiesing, 1991).

There are three important components to the processor's risk management strategy. These include the use of options, the ingredient/product price correlation, and the transformation relationship, each of which has been suggested in the literature, but have not been explicitly included in a comprehensive model of hedging. The correlation between input and product prices and factor intensities have a significant effect on risk management decisions. If product prices are highly correlated and/or if the input comprises a small component of costs, hedging demand is reduced substantially. Effectively, ingredient price changes would be offset by changes in product prices, thereby reducing hedging demand, i.e., changes in product prices would serve as a hedge against changes in input prices. In contrast, in markets in which the correlation between product and input prices is nil (as a result of industry structure and/or price regulations in some countries), hedging becomes more critical for risk management. The reason for this is that change in product prices would not offset the risk of changes in ingredient costs. Depending on the industry structure, output prices may or may not be closely related to input prices, and this relationship can have an important effect on risk management strategies.

2.3) Industry Practices: Input and Output Price Relations. The relationship between input and output prices are central to the problem and vary across industries. In the familiar case of grain merchandising and trading, input and output prices typically move contemporaneously and the transformation coefficient from input to output is one. As a result, hedging strategies

typically result in hedge ratios equal to one. However, these may differ from those of a processor who adjusts output prices on a quarterly or semiannual basis due to varying industry practices and/or in response to anticipated rival pricing decisions.¹

3. Hedging Models Using Futures and Options

A model of hedging by processors is developed in this section that includes futures and options and accounts for the correlation between product and ingredient prices and an explicit transformation relationship. The model uses a mean variance utility function and explicitly includes call options. A fixed proportion production function is assumed (a coefficient of transformation) and for simplicity, transaction costs are assumed nil. The model is first developed using only futures positions. Then, call options are added to the portfolio in a later section. Speculative and hedging demands for futures are derived for the first case, and in the second case the speculative and hedging demands for both futures and options are derived. Comparative statics are derived and compared and conclusions discussed.

3.1) Hedging Model Without Calls. Consider a firm that produces an output Q , in time period $t+n$, using an input X , that is purchased in time period t . We will assume that the firm produces the output using a fixed proportion production function so that

$$Q_{t+n} = c \cdot X_t, \quad (1)$$

where c is equal to the marginal rate of transformation from input X_t into output Q_{t+n} . The output is to be sold in time period $t+n$ at the price p_{t+n} . The input is to be purchased in time period t at the price r_t .

We assume that the firm has access to a futures market for the input and can take a position Z , in time period $t-m$, at a futures price f_{t-m} , and to be offset when the input is purchased at a futures price f_t . If Z_{t-m} is greater than (less than) zero, the firm is taking on a long (short) position in the futures market. The input price r_t is related to the futures price f_t by the following relationship:

$$r_t = f_t + b_t, \quad (2)$$

where b_t is the local cash basis for the input market. Assume that the firm knows that it will be required to produce a known and fixed amount of output \bar{Q}_{t+n} ; therefore, the amount of input required is fixed and known and is equal to

$$\bar{X}_t = \frac{\bar{Q}_{t+n}}{c}. \quad (3)$$

¹Appendix C provides a summary of recent examples on how some industry price leaders responded or intended to respond to the increase in commodity prices in the period from 2000 to 2003.

The firm's decision variable is the size of the futures position Z_{t-m} and must be made prior to knowing the firm's output price \tilde{p}_{t+n} and input price $\tilde{r}_t = \tilde{f}_t + \tilde{b}_t$. These unknown prices have the following distribution:

$$\begin{pmatrix} \tilde{p}_{t+n} \\ \tilde{f}_t \\ \tilde{b}_t \end{pmatrix} \sim N \left[\begin{pmatrix} \mu_p \\ \mu_f \\ \mu_b \end{pmatrix}, \begin{pmatrix} \sigma_{pp} & \sigma_{pf} & 0 \\ \vdots & \sigma_{ff} & 0 \\ \vdots & \vdots & \sigma_{bb} \end{pmatrix} \right]. \quad (4)$$

The covariance between the prices and the basis is assumed equal to zero. This is a reasonable assumption since the basis is generally composed of cost components (transportation, storage, quality premiums/discounts) that are independent of the level of price. It is assumed that the input futures and output prices are correlated with the correlation coefficient equal to

$$\rho_{pf} = \frac{\sigma_{pf}}{\sigma_p \sigma_f} \quad (5)$$

where σ_p and σ_f are equal to the square roots of the variances σ_{pp} and σ_{ff} .

Taken together, the profit of the firm can be represented by the following equation:

$$\tilde{\pi}_{t+n} = \tilde{p}_{t+n} \cdot \bar{Q}_{t+n} - \tilde{r}_t \cdot \bar{X}_t + (\tilde{f}_t - f_{t-m})Z_{t-m}, \quad (6)$$

or by substituting equations (1) through (3) into (6):

$$\tilde{\pi}_{t+n} = (c \cdot \tilde{p}_{t+n} - \tilde{f}_t - \tilde{b}_t) \bar{X}_t + (\tilde{f}_t - f_{t-m})Z_{t-m}. \quad (6')$$

The firm's optimal futures position, Z_{t-m}^* , is found by maximizing the following mean-variance utility function:

$$EU(\tilde{\pi}_{t+n}) = E[\tilde{\pi}_{t+n}] - \frac{\lambda}{2} \cdot \text{Var}[\tilde{\pi}_{t+n}], \quad (7)$$

where λ is the firm's constant absolute risk aversion (CARA) coefficient. Note that equations (4) and (6') imply that

$$E[\tilde{\mu}_{t+n}] = (c \cdot \mu_p - \mu_f - \mu_b) \bar{X}_t + (\mu_p - f_{t-m})Z_{t-m}, \quad (8)$$

and

$$\text{Var}[\tilde{\pi}_{t+n}] = \sigma_{pp} (c \cdot \bar{X}_t)^2 + \sigma_{ff} (Z_{t-m} - \bar{X}_t)^2 + \sigma_{bb} \cdot \bar{X}_t^2 + 2 \cdot \sigma_{pf} \cdot c \cdot \bar{X}_t \cdot (Z_{t-m} - \bar{X}_t). \quad (9)$$

Substituting (8) and (9) into (7) and taking the partial derivative with respect to Z_{t-m} gives

$$\frac{\partial EU(\tilde{\pi}_{t+n})}{\partial Z_{t-m}} = \mu_f - f_{t-m} + \lambda(\sigma_{ff} - c \cdot \sigma_{pf})\bar{X}_t - \lambda \cdot \sigma_{ff} \cdot Z_{t-m} . \quad (10)$$

Setting (10) equal to zero and solving for Z_{t-m} gives the optimal futures position

$$Z_t^* = \frac{\mu_f - f_{t-m}}{\lambda \cdot \sigma_{ff}} + \frac{\sigma_{ff} - c \cdot \sigma_{pf}}{\sigma_{ff}} \cdot \bar{X}_t, \quad (11)$$

or

$$Z_t^* = \frac{\mu_f - f_{t-m}}{\lambda \cdot \sigma_{ff}} + (1 - c \cdot \beta_{pf}) \cdot \bar{X}_t, \quad (11')$$

where $\beta_{pf} = \frac{\sigma_{pf}}{\sigma_{ff}}$ is the regression coefficient when regressing the futures price (f_t) on the output price (p_{t+n}).

Equation (11') indicates that the optimal futures position is the sum of the speculative and hedging demands. The first term on the right of the equals sign is the speculative component which depends on market bias [the firm's forecast of the futures price in period t (μ_f) and the current futures price (f_{t-m})], the risk aversion coefficient (λ), and the variance the futures prices (σ_{ff}). If bias is zero, the speculative component is nil and the demand for futures equals the hedging demand.

The hedging component equals the hedge ratio (ϕ) multiplied by the fixed cash position (\bar{X}_t) for the input. The hedge ratio (ϕ) can be less than, equal to, or greater than zero and is dependent upon the sign of the following relationship:

$$\phi \begin{pmatrix} > \\ = \\ < \end{pmatrix} 0 \Leftrightarrow 1 \begin{pmatrix} > \\ = \\ < \end{pmatrix} c \cdot \frac{\sigma_{pf}}{\sigma_{ff}}, \quad (12)$$

or

$$\phi \begin{pmatrix} > \\ = \\ < \end{pmatrix} 0 \Leftrightarrow 1 \begin{pmatrix} > \\ = \\ < \end{pmatrix} c \cdot \frac{\rho_{pf} \cdot \sigma_p \cdot \sigma_f}{\sigma_{ff}}, \quad (12')$$

or

$$\phi \begin{pmatrix} > \\ = \\ < \end{pmatrix} 0 \Leftrightarrow \sigma_f \begin{pmatrix} > \\ = \\ < \end{pmatrix} c \cdot \rho_{pf} \cdot \sigma_p. \quad (12'')$$

Equation 12'' implies that the firm will hedge the input (i.e., $\phi > 0$, long futures) if the standard deviation of the futures price exceeds the transformed standard deviation of the output price, which is the conventional representation of processor hedging. However, it may be optimal to take short futures positions (i.e., $\phi < 0$, short futures) if the standard deviation of the futures price is less than the transformed standard deviation of the output price. The transformed standard deviation of the output price will be higher for higher levels of the marginal rate of transformation c and for higher levels of the positive correlation between the output price and input futures price ρ_{pf} . Though not envisioned as a common practice, this could be interpreted as effectively hedging the output price.

These results illustrate the role and impact of input/product correlation. Though typically processors are thought to be long hedgers (short of ingredients and long futures), the size of their futures positions depend on a number of factors including the input/product price correlation ρ_{pf} and the marginal rate of product transformation c . The results (Equation 12) indicate that if ρ_{pf} is equal to zero, then the optimal hedge ratio is equal to one. Equation 12'' indicates that the optimal hedge ratio is zero if ρ_{pf} is equal to one, and when σ_f is equal to $c \cdot \sigma_p$; or equivalently, when the standard deviation of the futures price is equal to the transformed standard deviation of the output price. If the futures' standard deviation is greater (less) than the transformed output standard deviation, then the hedge ratio will be positive (negative). Equation 12' shows that when ρ_{pf} is less than zero, the optimal hedge ratio will always be positive, and in fact, will always be greater than one to compensate for the added risk from the output price.

3.2) Hedging Model With Calls. Consider the impact of adding European call options to the firm's portfolio. Suppose the firm can buy or sell a call option that has a strike price k and premium w . The option position will be placed concurrently with the futures position in time period $t-m$. The firm's profit can now be represented as:

$$\tilde{\pi}_{t+n} = \tilde{p}_{t+n} Q_{t+n} - \tilde{r}_t X_t + (\tilde{f}_t - f_{t-m}) Z_{t-m} + (\max(0, \tilde{f}_t - k) - w_{t-m}) R_{t-m} \quad (13)$$

where R_{t-m} is the firm's call option position (>0 for purchased, <0 for sold) in time period $t-m$. By substituting relations (1) through (3) into (13) gives:

$$\tilde{\pi}_{t+n} = \tilde{p}_{t+n} (c \cdot \bar{X}_t) - (\tilde{f}_t + \tilde{b}_t) \bar{X}_t + (\tilde{f}_t - f_{t-m}) Z_{t-m} + (\max(0, \tilde{f}_t - k) - w_{t-m}) R_{t-m}. \quad (13')$$

The distributional assumptions of equation (4) still apply. Equation (13') represents a profit function that is continuous, but nonsmooth, at the point where $f_t = k$. For cases where $f_t \leq k$, equation (13') can be written as:

$$\tilde{\pi}_{t+n} = \tilde{p}_{t+n} (c \cdot \bar{X}_t) - (\tilde{f}_t + \tilde{b}_t) \bar{X}_t + (\tilde{f}_t - f_{t-m}) Z_{t-m} - w_{t-m} R_{t-m}, \quad (13'')$$

and for cases where $f_t > k$, equation (13') can be written as:

$$\tilde{\pi}_{t+n} = \tilde{p}_{t+n} (c \cdot \bar{X}_t) - (\tilde{f}_t + \tilde{b}_t) \bar{X}_t + (\tilde{f}_t - f_{t-m}) Z_{t-m} + (\tilde{f}_t - k - w_{t-m}) R_{t-m}. \quad (13''')$$

Following Bullock and Hayes (1992), the mean of (13') can be written as:

$$E[\tilde{\pi}_{t+n}] = \alpha \cdot E[\tilde{\pi}_{t+n} | f_t \leq k] + (1 - \alpha) \cdot E[\tilde{\pi}_{t+n} | f_t > k], \quad (14)$$

where α is the probability that $f_t \leq k$ or $F(k)$. The variance of (13') can be written as:

$$\begin{aligned} \text{Var}[\tilde{\pi}_{t+n}] &= \alpha \cdot \text{Var}[\tilde{\pi}_{t+n} | f_t \leq k] + (1 - \alpha) \cdot \text{Var}[\tilde{\pi}_{t+n} | f_t > k] + \\ &\quad \alpha \cdot (1 - \alpha) \cdot \{E[\tilde{\pi}_{t+n} | f_t \leq k] - E[\tilde{\pi}_{t+n} | f_t > k]\}^2. \end{aligned} \quad (15)$$

For notational purposes, we use the '+' superscript to indicate a moment conditional upon $f_t > k$ and will use the '-' superscript to indicate a moment conditional upon $f_t \leq k$. Therefore, the conditional means of equation (13') can be written using (13'') and (13''') as:

$$E[\tilde{\pi}_{t+n} | f_t \leq k] = \mu_p^- \cdot c \cdot \bar{X}_t - (\mu_f^- + \mu_b) \bar{X}_t + (\mu_f^- - f_{t-m}) Z_{t-m} - w_{t-m} R_{t-m} \quad (16)$$

and

$$\begin{aligned} E[\tilde{\pi}_{t+n} | f_t > k] &= \mu_p^+ \cdot c \cdot \bar{X}_t - (\mu_f^+ + \mu_b) \bar{X}_t + (\mu_f^+ - f_{t-m}) Z_{t-m} - \\ &\quad (\mu_f^+ - k - w_{t-m}) R_{t-m}. \end{aligned} \quad (17)$$

Substituting (16) and (17) into (14), and noting that

$$\begin{aligned} \alpha \cdot \mu_p^- + (1 - \alpha) \cdot \mu_p^+ &\equiv \mu_p \text{ and} \\ \alpha \cdot \mu_f^- + (1 - \alpha) \cdot \mu_f^+ &\equiv \mu_f \end{aligned}$$

gives:

$$\begin{aligned} E[\tilde{\pi}_{t+n}] &= \mu_p \cdot c \cdot \bar{X}_t - (\mu_f + \mu_b) \bar{X}_t + (\mu_f - f_{t-m}) Z_{t-m} + \\ &\quad [(1 - \alpha) \cdot (\mu_f^+ - k) - w_{t-m}] R_{t-m}. \end{aligned} \quad (18)$$

For the variance in equation (15), we can derive the following:

$$\begin{aligned} \text{Var}[\tilde{\pi}_{t+n} | f_t \leq k] &= (c \cdot \bar{X}_t)^2 \cdot \sigma_{pp}^- + (Z_{t-m} - \bar{X}_t)^2 \cdot \sigma_{ff}^- + \bar{X}_t^2 \cdot \sigma_{bb} + \\ &\quad 2 \cdot c \cdot \bar{X}_t \cdot (Z_{t-m} - \bar{X}_t) \cdot \sigma_{pf}^-, \end{aligned} \quad (19)$$

$$\begin{aligned} \text{Var}[\tilde{\pi}_{t+n} | f_t > k] &= (c \cdot \bar{X}_t)^2 \cdot \sigma_{pp}^+ + (Z_{t-m} + R_{t-m} - \bar{X}_t)^2 \cdot \sigma_{ff}^+ + \bar{X}_t^2 \cdot \sigma_{bb} + \\ &\quad 2 \cdot c \cdot \bar{X}_t \cdot (Z_{t-m} + R_{t-m} - \bar{X}_t) \cdot \sigma_{pf}^+, \end{aligned} \quad (20)$$

and

$$E[\tilde{\pi}_{t+n} | f_t > k] - E[\tilde{\pi}_{t+n} | f_t \leq k] = c \cdot \bar{X}_t \cdot h_p + (Z_{t-m} - \bar{X}_t) \cdot h_f + R_{t-m} \cdot h_R, \quad (21)$$

where:

$$\begin{aligned}
h_p &= \mu_p^+ - \mu_p^- \geq 0, \\
h_f &= \mu_f^+ - \mu_f^- \geq 0, \\
h_R &= \mu_f^+ - k \geq 0.
\end{aligned}$$

Substituting (19), (20), and (21) into (15), gives the following representation for the variance of profits:

$$\begin{aligned}
\text{Var}[\tilde{\pi}_{t+n}] &= \sigma_{RR} \cdot R_{t-m} + \sigma_{ff} \cdot Z_{t-m} + 2 \cdot (\sigma_{ff} + c^2 \cdot \sigma_{pp} - 2 \cdot c \cdot \sigma_{pf} + \sigma_{bb}) \cdot \bar{X}_t^2 + \\
&2 \cdot [\sigma_{fR} \cdot Z_{t-m} + (c \cdot \sigma_{pR} - \sigma_{fR}) \cdot \bar{X}_t] \cdot R_{t-m} + 2 \cdot (c \cdot \sigma_{pf} - \sigma_{ff}) \cdot \bar{X}_t \cdot Z_{t-m},
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
\sigma_{ff} &\equiv \alpha \cdot \sigma_{ff}^- + (1 - \alpha) \cdot \sigma_{ff}^+ + \alpha \cdot (1 - \alpha) \cdot h_f^2, \\
\sigma_{pf} &\equiv \alpha \cdot \sigma_{pf}^- + (1 - \alpha) \cdot \sigma_{pf}^+ + \alpha \cdot (1 - \alpha) \cdot h_p \cdot h_f, \\
\sigma_{RR} &\equiv (1 - \alpha) \cdot \sigma_{ff}^+ + \alpha \cdot (1 - \alpha) \cdot h_R^2, \\
\sigma_{fR} &\equiv (1 - \alpha) \cdot \sigma_{ff}^+ + \alpha \cdot (1 - \alpha) \cdot h_f \cdot h_R, \\
\sigma_{pR} &\equiv (1 - \alpha) \cdot \sigma_{pf}^+ + \alpha \cdot (1 - \alpha) \cdot h_p \cdot h_R.
\end{aligned}$$

These variance – covariance terms are similar to those derived in Bullock and Hayes (1992). Substituting (17) and (22) into the expected utility function (7) and maximizing gives the following first-order conditions:

$$\begin{aligned}
\frac{\partial \text{EU}(\tilde{\pi}_{t+n})}{\partial Z_{t-m}} &= -\lambda \cdot \sigma_{ff} \cdot Z_{t-m} - \lambda \cdot (c \cdot \sigma_{pf} - \sigma_{ff}) \cdot \bar{X}_t + \\
\mu_f - f_{t-m} - \lambda \cdot \sigma_{fR} \cdot R_{t-m} &= 0
\end{aligned} \tag{23}$$

$$\begin{aligned}
\frac{\partial \text{EU}(\tilde{\pi}_{t+n})}{\partial R_{t-m}} &= -\lambda \cdot \sigma_{fR} \cdot Z_{t-m} - \lambda \cdot (c \cdot \sigma_{pR} - \sigma_{fR}) \cdot \bar{X}_t + \\
(1 - \alpha) \cdot h_R - w_{t-m} - \lambda \cdot \sigma_{RR} \cdot R_{t-m} &= 0.
\end{aligned} \tag{24}$$

Note that $(1 - \alpha) \cdot h_R$ is equal to the firm's fair market valuation of the call option. Solving (23) and (24) for the optimal futures and call option positions gives:

$$\begin{aligned}
Z_{t-m}^* &= \frac{\sigma_{RR} \cdot (\mu_f - f_{t-m}) - \sigma_{fR} \cdot [(1 - \alpha) \cdot h_R - w_{t-m}]}{\lambda \cdot \Gamma} + \\
&\left[1 - c \cdot \frac{\sigma_{pf} \cdot \sigma_{RR} - \sigma_{fR} \cdot \sigma_{pR}}{\Gamma} \right] \cdot \bar{X}_t,
\end{aligned} \tag{25}$$

$$R_{t-m}^* = \frac{\sigma_{ff} \cdot [(1-\alpha) \cdot h_R - w_{t-m}] - \sigma_{fR} \cdot (\mu_f - f_{t-m})}{\lambda \cdot \Gamma} + c \cdot \frac{\sigma_{fR} \cdot \sigma_{pf} - \sigma_{pR} \cdot \sigma_{ff}}{\Gamma} \cdot \bar{X}_t, \quad (26)$$

where

$$\Gamma = \sigma_{RR} \cdot \sigma_{ff} - \sigma_{fR}^2 \geq 0.$$

The first term in each demand equation represents the speculative component of demand, and the second term represents the hedging demand. The hedging demand is equivalent to the hedging

component in the model without options [equation (11')] since $\frac{\sigma_{pf} \cdot \sigma_{RR} - \sigma_{fR} \cdot \sigma_{pR}}{\Gamma} \equiv \frac{\sigma_{pf}}{\sigma_{ff}}$

(proof is in Appendix A). Also, the hedging demand for the call option is always equal to zero since $\sigma_{fR} \cdot \sigma_{pf} \equiv \sigma_{pR} \cdot \sigma_{ff}$ (proof is in Appendix B).

These results have a number of important conclusions. First, futures are more efficient at reducing variance; therefore, the hedging demand for options is always nil. The reason for this is that hedging with options will always be less effective because delta is less than one. Second, the hedging demand for futures is unaffected by the inclusion of options. Third, a non-nil speculative demand for options may be optimal, but only if there is a bias in the futures or options and would be interpreted as offsetting speculative demand for futures. Thus, generally, options are not useful for hedging purposes and would be used only for speculative purposes.

There are several specific interpretations of the use of futures and options and the role of bias. If there is a bias² in either market, futures and/or options become a component of the portfolio and would comprise the speculative component of demand. There are two forms of bias, one in each market. Options will only be used if one or both of these biases exist and affect the speculative demand for the instrument. Thus, options are used due to bias in futures or option pricing. If futures are biased, the speculative demand for futures changes and is offset in part by an opposite position in calls. If options are biased, call positions are non-nil and are offset by opposite positions in futures. In all cases, the hedging demand for options is nil.

4. Empirical Model and Results: Bread Baking

The hedging models with and without call options derived above were applied to the bread baking sector in which hedging of wheat purchases is a routine part of doing business. However, this varies substantially across countries and market segments, largely reflecting either

² Bias is important in market analysis. As an example of an interpretation, REFCO (2/14/00) noted “erosion to the 200 area is likely by late July ... the wheat market is overvalued...advise consumers to extend coverage out 6-8 months if wheat cop is threatened...”

price regulations in the former and/or industry structure and conduct [competitive rivalry] in the latter.

4.1) Industry. The bread baking industry is a major user of wheat and in the United States is highly competitive. Hedging is a routine part of risk management by most firms and makes extensive use of futures and options, as well as varying forms of flour contracts (fixed price, basis, margin and/or formula contracts tied to wheat futures, among other indicators). However, the price relationships are imperfect in part due to industry structure and that there are other ingredients that are related to the ingredient, product, and futures markets.

4.2) Data Sources, Manipulations, and Model Parameters. The model was defined for a prototypical bread baker producing 2 million pounds of white pan bread per month. The transformation rate of wheat to flour utilized was one bushel of wheat (60 lbs.) was assumed to produce 70 lbs. of white pan bread, (i.e., in our model, $c=70$). The risk aversion coefficient was $\lambda=.001$ and bias in the futures and options markets in our base case were assumed nil.

Price data were assembled on bread and wheat prices over the period January 2001 to September 2002. Time series on bread prices are only available monthly so the scope of the remaining data and analysis was monthly. Bread prices were defined as the U.S. city average price for white pan bread. Prices were obtained from the U.S. Bureau of Labor Statistics. Wheat prices were monthly average prices for MGEX (Minneapolis Grain Exchange) spring wheat futures and monthly average cash prices at Minneapolis for 14% Hard Red Spring (HRS) wheat. Daily wheat prices were obtained from MGEX for nearby futures and cash prices from which monthly averages were calculated. Simple distributions of these series are shown in Table 4.2.1. The correlation between these is relatively low at .249. While bread prices are variable, the relative risk of price changes in bread is not as great as in wheat.

Table 4.2.1. Price Distributions for the Bread Baking Simulations

	Bread	MGEX Futures	Mpls. Cash 14% Protein	Basis
	cents/lb	-----	\$/bu-----	
			-	
Mean	1.004	3.250	3.732	0.482
Std Dev	.011	.445	.401	.094
Correlation with Wheat Futures	.249	1.000	.981	
Base Case Values (Sept 2002)	1.016	4.876	5.245	.369

4.3) Results. The base case was developed assuming prices and distributions for futures and options are not biased (current futures price levels are expected to be unchanged and options are fairly valued). The variability is represented by the standard deviation and derived for the period January 2001 to September 2002 and used in the base case. Options were equal to 450c/b.

The base case and sensitivities are shown in Table 4.3.1. The hedging model without calls and with nil bias in options and futures results in a risk adjusted portfolio value (i.e., the EV) of \$1,633,760. Hedging demand is 16,526 bushels and speculative demand is nil due to assumption of nil bias. Since the baker requires 28,571 bushels of wheat to meet production demands, this implies an optimal hedge ratio of .58. When options are included (also with no bias), results are unchanged. This confirms the theoretical results in which if bias is nil in both futures and options, the speculative demand for futures would be nil, and both the speculative and hedging demand for options would be nil.

Some base case assumptions were relaxed to illustrate features of the model.

Strategic Implications of Hedging: The strategic implication of hedging demand can be illustrated with these results. Conventionally, ignoring the input/output correlation, a bread manufacturer would derive the flour component of their requirements, and the wheat equivalent of that component, and use that to derive the number of futures to buy to offset flour and wheat price risk. In this case, this would imply a futures demand of +28,571. This contrasts with our results with a futures demand of +16,526. The reason for this difference is that the effect of the input/output correlation is ignored in the former and, as shown here, results in the hedger buying a larger amount of futures (i.e., overhedging) than would be optimal. Fundamentally, this occurs because increases in wheat prices are partially offset by concurrent increases in bread prices, which implies that a smaller futures position would be necessary.

Correlations and Hedging Demand: If the correlation were nil, the hedging demand would be greater at 28,571 bushels (Table 4.3.1). This could be the case if the industry is highly concentrated and prices are sticky, if there are price regulations as in many importing countries, and/or would represent a case in which the baker is selling bread in forward contracts at fixed prices. Thus, in industries where either due to regulation or industry structure, the correlation between futures and output prices are nil, hedging demand increases and the EV decreases due to hedgers having greater risk. If the correlation between input and output prices were .5, the demand for futures would decrease, in this case to 4,384 bushels, implying a hedge ratio of .15. In all these cases, the hedging and speculative demand for options would be nil.

In an extreme case in which the correlation was one, the results and interpretations change radically. Strictly, this is the interpretation of the mean-variance portfolio in equation 12'. This suggests that the hedger would choose to use futures as a hedge of the output. In other words, looking forward the hedger would sell wheat futures, and remain short cash ingredients, as a hedge against decreases in bread prices. Though this is counterintuitive, it illustrates a special case in which a processor sells futures implicitly as a hedge against declines in the product price.

Table 4.3.1. Bread Baking Base Case and Sensitivity Results: Demand for Futures and Options

Scenario	<i>Futures Only</i>			<i>Futures and Call Options</i>				
	<i>EV</i>	<i>F-Spec.</i>	<i>F-Hedge</i>	<i>EV</i>	<i>F-Spec.</i>	<i>F-Hedge</i>	<i>O-Spec.</i>	<i>O-Hedge</i>
Base case (no bias)	1,633,760	0	16,526	1,633,760	0	16,526	0	0
Correl.=0	1,619,397	0	28,571	1,619,397	0	28,571	0	0
Correl.=.5	1,677,311	0	4,384	1,677,311	0	4,384	0	0
Correl.=1	1,851,054	0	-19,803	1,851,054	0	-19,803	0	0
Futures Bias: $F < F^*$ ($F = .95 \cdot F^*$)	1,637,801	48	16,526	1,665,091	16,003	16,526	-18,450	0
$F > F^*$ ($F = 1.05 \cdot F^*$)	1,629,743	-48	16,526	1,657,034	-16,003	16,526	18,450	0
Options Bias: $O < O^*$ ($O = .95 \cdot O^*$)				1,634,038	-1,608	16,256	2,008	0
$O > O^*$ ($O = 1.05 \cdot O^*$)				1,634,038	1,608	16,526	-2,008	0
Base Case Risk Aversion $\lambda = .005$	750,021	0	16,256	750,021	0	16,526	0	0
Futures Bias: $F < F^*$ ($F = .95 \cdot F^*$) $\lambda = .005$	754,232	10	16,526	759,690	3,201	16,526	-3,690	0

Legend: F and F* are current and expected futures prices; O and O* are current and expected option value in the future; F-F* and O-O* are measures of bias in futures and options, respectively. Positions are enumerated as: F-Spec and F-Hedge refer to speculative demand and hedging demand for futures, O-Spec and O-Hedge refer to the speculative and hedging demand for options.

These relationships are illustrated in Figure 4.1. The optimal hedge ratio declines (increases) as the correlation increases (declines). At some correlation (in this case about .6) the hedge ratio becomes negative. The lines (for the option overlays and for no options) show that EV (risk adjusted portfolio value) is unaffected by availability of options when examining effects of correlations between input futures and output prices.

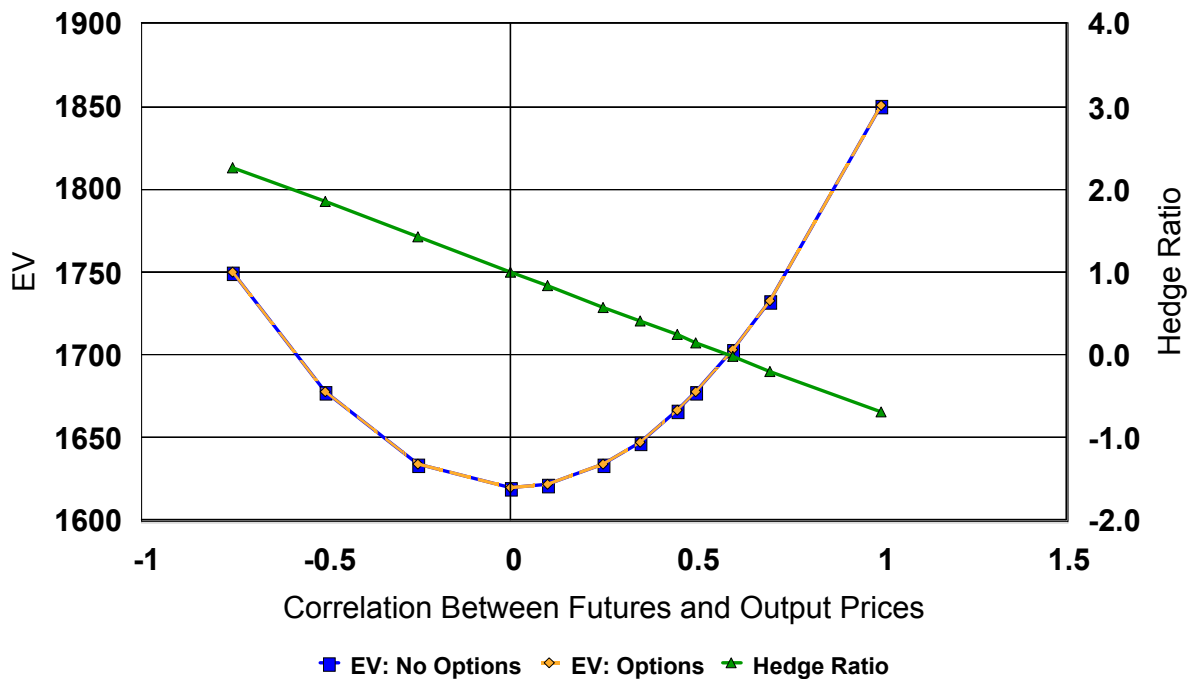


Figure 4.1. Effect of Correlation Between Input and Output Prices on EV (With and Without Options), and Optimal Futures Hedge Ratios

Bias: If futures and/or options are biased, the results change drastically. In each case, hedgers adjust their portfolios to include speculative demands for futures and options. In all cases, however, the hedging demand for options remains nil. If current futures were biased downwards 5% (e.g., $F < F^*$, or equivalently, if there was an upward bias in expectations), hedgers would respond. In the futures only model, this would result in supplementing the hedging demand with a speculative demand for futures of +48 bushels and the EV would increase relative to the base case. In the model with calls, the hedger would respond with a speculative futures position of +16,003 bushels and offset by a speculative options position of -18,450 bushels. The hedging demand for futures and options would be unchanged. Taken together, these positions imply a large long position in futures (the summation of hedging and speculative demand) which are offset in part by selling calls. Due to the call delta (in this case, $\Delta < .5$), the offsetting call position only partially offsets the increase in long futures.

The impact of futures bias on hedging and speculative demands and optimal hedge ratios and are illustrated in Figures 4.2 and 4.3, respectively. Results indicate that hedging demand for futures is unchanged by bias in the current futures. As current futures become more biased (either positively or negatively relative to expected futures), a larger speculative demand component is indicated with negative speculative positions when current futures are positively biased and with positive speculative positions when current futures are negatively biased. Futures bias impacts both the speculative demand for futures and options, but not the hedging demands. As the current futures bias increases, speculative demand for futures changes from long to short and would be offset by speculative option positions which change from short to long (Figure 4.3).

This portfolio would commonly be referred to as a “covered call write” (i.e., long futures and short calls) strategy in options trading. The change in payoffs of these positions is illustrated in Figure 4.4. These show that as the futures price at expiration declines, payoffs for the cash position increase; however, payoffs for the futures/options positions and net positions are kinked at the strike price \$4.50/bu, with the net position increasing as the futures price declined to \$4.50/bu and declining after. Figure 4.5 portrays the net wheat purchase cost for the short cash position that results from both a hedged and unhedged strategy.

Just the opposite occurs if current futures were biased upwards (e.g., $F > F^*$). In this case, hedgers would supplement their hedging demand with a short futures position, offset by a long call position. This is referred to as an “options protected short futures” (i.e., short futures and long calls) strategy.

Options bias is reflected here in terms of current options value (O) relative to the option's fair value O^* . This could be interpreted if a hedger's estimate of volatility is different from the volatility implied in current premiums. If current options are biased downward by 5%, the hedger would adjust the portfolio with +2008 calls, which would simultaneously be offset by a short speculative futures position of -1608. The hedging demand for futures would be unchanged and the EV would increase.

Risk Aversion: In the base case, the risk aversion coefficient was $\lambda = .001$. Increasing this to $\lambda = .005$, an increase in risk aversion, impacts the results. With nil bias, the futures and options positions are unchanged in both cases, but the EV declines. If futures are biased (current futures are biased downwards 5%), the values in the optimal portfolio change. In particular, in the futures only case, the speculative demand increases from 0 to +10 bushels and the EV increases. In the model with calls, the speculative demand for futures increases to +3,201 bushels and the speculative demand for options is -3690 calls. These changes in positions are much less than the base case risk aversion (less risk aversion than in this case), illustrating that more risk averse hedgers would be less inclined to pursue increased profits associated with comparable bias than less risk averse hedgers.

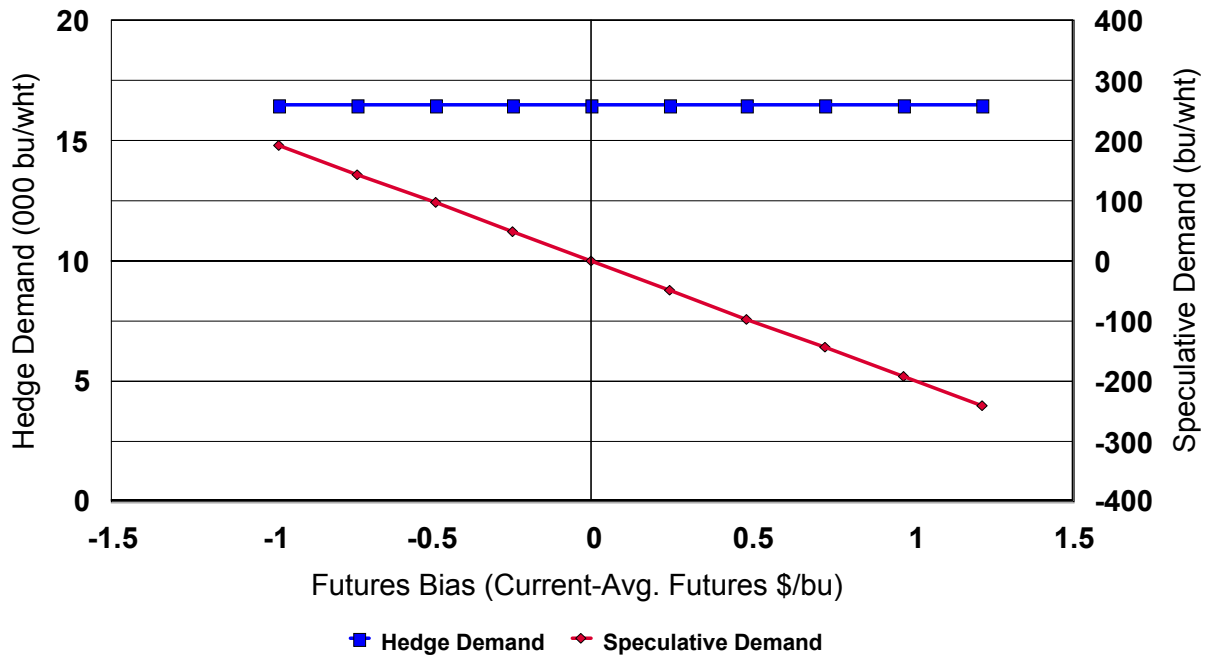


Figure 4.2. Effect of Futures Bias on Hedging and Speculative Demand (No Options)

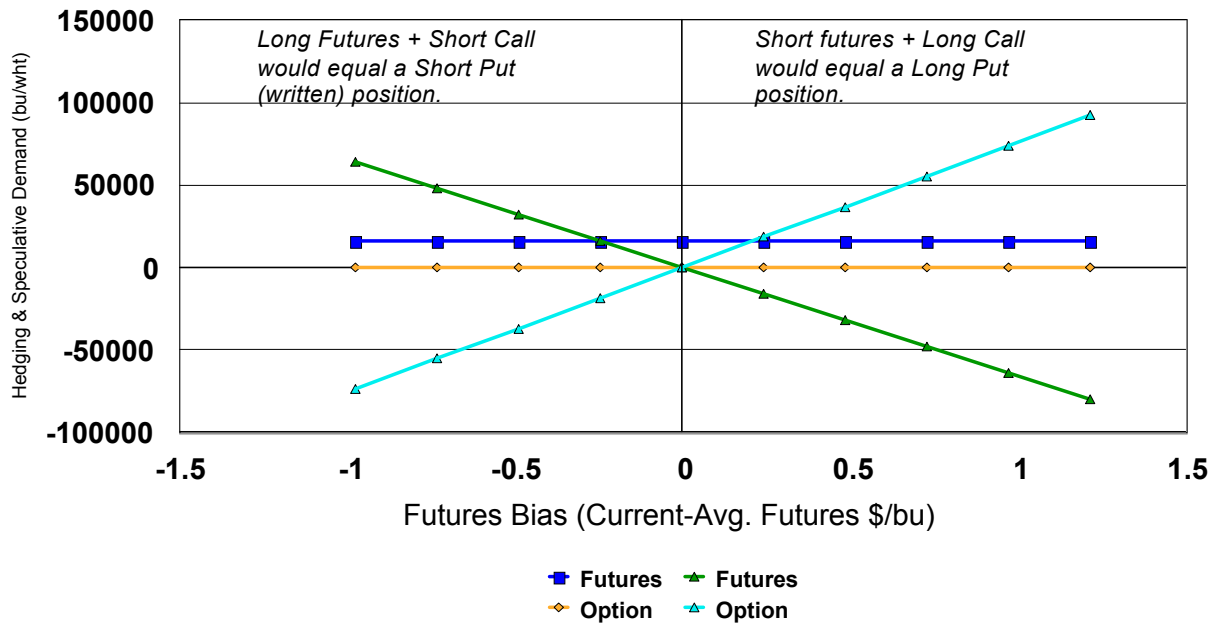


Figure 4.3. Effect of Futures Bias on Hedging and Speculative Demand (With Options)

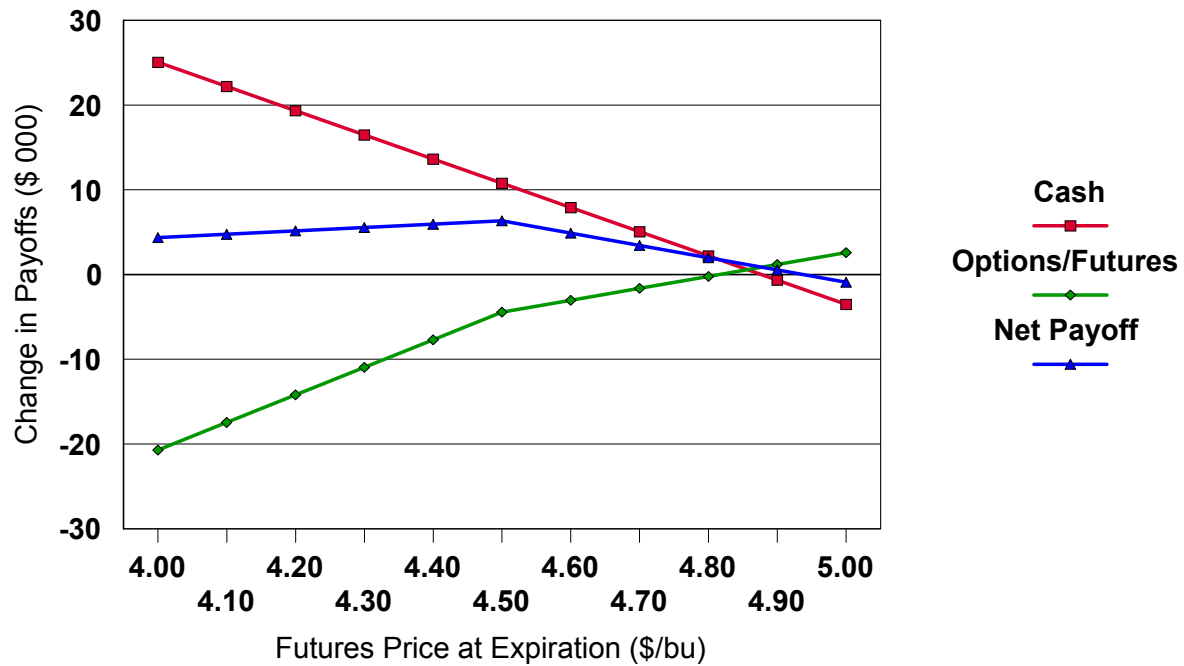


Figure 4.4. Change in Payoffs to Short Cash Position (Current Downward Futures Bias)

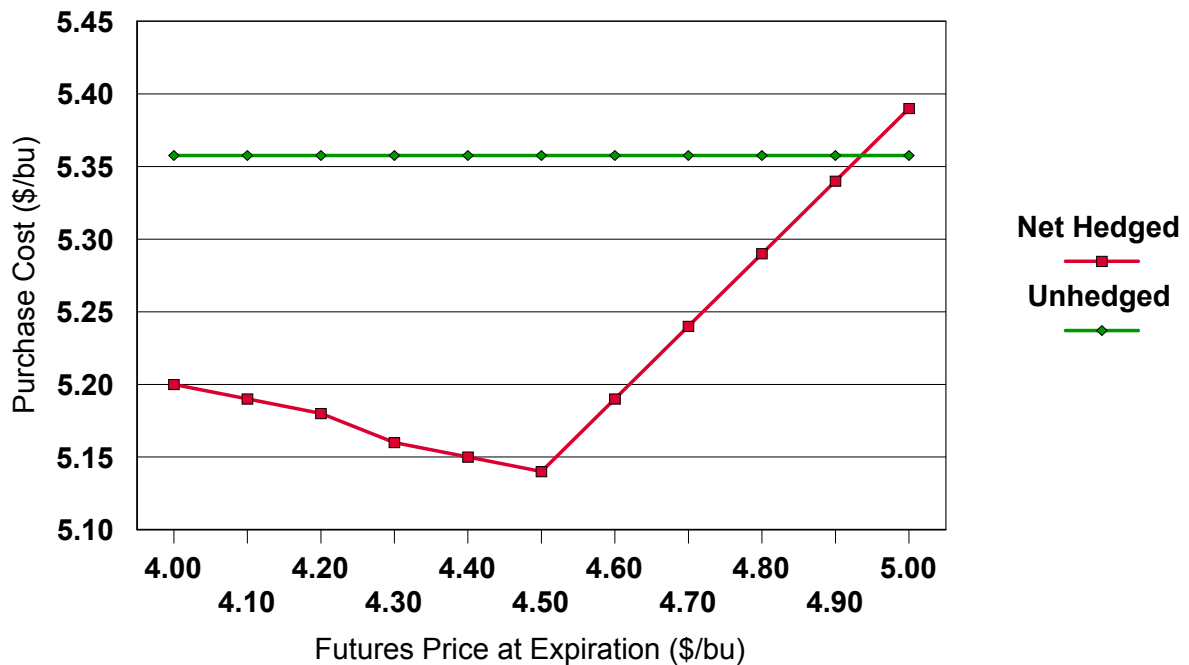


Figure 4.5. Unhedged and Net Hedged Purchase Cost for Short Cash Position (Current Downward Futures Bias)

Transformation Coefficient: The transformation coefficient has an impact on the components of hedging demand. In the base case the value was 70. Variations in this value are shown in Table 4.3.2 for the no bias case. As expected, as the transformation rate increases, the size of the hedging demand decreases (Figure 4.6).

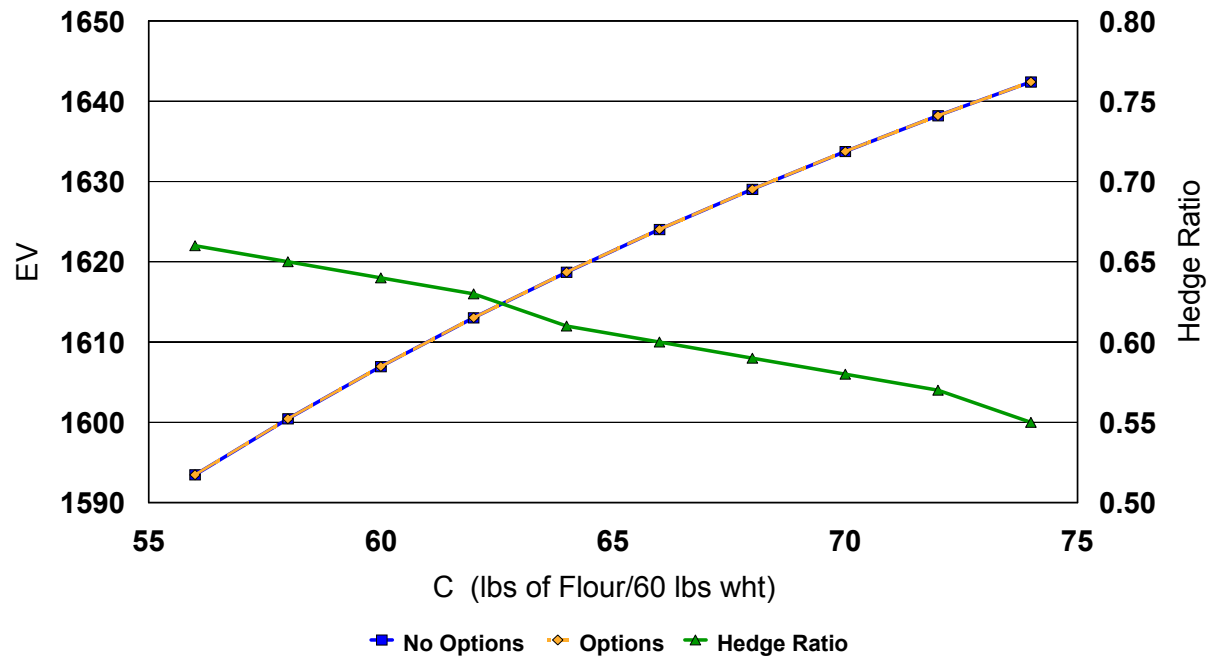


Figure 4.6. Effect of Conversion Rate (Output/Unit Input) on EV and Optimal Hedge Ratio (With and Without Options, No Bias)

Lagged Correlations: The correlation in the base case was the contemporaneous relation between wheat and bread prices. The timing of changes in wheat and bread prices is, however, unclear. One would expect that bread prices are impacted by wheat prices not only in the current period, but also in prior periods, reflecting the lag in product pricing decisions relative to changes in input prices. To illustrate these impacts, the correlations were derived for lagged time periods up to six months over the period from 1996 to 1998.

These are summarized in Table 4.3.3. It is important that considering these lags, both the standard deviations and the correlations change. The standard deviations in bread prices are greater at .02 (versus 0.011 in the base case), and the correlations between bread and futures increases with longer durations up to a 3-period lag. Taken together, this means the risk of price changes (notably bread price changes) is greater and the correlation which determines hedging effectiveness is larger. Both of these impact the optimal hedging strategies.

Table 4.3.2. Bread Baking Sensitivity Results - Impacts of the Transformation Coefficient, C

Scenario	<i>Futures Only</i>			<i>Futures and Call Options</i>				
	<i>EV</i>	<i>F-Spec.</i>	<i>F-Hedge</i>	<i>EV</i>	<i>F-Spec.</i>	<i>F-Hedge</i>	<i>O-Spec.</i>	<i>O-Hedge</i>
C=58	1,600,450	0	22,437	1,600,450	0	22,437	0	0
C=60	1,606,951	0	21,288	1,606,951	0	21,288	0	0
C=66	1,624,035	0	18,258	1,624,035	0	18,258	0	0
C=70 Base case (no bias)	1,633,760	0	16,526	1,633,760	0	16,526	0	0
C=74	1,642,412	0	14,982	1,642,412	0	14,982	0	0

Legend: F and F^* are current and expected futures prices; O and O^* are current and expected option value in the future; $F-F^*$ and $O-O^*$ are measures of bias in futures and options, respectively. Positions are enumerated as: F -Spec and F -Hedge refer to speculative demand and hedging demand for futures, O -Spec and O -Hedge refer to the speculative and hedging demand for options.

Table 4.3.3. Bread Baking Sensitivity Results - Lags: Demand for Futures and Options and Conversion Rate

	<i><u>Distribution Parameters (1996-1998)</u></i>		
	<i>Std. (futures)</i>	<i>Correlation</i>	<i>F-Hedge</i>
Lagged Correlation: Lag=0	0.72	0.15	22,348
1 Period Lag	0.73	0.31	15,638
2 Period Lag	0.74	0.47	9,082
3 Period Lag	0.74	0.49	8,217
4 Period Lag	0.73	0.44	10,266
5 Period Lag	0.71	0.34	13,805
6 Period Lag	0.69	0.35	13,222

The impacts of the lagged distributions on hedging strategies are evaluated assuming no bias. These results indicate the hedging demand varies depending on the hedge horizon. The hedging strategy implied in these results suggest that the optimal coverage for long futures positions changes from +22,348 for lag of zero, to +8,217 for a 3-period lag, and to +13,222 for a 6-period lag. These could be interpreted as holding a position of +13,222 six months forward, +13,805 five months forward, + 10,266 four months forward, etc., reducing that to +8,217 by three months, and during the month in which the product is produced, priced, and distributed, the position would be increased to +22,348. These results are likely reflective of the cumulative pricing decisions and inventory strategies in this industry. Thus, hedging strategies would be dynamic and adjusted by the correlation and the relationship between the standard deviations for the transformed output and lagged futures prices.

5. Summary and Implication

Processors make extensive use of futures and options which comprise an integral part of their price risk management strategies. While use of futures would be considered conventional, use of options has escalated and they are used in numerous combinations comprising their procurement portfolio. End-users use agricultural ingredients whose prices are volatile and, therefore, are an important source of risk, but also confront risks of product price changes which complicates the formulation of portfolios. In some industries, input and output prices are highly correlated, in others the correlations are less and in some the correlations may be nil, or vary depending on the time duration or a hedge, commonly referred as the hedge horizon. These relationships are important because the effect of changes in input prices on profits may be offset if there is a positive correlation with output prices.

The purpose of this study is to analyze the strategic impacts of input-output price relationships on end-users' demands for futures and/or options. An analytical model is developed based on mean-variance utility and extended to account for the impact of output prices and the inclusion of both futures and/or call options in the portfolio. This study makes several contributions to the literature on risk management in agriculture. First, its focus is on end-users and captures some unique characteristics of that market. Second, it explicitly captures the correlation between input-output prices on hedging strategies. Finally, it incorporates options into a portfolio model. The analytic model was applied to the bread baking industry, an important agribusiness processor, and which is interesting because of the relation between wheat prices, the primary ingredient, and bread prices. We show the optimal portfolio of futures and options and illustrate how this varies with several critical variables.

The results illustrate the role and impact of input/product correlation. Though typically processors are thought to be long hedgers (short cash and long futures), the size of their futures positions are dependent upon the input/product price correlation and the marginal rate of product transformation. The optimal hedge ratio is zero when the standard deviation of the futures price is equal to the transformed standard deviation of the output price. If the futures standard deviation is greater (less) than the transformed output standard deviation, then the hedge ratio will be positive (negative).

The theoretical model has a number of important conclusions. One is that futures are more efficient at reducing variance; therefore, the hedging demand for options is always nil. The

reason for this is that hedging with options will always be less effective because delta is less than one. Second, the hedging demand for futures is unaffected by the inclusion of options primarily for the aforementioned reason that options are not used for hedging demand. Third, a non-nil speculative demand for options may be optimal, but only if there is a bias in the futures or options.

These results suggest that options should be used by end-users only in special cases. One, as illustrated in these results, is where there is a bias in expectations regarding futures and/or option prices and their corresponding market value thus creating a speculative demand. Second, hedgers may not behave using a mean variance preference structure. Instead, hedgers may exhibit preferences more closely in line with a mean semivariance preference structure, where downside variability is reduced while upside variability is preferred. In fact, many hedgers may use percentiles rather than mean and variance in making their hedging decisions (such as using Value-at-Risk). This in itself would create a mean-semivariance preference structure. Third, options may be preferred if the firm confronts capital constraints in the financing of margin calls on futures hedges, while purchasing options requires no mark-to-market or margin requirements. This may also make options more attractive to lenders who finance risk management transactions. Finally, inclusion of transactions costs may impact the demand for options, but it is unlikely that the difference in futures and option commissions (less than a penny per bushel in most cases) would be enough to overwhelm the hedging demand for futures over options.

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APPENDIX A: PROOF A

$$\text{Proof that } \frac{\sigma_{pf} \cdot \sigma_{RR} - \sigma_{fR} \cdot \sigma_{pR}}{\Gamma} \equiv \frac{\sigma_{pf}}{\sigma_{ff}}$$

First, note that we can rewrite Γ as follows:

$$\begin{aligned} \Gamma &= \sigma_{ff} \cdot \sigma_{RR} - \sigma_{fR}^2, \\ &= \sigma_{ff} \cdot \sigma_{RR} - (\rho_{fR} \cdot \sigma_f \cdot \sigma_R)^2, \\ &= \sigma_{ff} \cdot \sigma_{RR} - \rho_{fR}^2 \cdot \sigma_{ff} \cdot \sigma_{RR}, \\ &= (1 - \rho_{fR}^2) \cdot \sigma_{ff} \cdot \sigma_{RR}, \end{aligned} \tag{A.1}$$

where ρ is the correlation coefficient. Also, note that

$$\begin{aligned} \sigma_{fR} \cdot \sigma_{pR} &= \rho_{fR} \cdot \sigma_f \cdot \sigma_R \cdot \rho_{pR} \cdot \sigma_p \cdot \sigma_R, \\ &= \rho_{fR} \cdot \rho_{pR} \cdot \sigma_f \cdot \sigma_p \cdot \sigma_{RR}, \end{aligned} \tag{A.2}$$

and

$$\begin{aligned} \rho_{pR} &= \frac{\sigma_{ff} \cdot \sigma_{pR}}{\sigma_{ff} \cdot \sigma_p \cdot \sigma_R}, \\ &= \frac{\sigma_{fR} \cdot \sigma_{pf}}{\sigma_f \cdot \sigma_R \cdot \sigma_p \cdot \sigma_f}, \\ &= \rho_{fR} \cdot \rho_{pf}. \end{aligned} \tag{A.3}$$

Substituting (A.3) into (A.2) gives

$$\begin{aligned} \sigma_{fR} \cdot \sigma_{pR} &= \rho_{fR}^2 \cdot \rho_{pf} \cdot \sigma_p \cdot \sigma_f \cdot \sigma_{RR}, \\ &= \rho_{fR}^2 \cdot \sigma_{pf} \cdot \sigma_{RR}. \end{aligned} \tag{A.4}$$

Therefore,

$$\begin{aligned} \sigma_{pf} \cdot \sigma_{RR} - \sigma_{fR} \cdot \sigma_{pR} &= \sigma_{pf} \cdot \sigma_{RR} - \rho_{fR}^2 \cdot \sigma_{pf} \cdot \sigma_{RR}, \\ &= (1 - \rho_{fR}^2) \cdot \sigma_{pf} \cdot \sigma_{RR}. \end{aligned} \tag{A.5}$$

Taking the result in (A.5) and dividing by the result in (A.1) gives

$$\frac{\sigma_{pf}}{\sigma_{ff}}.$$

(End of Proof).

APPENDIX B: PROOF B

Proof that $\sigma_{fR} \cdot \sigma_{pf} \equiv \sigma_{pR} \cdot \sigma_{ff}$

Note that

$$\begin{aligned}\sigma_{fR} \cdot \sigma_{pf} &= \rho_{fR} \cdot \rho_{pf} \cdot \sigma_f \cdot \sigma_R \cdot \sigma_p \cdot \sigma_f, \\ &= \rho_{fR} \cdot \rho_{pf} \cdot \sigma_R \cdot \sigma_p \cdot \sigma_{ff}, \\ &= \rho_{pR} \cdot \sigma_R \cdot \sigma_p \cdot \sigma_{ff}, \\ &= \sigma_{pR} \cdot \sigma_{ff}.\end{aligned}$$

(End of Proof).

APPENDIX C: INDUSTRY PRACTICES: INPUT AND OUTPUT PRICE RELATIONS

This appendix provides a summary of recent examples on how some industry price leaders responded or intended to respond to the increase in commodity prices in the period from 2000 to 2003. Each is a quote about a processor's pricing decision related to input price risks.

- During July 2000, white bread prices experienced a large increase nationwide. At the same time, flour prices remained relatively stable, but "substantial increases in the cost of diesel and gasoline fuel and natural gas, as well as in labor and labor-related expenses" resulted in a sharp rise of input costs (Sosland Publishing, August 22, 2000).
- Following large run-up in flour prices in spring 2001, "Earthgrains published a brochure detailing its need to pass along price increases to the consumer" (Sosland Publishing, April 3, 2001) "..... the cost of producing and delivering a loaf of bread has increased 6.8% in the past year the baking industry has been described as a penny business, meaning that even small changes in costs can have a big impact because they are multiplied by hundreds, thousands, and even millions of times" In this environment, retail price increases are needed and justified
- On June 5, 2001, General Mills announced price increases to offset increased ingredient costs and hoping that rivals will follow price hike lead (Merrill).
- Industry leader, IBC in late 2002 ... "higher baking ingredient costs have prompted IBC to raise prices on most of its products by an average of 3%." (Sosland Publishing, December 17, 2002). This followed Kraft foods ... indicating it would raise prices by 3% (Sosland Publishing, November 26, 2002) ... and Kellogg anticipating increases of 3-4% (Sosland Publishing, December 17, 2002).
- In Canada "Prices of bakery products in Canada rose 3.9% in November, this increase was due to the rise over the past three months in the price of wheat (Sosland Publishing, December 31, 2002).
- "We put our customer in the position of buying the right item from the right supplier at the right price." That synopsis describes the purchasing program offered by Connell Purchasing Services, Naperville, Ill., part of The Connell Co., according to J. Bryan Price, director of sales and marketing. He continued, "That does not always mean buying at the lowest price. It's whatever is going to meet that business need." (Sosland, November 6, 2001).
- "The importance of risk management increasingly has attracted the attention of management at the highest levels of companies in ways that were not the case in the past, Mr. Ritter," executive vice-president, Louis Dreyfus said. Taking the passive approach leaves management at risk he said. "Recent federal accounting standards applied to derivatives have brought the chief executive offices and chief financial offices into the

discussion for the first time,” he said. “There is also more and more accountability at the board level.”

“These companies now understand they are criticized if they don’t manage their energy exposure. And I think these companies believe that they are in the same way accountable if they don’t manage their agricultural price and volume exposures.” (Sosland, November 6, 2001).

- “Kraft foods Inc. announced Nov. 15th it will raise wholesale prices on some of its cookie and cracker brands, including Oreo and Ritz, by about 3%. Higher flour and cocoa prices precipitated increases, effective Dec. 30.” (Sosland Publishing, November 26, 2002).
- “David C. Nelson, director, food and agribusiness, Credit Suisse First Boston, in a research report issued Nov. 15th said the move by Kraft confirms earlier comments by other food companies that prices in 2003 should be examined on a brand-by-brand basis.” (Sosland Publishing, November 26, 2002).
- Concept 2 Bakers announced an increase in its prices for all frozen products attributed the price move to commodity price increases for both conventional and organic commodities The price increases are a direct result of poor growing conditions, reduced crop yields, increase worldwide demand and reduced hold over stocks of grain,” (Sosland Publishing, January 21, 2003).
- AIPC indicated that they had “great progress in implementing price increases through durum wheat cost pass-through agreements with customers” (Sosland Publishing, February 18, 2003).
- General mills announced it was “raising prices by 2% on its cereal brands to offset the rising cost of commodities such as cocoa, wheat and energy. Its rival, Kellogg Co., recently made a similar move.” (Merrill, February 18, 2003).