Measuring U.S. Agriculture Productivity: Primal vs. Dual Approaches

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Introduction

There is an ongoing discussion about whether U.S. agricultural productivity growth has recently slowed down. Differing views are supported by productivity estimates obtained from different datasets and econometric approaches, as well as primal versus dual representations of the underlying technology. This study contributes to the ongoing discussion by shedding light on how the choice of dual versus primal approaches, and the imposition of certain restrictions in estimation, affect the estimated productivity trends when the same dataset is used across models.

Plastina and Lence (2015) were the first to assess the effects of jointly imposing monotonicity and concavity conditions derived from economic theory on an aggregate representation of U.S. agricultural technology in the primal space. Previous applied production studies using flexible functional forms fell short of imposing both monotonicity and curvature conditions in estimation, and instead reported the proportion of the sample for which those conditions did not hold, warning readers about potential unknown biases introduced by those observations that did not conform to production theory.

Plastina and Lence (2015) found that not only the shapes but also the locations of the probability density functions (pdfs) of the parameter estimates were significantly affected by the imposition of those theoretical restrictions. As a result, output elasticities and multifactor productivity estimates also differed substantially between restricted and unrestricted models. Finally, for the estimates recovered from the dataset (Ball et al. 2004) to conform to production theory, both concavity and monotonicity at each data point had to be imposed in estimation.

The present study extends the analysis by Plastina and Lence (2015) into the dual space, and provides an encompassing comparison of productivity measures and technological characterizations of U.S. agriculture not only across restricted and unrestricted models, but also across primal and dual approaches.

The goals of the present study are twofold. The first goal is to analyze the effects on the estimated parameter pdfs caused by the restrictions stemming from production theory, for the case of a variable cost function representation of U.S. agricultural technology. The second goal is to evaluate the impact on multifactor productivity estimates of using dual versus primal representations of the agricultural technology.

Model

Since technology constrains the optimizing behavior of economic agents, one should be able to use an accurate representation of optimizing behavior to study the technology (Chambers 1996, p. 49). Assuming (1) the existence of a production function, $y = f(X)$, that shows the maximum output, y, attainable from an arbitrary vector $X \equiv [x_1, \cdots, x_n]$ comprising the levels of n inputs; (2) which satisfies monotonicity and weak essentiality in X; and (3) that the input requirement set, $B(y) = \{X: f(X) \geq y\}$, is closed, non-empty and convex; then a well-defined variable cost function $C = C(W, y)$ exists which exhibits the following characteristics (Chambers 1996):[1]

1. Non-negativity, $C = C(W, y) \geq 0$.
2. Non-decreasing in input prices, $W \equiv [w_1, \cdots, w_n]$. If any input price increases, cost must not decrease, $\nabla_W C(W, y) \geq 0$.
3. Concave and continuous in W. Concavity is a direct result of the cost minimizing behavior (input price changes generate opposite direction changes in input utilization, $\nabla W \cdot \nabla X \leq 0$), and it does not impose any condition on the underlying technology. Assuming that the cost function is twice differentiable with respect to input prices, concavity requires the matrix of second order derivatives of the cost function with respect to input prices, $H \equiv \nabla_W^2 C(W, y)$, to be negative semidefinite.

[1] If the input requirement set is convex and monotone, then the technology represented by the cost function will be identical to the true input requirement set. If the true input requirement set is non-convex or non-monotone, the derived input requirement set will be a convex and monotone version of the true set and, most importantly, the derived technology will have the same cost function as the true one (Varian 1992, Ch. 6).

4. Positively linearly homogeneous in W, $\lambda C(W, y) = C(\lambda W, y)$ for any $\lambda > 0$. This property implies that only relative prices matter to economically optimizing agents.
5. Non-decreasing in y.

The econometric estimation of the cost function requires selecting a specific functional form for the latter. In the present study, the following generalized quadratic cost function is used to represent the optimizing behavior:

(1) $$C(W, y, t) = \gamma_0 + \sum_{i=1}^{n} \gamma_i w_i + \gamma_t t + \gamma_y y + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} w_i w_j + \sum_{i=1}^{n} \gamma_{ti} w_i t + \sum_{i=1}^{n} \gamma_{yi} w_i y + \gamma_{yt} y t + \frac{1}{2} \gamma_{tt} t^2 + \frac{1}{2} \gamma_{yy} y^2,$$

where $\gamma_{ij} = \gamma_{ji}$ by Young's theorem. To allow for changes in the shape of the cost function through time, expression (1) incorporates a time trend, t, that enters the function in levels, interacted with inputs, and squared.

There are good reasons for employing the generalized quadratic function (1) for estimation purposes. Most importantly, it is a flexible functional form, both in the sense of being a second-order Taylor series (numerical) approximation to an arbitrary non-linear function, and in the sense of being a second-order differential approximation (with its function value, gradient, and Hessian equal to the corresponding magnitudes for any arbitrary general non-linear function evaluated at a certain level of its underlying arguments). In addition, the generalized quadratic allows for imposition concavity globally[2] in estimation, and is self-dual.[3]

In terms of the generalized quadratic, the aforementioned properties of the cost function can be expressed as a set of parametric restrictions, as follows. First, non-negativity requires that

(2) $$C(W, y, t) = \gamma_0 + \sum_{i=1}^{n} \gamma_i w_i + \gamma_t t + \gamma_y y + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} w_i w_j + \sum_{i=1}^{n} \gamma_{ti} w_i t + \sum_{i=1}^{n} \gamma_{yi} w_i y + \gamma_{yt} y t + \frac{1}{2} \gamma_{tt} t^2 + \frac{1}{2} \gamma_{yy} y^2 \geq 0.$$

[2] Alternative flexible functional forms, such as the translog, do not allow for the global imposition of concavity in estimation. Using filters, concavity can only be imposed locally in estimation (for a particular point in time, or at the means of the data).

[3] Self-duality will prove useful to expand the present analysis to include profit functions in future research.

Second, a cost function non-decreasing in W requires that

(3) $$\frac{\partial C(W,y,t)}{\partial w_i} = x_i(W,y,t) = \gamma_i + \sum_{j=1}^{n} \gamma_{ij} w_j + \gamma_{ti} t + \gamma_{yi} y \geq 0,$$

where the first equality holds by Sheppard's lemma, and $x_i(W,y)$ is the unique, cost minimizing demand for input x_i. Third, concavity requires the Hessian,

(4) $$H \equiv \nabla_W^2 C(W,y), = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & \ddots & \vdots \\ \gamma_{1n} & \cdots & \gamma_{nn} \end{bmatrix},$$

to be negative semidefinite. Fourth, homogeneity of degree 1 in W can be imposed in estimation through the following set of $n+1$ restrictions:

(5a) $\sum_{j=1}^{n} \gamma_{ij} w_j = 0$, for $i = 1, \ldots, n$

(5b) $$\gamma_0 + \gamma_t t + \gamma_y y + \gamma_{yt} yt + \frac{1}{2}\gamma_{tt} t^2 + \frac{1}{2}\gamma_{yy} y^2 = 0.$$

Finally, the cost function is non-decreasing in y if the marginal cost is non-negative:

(6) $$\frac{\partial C(W,y,t)}{\partial y} = \gamma_y + \sum_{i=1}^{n} \gamma_{yi} w_i + \gamma_{yt} t + \gamma_{yy} y \geq 0.$$

To analyze the effects of imposing cost function restrictions in the estimation of equation (1), we consider the following models:

- Model 1: Unrestricted estimation
- Model 2: Non-negativity constrains - i.e. conditions (2), (3), and (6) - imposed in estimation at data means.
- Model 3: Non-negativity constrains imposed at all data points in estimation.
- Model 4: Concavity imposed in estimation.
- Model 5: Concavity and non-negativity constrains imposed in estimation at data means.

- Model 6: Concavity and non-negativity constrains imposed at all data points in estimation.
- Model 7: Linear homogeneity, i.e. equations (5a) and (5b), imposed in estimation at data means.
- Model 8: Concavity and linear homogeneity imposed in estimation at data means.
- Model 9: Concavity, non-negativity constrains, and linear homogeneity imposed in estimation at data means.
- Model 10: Concavity, non-negativity constrains at all data points, and linear homogeneity at data means imposed in estimation.

More specifically, under non-negativity constrains at data means, conditions (2), (3), and (6) are satisfied when evaluated at the sample averages of the input prices and output levels, and the time variable. Non-negativity constrains at all data points is a far more stringent constraint, as it involves satisfying restrictions (2), (3), and (6) at each of the input prices and output levels, and the time values contained in the data set.

For each model, the elasticity of cost with respect to output, and the rate of disembodied technical change are calculated using expressions (7) and (8), respectively:

(7) $$\varepsilon_{CY} = \frac{\partial C(W,y,t)}{\partial y}\frac{y}{C(W,y,t)} = \left(\gamma_y + \sum_{i=1}^{n}\gamma_{yi}w_i + \gamma_{yt}t + \gamma_{yy}y\right)\frac{y}{C(W,y)},$$

(8) $$\varepsilon_{Ct} = \frac{\partial C(W,y,t)}{\partial t}\frac{1}{C(W,y,t)} = \left(\gamma_t + \sum_{i=1}^{n}\gamma_{ti}w_i + \gamma_{yt}y + \gamma_{yt}t\right)\frac{1}{C(W,y)}.$$

The elasticity of cost with respect to output is indicative of the returns to scale in production at the cost-minimizing bundle: if $\varepsilon_{CY} < 1$ production is characterized by increasing returns to scale; and if $\varepsilon_{CY} > 1$ production is characterized by decreasing returns to scale (Chambers 1988). The rate of disembodied technical change indicates the average percentage decline in costs due to increases in multifactor productivity.

Price elasticities for each variable input and the semi-elasticity of input demand with respect to time are calculated using expressions (9) and (10), respectively:

(9) $$\varepsilon_{x_i w_j} = \frac{\partial^2 C(W,y,t)}{\partial w_i \partial w_j} \frac{w_j}{\partial C(W,y,t)/\partial w_i} = \frac{\partial x_i(W,y,t)}{\partial w_j} \frac{w_j}{x_i(W,y,t)} = \frac{\gamma_{ij} w_j}{x_i(W,y,t)},$$

(10) $$\varepsilon_{x_i t} = \frac{\partial^2 C(W,y,t)}{\partial w_i \partial t} \frac{1}{\partial C(W,y,t)/\partial w_i} = \frac{\gamma_{ti}}{x_i(W,y,t)}.$$

We first analyze the differences in the estimated elasticities, semi-elasticities, and the rate of technical change across models to examine the implications of imposing cost function restrictions in the estimation of equation (1). Then, we compare the returns to scale and the rate of technical change obtained in this study against similar measures obtained from a generalized quadratic production function fit to the same data set (Plastina and Lence 2015).

Data

Models 1 through 6 are estimated by employing the official USDA panel dataset on agricultural production for the United States (USDA 2015, Table 23[4]). The dataset is described in Ball et al. (2004) and its main use is the calculation of TFP as the ratio of an index of output quantities to an index of input quantities. The panel was specifically developed to measure agricultural productivity. Earlier versions of the data were used by Morrison Paul et al. (2001), Huffman et al. (2002), and Wang et al. (2012) to evaluate agricultural productivity by means of a cost function; and by Plastina and Lence (2015) to evaluate productivity in primal space.

The dataset contains one aggregate agricultural output and n = 3 variable inputs (capital, labor, and materials) for each of the S = 48 contiguous states over the period 1960-2004, i.e., T = 45 annual observations. All quantities are measured as transitive implicit Fisher quantity indexes, or “EKS” indexes based on the work of Eltetö and Köves (1964), and Szulc (1964), calculated with price indexes with bases equal to unity in Alabama in 1996. The transitivity of the quantity indexes ensures that indexes are comparable across states and years.

[4] The dataset was updated on May 9, 2016, and the aggregate capital variable is no longer reported. Capital is disaggregated into Capital services excluding land and Land service flows in the new dataset.

The output quantity, y, measures the aggregate production of livestock, dairy, poultry, eggs, grains, oilseeds, cotton, tobacco, fruit, vegetables, nuts, and other miscellaneous outputs. Capital, $K \equiv x_1$, measures the service flows of real estate, durable equipment, and stocks of inventories. Labor, $L \equiv x_2$, measures the quality-adjusted amount of hired and self-employed labor. Materials, $M \equiv x_3$, include fertilizers, pesticides, energy and other miscellaneous inputs. Cost, $C \equiv C(W, y, t)$, is constructed as the summation of the products of input quantities and input prices. Summary statistics for output and the three inputs are reported in Table 1.

Estimation Methods

Estimation is conducted by setting up equations (1) and (3) as a seemingly unrelated regressions (SUR) system. The dependent variables in the four SUR equations are cost (corresponding to equation (1)), and capital, labor, and materials (corresponding to equation (3)). The system is estimated by imposing the restrictions in the regression coefficients stated in equations (1) and (3).[5] Because all four dependent variables are highly autocorrelated, the first lag of the corresponding dependent variable was added as an explanatory variable to each of the four equations, and their coefficients denoted by α_C, α_K, α_L, and α_M for cost, capital, labor, and materials, respectively.

The parameters of the SUR model are the 21 regression coefficients included in equations (1)-(3), the four coefficients corresponding to the lagged dependent variables, and the ten parameters involved in the covariance matrix of the regression residuals (consisting of four standard deviations σ_i and six correlation coefficients ρ_{ij}). We estimate the model's 35 parameters by employing Bayesian Hamiltonian Monte Carlo (HMC) sampling (Duane et al., 1987; Neal, 1994, 2011). The SUR HMC model is fitted using version 2.9.0 of the RStan program (Stan Development Team, 2015) and version 3.3.0 of the R software (https://www.r-project.org/).

[5] For example, the regression coefficient for the cross product of time and wages in the cost equation (1), γ_{tW_L}, is the same as the time coefficient in equation (3) for labor.

Bayesian techniques are quite useful for the present application, because they allow us to impose the desired estimation restrictions in a straightforward manner. Another advantage of the Bayesian approach is that it yields full posterior distributions for the parameters of interest. This feature is particularly useful when researchers try to characterize parameters with skewed posteriors, such as the parameters subject to concavity restrictions. Further, pdfs for the elasticities and the rate of technical change can be computed directly from the estimated pdfs for the underlying parameters, rather than by quadratic approximation (e.g., as in the delta method).

The Bayesian procedure is explained next by focusing on Model 6 (i.e., the most restrictive specification). Models 1 through 5 are estimated using the same method, except for the relaxation of the corresponding parameter constraints. Estimation proceeds by conditioning on the initial set of observations (i.e., the variable values observed at t = 1) (Lancaster 2004, Ch. 9). The priors are half-Cauchy for the standard deviations of the residuals (σ_i ~ Cauchy(0, 2.5) for σ_i > 0), LKJ for the correlation matrix of the residuals (Residual Correlation Matrix ~ LKJcorr(4)), and weakly informative normal for the lagged-dependent variable regression coefficients, as well as for the regression coefficients not included in the Hessian (γ_i ~ Normal(0, 5)). Since a negative semidefinite Hessian $H = [\gamma_{w_K w_K}, \gamma_{w_K w_L}, \gamma_{w_K w_M}; \gamma_{w_K w_L}, \gamma_{w_L w_L}, \gamma_{w_L w_M}; \gamma_{w_K w_M}, \gamma_{w_L w_M}, \gamma_{w_M w_M}]$ implies that the symmetric (3 × 3) matrix $\Omega \equiv -H$ is positive semidefinite, we compute H by first setting the same priors for Ω as for the covariance matrix, and then recovering the Hessian parameters from the relationship $H \equiv -\Omega$. That is, Ω = D(3) Corr(3) D(3), where D(3) is a (3 × 3) diagonal matrix and Corr(3) is a (3 × 3) of matrix correlation coefficients, so $[D(3)]_{ii}$ ~ Cauchy(0, 2.5) for $[D(3)]_{ii}$ > 0, and Corr(3) ~ LKJcorr(4). Finally, to ensure positive costs, capital, labor, materials, and output at all observation points, minimum values of coefficients γ_0, γ_{W_K}, γ_{W_L}, γ_{W_M}, and γ_Y are imposed so as to meet the positivity restrictions.[6]

[6] For example, the restriction $\gamma_y \geq -\min(\gamma_{yt}\, t + \gamma_{yw_K}\, w_K + \gamma_{yw_L}\, w_L + \gamma_{yy}\, y)$ is imposed to guarantee that condition (6) is met.

Results and Discussion

Estimation results for Models 1 through 6 for the state of Iowa are reported in Table 2. For each model, this table shows the means of the parameters of interest, as well as their standard deviations, medians, and 95% credible intervals.[7] For example, the unrestricted (i.e., Model 1) mean of parameter γ_Y equals -0.6251, with a standard deviation of 0.5701, a median equal to -0.6228, and a 95% credible interval ranging from -1.7344 to 0.4542.

It is evident from the figures in Table 2 (compare, e.g., the estimates of parameter γ_Y and the likelihood values across models) that the choice of model has only minor implications for the characterization of the production technology, and therefore for the policy recommendations stemming from it. This finding is in stark contrast to the conclusion in Plastina and Lence (2015) that imposing restrictions stemming from production theory in estimation of a generalized quadratic production function has a substantial effect on the characterization of U.S. agricultural production technology.

The point estimates of the coefficients for the lagged dependent variables range between $\alpha_C = 0.4831$ for Model 3, and $\alpha_C = 0.5334$ for Model 4; $\alpha_K = 0.9079$ for Model 3, and $\alpha_K = 0.9432$ for Model 4; $\alpha_L = 0.3419$ for Model 6, and $\alpha_L = 0.3678$ for Model 2; $\alpha_M = 0.3770$ for Model 3, and $\alpha_M = 0.4533$ for Model 4. In all instances, credible intervals are very tight and with lower bounds far from zero, providing a strong indication that adjustments occur over multiple periods. The derived demand for capital exhibits high inertia, but significantly less than 1 for all models but Model 3. The goodness of fit is consistent across Models 1 through 6, indicating that non-negativity and concavity restrictions are not too burdensome to the explanatory power of the model. It must be noted that, in general, the same qualitative results arise from Models 1 through 6.

The elasticity of cost with respect to output is positive and significantly lower than 1 in all models (except Model 4), indicating that agricultural production in Iowa is characterized by

[7] Credible intervals are the Bayesian analogs of confidence intervals. The upper and lower bound of the 95% credible intervals reported here are the 2.5% and 97.5% quantiles of the corresponding posterior distributions.

increasing returns to scale in the dual space (Table 3). This is an interesting finding given that Plastina and Lence (2015) report decreasing returns to scale for the U.S. aggregate in primal space across all models (those with concavity[8] imposed in estimation, and those where concavity was not imposed in estimation), using the same database.

The rate of disembodied technical change is significantly different from zero in all models, and the point estimates range between -0.74% for Model 2 and -0.84% for Model 3. Plastina and Lence (2015) report a 1.5% rate of technical change in primal space for Iowa, and 1.45% for the U.S. aggregate for 1960-2004.

As expected, the own-price elasticity of capital is negative and significantly different from zero, ranging from -0.1326 in Model 6 to -0.1512 in Model 1 (Table 4). The price elasticity of capital with respect to labor is negative, but not significantly different from zero in Models 1-6. The price elasticity of capital with respect to materials is positive and significantly different from zero for all models (except Model 4), with a point estimate ranging from 0.0564 in Model 4 to 0.0958 in Model 3.

The own price elasticity of labor is negative and significantly different from zero, ranging from -0.0866 in Model 4 to -0.0612 in Model 3 (Table 5). The price elasticity of labor with respect to capital is negative, but not significantly different from zero in Models 1-6. Similarly, the price elasticity of labor with respect to materials is negative, but not significantly different from zero in Models 1-6.

The own price elasticity of materials is negative and significantly different from zero for Models 4-6, ranging from -0.0859 in Model 4 to -0.0934 in Model 6 (Table 6). For Models 1-3, the point estimates of the own price elasticity of materials have the expected sign, but the 95% confidence intervals include the null value. The price elasticity of materials with respect to capital is positive (ranging from 0.0151 in Model 4 to 0.0258 in Model 3), and significantly different from zero in all Models except for Models 4 and 5. The price elasticity of materials with respect to labor is negative, but not significantly different from zero in Models 1-6.

[8] Imposing concavity on the production function rules out increasing returns to scale.

Succinctly, the price elasticities indicate that (1) labor demand is not strongly responsive to capital or material prices; (2) capital and materials are substitutes; and (3) own price effects on input demand dominate cross-price effects.

The semi-elasticities of input demands with respect to time suggest that technical change has not been Hicks-neutral. Instead, technical change been labor-saving ($\varepsilon_{Lt} < 0$ and significant across models in Table 5), material-saving ($\varepsilon_{Mt} < 0$ but not significant across models in Table 6), and capital-using ($\varepsilon_{Kt} > 0$ but not significant across models in Table 4). Plastina and Lence (2015) also report non-Hicks-neutral technical change, although they conclude that over time output has become more responsive to changes in materials and labor, and less responsive to changes in capital.

Concluding Remarks

The economic theory of producer behavior requires certain conditions to hold in order for a functional form to be representative of a production technology. Agricultural production studies are usually conducted using classical econometric methods that make it difficult, if not impossible, to impose such restrictions in flexible functional forms. Therefore, conditions required by economic theory need not hold in estimation. Using state-level panel data on U.S. agricultural production to fit a generalized quadratic cost function, we estimated six models characterized by different restrictions for Iowa. More specifically, Model 1 is unrestricted, whereas Models 2 through 6 impose respectively the following restrictions in estimation: non-negativity of the cost function at data means, non-negativity of the cost function at all data points, concavity, both concavity and non-negativity of the cost function at data means, and both concavity and non-negativity of the cost function at all data points.

Each model is estimated using Bayesian methods. A desirable feature of the proposed Bayesian procedure is that it greatly facilitates imposing concavity, non-negativity and homogeneity conditions. In addition, the procedure yields simulated parameter values from their posterior pdfs, which can be used to compute simulated pdfs for functions of such parameters, such as price elasticities, elasticities of scale, and technical change.

Contrary to what Plastina and Lence (2015) found in the primal space, imposing restrictions in the cost function does not qualitatively change the characterization of the underlying agricultural technology.

Disembodied technical change generated, on average, cost savings of about 0.8% per year over 1961-2004 in Iowa.

This is an ongoing work and we plan to estimate the model using all 48 states, and expand the analysis to include homogeneity of degree 1 for the cost function (Models 7-10). Finally, the analysis will be completed by comparing estimates of technical change on the dual space with estimates obtained in primal space, using the same database.

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Table 1. Descriptive statistics, 1961-2004

Indexes	Unit	Mean	Median	Standard Deviation	Minimum	Maximum
Cost value	Million $1996	9,291.1	10,860.5	3,557.2	3,427.4	14,127.6
Capital quantity	Million $1996	2,353.9	2,257.8	500.5	1,716.6	3,275.4
Labor quantity	Million $1996	5,127.1	5,309.9	1,544.8	2,541.7	7,918.6
Materials quantity	Million $1996	6,275	6,202	586	5,518	7,548
Price of Labor	Unitless*	0.641	0.802	0.363	0.148	1.105
Price of Capital	Unitless*	0.547	0.434	0.408	0.129	1.580
Price of Materials	Unitless*	0.894	1.057	0.341	0.359	1.319
Output quantity	Million $1996	13,034.6	12,866.2	1,828.9	10,138.5	16,362.1

*Ratio of prices in each state and year to corresponding price in Alabama in 1996.

Table 2. Parameter estimates for Models 1 through 6.

γ_0	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	3.7747	3.7386	2.1589	3.8845	3.7742	2.1664
Median	3.7587	3.8121	2.2238	3.9196	3.7425	2.2734
Stdev	3.7304	3.7258	2.4089	3.8359	3.6925	2.4022
Lower Bound^	-3.3771	-3.5357	-2.7400	-3.7407	-3.3768	-2.7496
Higher Bound^	10.8817	11.1129	6.7974	11.5088	11.2281	6.7864

γ_t	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.1570	0.1595	0.1211	0.1513	0.1550	0.1154
Median	0.1560	0.1579	0.1174	0.1506	0.1551	0.1121
Stdev	0.1139	0.1127	0.0783	0.1192	0.1127	0.0760
Lower Bound^	-0.0717	-0.0657	-0.0230	-0.0769	-0.0583	-0.0226
Higher Bound^	0.3823	0.3879	0.2823	0.3922	0.3882	0.2708

$\boldsymbol{\gamma_{tt}}$	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.0007	0.0007	0.0004	0.0010	0.0011	0.0006
Median	0.0008	0.0007	0.0004	0.0010	0.0011	0.0006
Stdev	0.0029	0.0028	0.0022	0.0029	0.0028	0.0021
Lower Bound^	-0.0051	-0.0050	-0.0038	-0.0048	-0.0046	-0.0036
Higher Bound^	0.0063	0.0061	0.0048	0.0065	0.0065	0.0046

$\boldsymbol{\gamma_{tWk}}$	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.0066	0.0065	0.0041	0.0078	0.0074	0.0049
Median	0.0068	0.0066	0.0044	0.0077	0.0073	0.0050
Stdev	0.0051	0.0051	0.0047	0.0049	0.0047	0.0039
Lower Bound^	-0.0038	-0.0040	-0.0059	-0.0018	-0.0020	-0.0031
Higher Bound^	0.0162	0.0164	0.0125	0.0178	0.0167	0.0124

$\boldsymbol{\gamma_{tWl}}$	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.0445	-0.0434	-0.0491	-0.0538	-0.0547	-0.0559
Median	-0.0447	-0.0433	-0.0490	-0.0536	-0.0550	-0.0566
Stdev	0.0204	0.0203	0.0209	0.0174	0.0180	0.0175
Lower Bound^	-0.0834	-0.0823	-0.0916	-0.0883	-0.0907	-0.0891
Higher Bound^	-0.0042	-0.0040	-0.0078	-0.0196	-0.0185	-0.0207

^Bounds of 95% Credible Interval

Table 2. Parameter estimates for Models 1 through 6 (continued).

γ_{tWm}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.0139	-0.0127	-0.0221	-0.0166	-0.0169	-0.0198
Median	-0.0131	-0.0124	-0.0211	-0.0175	-0.0181	-0.0207
Stdev	0.0200	0.0194	0.0205	0.0138	0.0140	0.0136
Lower Bound^	-0.0559	-0.0532	-0.0646	-0.0412	-0.0411	-0.0443
Higher Bound^	0.0237	0.0237	0.0157	0.0125	0.0132	0.0087

γ_{Wk}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.2215	0.2197	0.1526	0.2481	0.2392	0.1789
Median	0.2211	0.2213	0.1562	0.2452	0.2384	0.1798
Stdev	0.1663	0.1669	0.1595	0.1643	0.1605	0.1449
Lower Bound^	-0.1011	-0.1181	-0.1785	-0.0748	-0.0784	-0.1036
Higher Bound^	0.5465	0.5398	0.4506	0.5727	0.5448	0.4686

γ_{Wl}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	3.8243	3.7841	3.6529	3.7145	3.7169	3.5287
Median	3.8260	3.7754	3.6574	3.7158	3.7240	3.5335
Stdev	0.8893	0.8587	0.8082	0.9252	0.9044	0.8417
Lower Bound^	2.1075	2.1082	2.0535	1.8379	1.9308	1.8595
Higher Bound^	5.5868	5.5185	5.2481	5.5724	5.4691	5.1659

γ_{Wm}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	1.3104	1.3152	1.1005	1.2079	1.2113	1.1450
Median	1.3103	1.3092	1.1187	1.2050	1.2132	1.1290
Stdev	0.7739	0.7839	0.7711	0.7247	0.7137	0.6911
Lower Bound^	-0.2340	-0.2206	-0.4364	-0.1757	-0.1532	-0.1702
Higher Bound^	2.8484	2.8494	2.5658	2.6340	2.6655	2.5177

γ_{WkWk}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.5553	-0.5529	-0.5173	-0.5345	-0.5289	-0.4868
Median	-0.5546	-0.5554	-0.5222	-0.5343	-0.5324	-0.4891
Stdev	0.1196	0.1187	0.1130	0.1196	0.1213	0.1127
Lower Bound^	-0.7896	-0.7854	-0.7296	-0.7671	-0.7627	-0.6914
Higher Bound^	-0.3215	-0.3106	-0.2895	-0.2883	-0.2788	-0.2516

γ_{WlWl}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.7962	-0.8025	-0.8111	-0.5736	-0.5765	-0.6362
Median	-0.7874	-0.7967	-0.8061	-0.5647	-0.5746	-0.6292
Stdev	0.3139	0.3122	0.3119	0.3197	0.3213	0.3155
Lower Bound^	-1.4376	-1.4071	-1.4193	-1.2249	-1.2097	-1.2723
Higher Bound^	-0.1845	-0.1897	-0.2036	-0.0284	-0.0103	-0.0706

^Bounds of 95% Credible Interval

Table 2. Parameter estimates for Models 1 through 6 (continued).

γ_{WmWm}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.7028	-0.7280	-0.6190	-0.6029	-0.6121	-0.6553
Median	-0.7170	-0.7296	-0.6260	-0.5800	-0.5845	-0.6290
Stdev	0.4546	0.4405	0.4632	0.3432	0.3426	0.3198
Lower Bound^	-1.5799	-1.5613	-1.4802	-1.3175	-1.3644	-1.3383
Higher Bound^	0.2503	0.1740	0.3522	-0.0307	-0.0342	-0.1272

γ_{WkWl}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.0839	-0.0843	-0.0886	-0.0729	-0.0726	-0.0807
Median	-0.0846	-0.0844	-0.0882	-0.0733	-0.0717	-0.0805
Stdev	0.0508	0.0506	0.0493	0.0474	0.0482	0.0447
Lower Bound^	-0.1881	-0.1851	-0.1846	-0.1666	-0.1707	-0.1669
Higher Bound^	0.0142	0.0138	0.0049	0.0174	0.0191	0.0063

γ_{WkWm}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.2148	0.2144	0.2523	0.1483	0.1554	0.1877
Median	0.2149	0.2129	0.2502	0.1497	0.1574	0.1900
Stdev	0.0972	0.0958	0.0879	0.0873	0.0859	0.0766
Lower Bound^	0.0223	0.0252	0.0831	-0.0206	-0.0096	0.0286
Higher Bound^	0.4003	0.4049	0.4296	0.3102	0.3207	0.3366

γ_{WlWm}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.1536	-0.1677	-0.1322	-0.0398	-0.0349	-0.0469
Median	-0.1560	-0.1689	-0.1387	-0.0162	-0.0114	-0.0285
Stdev	0.2554	0.2536	0.2614	0.1449	0.1507	0.1544
Lower Bound^	-0.6506	-0.6708	-0.6311	-0.3761	-0.3890	-0.3921
Higher Bound^	0.3606	0.3212	0.3898	0.2065	0.2275	0.2239

γ_{Y}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.6251	-0.6231	-0.3693	-0.6427	-0.6311	-0.3772
Median	-0.6228	-0.6296	-0.3827	-0.6494	-0.6252	-0.4032
Stdev	0.5701	0.5680	0.3605	0.5868	0.5651	0.3593
Lower Bound^	-1.7344	-1.7462	-1.0280	-1.8034	-1.7689	-1.0592
Higher Bound^	0.4542	0.4948	0.3765	0.5110	0.4595	0.3633

γ_{YY}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.0460	0.0463	0.0254	0.0475	0.0468	0.0269
Median	0.0463	0.0465	0.0259	0.0478	0.0464	0.0286
Stdev	0.0462	0.0458	0.0293	0.0473	0.0455	0.0290
Lower Bound^	-0.0413	-0.0440	-0.0351	-0.0456	-0.0417	-0.0330
Higher Bound^	0.1343	0.1387	0.0786	0.1414	0.1397	0.0816

^Bounds of 95% Credible Interval

Table 2. Parameter estimates for Models 1 through 6 (continued).

γ_{Yt}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.0149	-0.0151	-0.0119	-0.0140	-0.0143	-0.0113
Median	-0.0148	-0.0150	-0.0116	-0.0140	-0.0144	-0.0110
Stdev	0.0092	0.0090	0.0061	0.0096	0.0091	0.0060
Lower Bound^	-0.0328	-0.0336	-0.0246	-0.0334	-0.0332	-0.0238
Higher Bound^	0.0039	0.0026	-0.0009	0.0050	0.0028	-0.0008

γ_{YWk}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.0115	0.0118	0.0162	0.0096	0.0101	0.0148
Median	0.0114	0.0116	0.0157	0.0096	0.0100	0.0147
Stdev	0.0096	0.0096	0.0091	0.0095	0.0092	0.0087
Lower Bound^	-0.0068	-0.0068	-0.0007	-0.0090	-0.0078	-0.0019
Higher Bound^	0.0307	0.0313	0.0348	0.0286	0.0283	0.0321

γ_{YWl}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.0050	0.0063	0.0246	-0.0006	0.0009	0.0214
Median	0.0047	0.0063	0.0242	-0.0018	0.0002	0.0202
Stdev	0.0517	0.0502	0.0483	0.0530	0.0518	0.0493
Lower Bound^	-0.0989	-0.0913	-0.0680	-0.1014	-0.0996	-0.0736
Higher Bound^	0.1090	0.1059	0.1229	0.1086	0.1064	0.1220

γ_{YWm}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.2167	0.2157	0.2534	0.2085	0.2103	0.2379
Median	0.2148	0.2134	0.2511	0.2067	0.2081	0.2374
Stdev	0.0589	0.0577	0.0551	0.0535	0.0537	0.0522
Lower Bound^	0.1094	0.1076	0.1521	0.1076	0.1070	0.1393
Higher Bound^	0.3396	0.3319	0.3675	0.3182	0.3201	0.3415

α_C	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.5146	0.5175	0.4831	0.5334	0.5314	0.5006
Median	0.5107	0.5148	0.4820	0.5298	0.5291	0.4969
Stdev	0.0764	0.0775	0.0675	0.0794	0.0782	0.0713
Lower Bound^	0.3702	0.3707	0.3538	0.3882	0.3854	0.3672
Higher Bound^	0.6734	0.6773	0.6146	0.6984	0.6879	0.6442

α_K	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.9273	0.9263	0.9079	0.9432	0.9399	0.9185
Median	0.9287	0.9271	0.9108	0.9436	0.9409	0.9218
Stdev	0.0324	0.0309	0.0253	0.0314	0.0285	0.0215
Lower Bound^	0.8627	0.8610	0.8497	0.8814	0.8811	0.8704
Higher Bound^	0.9886	0.9835	0.9464	1.0048	0.9894	0.9507

^Bounds of 95% Credible Interval

Table 2. Parameter estimates for Models 1 through 6 (continued).

α_L	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.3604	0.3678	0.3431	0.3530	0.3479	0.3419
Median	0.3586	0.3666	0.3420	0.3534	0.3490	0.3411
Stdev	0.1098	0.1073	0.1066	0.1063	0.1065	0.1032
Lower Bound^	0.1482	0.1612	0.1349	0.1364	0.1415	0.1417
Higher Bound^	0.5775	0.5835	0.5520	0.5650	0.5556	0.5421

α_M	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.4368	0.4432	0.3770	0.4533	0.4493	0.4066
Median	0.4390	0.4444	0.3799	0.4556	0.4469	0.4052
Stdev	0.1148	0.1136	0.1081	0.1072	0.1075	0.1021
Lower Bound^	0.2110	0.2225	0.1651	0.2441	0.2407	0.2120
Higher Bound^	0.6526	0.6626	0.5861	0.6586	0.6638	0.6078

σ_C	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.5019	0.5020	0.4955	0.5030	0.5022	0.5012
Median	0.4964	0.4959	0.4915	0.4977	0.4964	0.4978
Stdev	0.0589	0.0594	0.0556	0.0594	0.0581	0.0572
Lower Bound^	0.4022	0.4031	0.3992	0.4022	0.4048	0.4034
Higher Bound^	0.6310	0.6371	0.6174	0.6316	0.6298	0.6241

σ_K	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.0549	0.0549	0.0557	0.0549	0.0551	0.0559
Median	0.0542	0.0540	0.0551	0.0541	0.0545	0.0550
Stdev	0.0071	0.0074	0.0074	0.0070	0.0073	0.0075
Lower Bound^	0.0431	0.0429	0.0432	0.0431	0.0429	0.0440
Higher Bound^	0.0707	0.0719	0.0718	0.0707	0.0722	0.0734

σ_L	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.3769	0.3759	0.3790	0.3746	0.3747	0.3768
Median	0.3745	0.3715	0.3753	0.3712	0.3699	0.3718
Stdev	0.0437	0.0448	0.0445	0.0436	0.0448	0.0466
Lower Bound^	0.3020	0.3008	0.3020	0.2997	0.2991	0.2999
Higher Bound^	0.4747	0.4738	0.4790	0.4677	0.4724	0.4825

σ_M	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.3341	0.3323	0.3470	0.3230	0.3234	0.3332
Median	0.3289	0.3282	0.3429	0.3187	0.3194	0.3291
Stdev	0.0462	0.0458	0.0462	0.0430	0.0421	0.0435
Lower Bound^	0.2582	0.2556	0.2693	0.2516	0.2509	0.2578
Higher Bound^	0.4383	0.4362	0.4517	0.4197	0.4161	0.4257

^Bounds of 95% Credible Interval

Table 2. Parameter estimates for Models 1 through 6 (continued).

ρ_{CK}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.3201	0.3226	0.3634	0.3075	0.3112	0.3591
Median	0.3257	0.3364	0.3716	0.3151	0.3168	0.3675
Stdev	0.1476	0.1498	0.1387	0.1495	0.1474	0.1361
Lower Bound^	0.0178	0.0014	0.0663	-0.0113	0.0127	0.0665
Higher Bound^	0.5873	0.5902	0.6121	0.5698	0.5802	0.6074

ρ_{CL}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.4551	0.4512	0.4563	0.4243	0.4251	0.4332
Median	0.4644	0.4616	0.4674	0.4333	0.4364	0.4397
Stdev	0.1301	0.1308	0.1288	0.1368	0.1354	0.1328
Lower Bound^	0.1734	0.1698	0.1798	0.1320	0.1314	0.1645
Higher Bound^	0.6815	0.6748	0.6821	0.6639	0.6664	0.6652

ρ_{CM}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.5891	0.5893	0.6383	0.5728	0.5756	0.6214
Median	0.6050	0.6041	0.6537	0.5906	0.5947	0.6380
Stdev	0.1302	0.1307	0.1118	0.1376	0.1386	0.1219
Lower Bound^	0.2906	0.2860	0.3894	0.2645	0.2590	0.3413
Higher Bound^	0.8012	0.7959	0.8195	0.7925	0.7956	0.8095

ρ_{KL}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.0605	-0.0601	-0.0638	-0.0475	-0.0532	-0.0518
Median	-0.0625	-0.0601	-0.0650	-0.0471	-0.0549	-0.0526
Stdev	0.1464	0.1415	0.1418	0.1457	0.1416	0.1429
Lower Bound^	-0.3355	-0.3320	-0.3353	-0.3273	-0.3224	-0.3331
Higher Bound^	0.2282	0.2270	0.2086	0.2364	0.2271	0.2315

ρ_{KM}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.3028	0.3086	0.3956	0.2705	0.2811	0.3781
Median	0.3142	0.3172	0.4049	0.2780	0.2860	0.3851
Stdev	0.1946	0.1875	0.1668	0.1896	0.1819	0.1590
Lower Bound^	-0.1070	-0.0801	0.0385	-0.1193	-0.0859	0.0375
Higher Bound^	0.6496	0.6454	0.6864	0.6196	0.6099	0.6634

ρ_{LM}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.2601	0.2626	0.2369	0.2621	0.2535	0.2478
Median	0.2690	0.2663	0.2454	0.2655	0.2618	0.2516
Stdev	0.1410	0.1411	0.1439	0.1364	0.1392	0.1356
Lower Bound^	-0.0269	-0.0315	-0.0612	-0.0163	-0.0301	-0.0287
Higher Bound^	0.5194	0.5228	0.5015	0.5167	0.5090	0.4978

^Bounds of 95% Credible Interval

Table 2. Parameter estimates for Models 1 through 6 (continued).

LikelihoodP	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	176.5424	176.6852	173.4191	173.7588	173.4520	166.5135
Median	176.8824	177.1389	173.8097	174.0793	173.9746	166.9548
Stdev	4.5932	4.9389	4.5839	4.9116	5.4077	4.9978
Lower Bound^	166.6852	165.7644	163.4715	163.1750	161.3069	156.0331
Higher Bound^	184.6299	184.8873	181.5127	182.2938	182.4634	174.9589

^Bounds of 95% Credible Interval

Table 3. Cost elasticities for Models 1 through 6.

Elasticity of Cost with respect to Output

ε_{CY}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.240426	0.247187	0.28971	0.226974	0.233922	0.2826
Median	0.24264	0.246049	0.282801	0.228653	0.23174	0.275623
Stdev	0.108157	0.103564	0.089285	0.112803	0.10523	0.089941
Lower Bound^	0.022982	0.045575	0.133007	-0.00217	0.041418	0.125439
Higher Bound^	0.447214	0.454617	0.48358	0.439647	0.441744	0.471497

Semi-elasticity of Cost with respect to time

ε_{Ct}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.00741	-0.00736	-0.00837	-0.00758	-0.00773	-0.00827
Median	-0.00739	-0.0074	-0.00836	-0.00766	-0.00781	-0.00833
Stdev	0.002563	0.002576	0.002461	0.002389	0.00237	0.002219
Lower Bound^	-0.01241	-0.01226	-0.01314	-0.0121	-0.01209	-0.01236
Higher Bound^	-0.00236	-0.00227	-0.00355	-0.00273	-0.00298	-0.00383

^Bounds of 95% Credible Interval

Table 4. Capital elasticities for Models 1 through 6.

Own-price elasticity

ε_{Kw_K}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.15121	-0.15055	-0.14085	-0.14553	-0.14402	-0.13257
Median	-0.15102	-0.15122	-0.1422	-0.14549	-0.14497	-0.13318
Stdev	0.032576	0.032314	0.030766	0.032557	0.033019	0.030687
Lower Bound^	-0.215	-0.21385	-0.19868	-0.20887	-0.20768	-0.18825
Higher Bound^	-0.08754	-0.08457	-0.07882	-0.0785	-0.07592	-0.06851

Elasticity of Capital with respect to Labor Price

ε_{Kw_L}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.01952	-0.0196	-0.02061	-0.01694	-0.01688	-0.01878
Median	-0.01968	-0.01963	-0.02052	-0.01706	-0.01667	-0.01873
Stdev	0.011824	0.011779	0.011476	0.011033	0.011201	0.010405
Lower Bound^	-0.04375	-0.04305	-0.04294	-0.03874	-0.0397	-0.03881
Higher Bound^	0.003313	0.003199	0.001139	0.004037	0.004435	0.001475

Elasticity of Capital with respect to Price of Materials

ε_{Kw_M}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.081603	0.081442	0.095852	0.056359	0.059044	0.07132
Median	0.081659	0.080879	0.095067	0.056881	0.059809	0.072191
Stdev	0.036924	0.036396	0.033389	0.033166	0.032649	0.029094
Lower Bound^	0.008471	0.009578	0.03158	-0.00781	-0.00364	0.010851
Higher Bound^	0.15208	0.153856	0.163234	0.117871	0.121836	0.12788

Semi-Elasticity of Capital with respect to time

ε_{Kt}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.002807	0.002781	0.001733	0.003301	0.003124	0.002076
Median	0.002884	0.002825	0.001876	0.003286	0.003121	0.002107
Stdev	0.002183	0.002165	0.00199	0.002095	0.002009	0.001662
Lower Bound^	-0.0016	-0.00169	-0.0025	-0.00077	-0.00085	-0.0013
Higher Bound^	0.006901	0.006952	0.005304	0.007573	0.007085	0.005281

^Bounds of 95% Credible Interval

Table 5. Labor elasticities for Models 1 through 6.

Elasticity of Labor with respect to Capital Price

ε_{Lw_K}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.01049	-0.01054	-0.01108	-0.00911	-0.00908	-0.01009
Median	-0.01058	-0.01055	-0.01103	-0.00917	-0.00896	-0.01007
Stdev	0.006356	0.006332	0.006169	0.005931	0.006021	0.005593
Lower Bound^	-0.02352	-0.02314	-0.02308	-0.02083	-0.02134	-0.02086
Higher Bound^	0.001781	0.001719	0.000612	0.00217	0.002384	0.000793

Own-price elasticity

ε_{Lw_L}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.08501	-0.08569	-0.0866	-0.06124	-0.06155	-0.06792
Median	-0.08407	-0.08507	-0.08607	-0.06029	-0.06135	-0.06718
Stdev	0.033515	0.033339	0.033304	0.034133	0.034311	0.033682
Lower Bound^	-0.1535	-0.15024	-0.15155	-0.13078	-0.12917	-0.13584
Higher Bound^	-0.0197	-0.02026	-0.02174	-0.00303	-0.0011	-0.00754

Elasticity of Labor with respect to Price of Materials

ε_{Lw_M}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.02679	-0.02925	-0.02306	-0.00695	-0.0061	-0.00818
Median	-0.02721	-0.02947	-0.02419	-0.00283	-0.002	-0.00497
Stdev	0.044552	0.044233	0.045597	0.025271	0.02629	0.026933
Lower Bound^	-0.11349	-0.11701	-0.11008	-0.06561	-0.06786	-0.06839
Higher Bound^	0.062893	0.056023	0.067988	0.036025	0.039682	0.039057

Semi-Elasticity of Labor with respect to time

ε_{Lt}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.00868	-0.00846	-0.00959	-0.01049	-0.01067	-0.01089
Median	-0.00872	-0.00844	-0.00955	-0.01045	-0.01073	-0.01104
Stdev	0.003987	0.003961	0.004076	0.003396	0.003507	0.003416
Lower Bound^	-0.01627	-0.01605	-0.01786	-0.01723	-0.01769	-0.01738
Higher Bound^	-0.00081	-0.00078	-0.00153	-0.00383	-0.00361	-0.00404

^Bounds of 95% Credible Interval

Table 6. Materials elasticities for Models 1 through 6.

Elasticity of Materials with respect to Capital Price

ε_{Mw_K}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	0.021937	0.021894	0.025768	0.015151	0.015873	0.019173
Median	0.021952	0.021743	0.025557	0.015291	0.016079	0.019407
Stdev	0.009926	0.009784	0.008976	0.008916	0.008777	0.007821
Lower Bound^	0.002277	0.002575	0.00849	-0.0021	-0.00098	0.002917
Higher Bound^	0.040884	0.041361	0.043882	0.031687	0.032753	0.034378

Elasticity of Materials with respect to Price of Capital

ε_{Mw_L}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.0134	-0.01463	-0.01153	-0.00348	-0.00305	-0.00409
Median	-0.01361	-0.01474	-0.0121	-0.00142	-0.001	-0.00248
Stdev	0.022281	0.022121	0.022803	0.012638	0.013147	0.013469
Lower Bound^	-0.05675	-0.05852	-0.05505	-0.03281	-0.03394	-0.0342
Higher Bound^	0.031453	0.028017	0.034001	0.018016	0.019845	0.019533

Own-price elasticity

ε_{Mw_M}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.10016	-0.10375	-0.08822	-0.08592	-0.08723	-0.0934
Median	-0.10219	-0.10399	-0.08922	-0.08266	-0.08331	-0.08964
Stdev	0.064788	0.062778	0.06602	0.048908	0.048829	0.045573
Lower Bound^	-0.22517	-0.22252	-0.21095	-0.18777	-0.19446	-0.19074
Higher Bound^	0.035672	0.024801	0.050189	-0.00437	-0.00488	-0.01813

Semi-Elasticity of Materials with respect to time

ε_{Mt}	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Mean	-0.00222	-0.00202	-0.00352	-0.00264	-0.0027	-0.00315
Median	-0.00209	-0.00197	-0.00337	-0.00279	-0.00288	-0.0033
Stdev	0.003193	0.003093	0.00327	0.0022	0.002227	0.002163
Lower Bound^	-0.00891	-0.00848	-0.01029	-0.00657	-0.00656	-0.00706
Higher Bound^	0.00377	0.003775	0.0025	0.001999	0.002103	0.00139

^Bounds of 95% Credible Interval