A Theoretical Analysis of Multiproduct Mergers: Application in the Major Meat Sectors

Benjamin L. Sanderson
Mississippi State University
bls357@msstate.edu

Kalyn T. Coatney
Mississippi State University
k.coatney@msstate.edu

Bryon J. Parman
Mississippi State University
Bryon.Parman@msstate.edu

Jesse B. Tack
Mississippi State University
j.tack@msstate.edu

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1.0 Introduction

Beef, pork, and chicken are the three primary sources of meat-protein in the United States. Each sector has maintained separate supply-chains and processing facilities due to highly heterogeneous inputs, specialized production and processing. Since the beginning of increased concentration within each of these sectors (1970's), a vast majority of the mergers and acquisitions were sector specific. Increased sector concentration has long been a concern of producers. It was not until the proposed JBS/National Beef in 2009 that the Department of Justice (DOJ) significantly challenged a merger within any of the meat sectors, primarily based on regional buyer market power. Since the late 1990s, a new merger trend has emerged. Firms engaged in one meat sector, say beef processing, have been merging with or acquiring other meat product firms, primarily pork processors or poultry integrators. JBS, Cargill and Tyson control, in part, nontrivial portions of all three major meat sectors; beef, pork and chicken. No multiproduct meat merger has been challenged by the DOJ, including the most recent merger between JBS and Cargill Pork in 2015. JBS now holds 22 percent of beef, 20 percent of pork, and 18 percent of chicken processing capacities in the U.S. (Reuters, 2015; National Hog Farmer, 2015). As a result, new concerns of 'conglomerate' market power are arising (National Hog Farmer, 2015).

Traditional thought regarding multiproduct/conglomerate mergers is simply a change of ownership if in *sufficiently* unrelated product markets. Given a 'sufficiently close substitutes' requirement the DOJ uses to define the relevant product market (2010 Merger Guidelines), a multiproduct/conglomerate merger would not be easily challengeable. Kolasky (2001), articulated the now long standing position of DOJ that conglomerate mergers are unlikely to harm competition. Following the 2010 Merger Guidelines, a merger between firms across a wider set of products would naturally result in a low concentration index. Because only a small increase in production efficiency through economies of scale and/or scope would be required to offset any increases in market power, a multiproduct/conglomerate merger in the meat industry would be cause for little concern. However, consumers clearly view beef, pork and chicken as ‘relatively close’ substitutes (e.g. Capps Jr., 1989). Additionally, all three of these major meat industry sectors have long regarded each other as important rivals.

To analyze the potential effect of mergers across the beef, pork, and chicken sectors’ requires recognition that these meats are slightly differentiated submarkets within a larger differentiated “meat-protein market”. Therefore, multiproduct/conglomerate mergers require an alternative set of assumptions and analysis than traditional horizontal merger analysis. Most theoretical and empirical literature regarding mergers has assumed perfect, or nearly perfect, substitutes.

This study develops a flexible merger analysis model that assumes the relevant product market definition consists of a differentiated “meat protein” market, while maintaining within submarket differentiation. This modeling approach is generalizable to *j* submarkets with *n* firms within each submarket. Therefore, this modeling approach accounts for both within submarket and across submarket competition, and provides easy comparisons between increased market power and efficiency resulting from a broader product market specification. The theoretical foundations and assumptions of the model will be discussed, along with the necessary background and supporting legal context relevant to the meat-protein sector.

Additionally, beef, pork, and chicken are fairly concentrated markets individually. It will be shown that these conditions can lead to a market structure facilitating a single multiproduct firm’s ability to increase its market power in the meat-protein industry. The welfare effects of this market structure are largely unknown in this context. More specifically, it is unclear if there is market power and/or cost efficiencies to be gained from this type of merger. Sexton (2000) suggests that the market power gained from greater consolidation in agriculture is likely offset by efficiencies gained through technology and product marketing. However, he cautions readers that even small changes in market power may significantly change the allocation of welfare. To such end, possible economies of scope will be analyzed.

2.0 Literature

The existing literature addresses multiproduct mergers generally, but does not fully answer the questions posed in this paper. The current framework needs significant alteration. The new framework will consist of two components. The first component is within submarket mergers between n-firms, which will add up to be an n-firm oligopoly model. At the same time, there will also be k-firm across submarket mergers.

Because meat processing is heterogeneous between these three products, cost efficiencies are also heterogeneous. This point of view is furthered by Sexton and Zhang (2001) who state, "For example, although pork, beef, and poultry may substitute for one another in consumers' budgets, they do not substitute at all as inputs into a particular processing plant". Also due to production heterogeneity, the potential for economies of scope in a meat multiproduct merger must be addressed. Bouras (2007) suggests that higher economies of scope and higher degrees of substitutability will decrease collusion sustainability for a firm engaged in multiple meat products. This brings to focus the idea that economies of scope and substitutability of these products in demand needs to be further investigated.

Additionally, economies of scope will be incorporated into this analysis. This will identify the required efficiencies needed to offset increases in market power. This analysis will also preclude any economies of scale gained as a result of any merger.

For the "base" differentiated product model, the Bowley representative consumer model is used (Bowley, 1924). Dixit (1979) worked with the Bowley model in a Cournot differentiated product duopoly setting. Singh and Vives (1984) expanded this model to show how Bertrand and Cournot outcomes differ in a product-differentiated duopoly. They find that Bertrand competition shows more total surplus than Cournot competition in differentiated duopoly with linear demand. Further, they find that ideally firms would prefer Cournot competition when dealing with substitutes, and would prefer Bertrand competition when dealing in complements. Notably, Häckner (2000) expands to n firms in the Bertrand framework, albeit with some limitations due to the simplifying assumptions he uses. These assumptions include equal marginal cost of production for all products, each firm only producing one unique product, as well as other structural assumptions for mathematical simplification. Häckner, however, did not address welfare, leaving it for future research. Häckner finds that when expanded past duopoly, Singh and Vives' (1984) conclusions about Bertrand and Cournot competition do not hold, and one type of competition is no longer clearly more efficient than the other. Hsu and Wang (2005) pick up where Häckner left off, finding that at all levels of substitutability, welfare is higher under Bertrand competition than under Cournot competition.

3.0 Model Development

In this section, the Bowley differentiated products utility function, Bowley (1924), is altered to facilitate various merger scenarios (cases for comparison). Though the Shubik-Levitan utility function, is more appropriate when the analysis must consider the addition or subtraction of products (Theilen, 2012), it is assumed that for the instant case (meat protein), no additional major categories will be created or eliminated as a result of a merger. Additionally, attributes of the meat industry is explored to formulate which type of competition, Cournot or Bertrand, best describes competition.

3.1 Consumer Utility

The starting point for representing consumer utility in this analysis was first presented in Häckner (2000), which was an n-firm extension of the two product Bowley function (Bowley, 1924; Dixit, 1979; Singh and Vives, 1984). The assumptions, which will be relaxed in this analysis, are that the consumer identifies each product by the firm that produces the product, and each firm produces only one product. The Häckner representation, with minor notational adjustments here, is.

$$U(\mathbf{q}, Z) = \sum_{i=1}^{n} \alpha_i \, \mathrm{q}_i - \frac{1}{2}\left(\sum_{i=1}^{n} \beta_i q_i^2 + 2\theta \sum_{i \neq j} \mathrm{q}_i \mathrm{q}_j \right) + Z \, .$$

In this form of utility, the α_i's represent the representative consumer's reservation prices and the β_i's are independent inverse demand slope parameters. For simplicity, it is assumed the α_i's are symmetric. Varying the reservation prices not only complicates merger solutions, but also detracts from the within- and across-submarket substitutability impacts on mergers. Though the slope of inverse demands may vary, this analysis follows Häckner's simplifying assumption that the β_i's are symmetric and normalized to one. The parameter $\theta \in [-1, 1]$ represents a symmetric product substitutability, where a value of -1 indicates perfect complements, 0 independent, and 1 perfect substitutes. Because the current analysis is applied to the meat sector, only the region of substitutes will be considered. Finally, Z is a composite good.

The most important modification to the previous model is that utility can be further refined to include the possibility of sub-markets within the aggregate market. A submarket would entail any subset of products whose attributes are considered to be 'relatively close substitutes' within a wider sector of consumer products. For example, the meat protein sector is comprised of several potential submarkets, such as beef, pork, and chicken. Each submarket is in turn comprised of competing firms, each producing aggregate composite of similar products from animal carcasses. Even if the quality characteristics of the composite products produced by each firm are viewed as homogenous to the consumer, some differentiation may be established by well-known branding, such as Tyson, Pilgrims, and Sanderson chicken, likewise Tyson, Smithfield, and Hormel pork. Brand differentiation across beef firms is much weaker at the retail level, as these firms have yet to significantly brand their products. However, a small degree of indirect differentiation may be attained via product quality and service provided to the retailer, regardless of branding.

To facilitate the analysis of submarkets, product substitutability is further broken down into two major components: within-submarket and across-submarket. The within-submarket substitutability will be denoted by θ, while the across-submarket substitutability will be denoted by δ. The δ parameter measures how substitutable a within-submarket firm's product is with those products not included in the same submarket. Because the consumer views subsets of products to be closer substitutes than other subsets, it is logical to assume that within-submarket substitutability is greater than the across-submarket substitutability. This relationship is formalized as $0 < \delta < \theta < 1$. An analogous interpretation is that the difference between two differentiated beef products is less than difference between beef and chicken products. The resulting general representation of utility is

$$U(\mathbf{q}, Z) = \alpha \sum_{i=1}^{n} \mathrm{q}_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + 2\theta \sum_{i \neq j} \mathrm{q}_i \mathrm{q}_j + 2\delta \sum_{h \neq k} \mathrm{q}_h \mathrm{q}_k \right) + Z \; .$$

The utility identifies the $i \neq j$ firm combinations within-submarket and the the $h \neq k$ firm combinations across-submarkets.

In the merger analyses that follow, it is assumed the DOJ has stopped all within-submarket increases in concentration, leaving two firms within each submarket. Additionally, it is assumed there are only three relevant submarkets (beef, pork, and chicken). To begin, each firm is assumed to produce only one product within its respective submarket, therefore, six firm in total comprise sector competition. The resulting set of pairwise within-submarket product combinations identified by firm are {1,2}, {3,4}, and {5,6}. The resulting pairwise across-submarket combinations identified by firm are {1,3}, {1,4}, {1,5}, {1,6}, {2,3}, {2,4}, {2,5}, {2,6}, {3,5}, {3,6}, {4,5}, and {4,6}. The firm/product specific utility function is now

$$U = \alpha \sum_{i=1}^{6} q_i - \frac{1}{2}\begin{pmatrix} \sum_{i=1}^{6} q_i^2 + 2\theta\left(q_1q_2 + q_3q_4 + q_5q_6\right) \\ +2\delta\begin{pmatrix} q_1q_3 + q_1q_4 + q_1q_5 + q_1q_6 + q_2q_3 + q_2q_4 \\ +q_2q_5 + q_2q_6 + q_3q_5 + q_3q_6 + q_4q_5 + q_4q_6 \end{pmatrix} \end{pmatrix} + Z \; . \qquad (1)$$

Consumers maximize utility subject to a budget constraint $\sum_{i=1}^{6} p_i q_i + Z \leq I$, where I is income and the price for the composite good is normalized to one. The resulting system of firm/product specific linear inverse demands are

$$\begin{cases} p_1 = \alpha - q_1 - \theta q_2 - \delta\left(q_3 + q_4 + q_5 + q_6\right) \\ \cdots \\ p_6 = \alpha - q_6 - \theta q_5 - \delta\left(q_1 + q_2 + q_3 + q_4\right) \end{cases}$$

and corresponding system of linear demands

$$\begin{cases} q_1 = \dfrac{\alpha\left(1-2\delta+2\delta\theta-\theta^2\right)-\left(1+2\delta-4\delta^2+\theta\right)p_1-\left(4\delta^2-\theta-2\delta\theta-\theta^2\right)p_2-\left(\delta\theta-\delta\right)(p_3+p_4+p_5+p_6)}{\left(2\delta-\theta-1\right)\left(\theta-1\right)\left(1+4\delta+\theta\right)} \\ \cdots \\ q_6 = \dfrac{\alpha\left(1-2\delta+2\delta\theta-\theta^2\right)-\left(1+2\delta-4\delta^2+\theta\right)p_6-\left(4\delta^2-\theta-2\delta\theta-\theta^2\right)p_5-\left(\delta\theta-\delta\right)(p_1+p_2+p_3+p_4)}{\left(2\delta-\theta-1\right)\left(\theta-1\right)\left(1+4\delta+\theta\right)} \end{cases}$$

. (2)

3.2 Firm Competition

Shapiro (1989) stated, “The choice between a pricing game and a quantity game cannot be made on a priori grounds. Rather, one must fashion theory in a particular industry to reflect the technology of production and exchange in that industry.” Following this logic, competition in the meat processing sector, be it Cournot or Bertrand, is identified in relation to supply chain characteristics within each meat industry. Specifically, the structure of the supply chain directly impacts the level of control the processing firm has over the quantity produced.

To begin, the production cycles for the various live animal inputs varies significantly and there are various levels of production sectors from conception to slaughter. For instance, it takes approximately several years before beef cattle are ready for slaughter (USDA-ERS, 2016a). The major production sectors before processing are cow-calf producers, stocker operators, and feeders (MacDonald et al., 2000). It is only the cow-calf producers that set quantity in the market. In stark contrast, broiler chicken production takes about 50 days (MacDonald & McBride, 2009), and the production system is fully vertically integrated, hence quantity produced is directly controlled by the processor (Ollinger, 2000). Pork production is roughly between these two extremes. It takes swine roughly 6 months before being ready for slaughter (USDA-ERS, 2016b) and the supply chain, though highly vertically coordinated, is comprised of farrowing and finishing firms (MacDonald et al., 2000).

These production cycle and supply chain structures impact the processor's ability to control quantity in response to changing prices. It appears that there is no clear identification which best describes competition among all competitors in the meat processing industry. However, due to the long production cycle in one (beef) and the breeding of two (beef and pork) are largely controlled by upstream suppliers (for the exception of chicken), it is assumed the processors are more prone to Bertrand competition. This is in stark contrast to the prevalent assumption of Cournot competition in the Agricultural Economics literature, even for beef processors (Crespi et al., 2010). It is of note that it is possible to model this allowing for some firms to be Cournot competitors and others to be Bertrand competitors (Tremblay and Tremblay, 2011). However, this approach was not pursued for the sake of tractability.

Given the assumption of competition and q_i is the firm/product specific demand, firms maximize the profit objective function,

$$\pi_i = (p_i - c_i)q_i - F_i\,, \tag{3}$$

by choosing their optimal output price p_i^*, subject to the reaction of their within- and across submarket rivals. It is also assumed that firms have reached economies of scale within each submarket from previous mergers and thus experience constant marginal costs, c_i, within the relevant region of production. Fixed costs are denoted by F_i and are assumed to equal zero. To insure a solution exists, $\alpha > c_i$. This assumption is made such that the marginal cost of producing a product is not greater than the maximum the representative consumer would be willing to pay. Finally, α_i was previously assumed to be symmetric, and appears to be inconsistent with market price differentials across meat submarkets at relatively equal quantity demanded (Bentley, 2012).

Therefore, to maintain 'relatively' symmetric profitability across meat processors, it follows that c_i must also be assumed to be symmetric.

4.0 Merger Case Analysis

The following lists and describes the various merger cases most relevant to evaluating whether mergers across the various meat submarkets are more or less harmful than a merger within a submarket, which appears to be of greater importance to the DOJ. Also in this section, comparisons of the optimal prices, quantities and resulting profits are discussed.

4.1 Merger Case Descriptions

Case I: Base Case - No Merger

Case I is the base case consisting of three submarkets with two firms per submarket. This will be the baseline from which later cases can be compared. In the context of this problem, each submarket would represent a different meat protein submarket (e.g., beef, pork, or chicken) as a part of the larger meat-protein market. A visual representation of the market depicted in Case I is presented in Figure 1.

Case II: A Single Multiproduct Firm across all Three Submarkets

Case II depicts a scenario in which one firm has merged across each submarket, much like we see in these markets today, resulting in a multiproduct firm with contact in all three submarkets. Little evidence has shown that the Department of Justice would challenge such a merger because it would be viewed as a 'change of ownership' in a new market. This stems from the traditional stance that each meat product makes up its own relatively self-contained market. However, a pivotal contribution of this analysis is incorporating the analysis of a new, alternative market definition encompassing the demand related goods in the meat-protein sector (e.g. beef, pork, and chicken). Now, each submarket consists of an entity operated by the multiproduct firm and a fringe player, where the fringe player is only in that specific submarket. These relationships are shown in Figure 2.

Case III: Two Multiproduct Firms across all Three Submarkets

Case III depicts the next logical step from Case II. Figure 3 shows the new industry structure. If one firm is allowed to merge across submarkets unchallenged due to being a change of ownership, then it would be possible for the three fringe players to merge together and create a matching “conglomerate”, and they would certainly have unilateral incentive to do so because profits are strictly higher for the matching conglomerate than for any of the remaining individual firms, barring other regulatory restrictions placed on them.

Case IV: Monopolization of One Submarket

Case IV provides a “what-if” analysis into what the implications of allowing merger to monopoly might be in the meat-protein sector. Given the DOJ interpretation, it is virtually impossible for any submarket (considered the entire relevant market by DOJ convention) to merge to monopoly. However, this restriction will be removed in this analysis in order to see if any significant changes to this policy emerge as a result of a changed relevant market. In our analysis, the beef submarket will merge to monopoly (although it doesn’t particularly matter which submarket is chosen, due to the imposed symmetry across industries). In order to avoid functional form issues with the Bowley function and to maintain the slight differentiation between the two fresh beef products, the monopolized beef industry would still consist of two products. This coincides with real world conditions because often a merged firm maintains the brand they have purchased, so that they may not risk losing those consumers who would display brand loyalty to the acquired brand. An example this would be Tyson Foods, Inc. maintaining the IBP brand after purchase. A visual of Case IV is provided in Figure 4.

4.2 Overall Price, Quantity, and Profit Results and General Comparisons

The results for each case are provided in tables 1 through 4. Comparisons between the various merger cases discussed.

Case I: Base Case - No Merger

The optimal prices, quantities, and resulting profits are shown in Table 1. Case I results in all firms charging the same price due in part to the assumptions used. As expected, each product's optimal quantity is equivalent the others'. Finally, optimal profits can be calculated. Optimal profits, again equal for all firms in Case I.

Case II: A Single Multiproduct Firm across all Three Submarkets

Mathematically, Case II is different from Case I in that the multiproduct firm is no longer maximizing profit for each product individually. Instead, they are now maximizing the joint profit for all products they produce. These results are shown in Table 2. The multiproduct firm's price for its goods (good from beef firm 1, pork firm 1, and chicken firm 1) is shown by (8). The fringe players (beef firm 2, pork firm 2, and chicken firm 2) now optimally charge the price shown in (9). Comparisons between $p^*_{1,3,5}$ as compared to $p^*_{2,4,6}$ reveal that the price charged by the multiproduct firm is always greater than the price charged by the fringe player, in any submarket. Not surprisingly, comparisons also reveal that prices are unilaterally higher than in Case I. Notice again that due to the assumptions previously stated, each fringe firm is symmetric. Comparisons show that the multiproduct firm will also optimally have a lower output quantity than the fringe firm. This occurs because the multiproduct firm is better able to take advantage of the substitutability between the different meat-protein products. The multiproduct firm's profits for all three products it produces are larger than the profit of the fringe firms producing one product. However, per product profits of the multiproduct firm are lower than the fringe player. However, even the profit for the fringe player is greater than the premerger profit for any given firm.

Case III: Two Multiproduct Firms across all Three Submarkets

In Case III both conglomerate firms are now maximizing joint profits across all their products. Results from Case III are shown in Table 3. This case results in prices again being constant across all firm/product combinations. Similarly, the same price being charged for each product/firm combination results in the same quantity for each product/firm combination. As a result, profit for conglomerate 1 and conglomerate 2 are equal to each other, shown in (16).

Comparisons of results yield several important results. First, all prices in case III are higher than both the premerger case (Case I) and the case with one multiproduct firm and fringe competitors (Case II). Another important result found by looking at profit comparisons was that profits for either conglomerate firm are higher than for the multiproduct firm in Case II. This demonstrates that given one firm has merged to become a multiproduct firm in all three submarkets, there is an incentive for the remaining firms to merge and form a multiproduct firm (resulting in case III). This coincides with the observations made in the U.S. meat-protein sector. Observation of firms merging in this nature served as a motivating force for creation of this paper. However, for some substitutabilities, δ at its highest range (0.75) and θ near its lower bound (0.75), a structure with two conglomerates is the second most profitable structure.

Case IV: Monopolization of One Submarket

Case IV allows for insight into whether a merger to monopoly (Case IV), multiproduct firm (Case II), or conglomerates (Case III) would be more or less competitive under the market definition given for this analysis. In Case IV, the beef submarket monopolist is maximizing joint profits across his two beef products. The other firms are maximizing their profits. The prices for the non-monopolized players are symmetric. These results are shown in Table 4.

Comparisons for Case IV reveal some very interesting findings. First, all prices in this case are greater than those in the premerger case (Case I). Profits for the monopolist are greater than for any non-monopolized individual in the Case IV meat-protein market. The most interesting discovery made during the comparisons for Case IV was that no clear, generalizable result was obtainable for comparing the prices and profits of the monopolist against both a multiproduct firm (Case II) and conglomerate firm (Case III). For the price of beef good 1, it depended on the interactions and values of δ and θ as to whether it was higher under the monopolist scenario or multiproduct scenario. The same parallel existed between the price of beef good 1 comparing the price under the monopolist scenario against the price of beef good 1 against the conglomerate scenario. Profits also followed this pattern. Given some values of δ and θ, profit for the monopolist may be higher than those of the multiproduct firm or conglomerate firm, respectively. For other values of δ and θ, the multiproduct firm or conglomerate firm would have higher profit than the monopolist.

4.2.1 Submarket Monopolization versus a Single Multiproduct Firm Case Comparison

The primary focus of this analysis, and for the comparison of the two market definitions are the actual outcomes. When firms are not considered to be part of the same relevant market, as is with the DOJ interpretation of these markets, it is obvious that a monopolist lessens competition significantly. However, given the interpretation of the relevant market introduced here, a new comparison opportunity arises. The new definition of a relevant market begs the question, “Is monopolization of a submarket better or worse for competition than a multiproduct firm for competition?" To facilitate this comparison, a multiproduct firm structure will be compared against structure with a monopolist in one submarket.

To obtain usable results in this case, further restrictions were necessary. Without these restrictions, the exact nature of this relationship between the scenarios is much less clear and not easily justifiable mathematically. However, relatively clear results arise once further restrictions are put into place. For the reader's convenience and ensured clarity, both the new assumptions and the previously mentioned assumptions will be stated. Again, the $\beta_i 's$ are symmetric, equal to one, and removed from the model. The $\alpha_i 's$ are equal to each other, resulting in a single α parameter, which is greater than cost. More explicitly, $\alpha > c$. The major change in assumptions comes with the substitutability parameters. In this paper, the original assumption for the relationship of the parameters was $0 < \delta < \theta < 1$. This part of the analysis requires that $0 < \delta < 0.75 < \theta < 1$ and that the two parameters be significantly different from each other. The second part of this condition is not easily displayed in a tractable form resulting from the extreme, non-linear form of the actual restriction. However, when displayed as a 3D graphic, the required relationship becomes much clearer.

First, the price relationship between a product produced by both the submarket monopolist and the multiproduct firm (each reacting to the other firms in that scenario, respective to that scenario) is shown in Figure 5. Note that this is only for one of the goods each produces. Under these restrictions, the submarket monopolist will always charge a price that is higher than the multiproduct firm for the product each would produce in their respective scenarios, unless it is within the range of substitutabilities that are significantly close to one another. The vertical dimension of the graph is the price each would charge. Notice how the price charged increases relatively more for the submarket monopolist firm when the products are less substitutable across submarkets. Although in absolute terms, both scenarios allow the firm to increase price more as across submarket substitutability decreases. The same is true within-submarket; firms can charge higher prices as within-submarket substitutability decreases.

Figure 6 shows the relationship between output quantities of the submarket monopolist and the multiproduct firm. Again, note that is for only one of the products each produces, respectively. The axes in Figure 6 are of the same dimensions and of the same view as in Figure 5. Notice that the multiproduct firm is producing more of this one good in this submarket than the submarket monopolist is (except within the area already discussed as not being significantly close between within-submarket substitutability and across-submarket substitutability). This is due to the fact that the multiproduct firm is maximizing his profits across separate submarkets while the submarket monopolist is maximizing profits across two products within the same submarket.

Perhaps a more revealing comparison is between the total profits each would earn. This is particularly revealing because the submarket monopolist is only producing two products (in the same submarket) while the multiproduct firm is producing 3 total products (one in each of the three submarkets). Figure 7 and Figure 8 show this comparison. As can be seen in Figure 7 and again in Figure 8, no specific firm/industry structure is more profitable unilaterally. Instead, it depends on the substitutabilities; both within-submarket and across submarket. Most notable from this graph is the realization that when products are nearly independent across submarkets and nearly identical within-submarket (i.e., values close to zero for δ, and values close to one for θ), then the multiproduct firm is able extract substantially more profit than the submarket monopolist. At near independence across submarkets, and near our lower bound within-submarket market for θ (near 0.75), both firms make almost identically high profit, but the submarket monopolist firm still has a slightly higher profit. This follows intuition quite well. Figure 8 clearly shows how the relationship between the substitutabilities parameters affect profits. Interestingly, it can be seen that there are some instances in which as long as the within-submarket substitutability is high enough, there is no situation in which the multiproduct firm can attain higher profits than the submarket monopolist. Additionally, for low enough levels of across-submarket substitutability and regardless of the within-submarket substitutability, the submarket monopolist firm can earn higher profits than the multiproduct firm. Again, this may be attributed to the difference total number of products produced by firm.

More importantly, this allows for some justifiable and informed understanding of the current DOJ market definition stance. There are still situations in which a submarket monopolist can earn higher profits than a multiproduct firm, even in this much more broadly defined market. This lends itself towards agreement that, at least for some values of θ and δ, merger to monopoly in one meat-protein market will still be more harmful that the emergence of multiproduct meat-protein firms. Given this, there is some justification of the DOJ stance that these are three entirely separate markets. However, it is still important to remember that given the reinterpreted market definition, there is a significant continuum of instances in which a multiproduct firm can earn higher profit than a submarket monopolist, which in turn, gives some credence to this hypothesized relevant market definition.

4.3 Welfare Results

Perhaps the most important economic evaluation for any merger analysis is the calculation of welfare, both consumer and producer surplus. This allows for the net effects of the merger to be calculated in aggregate. Practically, this yields a justifiable set of results, in terms of pure economics. However, this misses much in the larger realm of antitrust analysis. Kirkwood and

Lande (2008) show that antitrust laws intended and courts have consistently upheld that the purpose of these laws is consumer protection, not efficiency. Kirkwood and Lande also provide information on the ‘traditional’ view focused on the efficiency argument [this view was championed by Williamson (1968)], but make clear that the court system has not embraced this view, instead opting for the consumer protection argument. Further, Zerbe (2015) finds the following, “This nevertheless means that in at least 1,478 cases, or ninety-eight percent of all federal antitrust cases, consumer protection was the overriding concern.”

This research is focused on the positive economic implications associated with merger analysis, thus, normative economic analysis into the merits of these welfare standards will not be discussed. Rather, results reflecting both modes of thought will be presented. Total welfare results will be presented, noting both producer and consumer surplus. This will allow for comparisons of total welfare, consumer surplus, and producer surplus. This article is merely showing the welfare implications in the traditional economic sense, whilst carefully noting that the broader, largely legal context of antitrust analysis does not fully coincide with this.

To calculate consumer and producer surplus, the methods of Chung et al. (2013) are utilized. They performed welfare calculations based on a modified Bowley functional form of consumer utility, from which their calculations may be adapted to fit the model extension shown in this paper. Producer surplus (PS) is simply the total industry profits (for a given case). Because the underlying utility function is that of the representative consumer, consumer surplus (CS) can be calculated in the manner shown in expression (6).

$$CS = U - \sum_{i=1}^{6} p_i q_i \tag{4}$$

Total surplus, or welfare, is simply $TS = CS + PS$. The following tables summarize these calculations. Tables 1-4 show welfare calculations for Cases I-IV.

These tables summarize welfare implications. Again, we will compare using both standards: consumer surplus and total surplus. An interesting finding is that under both welfare standards, but one that is hardly surprising, is that surplus (by either measure) in every merger scenario was always lower than the base case. This is intuitive, and follows logically. It is also noteworthy that the qualitative comparisons between the two welfare standards did not vary except in one case.

The only case in which the two merger standards were not in agreement was when comparing welfare between Case II (a multiproduct firm in all three submarkets, with a fringe competitor in each submarket) and Case III (two conglomerates). Using the consumer surplus standard, consumer surplus is strictly greater in Case II than in Case III. Conversely, when using the total welfare standard, Case II almost always results in a greater total surplus than Case III. However, there is a miniscule area, (not visible graphically nor easily calculable) in which total surplus is greater for Case III. Other interesting comparisons also arise. For both welfare standards, Case II shows higher welfare than Case IV, except when the two substitutabilities are sufficiently close to one another. Similarly, in Case III welfare is higher (under both measures) than in Case IV unless the two substitutabilites are sufficiently close to one another.

These comparisons serve to highlight two important issues. The first is that under this modeling mechanism, mergers without efficiency reduced welfare compared to the base case, regardless of whether it was measured using the consumer surplus standard or the total welfare standard. This pattern logically follows given the analysis mechanism. Second, these comparisons qualitatively show the impacts of different mergers on welfare, allowing some relative rankings to emerge.

5.0 Economies of Scope

In their text, "Contestable Markets And the Theory of Industry Structure", Baumol, Panzar, and Willig (1982) explain economies of scope as,

> "…there is also the possibility that cost savings may result from simultaneous production of several different outputs in a single enterprise, as contrasted with their production in isolation, each by its own specialized firm. That is, there may exist economies resulting from the scope of the firm's operations."

Case II, as presented earlier, showed the effects of a firm who has merged across all three submarkets. Given this, it is clear that market power has been gained absent some force to pass these gains on to consumers. Early assumptions of this model precluded economies of scale as a possible factor. Economies of scope were mentioned as a possible source of market power neutralization, as is possible given that under this case that the merged firm is now producing three products. Case I and Case II will now be revisited with a focus on incorporating economies of scope into the analysis and required cost reductions required for the merger to keep prices at the premerger levels.

At first glance, it may seem as if even if a merged firm produces a product in three sub-markets, by this definition and the previous modeling there are no economies of scope to be gained because they functionally being produced independently (with just a joint profit maximization). This is a true statement when cost is simply a single parameter, "c". However, to incorporate this concept, the cost parameter will be redefined for each firm. A new restriction arises to satisfy the functional form, $\alpha > k + m$.

$$c = k + m$$

Parameter k is product specific costs. These costs are unique to the product being produced and cannot be reduced/increased due to scope economies or diseconomies (although considered symmetric for all products in this analysis). The parameter m represents the general business costs common to all products produced, denoted as m for “management” cost. This includes things like overhead management costs, corporate salaries, product distribution, marketing, other costs at the corporation level, etc.

The merged firm takes on a new cost structure is

$$\tilde{c}_i = \sum_{i=1}^{n} k_i + \lambda \sum_{i=1}^{n} m_i \, , \qquad (5)$$

where k and λ are assumed symmetric.

The λ parameter is a scalar that measures the management cost reductions/increases resulting from the merger. The parameter λ can take values between $0 < \lambda < \infty$. When $0 < \lambda < 1$, economies of scope exist for the merged firm. At $\lambda = 1$, no economies of scope exist. Note that when a firm is only producing 1 product, the λ parameter is not present because scope inherently requires that the firm produce multiple products. When $1 < \lambda < \infty$, diseconomies of scale are present. This means that producing multiple products is more costly to the firm than if the products were produced by separate firms in the industry.

With this change, the exercise presented in Chapter 4, Case I was replicated, obtaining the optimal prices charged by all firms and the resulting output quantities. In Case II, where one firm has merged across the three submarkets and a fringe player remains in each submarket, is also redone using this notation. The question being addressed here is how much economies of scope need to be present in order for the post-merger prices to be equal to the premerger prices, thus creating a welfare neutral merger. To do this, $p^*_{n=1,3,5}$ (the multiproduct case price) was set equal to $p^*_{1,\dots,6}$ (the premerger case price). When these two are equal, the resulting merger is welfare neutral.

The parameter λ takes on the value shown in (9) when the two prices are equal.

$$\lambda^* = \frac{m(2+2\delta-\theta)\left(-1-2\delta+4\delta^2-\theta\right)-2(k-\alpha)\delta(-1+\theta)^2}{m(1+2\delta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)} \tag{6}$$

Comparative statics reveal some important results. First we find the partial derivative of λ^* with respect to across-submarket substitutability, $\frac{\partial \lambda^*}{\partial \delta} < 0$. This demonstrates that an increase in across-submarket substitutability lowers the required λ^* for the merger to be welfare neutral. The partial derivative of λ^* with respect to within-submarket substitutability was also found, $\frac{\partial \lambda^*}{\partial \theta} > 0$. From this it can be seen that increasing substitutability across submarkets increases the required λ^* for welfare neutrality. These statics allow for greater understanding of the nature of these substitutability parameters. Other comparative statics yield interesting results. For instance, $\frac{\partial \lambda^*}{\partial \alpha} < 0$, which means that somehow increasing the maximum willingness-to-pay to the consumer, lowers the required λ^* to be welfare neutral. Conversely, if cost were to increase (increases in either k or m, then the required λ^*needed for welfare neutrality would increase because $\frac{\partial \lambda^*}{\partial m} > 0$ and $\frac{\partial \lambda^*}{\partial k} > 0$.

6.0 Conclusions and Policy Implications

This paper has shown several important things. First, it has hypothesized a new market definition for use when analyzing products in demand related goods. Second, it has further characterized the effects of within- and across-submarket substitutabilities, and the role they play in affecting multiproduct-mergers of demand related goods. Third, this has shown a mathematically tractable model suitable for analyzing mergers similar in nature to this problem. Fourth, it has been shown that there is some justification to the DOJ's view that merger with submarkets can cause harm, and should be looked at. However, this paper also highlights the importance of recognizing that mergers across submarkets may raise competitive concerns if there are not enough economies of scope to offset their impact.

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Table 1: Prices, Quantities, and Profits for Case I

Price	$p^{*}_{i=1,\ldots,6} = \dfrac{\alpha\left(-1+2\delta-\theta\right)\left(-1+\theta\right)+c\left(1+2\delta-4\delta^{2}+\theta\right)}{2-4\delta^{2}+\theta+2\delta\theta-\theta^{2}}$
Quantity	$\underset{i=1,\ldots,6}{q_i^{*}} = \dfrac{\left(\alpha-c\right)\left(-1-2\delta+4\delta^{2}-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^{2}-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}$
Profit	$\underset{i=1,\ldots,6}{\pi_i^{*}} = \dfrac{\left(c-\alpha\right)^{2}\left(-1+2\delta-\theta\right)\left(-1-2\delta+4\delta^{2}-\theta\right)\left(1-\theta\right)}{\left(1+4\delta+\theta\right)\left(2-4\delta^{2}+\theta+2\delta\theta-\theta^{2}\right)^{2}}$

Table 2: Prices, Quantities, and Profits for Case II

Price	$\underset{i=1,3,5}{p^*_{mp}} = \frac{\alpha(-1+\theta)\left(-12\delta^2+2\delta(2+\theta)+(1+\theta)(2+\theta)\right)+c\left(24\delta^3-(1+\theta)(2+\theta)-2\delta(4+5\theta)\right)}{24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta\left(-6+(-4+\theta)\theta\right)}$ $\underset{i=2,4,6}{p^*_{f}} \frac{-\alpha(-1+2\delta-\theta)(-1+\theta)(2+6\delta+\theta)+c(1+2\delta)\left(6\delta(-1+2\delta)-(1+\theta)(2+\theta)\right)}{24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta\left(-6+(-4+\theta)\theta\right)}$
Quantity	$\underset{i=1,3,5}{q^*_{mp}} = \frac{(\alpha-c)(1+2\delta)\left(12\delta^2-2\delta(2+\theta)-(1+\theta)(2+\theta)\right)}{(1+4\delta+\theta)\left(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta\left(-6+(-4+\theta)\theta\right)\right)}$ $\underset{i=2,4,6}{q^*_{f}} = \frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)(2+6\delta+\theta)}{(1+4\delta+\theta)\left(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta\left(-6+(-4+\theta)\theta\right)\right)}$
Profit	$\underset{i=1}{\pi^*_{mp}} = \frac{3(c-\alpha)^2(1+2\delta)(1-\theta)\left(-12\delta^2+2\delta(2+\theta)+(1+\theta)(2+\theta)\right)^2}{(1+4\delta+\theta)\left(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta\left(-6+(-4+\theta)\theta\right)\right)^2}$ $\underset{i=2}{\pi^*_{f}} = \frac{(c-\alpha)^2(-1+2\delta-\theta)\left(-1-2\delta+4\delta^2-\theta\right)(1-\theta)(2+6\delta+\theta)^2}{(1+4\delta+\theta)\left(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta\left(-6+(-4+\theta)\theta\right)\right)^2}$

Table 3: Prices, Quantities, and Profits for Case III

Price	$\underset{i=1,\ldots,6}{p^*_{i,c}} = \frac{c+\alpha+2c\delta-\alpha\theta}{2+2\delta-\theta}$
Quantity	$\underset{i=1,\ldots,6}{q^*_{i,c}} = \frac{(\alpha-c)(1+2\delta)}{(2+2\delta-\theta)(1+4\delta+\theta)}$
Profit	$\underset{i=1,\ldots,6}{\pi^*_{i,c}} = \frac{3(c-\alpha)^2(1+2\delta)(1-\theta)}{(-2-2\delta+\theta)^2(1+4\delta+\theta)}$

Table 4: Prices, Quantities, and Profits for Case IV

Price	$\underset{i=1,2}{p^{*}_{i,bm}} = \frac{1}{2}\left(c+\alpha+\frac{4(c-\alpha)(-1+\delta)\delta(1+4\delta+\theta)}{-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta)}\right)$ $\underset{i=3,4,5,6}{p^{*}_{i,nm}} = \frac{-\alpha(-1+2\delta-\theta)(-1+\theta)(1+3\delta+\theta)+c(-1+\delta)(1+2\delta+\theta)(1+4\delta+\theta)}{-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta)}$
Quantity	$\underset{i=1,2}{q^{*}_{i,\mathrm{bm}}} = \frac{(\alpha-c)(1+2\delta+\theta)\left(4\delta^2+(-2+\theta)(1+\theta)+\delta(-6+4\theta)\right)}{2(1+4\delta+\theta)\left(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta)\right)}$ $\underset{i=3,4,5,6}{q^{*}_{i,\mathrm{nm}}} = \frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)(1+3\delta+\theta)}{(1+4\delta+\theta)\left(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta)\right)}$
Profit	$\underset{i=1,2}{\pi^{*}_{bm}} = \frac{(c-\alpha)^2(1-2\delta+\theta)(1+2\delta+\theta)\left(4\delta^2+(-2+\theta)(1+\theta)+\delta(-6+4\theta)\right)^2}{2(1+4\delta+\theta)\left(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta)\right)^2}$ $\underset{i=3,4,5,6}{\pi^{*}_{i,\mathrm{nm}}} = \frac{(c-\alpha)^2(1-2\delta+\theta)\left(-1-2\delta+4\delta^2-\theta\right)(-1+\theta)(1+3\delta+\theta)^2}{(1+4\delta+\theta)\left(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta)\right)^2}$

Table 5: Case I Welfare Calculations

Consumer Surplus	$\dfrac{\begin{pmatrix} 3c^2\left(1+2\delta-4\delta^2+\theta\right)^2-6c\alpha\left(1+2\delta-4\delta^2+\theta\right)^2 \\ +3\alpha^2\left(1+2\delta-4\delta^2+\theta\right)^2+z\left(1+4\delta+\theta\right)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2 \end{pmatrix}}{\left(1+4\delta+\theta\right)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2}$
Producer Surplus	$\dfrac{6\left(c-\alpha\right)^2\left(-1+2\delta-\theta\right)\left(-1-2\delta+4\delta^2-\theta\right)\left(1-\theta\right)}{\left(1+4\delta+\theta\right)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2}$
Total Surplus	$\dfrac{\begin{pmatrix} (-6\left(c-\alpha\right)^2\left(-1+2\delta-\theta\right)\left(-1-2\delta+4\delta^2-\theta\right)\left(-1+\theta\right) \\ +3c^2\left(1+2\delta-4\delta^2+\theta\right)^2-6c\alpha\left(1+2\delta-4\delta^2+\theta\right)^2+3\alpha^2\left(1+2\delta-4\delta^2+\theta\right)^2 \\ +z\left(1+4\delta+\theta\right)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2) \end{pmatrix}}{\left(1+4\delta+\theta\right)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2}$

Table 6: Case II Welfare Calculations

Consumer Surplus	$z+\frac{3(c-\alpha)^2(1+2\delta)\begin{pmatrix} -144\delta^4+288\delta^5+2\delta(1+\theta)(2+\theta)(7+5\theta)+(2+3\theta+\theta^2)^2 \\ -24\delta^3(6+\theta(8+\theta))-2\delta^2(-13+\theta(-9+\theta(3+\theta))) \end{pmatrix}}{(1+4\delta+\theta)\left(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta)\right)^2}$
Producer Surplus	$\frac{3(c-\alpha)^2(-1+\theta)\begin{pmatrix} -(-1+2\delta-\theta)(-1-2\delta+4\delta^2-\theta)(2+6\delta+\theta)^2 \\ -(1+2\delta)\left(-12\delta^2+2\delta(2+\theta)+(1+\theta)(2+\theta)\right)^2 \end{pmatrix}}{(1+4\delta+\theta)\left(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta)\right)^2}$
Total Surplus	$z+\frac{\begin{pmatrix} 3(c-\alpha)^2(576\delta^6-576\delta^5(-1+\theta)-(1+\theta)^2(2+\theta)^2(-3+2\theta) \\ +96\delta^4(-6+(-4+\theta)\theta)-2\delta(1+\theta)(2+\theta)(-21+\theta(-7+\theta(9+\theta))) \\ +4\delta^3(-103+\theta(-39+\theta(51+19\theta)))+2\delta^2(43+\theta(85+\theta(41-\theta(5+2\theta))))) \end{pmatrix}}{(1+4\delta+\theta)\left(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta)\right)^2}$

Table 7: Case III Welfare Calculations

Consumer Surplus	$z+\frac{-6c\alpha\left(1+2\delta\right)^{2}+3\left(c+2c\delta\right)^{2}+3\left(\alpha+2\alpha\delta\right)^{2}}{\left(-2-2\delta+\theta\right)^{2}\left(1+4\delta+\theta\right)}$
Producer Surplus	$\frac{6\left(c-\alpha\right)^{2}\left(1+2\delta\right)\left(1-\theta\right)}{\left(-2-2\delta+\theta\right)^{2}\left(1+4\delta+\theta\right)}$
Total Surplus	$z+\frac{3\left(c-\alpha\right)^{2}\left(1+2\delta\right)\left(3+2\delta-2\theta\right)}{\left(-2-2\delta+\theta\right)^{2}\left(1+4\delta+\theta\right)}$

Table 8: Case IV Welfare Calculations

Consumer Surplus	$z+\dfrac{\begin{pmatrix}(c-\alpha)^2(1+2\delta+\theta)(480\delta^5+48\delta^4(-11+7\theta)+4\delta^2(1+\theta)(43+4(-11+\theta)\theta)\\ +6\delta(1+\theta)^2(16+(-6+\theta)\theta)+8\delta^3(-31+10(-6+\theta)\theta)+(1+\theta)^3(12+(-4+\theta)\theta))\end{pmatrix}}{4(1+4\delta+\theta)(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta))^2}$
Producer Surplus	$\dfrac{\begin{pmatrix}(c-\alpha)^2(1-2\delta+\theta)(32\delta^5+368\delta^4(-1+\theta)+(1+\theta)^3(12+(-12+\theta)\theta)\\ +2\delta(1+\theta)^2(48+5(-10+\theta)\theta)+8\delta^3(-7+34(-1+\theta)\theta)+4\delta^2(1+\theta)(51+2\theta(-35+9\theta)))\end{pmatrix}}{2(1+4\delta+\theta)(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta))^2}$
Total Surplus	$z+\dfrac{\begin{pmatrix}(c-\alpha)^2(832\delta^6+64\delta^5(15-4\theta)+(1+\theta)^4(36+\theta(-28+3\theta))+4\delta(1+\theta)^3(66+\theta(-49+6\theta))\\ +16\delta^4(-96+\theta(-4+9\theta))+4\delta^2(1+\theta)^2(97+3\theta(-34+11\theta))+16\delta^3(1+\theta)(-52+\theta(-16+23\theta)))\end{pmatrix}}{4(1+4\delta+\theta)(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta))^2}$

Table 9: Economies of Scope calculations

Case I Prices	$p^{*}_{1,\dots,6}=\dfrac{\alpha\left(-1+2\delta-\theta\right)\left(-1+\theta\right)+k\left(1+2\delta-4\delta^{2}+\theta\right)+m\left(1+2\delta-4\delta^{2}+\theta\right)}{2-4\delta^{2}+\theta+2\delta\theta-\theta^{2}}$
Case I Quantities	$q^{*}_{1,\dots,6}=\dfrac{\left(\alpha-k-m\right)\left(-1-2\delta+4\delta^{2}-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^{2}-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}$
Case I Profits	$\pi^{*}_{1,\dots,6}=\dfrac{\left(k+m-\alpha\right)^{2}\left(1-\theta\right)\left(8\delta^{3}+\left(1+\theta\right)^{2}-4\delta^{2}\left(2+\theta\right)\right)}{\left(1+4\delta+\theta\right)\left(2-4\delta^{2}+\theta+2\delta\theta-\theta^{2}\right)^{2}}$
Case II Prices	$p^{*}_{n=1,3,5}=\dfrac{\left(\begin{array}{l}\alpha\left(-1+\theta\right)\left(-12\delta^{2}+2\delta\left(2+\theta\right)+\left(1+\theta\right)\left(2+\theta\right)\right)+k\left(24\delta^{3}-\left(1+\theta\right)\left(2+\theta\right)-2\delta\left(4+5\theta\right)\right)\\+m\left(8\delta^{3}\left(1+2\lambda\right)-\left(1+\theta\right)\left(\theta+2\lambda\right)+4\delta^{2}\left(-1+\theta+\lambda-\theta\lambda\right)-2\delta\left(1+2\theta+3\left(1+\theta\right)\lambda\right)\right)\end{array}\right)}{24\delta^{3}-12\delta^{2}\left(-1+\theta\right)+\left(-2+\theta\right)\left(1+\theta\right)\left(2+\theta\right)+2\delta\left(-6+\left(-4+\theta\right)\theta\right)}$ $p^{*}_{n=2,4,6}=\dfrac{\left(\begin{array}{l}-\alpha\left(-1+2\delta-\theta\right)\left(-1+\theta\right)\left(2+6\delta+\theta\right)+k\left(1+2\delta\right)\left(6\delta\left(-1+2\delta\right)-\left(1+\theta\right)\left(2+\theta\right)\right)\\+m\left(1+2\delta\right)\left(-2\delta\left(2+\lambda\right)+4\delta^{2}\left(2+\lambda\right)-\left(1+\theta\right)\left(2+\theta\lambda\right)\right)\end{array}\right)}{24\delta^{3}-12\delta^{2}\left(-1+\theta\right)+\left(-2+\theta\right)\left(1+\theta\right)\left(2+\theta\right)+2\delta\left(-6+\left(-4+\theta\right)\theta\right)}$
Case II Quantities	$q^{*}_{n=1,3,5}=\dfrac{\begin{array}{l}\left(1+2\delta\right)(k\left(-1+\theta\right)\left(-12\delta^{2}+2\delta\left(2+\theta\right)+\left(1+\theta\right)\left(2+\theta\right)\right)-\alpha\left(-1+\theta\right)\left(-12\delta^{2}+2\delta\left(2+\theta\right)+\left(1+\theta\right)\left(2+\theta\right)\right)\\+m\left(8\delta^{3}\left(-1+\lambda\right)-4\delta^{2}\left(-1+\theta\right)\left(1+2\lambda\right)+2\delta\left(1+2\theta+\left(-3+\left(-1+\theta\right)\theta\right)\lambda\right)+\left(1+\theta\right)\left(\theta+\left(-2+\theta^{2}\right)\lambda\right)\right))\end{array}}{\left(-1+\theta\right)\left(1+4\delta+\theta\right)\left(24\delta^{3}-12\delta^{2}\left(-1+\theta\right)+\left(-2+\theta\right)\left(1+\theta\right)\left(2+\theta\right)+2\delta\left(-6+\left(-4+\theta\right)\theta\right)\right)}$ $q^{*}_{n=2,4,6}=\dfrac{\left(1+2\delta-4\delta^{2}+\theta\right)\left(k\left(-1+\theta\right)\left(2+6\delta+\theta\right)-\alpha\left(-1+\theta\right)\left(2+6\delta+\theta\right)+m\left(-2+4\delta^{2}\left(-1+\lambda\right)+\theta\left(\theta+\lambda\right)+2\delta\left(-4+\lambda+\theta\left(2+\lambda\right)\right)\right)\right)}{\left(-1+\theta\right)\left(1+4\delta+\theta\right)\left(24\delta^{3}-12\delta^{2}\left(-1+\theta\right)+\left(-2+\theta\right)\left(1+\theta\right)\left(2+\theta\right)+2\delta\left(-6+\left(-4+\theta\right)\theta\right)\right)}$

Figure 1: Case I Industry Structure

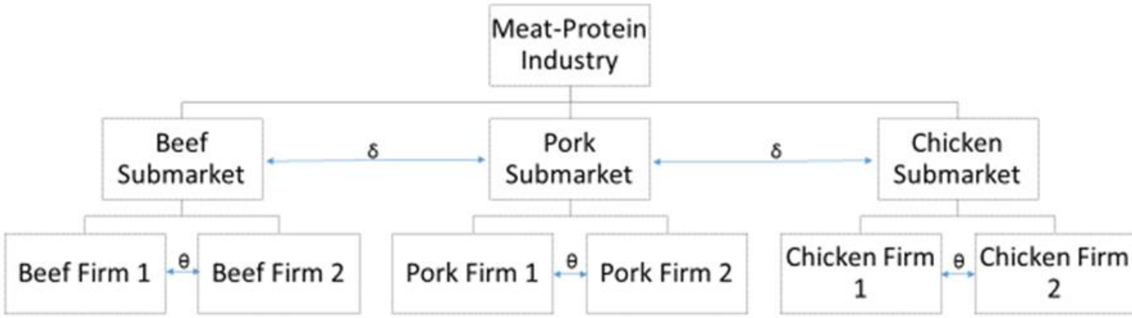


Figure 2: Case II Industry Structure

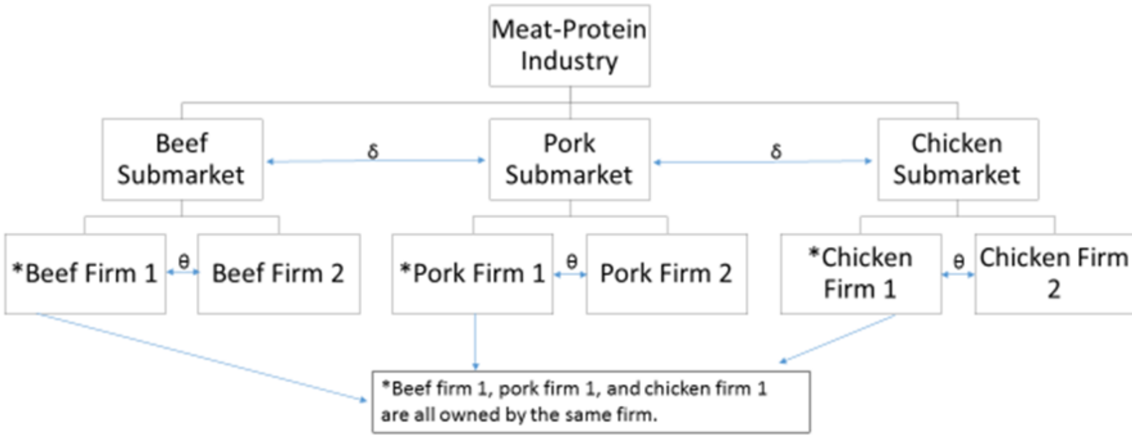


Figure 3: Case III Industry Structure

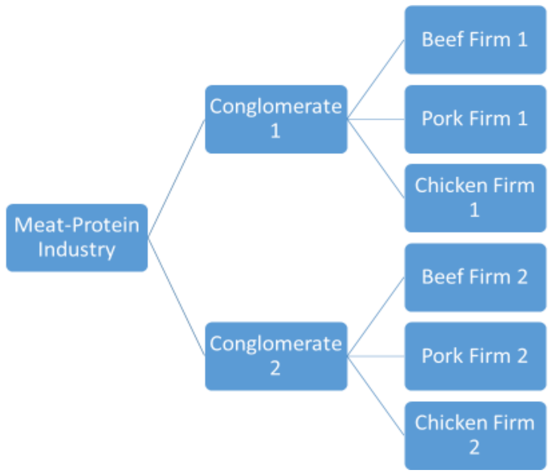


Figure 4: Case IV Industry Structure

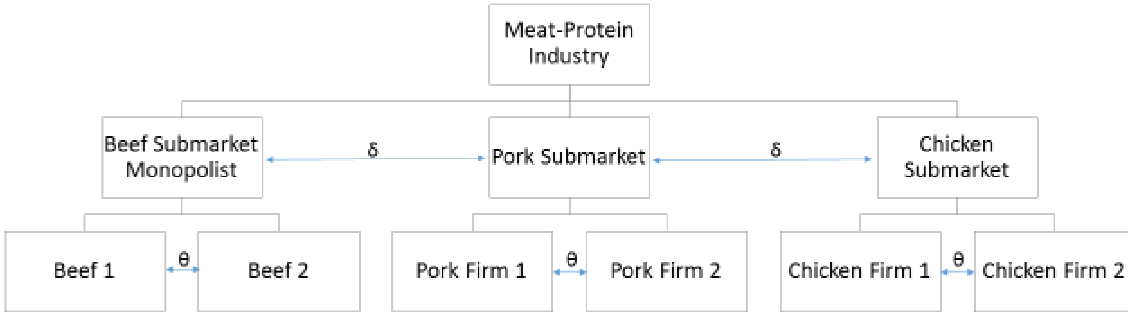
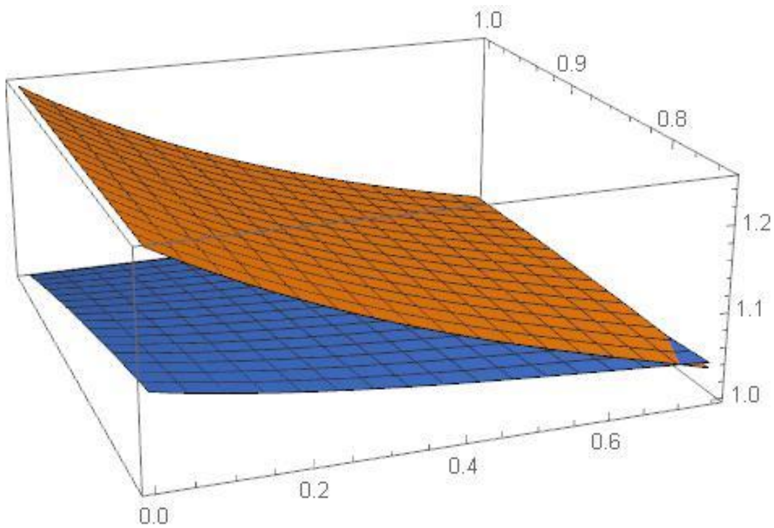
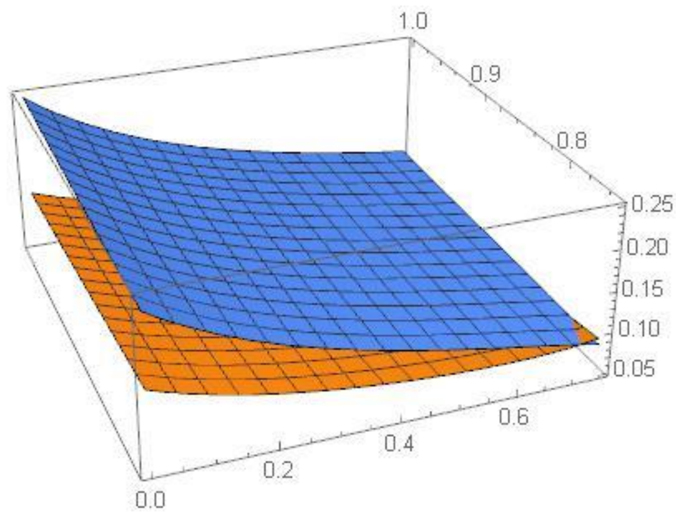


Figure 5: Comparison of the Prices Charged by a Sub-market Monopolist and a Multiproduct Firm



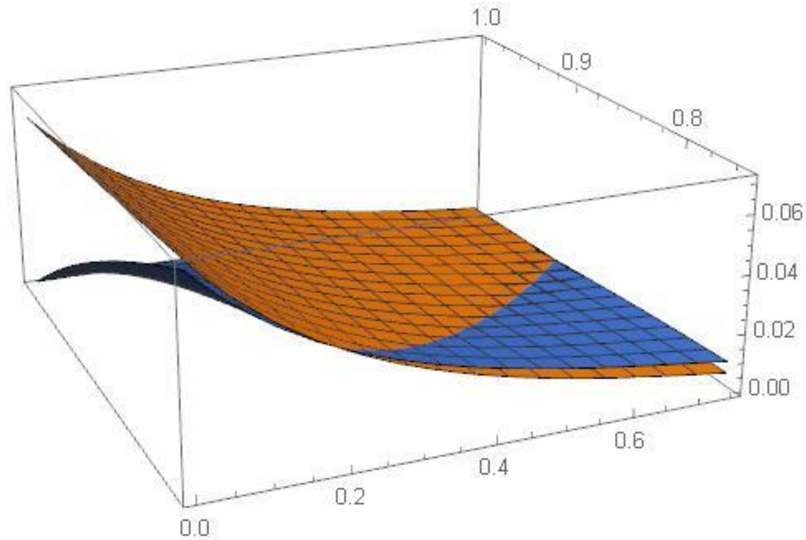
Note: Submarket-monopolist is shown in orange. Multiproduct firm is shown in blue. To aid in graphing, α=1.5 and c=1.

<u>Figure 6: Comparison of Output Quantities for One Product for a Sub-market Monopolist and a Multiproduct Firm</u>



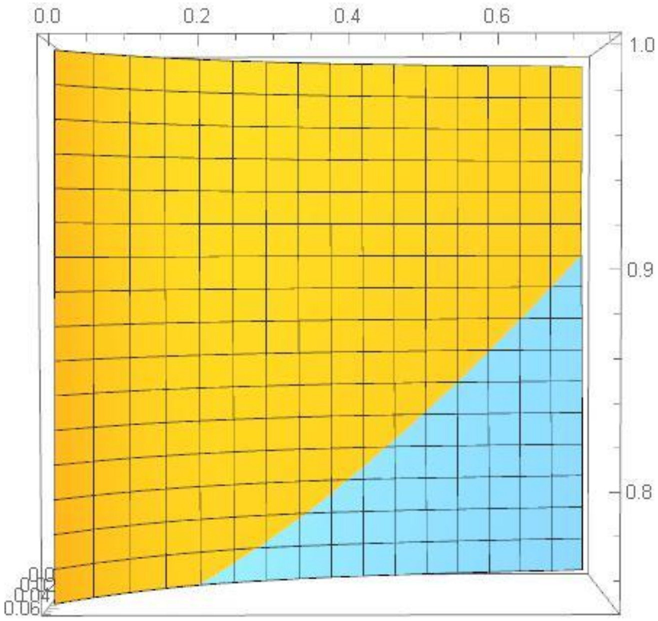
Note: Submarket-monopolist is shown in orange. Multiproduct firm is shown in blue. To aid in graphing, α=1.5 and c=1.

Figure 7: Comparison of Total Profits for a Sub-market Monopolist vs. Multiproduct Firm



Note: Submarket-monopolist is shown in orange. Multiproduct firm is shown in blue. To aid in graphing, α=1.5 and c=1.

Figure 8: Vertical View Comparison of Total Profits of Submarket Monopolist and Multiproduct Firm



Note: Submarket-monopolist is shown in orange. Multiproduct firm is shown in blue. To aid in graphing, α=1.5 and c=1.