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# Examining pricing mechanics in the poultry value chain - empirical evidence from Pakistan

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## **Introduction**

Fluctuations in prices of agricultural products have a large impact on the welfare of consumers and farmers. Especially in less developed countries, characterized by heavy dependence on domestic production of food based commodities and imperfect or even nonexistent risk sharing markets (futures contracts and crop insurance etc). Despite, the emergence of a vast empirical and theoretical literature on the dynamics of agricultural product prices in the aftermath of the 2008 food price crisis, many important research problems are yet to be resolved. In particular, one finds significant disagreements within economists regarding an adequate theory of the underlying mechanisms behind the often large fluctuations in the prices of agricultural products over time, in spite of relatively stable fundamentals. Given the relatively inelastic demand for agricultural products (particularly food based commodities) even small changes in supply can lead to large fluctuations in prices (Tomek 2000). While, compared to the supply-side dynamics of non-agricultural commodities, the analysis of the supply of agricultural products is greatly complicated by interlinked value chains, uncertainty caused by the biological nature of the production process and time lags between the decision to produce and the realization of production plans.

Nevertheless, at the expense of generalization, theories of mechanisms behind price fluctuations can be broadly classified into two categories. The theory of exogenous price fluctuations, developed by Muth (1961), Deaton & Laroque (1996, 1992) and Miranda & Glauber (1993) etc. amongst others, argues that the observed fluctuations are a result of the impact of exogenous factors on farm supply e.g. random weather and disease shocks or seasonality of production. The competitive storage model along with its different variants, extensively reviewed by Gouel (2012), is the workhorse behind the development of this theory. In these models, consistent with the rational expectation hypothesis, the distribution of shocks conditional on information available in the current period is assumed to be common knowledge. Therefore, prices fluctuate around the long-run equilibrium level due to random shocks and not any systematic changes in farmers' behavior. Though, very much within the tradition of modern neoclassical economics, however in contrast to real world prices, which are usually characterized by irregular fluctuations, high first order autocorrelation, positive skewness and small kurtosis, price data generated from these types of models exhibits low first order autocorrelation, high skewness and kurtosis (Mitra and Boussard 2012).

On the other hand, the theory of endogenous fluctuations can be traced back to the early work of Ezekiel (1938) on cobweb cycles. In cobweb models, expectations of future prices based on past prices (i.e. backward looking expectations) leads to periods of oversupply and undersupply, which results in periodic cycles of low and high prices, respectively. Yet, with linear farm supply and demand functions, dynamics in a simple cobweb model are limited to three unrealistic scenarios i.e. rapid convergence to steady state, two-period cycles of high-low prices (negatively auto-correlated prices), and explosive oscillations (divergence that results in negative or infinite prices). Clearly, neither pattern fits the stylized facts associated with real world price data. However over the past decades, several refinements of the original cobweb model (Finkenstadt and Kuhbier 1992, Boussard 1996 and Hommes 1998), commonly known as

chaotic<sup>1</sup> cobweb models of agricultural markets, have resuscitated interest in the theory of endogenous price fluctuations. This stream of literature, address key shortcomings of the cobweb model (i.e. negatively auto-correlated prices and convergence to equilibrium) by introducing nonlinearities into the demand/supply functions (Lichtenberg & Ujihara 1989, Day and Hanson 1991, Hommes 1994 and Finkenstädt 1995) without changing the essential features of the original model i.e. backward looking expectations and production lags. As a result, compared to simple cobweb models, the price trajectories derived from chaotic cobweb models display irregular and random behavior, consistent with the observed price fluctuations in real world data. Unfortunately, in contrast to real world data, these price trajectories are often typified by negative skewness and low first order autocorrelation.

Given the adverse sociopolitical ramifications of price fluctuations and the high priority given to agricultural product prices in government agendas across the world, disagreement about an underlying theory of price fluctuations is not merely an academic debate but it has direct policy implications. Because the nature of policy interventions designed to mitigate price fluctuations hinges upon adherence to a particular theory of agricultural price fluctuations. Policies based on the first theory of price fluctuations (exogenous fluctuations) discourage direct interventions in markets and instead push for the diversification of production at the farm level together with maintaining appropriate levels of inventory in storage (when possible) at the aggregate level. Whereas policies designed in light of the second theory (endogenous fluctuations) endorse market interventions through planned production or production quotas at an aggregate level in order to stabilize prices.

In light of the abovementioned debates about the theory of price fluctuations and its policy relevance in less developed countries, we examine the underlying mechanisms behind the price dynamics in the Pakistan poultry sector. Currently, the \$5 billion industry, which provides employment to approximately 1.5 million people and contributes 1.3% towards the GDP of Pakistan, is at the brink of a crisis (Government of Pakistan 2014). Many poultry farmers have been forced to shut down their business due to persistent fluctuations in the prices of intermediate poultry products i.e. chicks and broilers, especially, small liquidity constrained farmers. For instance between 2011 to 2013, over 2500 poultry farms were shut down in Punjab (10% of total farms in Punjab), the hub of poultry production (The Nation 2013). Whereas the existing poultry farmers face a lot of risk due to high uncertainty involved in predicting future cash-flows and hence formulating optimal production plans. Given these settings, our paper makes several timely contributions to the literature within the realm of applied economics.

First, based on extensive fieldwork, we document the organization of production and the price discovery process in the poultry sector in Pakistan<sup>2</sup>. In doing so we shed light on the often

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<sup>1</sup> Wernndl (2009) defines chaos as “chaotic systems are deterministic systems showing irregular, or even random, behavior and sensitive dependence to initial conditions,” which “means that small errors in initial conditions lead to totally different solutions.”

<sup>2</sup> These findings are based on a research project (PI: Muhammad Imran Chaudhry) funded by the Pakistan Poultry Association. This project involved detailed, structured interviews with key stakeholders in the poultry value chain during field work in Pakistan from April-2015 to October-2015. Under the supervision of the PI, a marketing research company was hired to interview approximately 50 major players in the poultry sector including grandparent stock companies, breeders, hatcheries, broiler farmers and retailers. The interview questionnaire is given in Exhibit-1. The primary purpose of this exercise was to understand the organization of production and the price discovery

poorly understood mechanics of agricultural value chains in less developed countries from a purely descriptive standpoint. Second, after taking into account the decision making practices at the level of the farm, we develop a stylized dynamic model to simultaneously capture the optimizing behavior of chick and poultry farmers within the prevalent institutional environment. We explicitly model the interlinked structure of the agricultural value chain and the resulting interdependencies between upstream (chick) farmers and downstream (broiler) farmers, an important aspect of agricultural production that has been overlooked in the theoretical literature on chaotic cobweb models. Third, under very general assumptions about the behavior of cost and demand functions, we derive empirically testable hypothesis about farmers' expectation regime from the underlying model as a system of coupled difference equations. Thereafter, we employ a unique, hand collected dataset comprising of weekly farm-gate prices of chicks and broilers in Pakistan from January-2008 to June-2015 to fit different time series models. Our empirical analysis reveals that the behavior of poultry prices in Pakistan is broadly consistent with the theory of endogenous price fluctuations.

Lastly, we employ analytical methods and numerical tools to examine the dynamical behavior of the underlying system of time-delay difference equations derived from our theoretical model under the naïve expectations hypothesis. First, we prove the existence of a unique equilibrium state. Second, we compute the system's eigen values to show asymptotic stability, in the special case of quadratic costs and a linear retail demand curve for broilers, whereby the underlying model reduces to a system of linear time-delay difference equations. Interestingly, Hale et al. (1985) stability criterion for time delay systems reveals that this linear system is unstable independent of time-delays i.e. delays, usually associated with instability, actually stabilize the underlying model. Thereafter, for the purpose of simulations, the model is calibrated with (convex) power functions to represent the production costs of both types of farmers, an appropriate linear retail demand curve for broilers and reasonable values for model parameters derived from industry reports. We examine the price dynamics in several interesting cases, including the special case of a linear system of time-delay difference equations and the more general case of system of nonlinear time-delay difference equations.

The simulations reveal that the model reproduces the patterns observed in the actual poultry price data i.e. cyclical behavior, positive first order correlation, low skewness and negative kurtosis. Likewise, consistent with the prior literature, we document chaotic behavior in the presence of nonlinearities in the underlying system. We also implement the BDS test (Brock et. al 1987 and Brock et. al 1996) in order to characterize any chaotic dynamics in the actual price data. Interestingly, although non-chaotic in the strictest sense due to the presence of stable limit cycles, simulation results from the linear model show a markedly high sensitivity to initial conditions and small changes in model parameters. This type of behavior, at times referred to as "thin chaos", may arise in linear time-delay models due to the intricate dynamics and complexities generated by time-delays in the feedback mechanisms.

These findings have important ramifications vis-à-vis the literature on endogenous price fluctuations. First, we try to address an important gap in the theoretical literature on chaotic cobweb markets, which has largely ignored the modeling of vertically interlinked agricultural

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process, in particular the structure of the poultry supply chain, information flows and the economics of decision making at the level of the farm.

markets and asymmetric production lags. In doing so, we find that naïve expectations can lead to complicated and even chaotic fluctuations in a vertically linked agricultural supply chain with asymmetric production lags. Second, we seek to bridge the gap between theory and empirics in the endogenous price fluctuations literature by employing farmer survey responses, empirical analysis and numerical simulations along with a broader understanding of the domestic institutional environment to make a compelling case for the existence of cobweb cycles in less developed countries.

The remaining paper is organized as follows. Based on extensive fieldwork comprising of structured interviews with poultry farmers, opinion surveys and market visits in Pakistan, we describe the mechanics of the poultry supply chain, the price formation mechanisms and the stylized facts associated with behavior of farm-gate prices of chicks and broilers in section-1. Section-2 explains the importance of future price expectations in the analysis of agricultural commodities. We provide a brief summary of the literature on cobweb cycles and highlight key assumptions and criticisms of cobweb models. Thereafter, in light of the literature on expectations and the responses of poultry farmers in Pakistan, we use the framework of bounded rationality to present arguments in favor of the optimality of backward looking (naïve) expectations in our institutional environment. In section-3, findings from the previous sections are used to motivate and develop a stylized dynamic model based on profit maximization by upstream (chick) and downstream (broilers) farmers in an interlinked poultry supply chain. Solving the model under the naïve expectation hypothesis allows us to derive a coupled-system of difference equations for chick and broiler prices along with the related empirically testable comparative static results.

Section-4 describes our empirical strategy and discusses the estimates from different time series models vis-à-vis cobweb cycles. In section-5 we use analytical and numerical methods to highlight some important characteristics of the underlying dynamical system. The system of difference equations is calibrated and simulated under different scenarios in order to compare the statistical properties of the simulated data with that of the actual data. And standard numerical techniques, commonly used in the analysis of chaotic systems, are employed to study the behavior of orbits generated by our theoretical model. Finally, a discussion of the limitations of the underlying economic model is followed by policy recommendations in a brief conclusion.

## **1-Pakistan Poultry Sector: Organization of Production, Price Discovery & Price Dynamics**

### **1.1. Background-Poultry Industry in Pakistan**

Pakistan is the 6<sup>th</sup> most populous country in the world and is categorized as a low income agrarian economy (World Bank 2012). Agriculture is the 2<sup>nd</sup> largest sector of the economy, accounting for over 21% of GDP and providing employment to 45% of the total labor force (Pakistan economic Survey 2010). Livestock is the largest subcomponent of agriculture, contributing 11.6% towards the national GDP during 2010-12. Fueled by private sector investments and rapid mechanization, the poultry industry is the most vibrant segment of the livestock sector, with an investment of \$2 billion and an annual turnover of approximately \$7 billion, it generates employment for 1.5 million people (Government of Pakistan 2014). The poultry industry is also by far the most organized agricultural sector in Pakistan, the interests of

poultry industry are represented by the Pakistan Poultry Association (a non-profit business association comprising of key stakeholders in the poultry value chain) at the national level.

Commercialization of poultry production began in 1964 in major cities like Karachi, Lahore, Faisalabad, Rawalpindi, and Hyderabad at the behest of the federal government. From there on the industry has witnessed rapid growth primarily driven by the private sector enterprise and facilitated by favorable government policies e.g. cheap credit, tax exemptions and generous subsidies on the import of breeding stock, farm equipment and poultry feed (Hussain et al. 2015). As a result, the share of traditional poultry farming (predominantly in rural areas) has steadily declined to less than 10% over time. The production practices at modern commercial poultry farms are increasingly consistent with global poultry industry standards. In fact Pakistan's breeder population is considered to be amongst the top 10 in poultry industries worldwide (USDA 2010). To sum up, private sector involvement has fueled the rapid transition from low productivity, subsistence poultry farming to high productivity, technology intensive commercial poultry farming methods.

At the same time, due to the shortages in the supply of beef and lamb caused by the continued adherence to traditional production methods in rural areas; the share of poultry in total meat production in Pakistan has consistently increased over the past decades. According to conservative estimates, poultry products now contribute approximately 30-40% towards total meat production in Pakistan and it is also relatively cheaper (substitution effect) compared to beef/lamb etc. However as highlighted in table-1 below, despite the growth of the poultry sector, per capita consumption of poultry chicken is low compared to international standards. Furthermore, with rising incomes and growth in an already large population (income effect), the demand for poultry products is expected to increase substantially over the next decades. Therefore, from a food security perspective, equivalent expansion in poultry production is critical to meet the increasing demand for poultry products in future and fulfill the nutritional deficiencies, especially amongst the lower strata of the society.

Table-1: Production & Consumption of Poultry Meat in Pakistan

	2006	2007	2008	2009	2010	2011	2012
Total Production (KT)	521	564.3	575.1	517.7	539.7	563.7	589.6
Total Consumption(KT)	521.5	569.5	574.7	517.9	540.1	564.4	590.7
Per Capita Consumption(Kg)	3.1	3.3	3.2	2.9	2.9	3.0	3.1

Source: OECD-FAO Agricultural Outlook 2010-2019. KT denotes kilotons and Kg stands for kilograms.

It is interesting to note that unlike developed countries, due to the lack of cold storage facilities on one hand and preference of consumers for fresh chicken on the other<sup>3</sup>, the poultry sector in Pakistan is primarily a live bird market as opposed to chilled/frozen bird market. Consequently, international trade in poultry products (chicks and broilers) plays a negligible role in Pakistan as shown by the relatively equal measures of production and consumption in Table-1. At the same time, export of chicken along the adjoining borders of India, Iran and Afghanistan is officially banned. These observations have important ramifications vis-à-vis price volatility, because in the absence of inventory and international trade, even small changes in supply can

<sup>3</sup> A primary driver of preferences for freshly slaughtered chicken is a high concern about the halal status of chicken amongst consumers in Pakistan, a predominantly Muslim country.

result in large price fluctuations. We take into account these features of the poultry sector in the formulation of our theoretical model in section-3.

## 1.2.Key Features of the Value Chain

Despite gains in production efficiency, the organizational structure of the poultry sector poses several challenges to sustained growth in poultry production. First, unlike poultry farming in the US, the poultry sector in Pakistan is neither vertically integrated nor characterized by contract farming agreements. Second, in contrast to the pervasiveness of integrators<sup>4</sup> in developed countries, the supply chain in Pakistan comprises of several firms operating independently of each other, often based on shortsighted goals. Third, institutional voids and an opaque information environment, further convolute production decisions at the farm level. We discuss each of the aforementioned factors in the following paragraphs.

The key players involved in the poultry supply chain can be categorized into: grandparent stock companies, parent stock companies/breeders, hatcheries and broiler farmers. The grandparent stock companies import broiler grandparent stock and rear it to produce parent stock or broiler breeders<sup>5</sup>. However, as is often the case in developing countries, artificial scarcity created by trade quotas makes import licenses valuable and a source of monopoly rents, consequently licenses are contested through political connections and kickbacks (Krueger 1974). Likewise in Pakistan, in 2008 only 2 companies imported grandparent stock and the price of was fixed at Rs. 250/bird. In 2014, this number increased to 4 and as a result the price of parent stock declined to a fixed price of Rs. 150 per bird. Parent stock chicks purchased from grandparent stock companies are reared to produce fertilized eggs by the breeders. Thereafter, the fertilized eggs are sold to hatcheries and incubated in order to produce day old chicks. The hatcheries sell the day old chicks to broiler farmers, who grow them into broilers that are eventually sold to retailers via commission agents<sup>6</sup>.

Geographically, production is concentrated in the northern part of Pakistan, in particular Punjab, the most populous and economically productive province of Pakistan. The southern part of Pakistan, in particular Karachi and adjoining areas represent the other production belt. The poultry sector in northern Punjab accounts for approximately 70% of the total production and it is relatively more capital intensive, due to the relatively higher percentage of control sheds compared to the poultry farms in the southern part of Pakistan. Apart from certain degree of integration at the level of breeders and hatcheries, the value chain is fragmented and spanned by thousands of independent poultry farms. According to PPA (2015), there are more than 25,000 poultry farms spread over the two production belts, while estimates of production data show

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<sup>4</sup>Integrators are firms that have expanded their operations to span the entire poultry value chain i.e. beginning with acquisition of parent stock, to raising parent stock and chicks, to rearing broilers and even selling to consumers.

<sup>5</sup>Famous brands of Grand Parent stock being imported in Pakistan include Hubbard and Ross.

<sup>6</sup>The focus of this paper is gaining an understanding of the underlying causes behind the price fluctuations in the farm-gate prices of day old chicks and broilers i.e. the production value chain as opposed to price transmission from the farm sector to the retail sector. Therefore, we digress from a detailed description of the marketing channel. Nevertheless, very briefly, based on fieldwork in Pakistan we found that unlike the marketing channel in the developed countries which is dominated by large retailers and processors, the retailers don't enjoy any market power in Pakistan. In fact, retailers comprise of small setups scattered around towns, which primarily slaughter live chicken for consumers on demand and possess little bargaining power vis-à-vis poultry farmers, empirical estimates from standard price transmission models support these observations.



parent stock placement of 11 million along with chick and broiler production of 1.1 billion and 1.05 billion, respectively, in 2015.

Despite the relatively large size of the poultry sector, there is a complete lack of coordination regarding production decisions at different stages of the poultry value chain. Moreover, there is no timely source of official data on key variables like parent stock placement by breeders or production at hatcheries at the association level or regionally. Decision making at the farm level across the value chain is further complicated by the lack of futures markets on poultry products, in particular broilers. While, the weak institutional environment i.e. poor contract enforcement etc. means that production cannot be pre-contracted. Since contracts carry little value in situations of adverse price movements e.g. if market price of chicks falls below the contracted chick price, many broiler farmers will renege on their contracts with hatcheries. To make matters worse, an underdeveloped marketing channel means that farmers cannot preempt changes in demand and adjust production accordingly based on pre-order information received from retailers. We reiterate these issues in the subsection on poultry farm economics.

To summarize, the price of parent stock is fixed due to the monopoly rents generated by import licenses held by grandparent stock companies. Nevertheless, the relatively large number of chick and broiler farmers diminishes any market power concerns further downstream.<sup>7</sup> Even though production is concentrated in northern Punjab, the poultry sector is not vertically integrated despite the fact that the production of chicken involves various intermediate and closely interlinked production processes. We now turn our attention towards the mechanical aspects of the broiler chicken production cycle.

### 1.3.Mechanics of the Production Cycle

Compared to other agricultural commodities i.e. crops and livestock, a unique feature of poultry value chain is the relatively short duration of the production cycle, which is most appropriately measured in weeks compared to months and sometimes even years in case of the former categories. Moreover, in the absence of vertical integration or contract farming, another interesting feature of the production cycle arises from the interdependencies between upstream and downstream farmers created by the interlinked and sequential nature of the production process. Therefore, unlike the previous theoretical literature on cobweb models, one cannot merely focus on price fluctuations in the final downstream product whilst ignoring the price dynamics in the upstream product. We specifically model these interdependencies in our paper, for further details please refer to section-3.

In the first stage, breeders purchase the parent stock from the grandparent stock companies at a predetermined, fixed price.<sup>8</sup> After an unproductive rearing period of 25 weeks, the parent stock enters production for approximately 66 weeks and produces on average 120 fertilized eggs in its life cycle. However, productivity declines after an initial surge, and if chick prices are low, the parent stock is sometimes strategically culled i.e. sold in the broiler market, to take advantage of high broiler prices even before the end of its productive life. Based on feedback received from

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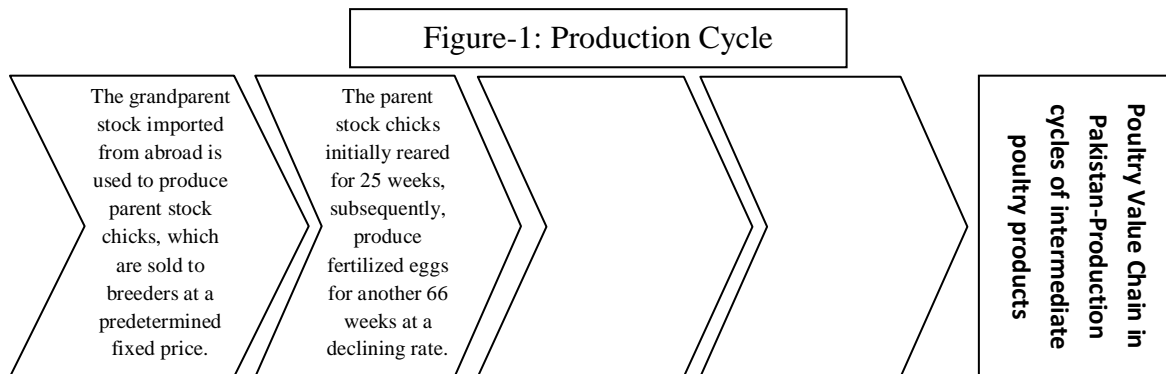
<sup>7</sup> Based on data provided by PPA, the eight firm-concentration ratio is approximately 50% and 10%, respectively in the hatchery and broiler subsector of poultry value chain.

<sup>8</sup> There is no domestic production of the grandparent stock and it is simply imported from abroad, therefore we begin the description of the production cycle from the parent stock.

poultry farmers in Pakistan, the average productive period of parent stock is around 60 weeks. Breeders in Pakistan maintain parent stock at different stages of the lifecycle at any given time to smooth cash flows and on average buy new parent stock chicks thrice a year. Note that the fertilized eggs produced by the parent stock can be technically sold in the table egg market, although this rarely happens in practice due to price considerations.

The hatcheries purchase fertilized eggs from breeders and incubate the fertilized eggs in controlled environment sheds for exactly 3 weeks. Note that unlike breeders, at any given time, hatcheries possess a single batch of fertilized eggs, all of the same age. Although some farmers have multiple sheds and purchase fertilized eggs continuously, nevertheless fertilized eggs for a given hatchery will be purchased only once in a 3-week cycle. The eggs hatch on the 22<sup>nd</sup> day and the resultant chicks must be sold as soon as possible, in order to realize cash flows and prepare the shed for the incubation of a new batch of fertilized eggs. According to chick farmers, after the 22<sup>nd</sup> day, probability of chick mortality increases drastically with time; therefore they are in a rush to sell the chicks to broiler farmers.

The broiler farmers' directly purchase the chicks from the hatcheries<sup>9</sup>. The chicks are grown into broilers and reach market weight in approximately 6-7 weeks depending on the type of shed, quality of feed and the season. Like hatcheries, at any time, broilers at a given broiler farm are all of the same age and are sold to retailers via commission agents as soon as they reach market weight, because the ratio of feed-cost to weight-gain and the mortality rate increases significantly after this weight. Recall, that the market for poultry chicken in Pakistan is primarily a live bird market therefore, broiler farmers have no other choice but to sell broilers at the prevalent market rate. Thereafter, broiler farmers buy the next batch of chicks and the cycle continues. Figure-1 summarizes the production mechanics of the poultry value chain i.e. production processes of the intermediate poultry products along with relevant life cycles. We describe the price discovery process in the next subsection, followed by a visual depiction of the actual price data.



#### 1.4. Price Discovery Process

In contrast to a formal theory of price determination, based on the principles of optimization by agents (e.g. for producers marginal revenue equals marginal costs) and equilibrium (prices

<sup>9</sup> There is no intermediary between the chick farmers and broiler farmers in the Northern production belt, while in southern production belt commission agents act as an intermediary between hatcheries and broiler farmers. As mentioned later, our price dataset concerns the northern production belt, therefore we make the above statement.

adjust so that markets clear), the price discovery process deals with the “mechanics” of pricing. In particular, it describes the institutional arrangements, information flows and methods employed by buyers and sellers to arrive at transaction prices (Tomek & Robinson 2003). Unsurprisingly, price discovery process depends on the economic settings, institutional environment, technological advancements and transactions costs. We describe these factors along with their impact on price discovery in the following paragraphs.

The chicken market in Pakistan is primarily a live bird market with approximately 98% of the demand for chicken met by freshly slaughtered broilers and an estimated 5 million broilers were slaughtered everyday in Pakistan during the past year (Haq 2014). Therefore, markets for intermediate poultry products, especially the chick and broiler markets are very active. In the absence of cold storage facilities, vertically integration or pre-contracted production, the current supply and demand situation primarily determines the market clearing price for both chicks and broilers. The impact of current supply on market prices is further amplified by the fact that both chicks and broilers fall within the category of highly perishable products and must be sold off as soon as “ready”, often on the same day, for reasons explained above. Consequently, poultry farmers cannot “hold” supply in lieu of adverse price movements and have to sell the production at the prevailing market price. Therefore, as opposed to short-run, *very* short-run supply determines equilibrium prices. At the same time because of the interlinked nature of the production process, the supply of chicks and broilers is mutually interdependent; therefore, chick prices are affected by broiler prices and vice versa.

The government plays a minimal role in the price determination processes in poultry markets in Pakistan. And prices adjust to ensure that markets clear i.e. demand equals supply in the broiler and chick markets. The market clearing farm-gate prices, for both chicks and broilers, are circulated on a daily basis by the Pakistan Poultry Association (PPA). These prices are finalized after reviewing the *going* rate at poultry auction markets known as *Mandis* in northern Punjab and consulting large hatcheries and broiler farmers active in the chick/broiler markets on a given day. The *Mandis* serve as congregation point for potential buyers and sellers of chicks and broilers from different locations across Pakistan. The availability of good road networks in the production hub of northern Punjab facilitates the participation of large number of buyers and sellers from different adjoining areas. The continuous interaction between buyers and sellers transpires into the rapid aggregation of information on demand and supply into prices, leading to the determination of equilibrium, market clearing prices.

The PPA rate serves as the reference rate for the transactions between both chick and broiler farmers and broiler farmers and retailers across Pakistan. Given the relatively short poultry production cycle, information flows pertaining to the prevailing prices spread quickly due to significant improvements and expansion in the information and communications technologies (ICT) in Pakistan over the last decade. Additionally, due to the improvements in the road and transportation network, many arbitrageurs seek to exploit regional price differentials. Consequently, large regional price differentials net of transportation cost are quickly arbitrated away. Although negotiations and bargaining on transaction price takes place between counter parties, based on payment terms and sometimes even product quality, however the competitive nature of poultry sector in lieu of the large number of hatcheries, broiler farmers and retailers along with the standardized nature of the underlying product means that major deviations from the official PPA prices are uncommon.

In summary, broiler and chick prices are, by and large, determined by the current (or *very* short-run) supply situation. *Mandis*, arbitrageurs, good road networks and widespread use of ICTs, facilitate the aggregation of demand/supply information into prices and leads to rapid price adjustments across different regions engaged in poultry production. Therefore, the market clearing price documented and disseminated by the Pakistan Poultry Association, is representative of price levels at which broilers and chicks are bought and sold in Pakistan, and thus reflective of the overall the demand and supply situation.

### 1.5. Price Data-Styled Facts

We now highlight the stylized facts associated with the prices of broilers and chicks. The price data was provided by Pakistan Poultry Association and comprises of daily, market clearing farm-gate prices for chicks and broilers in Punjab (PPA-North Region), the production hub of poultry and the most populous province of Pakistan, from Jun-2008 to Jun-2015. According to Pakistan Poultry Association, these prices are representative of prices in other parts of Pakistan and are used by the Federal government in computations of price levels (CPI, inflation etc). Given that the production cycle of poultry is most aptly measured in weeks, we converted daily prices into average weekly prices for the sake of consistent estimation and intuitive interpretation<sup>10</sup>. Summary statistics are shown in Table-2.

Table 2: Summary Statistics of Broiler & Chick Prices

	Max	Min	Mean	Std-Deviation	Coefficient of Variation	Skewness	Kurtosis
Broiler	173.29	58.00	110.37	23.14	0.21	0.21	-0.48
Chicks	74.00	4.29	36.29	16.36	0.45	0.26	-0.65

The summary statistics are based on average weekly prices from June 2008 to June 2015. The data was sourced from Pakistan Poultry Association (North). All prices are in nominal units of the local currency.

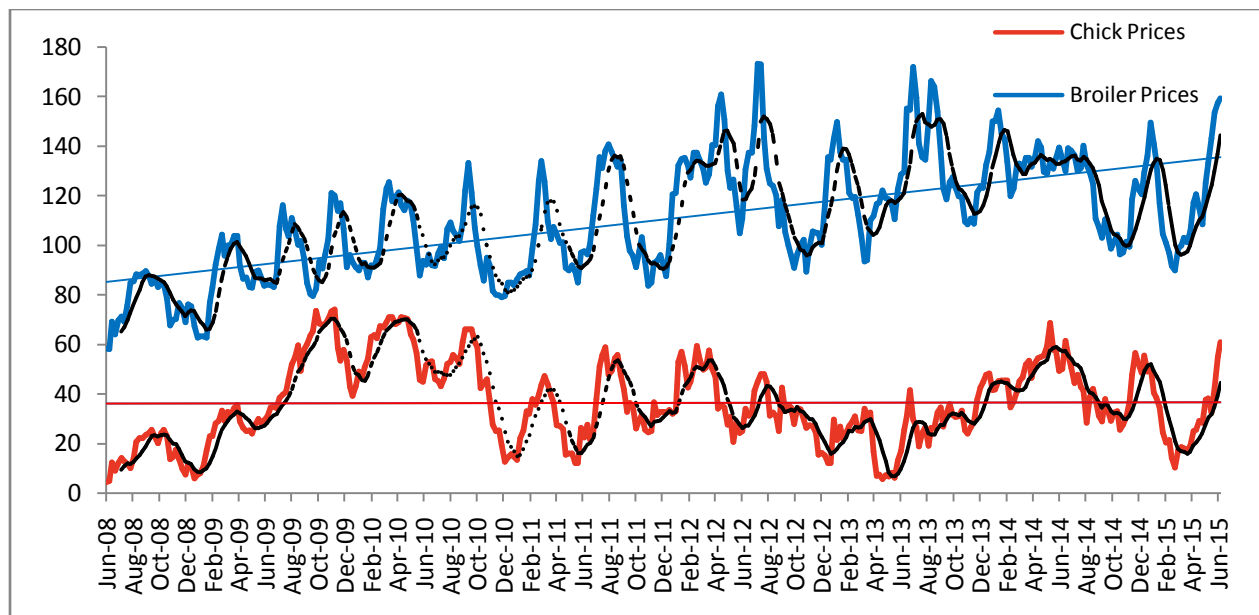
Table-2 highlights the variation in both price series. The difference between minimum and maximum price for broilers and chicks is particularly noteworthy. Using the coefficient of variation to normalize average price variability (measured by the standard deviation), we find that on average the variability in chicks prices is approximately twice that of broiler prices. Both price series depict low skewness and negative kurtosis, compared to the positive kurtosis displayed by most agricultural commodities. Taken together, these observations imply that both price series are characterized by a relatively flat probability distribution i.e. fatter tails (frequent peaks and troughs).

The relatively long coverage of the dataset (8 years) allows us to observe the dynamics of price fluctuations. Figure-2 shows the evolution of broiler prices and chick prices over time with (nominal) price in Pakistan Rupees on the vertical axis. The solid lines show a simple time trend of each price series, whereas the dotted black line denotes a six-week moving average<sup>11</sup>.

Figure-2 Time Series Line Plots of Broiler & Chick Prices

<sup>10</sup> Section-4 provides several reasons for converting daily prices into weekly prices.

<sup>11</sup> As the forthcoming analysis will show, the broiler prices play a fundamental role in the production decisions of both chick and broiler farmers. Therefore we used 6-week moving averages, the approximate length of the broiler production cycle, to analyze the variability in broiler and chick prices.



First, and foremost we note that there has been incredible variation in the prices of poultry products that shows no sign of dampening, in fact fluctuations have actually increased over time. A simple linear trend line, drawn for each price series reveals that broiler prices have shown a positive time trend on average but chick prices have remained largely stagnant over this period. Second both series show very strong cyclical behavior i.e. prices repeatedly rise and fall in cycles, often multiple times in a given year. However, the “amplitude” and “frequency” of these cycles is not constant and changes randomly over time. Note that, these cycles cannot be explained by seasonal dummies based on the Gregorian calendar<sup>12</sup>. This is intuitive, since chick and broilers are continuously produced throughout the year, whilst, demand for chicken is fairly stable within a certain price range. The dotted lines, representing the 6-week moving average, lend support to the abovementioned observations and demonstrate that a systematic component as opposed to mere noise is driving the cyclical behavior. In a nutshell, both prices series are characterized by excessive variability and cyclical behavior that, as we will argue later, cannot be explained by fundamentals and may perhaps be a manifestation of chaotic cobweb cycles.

## 2- Literature Review-Cobweb Models: Key Assumptions, Criticisms & Refinements

### 2.1 Cyclical Behavior of Agricultural Product Prices

In theory, a price cycle is defined as a price pattern that repeats itself over time with a fixed period and amplitude. But this idealistic definition of cyclical behavior seldom holds in reality due to the interaction of several systematic components in agricultural product prices e.g. seasonality, random shocks and deterministic time trends. As a result, the period and amplitude of price cycles in agricultural commodities usually varies from one cycle to another due to the presence of the abovementioned systematic components. Therefore, exact empirical

<sup>12</sup>In the empirical literature price cycles driven by seasonality are usually assumed to be deterministic and thus relatively easy to identify based on the Gregorian calendar within a simple regression framework. In our case, monthly dummies based on the Gregorian calendar could not explain the cyclical behavior. Other types of seasonality are discussed in section-4.

identification of cycles i.e. isolating the stage that a cycle is in at any given time, is almost impossible in practice. Nevertheless, price cycles in agricultural commodities usually depict (qualitatively) repetitive behavior which is characterized by periods of rising prices followed by periods of declining prices and vice versa.

The literature has provided two major explanations to explain the underlying causes behind cyclical behavior of prices and the associated price volatility. The theory of exogenous price fluctuations originally developed by Muth (1961) and vastly improved upon by later researchers (Deaton & Laroque 1992, Deaton & Laroque 1996, Chambers & Bailey 1996 and Osborne 2004) is based upon different models of competitive storage under the rational expectation hypothesis. According to this theory, random shocks like droughts and epidemics lead to a temporary reduction in supply and hence high prices. Whilst, good weather shocks and low incidence of diseases may lead to a supply glut and hence low prices. However, given that the occurrence of such shocks is rare, price cycles caused by external shocks dampen over time as production returns to normal levels and prices converge to levels consistent with long run equilibrium in the absence of new shocks. Similarly, price cycles may be driven by seasonality of production (Miranda & Glauber 1993b and Lowry et. al 1987). For example, the production of certain crops is tied to particular seasons i.e. periodic production and prices gradually increase after the end of the production season due to lack of new supplies entering the market.

Gouel (2012) provides a detailed review of the price dynamics and key features of different versions of the competitive storage model employed in agricultural economics literature. Consistent with the rational expectation hypothesis, farmers are assumed to possess complete albeit imperfect knowledge of the underlying structure of the aggregate supply and demand equations along with the relevant information on variables related to prices in these models. Although uncertain about the actual realization of prices, farmers know the probability distribution of future prices and hence the mean price. Therefore, price cycles are a short term phenomena and represent deviations from the long run equilibrium level due to the incidence of external shocks or seasonality of production and not any systematic changes in the production decisions of farmers. However, the stylized facts associated with real world price data, in particular large first-order autocorrelations, positive skewness and kurtosis cannot be replicated by price fluctuations generated from models of competitive storage. In fact, in the absence of storage, price data generated from these models is characterized by unrealistically low first order correlation and large negative skewness. This brings us to another theory of price cyclicity i.e. endogenous fluctuations.

## 2.2 The Cobweb Model-Underlying Phenomena & Key Assumptions

The cobweb model is a popular conceptual framework to explain persistent cyclical movements in the prices of agricultural products. The original model, due to the seminal work of Ezekiel (1938), challenged the notion in neoclassical economics that production/prices converge to equilibrium following random shocks and instead showed that systematic forecast errors can lead to periods of over and undersupply, resulting in cyclical price fluctuations. However due to its simplistic nature, the original cobweb model was merely a pedagogic tool with limited applicability to real agricultural markets, although later work by Nerlove (1958) addressed some of the early criticisms directed at the original cobweb model. Nonetheless, despite not being the mainstream approach, simple cobweb models symbolized an alternative view of price

fluctuations i.e. theory of endogenous price fluctuations till the early 1990s. However, recent advancements in the analysis of chaotic systems and the emergence of chaos theory has resuscitated interest of economists in cobweb type models of commodity prices (Mitra & Boussard 2012, Westerhoff & Wieland 2010, Schenk-Hoppé 2004 and Lundberg et. al 2015). We put off a discussion of this stream of literature for now, in order to first lay out the essential features of a simple cobweb model.

In a simple cobweb model, quantities and prices are determined sequentially in a linked causal chain due to the time lags between the decision to produce and realization of actual production plans. The underlying phenomenon is intuitive, farmers increase (decrease) planned future production in view of high (low) current prices under the presumption that high (low) prices today will translate into high (low) prices in the next period. This results in a supply glut (shortage) when planned production is realized in the next period, leading to low (high) prices. Therefore, simple cobweb models generate periodic oscillations of high and low prices, resulting in negatively auto-correlated prices.

Like all models, the cobweb model of price fluctuations relies on some key assumptions. First, a time lag must exist between the decision to produce and the realization of actual production. Evidently, in the case of agricultural commodities, this assumption almost always holds in reality due to the biological nature of the production process. Second, producers are price takers in the output market and current prices are primarily determined by the realization of current production i.e. absence of inventories and trade in the given agricultural commodity. Though true in our specific institutional environment, this assumption may not hold in other agricultural commodities like grains etc which can be easily stored and imported/exported. Lastly and most controversially, producers in cobweb models are assumed to base their production decisions on current prices (naïve expectations) or a weighted average of current and past prices (adaptive or quasi-naïve expectations). In other words, producers use information embedded in today's prices to forecast future prices in a cobweb model or put more simply producers simply expect current (or recent past) prices to continue in the next period (extrapolative or backward looking expectations).

Many have criticized the assumption of naïve expectations or backward looking expectations in cobweb models. The standard argument is that farmers can adopt counter cyclical strategies i.e. plan to produce less (more) when current prices are high (low). In doing so they not only earn superior profits but their counter cyclical production strategies will also dampen the price cycles. But the issue is not as simple as it seems, because the development and evolution of price cycles is difficult to predict ex-ante due to the interactions between different systematic components in agricultural prices i.e. seasonality, time trends and random shocks (Tomek & Robinson 2003). Moreover, as argued above, the period and amplitude of price cycles changes over time in response to interaction of many factors. Therefore, though straightforward in theory, successful adoption of countercyclical strategies is not easy to implement in practice due to an inadequate knowledge about future trajectories of price cycles. Moreover, if the price series depict chaotic behavior than adopting a simple countercyclical strategy does not guarantee superior profits due to the inherent randomness in prices, especially in our settings, where production cannot be stored and has to be sold at the prevailing market rate.

### 2.3 Importance of Expectation Regimes in Agricultural Economics

The specification of an expectation regime is a key ingredient of dynamic models in neoclassical economics<sup>13</sup>, even more so in agricultural economics due to the time lag between production decisions and actual realization of agricultural output. Consequently, expectations (i.e. forecasts of future output prices) play a key role in the current production decisions of farmers and as a result much of the analysis pertaining to the supply of agricultural commodities revolves around future (output) price expectations of farmers (Sulewski et.al 1994). Of course, there is a direct link between producers' formulation of expectations and the dynamics of agricultural product prices (volatility, cyclicalities etc). Because if production decisions in the current period depend on the expectations of output prices in the next period, then expectations of future output prices today will have a large impact on the future realization of actual prices (assuming minimal effect of inventories and trade).

Given the significance of expectations in economic models, unsurprisingly, one finds disagreements between economists regarding the "correct" formulation of expectation regimes. However, since expectations about future prices cannot be directly observed, there is no easy resolution to this debate. Nonetheless, the unobservability of expectations has not hindered economists from employing several different methodologies including econometric techniques, direct surveys and laboratory experiments to empirically test hypothesis related to the different expectation regimes e.g. rational expectations, quasi-rational expectations, adaptive expectations or naïve expectations. Before reviewing the empirical literature on expectations, we briefly highlight the key features and implicit assumptions of the rational expectation hypothesis.

### 2.3.1 Rational Expectation Hypothesis-Key Features, Implicit Assumptions & Empirical Tests

Few would doubt that the rational expectation hypothesis, originally formulated by Muth (1961), is the dominant approach in the economics literature to formulate expectations. However, the popularity of the rational expectation hypothesis does not stem from an accumulated body of empirical evidence but instead it is based on the logical structure of rational expectation hypothesis that allows economists to formulate and solve economic models (Lovell 1986). Nevertheless, the emergence of the bounded rationality framework has increasingly challenged the dominance of the rational expectation hypothesis, especially in the aftermath of the recent financial crisis.

According to Muth (1961), expectations that are consistent with the underlying economic model are rational i.e. the subjective expectation of economic agents equals the mathematical expectation in the underlying economic model conditional on the available information. However, as many economists have pointed out, the requirements of the abovementioned consistency requires stringent and often unrealistic assumptions. First, agents have perfect knowledge of the structure of the model and equilibrium relationships derived from the underlying model are used to forecast endogenous variables. Second, acquisition and processing of information is costless. Whilst, the information set of agents contains information on all available variables that are thought to influence future prices, including endogenous variables, exogenous processes (the probability distribution of random shock) and expectation of other agents etc (Irwin & Thraen 1994). A corollary of this representational scheme is that agents know the probability distribution of future prices (hence the mean price) even though the actual

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<sup>13</sup> The theoretical foundations of modern macroeconomics, developed by renowned economists like Lucas, Sargent & Barro in the 1980s, draws heavily on the rational expectations hypothesis proposed by Muth (1961).



realization of future prices is unknown. Consequently, forecasts of agents in a rational expectations equilibrium converge towards actual realizations.

From the implementation standpoint, apart from the computational difficulties, the validity or “rationality” of prices derived from the rational expectation equilibrium requires that the underlying economic model is correctly specified vis-à-vis the information set of agents i.e. variables included in the demand and supply equations (Irwin & Thraen 1994). Therefore, predictions from rational expectation models are sensitive to the underlying structure of economic models and any empirical test of the rational expectation hypothesis is also a joint test of the underlying model (the joint hypothesis problem). Consequently, many researchers simply employ a futures market quote (assuming futures markets are efficient and aggregate all relevant information) to operationalize the rational expectations hypothesis in empirical research or use forecasts from an econometric model (quasi-rational expectations).

Although firmly embedded within the tradition of neoclassical economics, the rational expectation hypothesis has little empirical support. Empirical work on expectations can be divided into two broad categories i.e. indirect tests based on structural econometric models and direct tests based on (model free) survey results<sup>14</sup>. Irwin & Thraen (1994) present a detailed review of empirical tests of rational expectation hypothesis in agricultural economics and find that tests based on econometric models fail to offer any consensus regarding the verification or falsification of the rational expectation hypothesis, despite spanning an array of agricultural commodities and different time periods.

For example, Shonkwiler & Emerson (1982) find that expectations of farmers in the US tomato sector are consistent with rational expectations but Shonkwiler & Spreen (1982) find that expectations of farmers in the US lettuce market are consistent with naïve expectations. Likewise, while Goodwin & Sheffrin (1982) make a strong case for rational expectations in the US broiler industry, while Antonovitz & Green (1990) reject the rational expectation hypothesis in the US beef cattle market. According to Irwin & Thraen (1994) the variability in the results of empirical tests of the rational expectation hypothesis is driven by the low power of statistical tests and specification searching by researchers.

More recent work presents a similar picture. For instance, Chavas (1999a) estimates a dynamic structural model of profit maximization by farmers in the US poultry sector and finds that expectations of approximately 90% of poultry farmers are consistent with backward looking expectations. Based on a similar approach, Chavas (1999b & 2000) find that a large proportion of US beef and pork producers’, respectively behave naively i.e. base future production decisions only on the most recently observed market prices, while a significant minority conforms to quasi rational expectations. Our paper also contributes towards this stream of empirical literature i.e. empirical test of farmers’ expectation regimes based on a structural economic model.

However, direct tests of expectations i.e. empirical analysis of survey data (checking for unbiasedness and orthogonality of forecasts) reveals a less ambiguous picture of expectation formation. Based on analysis of micro survey data on exchange rate expectations of financial

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<sup>14</sup> “Model-free” means that a survey measure of expectations is independent of an econometrician's specification of the market's structure. This characteristic of survey expectations is attractive, and leads some economists to argue that direct tests of rationality are superior to tests based on structural econometric models” Irwin & Thraen 1994 pg.

institutions, Ito (1990) rejects the rational expectation hypothesis but finds significant heterogeneity within the expectations regimes of different agents. Kawasaki and Zimmermann (1986) arrive at similar conclusions, using survey data from manufacturing firms about future demand forecasts. After reviewing several empirical studies based on surveys pertaining to expectation formation, Lovell (1986) concludes "...it seems to me that the weight of empirical evidence is sufficiently strong to compel us to suspend belief in the hypothesis of rational expectations, pending the accumulation of additional empirical evidence."

This view is mirrored in the agricultural economics literature as well, Irwin & Thraen (1994) arrive at the conclusion that the overall results of survey studies favor rejection of the rational expectation hypothesis. For example Beach et. al (1995) find that expectations of vegetable farmers are inconsistent with the rational expectation hypothesis. Likewise, Runkle (1991) finds that hog farmers breeding decisions are not rational, similarly Irwin et. al (1992) reject the rational expectations hypothesis after analyzing the expectations of hog and cattle farmers in the US. The burgeoning behavioral and experimental economics literature echoes a similar view. For example, Heemeijer et al., (2009) finds that that subjects prefer to use simple rule of thumb which are inconsistent with the rational expectation hypothesis to forecast future prices. Hey (1994) finds that though subjects "try to" behave rationally but their expectations bear a clear resemblance to backward looking expectation regimes. The abovementioned findings are consistent with anecdotal evidence, which suggests that farmers normally use the most recently received price to predict prices in the next period as opposed to forecasts from complicated models.

#### 2.4 Naïve Expectations or Rational Expectations-Towards a resolution

Given the fairly strong assumptions of the rational expectation hypothesis, it is not surprising to find a lack of empirical support for the rational expectation hypothesis. Undoubtedly the rational expectation hypothesis has a certain appeal that draws from its logical structure and internal consistency, which allows economists to build elegant models and derive strong analytical results. However, the assumptions of the rational expectation hypothesis i.e. perfect knowledge of the underlying structure of the data generating process and costless acquisition/processing of available information simply do not hold in reality. Perfect knowledge of an underlying model, that is essentially unobservable, overestimates the processing ability of economic agents. For example, the experimental economics literature has clearly shown that agents prefer simple rules of thumbs to forecast prices as opposed to complicated models. This is even more likely in the case of the largely uneducated farmers in less developed countries, completely unfamiliar with basic concepts of mathematical modeling and statistical forecasting. More importantly, often accurate information on stocks, inventories and competitors production plans is simply not available, especially in less developed countries.

Second, as argued by Grossman and Stiglitz (1976), if the costs of acquiring and processing information exceed the perceived benefits of better forecasts, then even rational actors will choose not to employ all the available information to forecast future variables. An alternative and more realistic view, the bounded rationality framework, argues that economic agents possess limited information processing ability and thus, weight the costs and benefits of acquiring additional information whilst formulating expectations about future outcomes. This is in contrast to models based on rational expectations (Miranda & Helmberger 1988), where only

the benefits of additional information vis-à-vis expectations are considered whilst ignoring the cost of collecting and processing information. Interestingly, in their seminal work, Brock and Hommes (1997) show that forecasts based on naïve expectations can be both rational and optimal if acquisition and processing of information is costly. Moreover, they show that several expectation regimes are potentially rational in an environment where agents have heterogeneous information processing costs.<sup>15</sup> We further elaborate these points in section-3.1.

In summary, in light of the empirical literature on expectations and anecdotal evidence derived from fieldwork and surveys in Pakistan, the assumption of naïve expectations does not seem unrealistic or “irrational”. In fact, the suitability of a given expectation regime hinges upon the specific institutional environment and background of the underlying research problem. Therefore, a solid understanding of the context of the research problem is essential to correctly specify an appropriate expectation regime. This understanding should help researchers’ answer the fundamental question: What types of information are commonly available to farmers at the time of making production decisions? Few would argue with the fact that current prices are the only source of reliable information easily available to poultry farmers in less developed countries plagued by numerous institutional voids and information asymmetries. Additionally, low levels of human capital (poor literacy rates compounded by limited opportunities of quality education) mean that prices also represent a category of information that is relatively “cheaper” to process, compared to information on stock levels, aggregate production and demand. In short, while, rational expectations hypothesis may be valid in certain economic environments it does not seem to be a likely expectation formulation method in our settings. We reiterate features of the economic environment that lend support to the assumption of naïve expectations in the poultry sector in Pakistan in section-3

## 2.5 Price Dynamics in Chaotic Cobweb Models

As mentioned before, price dynamics in original cobweb model are limited to three simple but unrealistic cases i.e. explosive divergence, periodic cycles and convergence to a steady state. Explosive divergence, leading to negative prices is obviously not feasible. While, deterministic cycles with fixed periods can be exploited by farmers pursuing counter cyclical production plans which will lead to dampening of the price cycles. Convergence to a steady state, though consistent with the neoclassical economics theory, contradicts the real world price data which is best characterized by quasi-cyclical behavior.

However, researchers have developed several refinements of the original cobweb models over the past decades that depict price dynamics consistent with the real world data. This stream of literature, popularly known as chaotic cobweb cycles, introduces non-linearities into the supply/demand equations whilst keeping the essential features of the original cobweb model i.e. backward looking expectations and production lags. Analogous to real world prices, the price dynamics generated from chaotic cobweb models depict seemingly random behavior, even though the models are entirely deterministic. Another interesting feature of chaotic cobweb models is their sensitivity to initial conditions, whereby small perturbations to parameters or

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<sup>15</sup>As mentioned above, industry studies also indicate that commodity producers have heterogeneous price expectations. This is not surprising given that different producers possess different information and have different costs associated with information collection and processing (Burton & Love 1996). However, we do not explicitly account for this aspect in our paper.

initial values results in qualitatively different price trajectories. However, data generated by chaotic cobweb models shows low first order autocorrelations and negative skewness, in contrast to high first order autocorrelation and positive skewness (occasional spikes) in the real price data. Researchers have proposed several explanations to explain the differences between data simulated from chaotic cobweb models and actual prices including slow adjustment towards optimal production in response to changes in prices (Onozaki et. al 2000), heterogeneous expectations (Chavas 1999 and Chavas 2000), risk aversion (Boussard 1996) and storage (Mitra & Boussard 2012).

Several other factors like, prices of substitutes and complements, consumer preferences, institutional factors like tariffs and production technology also have a nontrivial impact on the cyclical behavior of actual prices of agricultural products. For example, Westerhoff and Wieland (2010) develop a cobweb commodity model that accounts for the interaction between consumers, producers and heterogeneous speculators to reproduce price dynamics which mimic the cyclical price movements in actual commodity markets. Similar to our approach, Dieci and Westerhoff (2012), set up a model with two interacting linear cobweb markets, whereby farmers choose to produce either good based on profit differentials between the markets in any given period. Non-linearities in their model are endogenously generated by allowing suppliers to “switch” between markets. They find that interacting cobweb markets contributes to the price cyclicity observed in real world data. Lundberg et al. (2015) arrive at similar conclusions in a model of interacting cobweb markets with land-use competition between food and bioenergy crops.

Though, horizontal linkages between agricultural markets have been studied in the burgeoning theoretical literature on cobweb models but mutual interdependencies between upstream farmers and downstream farmers, an important aspect of agricultural production, has been largely overlooked. In addition to the empirical contribution, we try to address this gap in the theoretical literature by developing a simple model to capture the vertical linkages between upstream and downstream farmers in agricultural value chains under the naïve expectation hypothesis. Although specific to the poultry industry, our model can be modified to analyze other vertically linked cobweb markets especially livestock markets. An interesting feature of the vertically linked agricultural value chains is the interlinked structure of production and the associated interdependency between upstream and downstream farmers. This means that actual prices of upstream products are a function of expected prices of both downstream and upstream products. Also, in contrast to the theoretical literature on horizontally linked agricultural markets, our model has asymmetric production lags (also known as time delay system) due to the differences in the length of production cycles of upstream (chicks) and downstream farmers (broilers). The details of the model are provided in the next section.

To summarize, in this section we briefly reviewed the theoretical literature on price fluctuations in agricultural market. Thereafter, in light of the relevant empirical literature on the formation of expectations we argued that the observed fluctuations in prices of poultry products in Pakistan are a manifestation of cobweb cycles. Lastly, we highlighted important features of chaotic cobweb markets and important contributions in this area. In the next section we formulate a stylized dynamic model to simultaneously capture the optimizing behavior of chick and poultry farmers within the prevalent institutional environment under the naïve expectation hypothesis.

### **3-Theoretical Framework: Endogenous Price Fluctuations in an Interlinked Agricultural Market**

Armed with an understanding of the institutional environment, the mechanics of the production cycle and the price discovery process in the poultry sector in Pakistan along with an overview of the broader literature on cobweb cycles, we turn our attention towards understanding the production decisions of poultry farmers at the farm level.

#### **3.1 Poultry Farm Economics-Some Key Observations from Farmer Surveys**

First of all, based on fieldwork in Pakistan we found that the price of parent stock does not play a significant role in the short-term production decisions of farmers in the poultry value chain for several reasons. First, for reasons described above, breeders are generally cash rich companies with market power and hence produce at optimal capacity regardless of market prices in the short run. Accordingly, the price of parent stock remained fixed at 250/unit between 2008-2014, and decreased to 150/unit afterwards due to entry of additional grandparent stock companies. Second, as described in section-1, the lifecycle of parent stock is approximately 100 weeks, compared to 3 weeks and 6-7 weeks for chicks and broilers, respectively. Therefore unlike hatcheries and broiler farmers, breeders or parent stock farmers, cannot adjust production levels over short intervals in lieu of price signals. Therefore, one can conclude that the observed price fluctuations in the short-run are not stemming from the decisions of parent stock farmers since the price of parent stock is fixed during this interval and parent stock production is fairly stable over the short run. Although, we acknowledge that parent stock placement has an impact on poultry prices in the long run but it is not important in explaining short run price dynamics. Moreover, on average, the cost of parent stock procurement represents approximately 5% of the total production cost of breeders. Therefore, we believe that overlooking parent stock dynamics in our theoretical model does not impact the validity of our conclusions.

Feed costs, comprising approximately 60% of the total production costs, are a major component of production expenditures incurred by breeders and broiler farmers<sup>16</sup>. For broiler farmers, production costs are primarily driven by the rearing expenses incurred on chicks and hence tied to the number of chicks procured at the beginning of the production cycle. Whereas, for chick farmers, production costs are driven by the number of fertilized eggs incubated in hatcheries. The cost of fertilized eggs is correlated with the production costs of breeders, which largely comprises of the rearing cost of parent stock. Based on the responses of poultry farmers in our survey, we observed that although a fundamental determinant of profitability, feed costs are not the source of price fluctuations in the industry. First, although feed costs witnessed an upward trend over the past decade but compared to the large price fluctuations over short periods in the actual chick and broiler prices, changes in feed prices were not marred by volatility. Second, poultry farmers maintain a reasonable inventory of feed stock at any given time (usually for one production cycle and at times for several months), limiting the impact of contemporaneous changes in feed costs on production decisions in the short run. Lastly, due to

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<sup>16</sup> More specifically, for breeders variable costs comprise of feed (60%), parent stock price (5%), vaccinations and medicine (10%), while fixed costs (25%) include rent of controlled sheds, overheads and wages (5%). In the case of broiler farms, variable cost comprises of chick price (30%) and feed (60%), while fixed costs (10%) include rent of controlled sheds, overheads and wages.

the competitive nature of poultry markets, increases (decreases) in production costs translate into lower (higher) production and hence high (low) prices.

Due to the biological nature of the production process, production decisions (quantity supplied in future) are determined to a large extent by expectations about future prices today. Given the relatively large market for chicken products in Pakistan, the lack of cold storage facilities and limited demand for frozen chicken, means that production of poultry products takes place all year round. For instance, approximately 5 million broilers are slaughtered everyday in Lahore, the provincial capital of Punjab (poultry production hub), to meet demand. However, future prices are highly uncertain due to the absence of vertical integration, production planning via regional cooperatives or associations, pre-contracted production and agricultural futures markets<sup>17</sup>. Weak contract enforcement and incomplete risk markets further complicate the production decisions at the farm level. While, an opaque information environment characterized by lack of timely and accurate data on key variables like parent stock levels, incubated fertilized eggs, broiler placement and consumer demand, limits the usefulness of formal forecasting models. In an uncertain economic environment plagued with numerous institutional voids, farmers in the poultry value chain are very secretive about individual production plans. Consequently, coordination or communication between key players in the supply chain regarding production decisions is almost nonexistent.

Based on the responses of farmers during structured interviews, we observed that chick farmers look at current prices of chicks and broilers whilst formulating future production plans i.e. current prices of chicks and broilers served as a proxy for expected price of chicks and broilers in future. Their underlying logic was that if chick prices are high (low) today, then chick prices are expected to remain high (low) at the time of realization of planned production due to relatively short chick production cycle. On the other hand, higher (lower) broiler prices today were viewed as an indicator of higher (lower) broiler prices once the chicks hatched 3 weeks from now, implying a higher (lower) demand for chicks and thus higher (lower) chick prices. Likewise, in addition to chick prices, broiler farmers employed current broiler prices as a proxy for expected future price of broilers, whilst formulating their production plans.<sup>18</sup> As a result, broiler farmers increased (decreased) planned production, in view of high (low) current broiler prices under the expectation that the current high (low) price levels will continue into the following weeks. Unsurprisingly, actual chick prices also influence the production decisions of broiler farmers.<sup>19</sup> For example, if chick prices are exorbitantly high, liquidity constrained broiler farmers are compelled to buy a lower number of chicks, eventually leading to lower production

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<sup>17</sup> In agricultural cooperatives, pervasive in US and other developed countries, retailers generally submit future procurement orders to cooperatives based on forecasted demand. This information is quickly relayed to farmers in the value chain, so that they may adjust production plans accordingly.

<sup>18</sup> Note that broiler farmers face no uncertainty regarding chick prices, since they can observe chick prices at the time of production, whilst, chick farmers face uncertainty regarding both chick and broiler prices at the time of production.

<sup>19</sup> Credit constraints are a likely explanation for the impact of chick prices on broiler farmers' production decisions. For example, one broiler farmer remarked that chick prices were a fundamental driver of his production decisions. If chick prices were low he simply bought more chicks and hence produced more broilers and vice versa. Credit plays a key role in business model of poultry farmers and latest data shows that banks made net loans of Rs.4 billion in 2011 to the poultry sector and distributed loans of around Rs.3 billion in 2012, a fairly large amount given the size of the poultry sector. But over the last two years financing to poultry farms has fallen short of actual needs. (Dawn 2012)

of broilers and hence high broiler prices at the end of production cycle. Though not immediately obvious, the consistency of these observations with the naïve expectations hypothesis is clearly crystallized upon examination of the equilibrium relationships derived from the forthcoming theoretical model.

However a key question remains: under what conditions would rational agents choose backward looking expectations over forward looking expectations? Or in other words under what conditions are naïve expectations consistent with economic rationality? In order to answer this question we need to examine the relative costs and benefits of different expectation regimes. Looking at costs first, one can easily arrive at the conclusion that the cost of acquiring relevant information is very high in the prevalent institutional environment due to the absence of future markets on poultry products, limited communication and coordination between poultry farmers and lack of access to reliable data on key variables driving demand/supply. In addition to the opaque information environment, low levels of human capital, due to the lack of access to quality education and limited exposure to basic forecasting models, mean that the cost of processing information is also relatively high. Clearly, under such a situation the relatively high costs of formulating “better” forward-looking expectations outweigh its perceived benefits. Moreover, as we argue in section-6, if price trajectories are characterized by chaotic behavior, “better” forward-looking expectations offer minimal incremental benefits over “cheap” backward looking expectations. The aforementioned arguments highlight that in the presence of high information costs and chaotic price trajectories, naïve expectations are consistent with both rationality and optimality within a framework of bounded rationality. Consequently, the net benefits of simple forecasting rules of thumb based on backward looking expectation may exceed the net benefits of “expensive” alternatives based on forward looking expectations. Even though, the former method clearly neglects important information related to the dynamics of agricultural prices.

Furthermore, note that unlike other agricultural commodities, the production cycle of poultry products is very short i.e. a few weeks. Given such a short window, the assumption that current prices persist into the future seems quite reasonable. Moreover, continuously updating forecasts over such a short window based on forward looking expectations is both impractical and exorbitantly expensive. Therefore, the materialization of naïve expectations and the associated cobweb cycles is very likely in agricultural products like poultry that possess short production cycles compared to crops etc. that are characterized by relatively long gestation periods. Lastly, many economists acknowledge that assumptions of the rational expectation hypothesis are violated even in highly developed countries (Chavas 1999a, 1999b & 2000). Clearly these stringent assumptions do not hold in the weak institutional environment of less developed countries. Likewise, there is nothing in the empirical literature that supports the pervasiveness of rational expectations/forward looking expectations over naïve expectations. In fact, the balance of empirical evidence seems to favor the latter category.

In the following sections, we use economic modeling, statistical testing and numerical simulations to provide quantitative evidence to support the preceding analysis that has largely relied on qualitative evidence (prior literature, stylized facts, and anecdotal evidence based on surveys) to argue that price fluctuations in Pakistan poultry sector are a manifestation of cobweb cycles.

### 3.2 Poultry Value Chain-A Simple Model of Vertically Linked Cobweb Markets

In this subsection, we develop a stylized model to capture the optimizing behavior of chick and broiler farmers given the abovementioned institutional features of the Pakistan poultry sector. An interesting feature of the poultry value chain is the interlinked profit functions along with the mutual interdependency between upstream and downstream farmers. These vertical relationships between upstream and downstream agents have been examined extensively in the agricultural economics literature albeit under very different settings. Whereby, at any given point, the demand for intermediate products is derived from the solution to the profit maximization problem of agents further downstream in the value chain (Hicks, 1956; Gardner 1975). Given the constraint that price data as opposed to quantity data is generally available at reasonable frequencies in less developed countries, we use the abovementioned setup in a dynamic environment to derive empirically testable hypothesis on chick (upstream) and broiler (downstream) price series under the naïve expectations hypothesis.

Let,  $\omega_t^C$  denote actual chick price at time  $t$  and  $\tilde{\omega}_{t+l}^C$  represent the expectation of future chick prices  $l$ -periods (in other words a  $l$ -period forecast at time  $t$ ) from now, formulated at time  $t$ . Evidently, under naïve expectations:  $\tilde{\omega}_{t+l}^C = \omega_t^C$ . Similarly,  $p_t^B$  denotes the actual broiler price at time  $t$  and  $\tilde{p}_{t+l}^B$  represents expected price of broilers  $l$  periods from now formulated at time  $t$ . Time lags in our model capture the fact that poultry farmers cannot respond immediately to price signals, instead the response time/delay is a function of the length of the production cycle. Of course the market situation may change over the duration of the production cycle, introducing *ex-ante* uncertainty into the model. For expository clarity, we assume  $\Delta t = 3$  weeks (the chick production cycle) and revert to the original time configurations later in the empirical section.  $q_t^C$  and  $q_t^B$  represent the corresponding levels of production at time  $t$  for chick and broiler farmers, respectively. The cost functions for chick and broiler farmers are given by  $C_1$  and  $C_2$ , respectively, where  $C_i$  is a continuously differentiable, convex function i.e.  $C_i' > 0$  and  $C_i'' > 0$ . Both chick and broiler markets are competitive thus farmer are merely price takers. Lastly,  $N_1, N_2$  and  $N_3$  denote the total number of chick farmers, broiler farmers and retailers in the sector, respectively<sup>20</sup>.

As is often the case with dynamic models involving multiple agents and interlinked payoffs, we proceed to solve for the equilibrium backwards rather than forwards. Notice that due to the biological nature of the broiler production i.e. rearing of chicks into broilers, production plans need to be formulated in  $t + 1$  but will only be realized in  $t + 3$  (recall that the production cycle of broiler is six weeks i.e.  $t = 2$  periods since we assumed  $\Delta t = 3$ ). Therefore, at time  $t + 1$ , a representative broiler farmer solves the following profit maximization problem:

$$\text{Arg Max}_{(q_{t+3}^B)} \pi_{t+3} : q_{t+3}^B \tilde{p}_{t+3}^B - q_{t+1}^C \omega_{t+1}^C - C_2(q_{t+1}^C)$$

subject to

$$q_{t+3}^B = k q_{t+1}^C, 0 < k < 1$$

In words, the broiler farmer has to decide how many broilers to produce given the price of chicks at time  $t + 1$  and subject to a simple fixed proportions production technology that

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<sup>20</sup> In the section on numerical simulations, we will use the relative size of counter parties in a given market to incorporate the effects of any bargaining power on price fluctuations.



converts chicks purchased at time  $t + 1$  into broilers at time  $t + 3$  at a conversion rate of  $k$ . The technology parameter is assumed to be fixed and reliable estimates of the conversion ratio are easily available from industry reports. Production costs are a convex function of the number of chicks purchased at time  $t + 1$ . Where production costs comprises largely of feed costs. As mentioned before the broiler market in Pakistan is a live bird market, and in the absence of vertical integration or production contracts broiler farmers have to sell their produce at the going market rate. Therefore, given the time lag between formulation of production plans and realization of production, the expectation of future broiler prices plays a key role in determining the optimal level of broiler production.

Assuming  $C_2'$  is invertible, we can simply substitute in the broiler production function into the profit equation and use the Hotelling Lemma to get the optimal supply curve for broiler farmers<sup>21</sup>:

$$\frac{\partial \pi_{t+3}}{\partial q_{t+3}^B} = \tilde{p}_{t+3}^B - \frac{\omega_{t+1}^C}{k} - C_2' \left( \frac{q_{t+3}^B}{k} \right) = 0 \Rightarrow q_{t+3}^B = k(C_2')^{-1} [k\tilde{p}_{t+3}^B - \omega_{t+1}^C]$$

Note that since  $C_2$  is convex, so  $C_2'$  is an increasing function by definition and the inverse of an increasing function is also increasing. As a result, it is straightforward to observe that the quantity of broilers produced is increasing in price expectations in period  $t + 3$  and decreasing in the price of chicks i.e. the input price.

As mentioned before, in our institutional environment, there are no future contracts on either broilers or chicks, the poultry sector is not vertically integrated and even binding contracts are seldom enforceable in LDCs. Given these settings and in light of the literature reviewed in section-2 along with the anecdotal evidence collected from field work (interviews and surveys of poultry farmers) in Pakistan, naïve expectations seems a reasonably justified expectation regime in the poultry sector. Interestingly, the academic literature lends support to this expectation regime in the poultry sector e.g. even in a highly developed country like U.S, Chavas (1999a) empirically estimates a structural model based on joint profit maximization and finds that approximately 91% of poultry farmers expectations are consistent with naïve expectations. By definition, the naïve expectation hypothesis implies that  $\tilde{p}_{t+3}^B = p_{t+1}^B$  here i.e. current prices are expected to continue into the future. We substitute this into the optimal supply curve of a representative farmer, assume homogeneity and aggregate over the  $N_2$  broiler farmers in the poultry sector to arrive at the aggregate broiler supply function at time  $t + 3$ :

$$Q_{t+3}^{B,S} = \sum q_{t+3}^B = kN_2(C_2')^{-1} [kp_{t+1}^B - \omega_{t+1}^C]$$

Assuming a negatively sloped broiler retail demand curve for  $N_3$  homogenous retailers is given by  $F(p_{t+3}^B)$  where  $F' < 0$ , we can define aggregate demand for broilers at time  $t + 3$  as  $Q_{t+3}^{B,D} = N_3 F(p_{t+3}^B)$ . Now recall that the chicken market in Pakistan is predominantly a live bird

<sup>21</sup> Note that the convexity of cost function insures that the first order condition is a sufficient condition for optimality since  $\frac{\partial \pi_{t+3}}{\partial q_{t+3}^B} \geq 0$  if  $(k\tilde{p}_{t+3}^B \geq C_2'(q_{t+3}^B/k) + \omega_{t+1}^C)$  and  $\frac{\partial^2 \pi_{t+3}}{\partial q_{t+3}^B{}^2} < 0$ .

market with limited cold storage facilities. In the absence of inventory or trade, the current demand and supply situation determines the market clearing price, therefore we simply equate aggregate quantity demanded and supplied for broilers in time  $t + 3$  i.e.  $Q_{t+3}^{B,D} = Q_{t+3}^{B,S}$ , to get:

$$N_3 F(p_{t+3}^B) = kN_2 (C_2')^{-1} [kp_{t+1}^B - \omega_{t+1}^C]$$

Assuming  $F$  is invertible and after some simplifications, we get the following recursive broiler price equation:

$$p_{t+3}^B = F^{-1} \left( \frac{kN_2}{N_3} (C_2')^{-1} (kp_{t+1}^B - \omega_{t+1}^C) \right)$$

Since this equilibrium relationship holds for all time  $t$ , we can express it as time delay difference equation given by:

$$p_t^B = F^{-1} \left( \frac{kN_2}{N_3} (C_2')^{-1} (kp_{t-2}^B - \omega_{t-2}^C) \right) \quad (1)$$

Note that since  $F$  is a decreasing function,  $F^{-1}$  is also a decreasing function. Using this result along with the fact that  $(C_2')^{-1}$  is an increasing function, it is straightforward to arrive at the following comparative static results via the chain rule<sup>22</sup>:

- I.  $\frac{\partial p_t^B}{\partial p_{t-2}^B} < 0$
- II.  $\frac{\partial p_t^B}{\partial \omega_{t-2}^C} > 0$

Both results are intuitive. Result-I represents the standard cobweb phenomena i.e. assuming naïve expectations, if prices were high when production decisions were made (time  $t - 2$ ), then prices will be low in time  $t$  due to the resulting supply glut and vice versa. The vertically linked nature of the poultry production process is driving Result-II, e.g. if price of chicks was high when production decisions were made (time  $t - 2$ ) then broiler farmers will reduce procurement of chicks, leading to lower production of broilers in time  $t$  and hence higher broiler prices.

In order to close the model and fully specify the dynamics of broiler prices, we need to understand the dynamics of chick prices. Since the dynamics of upstream prices play a key role in determining the supply of the final downstream product i.e. broiler farmers cannot produce broilers without chicks! Consequently, prices at different levels of the value chain are interlinked, representing a coupled system of difference equations. A failure to account for this facet of agricultural markets can lead to erroneous conclusions.

We follow essentially the same steps to derive the difference equation for chick prices. Proceeding backwards, the chick farmers profit maximization problem at time  $t$  can be written as:

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<sup>22</sup> The composition of a decreasing function and an increasing function is always a decreasing function.

$$\text{Arg Max}_{(q_{t+1}^C)} \pi_{t+1} : q_{t+1}^C \tilde{\omega}_{t+1}^C - C_1(q_{t+1}^C)$$

Chick farmers simply need to decide how much chicks to produce next period. In light of the arguments mentioned in the previous subsection, we do not account for parent stock dynamics in our model and simply assume unconstrained production of chicks with a production lag of 1 period (i.e. 3 weeks). Assuming  $C_1'$  is invertible, first order condition yields the optimal supply curve for the chick farmers:

$$\frac{\partial \pi_{t+1}}{\partial q_{t+1}^C} = \tilde{\omega}_{t+1}^C - C_1'(q_{t+1}^C) = 0 \Rightarrow q_{t+1}^{C,S} = (C_1')^{-1}(\tilde{\omega}_{t+1}^C)$$

As expected the quantity of chicks supplied in period  $t + 1$  is increasing in the expected price of chicks in the next period because  $(C_1')^{-1}$  is increasing function. Now assuming chick farmers are homogenous and summing over the  $N_1$  chick farmers we get the aggregate supply curve of chicks under naïve expectations ( $\tilde{\omega}_{t+1}^C = \omega_t^C$ ):

$$Q_{t+1}^{C,S} = \sum q_{t+1}^{C,S} = N_1 (C_1')^{-1}(\omega_t^C)$$

As pointed out previously, chick prices and broiler prices are closely interrelated. Anecdotal evidence based on field work also suggests that broiler prices play a key role in the current production decisions of chick farmers. This is not surprising since higher broiler prices will evidently have a positive effect on the willingness to pay of broiler farmers for chicks. The resultant higher demand for chicks will translate into higher chick prices. Consequently, expectations about the (derived) demand for chicks at time  $t + 1$ , formulated in period  $t$  impact the optimal production decision of chick farmers. Therefore, following the standard method for analyzing vertically linked markets, we solve the broiler farmers profit maximization problem to derive the input (chick) demand at time  $t+1$ . Substituting the production function into the profit equation and applying the shepherd's lemma, the optimal demand for chicks is given by:

$$\frac{\partial \pi_{t+3}}{\partial q_{t+1}^C} = k\tilde{p}_{t+3}^B - \omega_{t+1}^C - C_2'(q_{t+1}^C) = 0 \Rightarrow q_{t+1}^{C,D} = (C_2')^{-1}(k\tilde{p}_{t+3}^B - \omega_{t+1}^C)$$

Clearly, the demand for chicks increases with the expected broiler prices in time  $t+3$ . Since, chick farmers anticipate expected demand for chicks by broilers farmers at the time of formulating production decisions i.e. time  $t$ , therefore, under naïve expectations we have  $\tilde{p}_{t+3}^B = p_t^B$ . Thus, the aggregate demand for chicks in time  $t+1$  under the naïve expectation hypothesis is given by:

$$Q_{t+1}^{C,D} = \sum q_{t+1}^{C,D} = N_2 (C_2')^{-1}(kp_t^B - \omega_{t+1}^C)$$

Because, chick farmers sell all their chicks to broiler farmers, we simply use the market clearing condition and equate aggregate quantity demanded of chicks with aggregate quantity supplied at time  $t+1$  i.e.  $Q_{t+1}^{C,S} = Q_{t+1}^{C,D}$ :

$$N_1 (C_1')^{-1}(\omega_t^C) = N_2 (C_2')^{-1}(kp_t^B - \omega_{t+1}^C)$$

After some algebra and re-indexing of the time subscripts as before, (since the equilibrium relationship holds for all time  $t$ ) we get the following difference equation for chick prices:

$$\omega_t^C = kp_{t-1}^B - C_2' \left( \frac{N_1}{N_2} (C_1')^{-1} (\omega_{t-1}^C) \right) \quad (2)$$

Since the composition of two increasing functions is always an increasing we arrive at the following intuitive comparative static results for the chick prices difference equation:

$$\begin{aligned} \text{III.} \quad & \frac{\partial \omega_t^C}{\partial \omega_{t-1}^C} < 0 \\ \text{IV.} \quad & \frac{\partial \omega_t^C}{\partial p_{t-1}^B} > 0 \end{aligned}$$

Result-III is the standard outcome in cobweb models, i.e. under the naïve expectation hypothesis, if prices were high at the time production plans were formulated than prices will be low at the end of the production period due to an oversupply and vice versa. In a vertically linked value chain, result-IV represents the positive effect of high broiler prices on the demand for chicks and vice versa. More specifically, broiler farmers are willing to pay higher prices to chick farmers in view of high broiler prices in the previous period, under the (naïve) expectation of benefiting from these higher broiler prices in future. Likewise, chick farmers can successfully bargain over higher chick prices if broiler prices were high in the previous period.

Together equation (1) & (2) represent a coupled system of time-delay difference equations that determine the trajectory of prices under the naïve expectation hypothesis. Since we do not account for the long run profit considerations, whereby the number of chick farmers, broiler farmers and retailers are endogenously determined by the zero profit or free entry condition, thus, Model-A is a representation of short run price dynamics only:

$$\text{Model A - } \begin{cases} p_t^B = F^{-1} \left( \frac{kN_2}{N_3} (C_2')^{-1} (kp_{t-2}^B - \omega_{t-2}^C) \right) & (1) \\ \omega_t^C = kp_{t-1}^B - C_2' \left( \frac{N_1}{N_2} (C_1')^{-1} (\omega_{t-1}^C) \right) & (2) \end{cases}$$

Like all economic models, this model is a simplified depiction of reality that aims to only capture the essential features of the underlying phenomenon in order to highlight important mechanisms driving price fluctuations. Whilst, ignoring auxiliary yet potentially important factors like capacity constraints, adjustment costs, market power, farmer risk averseness or price stickiness. Nevertheless, given the data limitations, we feel that our modeling approach captures key aspects of agricultural value chains i.e. vertical linkages, production lags and price uncertainty. At the same time, it provides us with a parsimonious framework to specify and interpret results from an empirical model. More specifically, we seek to empirically test whether relationships I-IV are present in the actual data, where their presence lends support to the theory of endogenous price fluctuations in the Pakistan poultry sector. Therefore, in the next section, we use actual data to estimate the aforementioned coupled system of time delay difference equations in order to empirically evaluate the validity of results I-IV and thus determine whether or not the observed price fluctuations are consistent with the cobweb phenomena.

#### **4-Empirical Evidence-Naïve Expectations & Cobweb Cycles in the Pakistan Poultry Sector**

Up till now, we have presented some qualitative evidence (unique institutional environment of the poultry sector, responses of poultry farmers during structured interviews and stylized facts associated with poultry price data in Pakistan) that lends support to the theory of endogenous price fluctuations (i.e. existence of cobweb cycles) in the Pakistan poultry sector. The primary objective of this section is to formulate an appropriate econometric model to evaluate the validity of this theory empirically. To this end, we first analyze the time series properties of the data. Thereafter, we develop an econometric methodology to statistically test whether or not the actual price data conforms to the predictions (results I-IV) made by our stylized model of price dynamics in a vertically linked agricultural value chain under the naïve expectation hypothesis.

The unique, nature of our dataset lends itself perfectly for this type of analysis. Because, estimates from econometric models of price dynamics based on high-frequency data are generally considered to be more reliable compared to estimates based on low-frequency data (Von-Crammon Taubadel & Loy 1996). The original data comprised of daily prices, however, given the length of the poultry production cycle, using daily data would lead to an exponential rise in the number of estimated parameters<sup>23</sup>. Since the production cycle can be easily divided into weekly increments i.e. 3 weeks for chicks and approximately 6-7 weeks for broilers, we converted the daily prices into average weekly prices. In addition to generating smoother data, aggregation over weekly periods is also more intuitive. Because, farmers are more likely to use prices over the past few days to formulate (naïve) expectations about future prices as opposed to merely the prices on a given day.

The nature of the production cycle rules out monthly aggregation. Because in that case the frequency of aggregation i.e. months will not match the duration of the production cycle, masking the production dynamics along with the associated production decisions at the farm level. Additionally, with monthly aggregation it will be difficult to pinpoint the “current” price farmers use to form future expectations. Perhaps, the frequency mismatch between the length of production cycle and data aggregation is a key factor behind the (incorrect) rejection of the naïve expectations hypothesis in previous work on price fluctuations in some agricultural markets. Clearly, data at annual or quarterly frequency is not suitable for the evaluation of the naïve expectation/cobweb cycle hypothesis. Since, low frequency data masks the uncertainty faced by farmers and the dynamics of production decisions at the farm level, while, knowledge of both is essential to devise an appropriate empirical test to indentify cobweb cycles.

##### **4.1 Time Series Properties of the Data-Tests for Stationarity & Cointegration**

It is well known that prices, especially prices of agricultural products are characterized by non-stationary behavior i.e. time varying mean, variance or covariance. Therefore, we need to carefully evaluate the time series properties of the data before specifying an econometric model to empirically test results I-IV. Since, OLS estimates based on non-stationary data are usually

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<sup>23</sup> For example in an unrestricted model with daily data, we would have to estimate more than 100 parameters, severely compromising the consistency of the estimates. At the same time there was minimal day to day variation in daily prices, e.g. there is no price change on consecutive days for more than 50% of the data in both chick and broiler price series, therefore we do not lose significant information by averaging over weekly intervals.

spurious at the same time standard regression diagnostics are no longer valid. Therefore, we first need to establish whether chick and broiler prices are stationary or non-stationary. In the case of the latter, we also need to check the order of integration of both series to determine whether or not both series are cointegrated. Once we have this information, an appropriate econometric model can be specified to empirically test results I-IV.

In view of the low power of the augmented Dicky Fuller (ADF) unit root test, Elliott et al (1996) proposed the Dicky Fuller-Generalized Least Square (DF-GLS) unit root test. Their test is identical to the ADF unit root test except that the underlying data is transformed using a generalized least squares (GLS) regression before performing the ADF test. The theoretical literature has shown that DF-GLS unit root test possesses significantly higher power and efficiency compared to the simple ADF unit root test (Ng and Perron 2001 & Perron and Ng 1996). Therefore, we report the test statistic from DF-GLS unit root test instead of the standard ADF unit root test in Table-3<sup>24</sup>. The DF-GLS unit root test shows that both chick and broiler prices are non-stationary i.e. possess unit roots in levels but are stationary in first differences. This conclusion is supported by a visual analysis of chick and broiler prices line plots presented in figure-1.

Given that both series are I(1), we employ the Johansen & Juselius (1990) co-integration test to determine if chick and broiler prices are cointegrated. It is a maximum likelihood ratio test based on the maximal eigen value or the trace of the coefficient matrix of the underlying vector autoregressive (VAR) model. The max and trace statistic, reported in panel-B, show that both price series are cointegrated i.e. move together in the long run<sup>25</sup>. This result is intuitive, since one would expect a relationship between input (chick) and output (broiler) prices in the long run.

**Table 3-Time Series Properties of Data**

<i>A-Test for Stationarity: Dicky Fuller-Generalized Least Square (DF-GLS) Unit Root Test</i>		
	Levels	First Difference
Broiler Prices ( $p_t^B$ )	-2.16	-10.23***
Chick Prices ( $\omega_t^C$ )	-1.75	-6.57***
<i>B-Test for Cointegration: Johansen (1995) Cointegration Test</i>		
	Trace Statistic ( $r = 1$ )	Max Statistic( $r = 1$ )
Broiler prices ( $p_t^B$ ) & Chick prices ( $\omega_t^C$ )	12.14**	12.14**

All statistical tests are based on average weekly prices from June 2008 to June 2015. The data was sourced from Pakistan Poultry Association (North). All prices are in nominal units of the local currency. Note that the null hypothesis in the DF-GLS test is presence of a unit root. Therefore, accepting the null corresponds to non-stationarity while rejecting the null corresponds to stationarity. The Ng-Perron modified Akaike information criterion (MAIC) was used to determine the optimal number of lags in autoregressive models for the DF-GLS unit root test. A restricted trend specification was used for the Johansen (1995) cointegration test based on the Pantula principle (1989), while, the optimal number of lags in the underlying model were based on commonly used information criterion (SBIC & HQIC). Note that  $r$  denotes the number of cointegrating vectors under the null, in bivariate system  $r$  is

<sup>24</sup> Based on the ADF test, broiler price series was marginally stationary in levels. This conclusion seemed erroneous because the autocorrelation function showed that the autocorrelations don't die out as the lags are increased. A clear violation of:  $\lim_{k \rightarrow \infty} Cov(x_t, x_{t-k}) = 0$ , an important property of stationary data. Visual inspection of the line plots in figure-2, also lead one to doubt the conclusion of stationarity.

<sup>25</sup> Neither statistic follows a chi-square distribution but critical values are provided by Johansen & Juselius (1990) paper.

In light of these findings, an empirical model in price levels is not a viable strategy. A simple vector autoregressive (VAR) model in first differences seems an appropriate choice given the equilibrium relationships derived in the previous section represent a coupled system of difference equations of chick and broiler prices. But, as we argue later, a bivariate vector error correction model (VECM) is our preferred specification.

#### 4.2 Identification & Empirical Strategy

In the previous section we used a simple dynamic model of profit maximization with vertical linkages between upstream and downstream farmers to derive difference equations for chick and broiler prices (equations I & II) under the naïve expectation hypothesis. In order to empirically test the comparative static results (I-IV) in an OLS framework we calibrate the underlying model with a quadratic cost function for both type of farmers and a negatively sloped linear retail demand curve for broilers. These functional forms are commonly used in research on cobweb markets in the agricultural economics literature (Onozaki et al 2000; Dieci and Westerhoff 2009; Dieci and Westerhoff 2012). The comparative static results (I-IV) can also be identified with other more general class of functions satisfying the relevant assumptions by taking a first order Taylor series expansion of the resulting nonlinear system of equations. However, we avoid this approach for the sake of simplicity. Details of the derivation are relegated to appendix-1, but it is easy to see that the comparative static results I-IV are well identified in the resulting system of difference equations<sup>26</sup>:

$$\text{Model B} \begin{cases} p_t^B = \frac{a}{b} - \frac{N_2 k^2}{2bN_3} p_{t-2}^B + \frac{N_2 k}{2bN_3} \omega_{t-2}^C & (3) \\ \omega_t^C = k p_{t-1}^B - \frac{N_1}{N_2} \omega_{t-1}^C & (4) \end{cases}$$

Where  $a$  denotes the extent of the market and  $b$  represents sensitivity to price in the linear retail demand function of broilers, all other variables are as defined above.

Since we are primarily interested in the verification or falsification of results I-IV as opposed to point estimates, we can estimate the following well identified system to empirically evaluate the existence of cobweb cycles in the poultry sector after reverting to the actual time configurations based on weekly data<sup>27</sup>:

$$\begin{cases} p_t^B = \alpha + \beta_{p,7}^B p_{t-7}^B + \beta_{\omega,7}^B \omega_{t-7}^C + \epsilon_t^B \\ \omega_t^C = \beta_{p,3}^C p_{t-3}^B + \beta_{\omega,3}^C \omega_{t-3}^C + \epsilon_t^C \end{cases}$$

<sup>26</sup> All variables in the above equations are strictly positive, therefore the original comparative static results (I-IV)

hold true i.e.  $\frac{\partial \omega_t^C}{\partial \omega_{t-3}^C} < 0, \frac{\partial \omega_t^C}{\partial p_{t-3}^B} > 0, \frac{\partial p_t^B}{\partial p_{t-7}^B} < 0$  &  $\frac{\partial p_t^B}{\partial \omega_{t-7}^C} > 0$

<sup>27</sup> Recall that the length of the chick production cycle is 22 days (approximately 3 weeks), whereas the length of the broiler production cycle is 6-7 weeks or twice the length of the chick production cycle. We assume a 7 week production cycle for broilers since it results in a better fit of the model compared to a 6 week cycle.

Where, the subscript of the parameters represents the underlying lag, while the superscript identifies the relevant price equation ( $B$  for broiler price equation and  $C$  for chick price equation). Since, the price data is non-stationary in levels; we cannot estimate this system in levels due to the possibility of spurious estimates. Additionally, standard regression diagnostics are no longer valid if the underlying variables are  $I(1)$ . Therefore, we simply take first differences of the above-mentioned system and instead estimate a model in first differences. Even though, we lose significant information due to first differencing, but this loss is offset by the gains derived from the reliability of parameter estimates based on stationary data. At the same time the basic intuition behind results I-IV remains intact in a model specified in first differences<sup>28</sup>.

$$\begin{cases} \Delta p_t^B = \beta_{p,7}^B \Delta p_{t-7}^B + \beta_{\omega,7}^B \Delta \omega_{t-7}^C + \Delta \epsilon_t^B \\ \Delta \omega_t^C = \beta_{p,3}^C \Delta p_{t-3}^B + \beta_{\omega,3}^C \Delta \omega_{t-3}^C + \Delta \epsilon_t^C \end{cases}$$

More specifically, parameter estimates of  $\beta_{p,7}^B < 0$ ,  $\beta_{\omega,7}^B > 0$ ,  $\beta_{p,3}^C > 0$  and  $\beta_{\omega,3}^C < 0$  will lend support to the existence of cobweb cycles. Obviously, other factors beyond expectations of chick and broiler prices impact the production decisions of poultry farmers and hence actual prices. To improve the fit of the model and reduce the likelihood of omitted variable bias, we describe the key factors influencing poultry prices along with their relevant proxies in our empirical model in the next subsection.

#### 4.3 Model Specification-Incorporating Additional Explanatory Variables

In the section on poultry farm economics, we documented that feed costs comprise a large portion of production costs in the poultry sector. The price of poultry feed is linked to prices of primary agricultural commodities, whilst overheads at poultry farms are primarily driven by energy and labor costs. Poultry farmers decide how many chicks/broiler to produce based on estimates of rearing costs at the beginning of the production cycle and usually procure an adequate amount of inventory to cover feeding requirements for the duration of the production cycle. In the absence of data on poultry feed costs, which unarguably varies from farm to farm depending on management practices and quality of feed. We use the sensitive price indicator (SPI) at the beginning of a given production cycle as a (noisy) proxy for production costs (or more aptly rearing costs) during the production cycle in our empirical model<sup>29</sup>. The sensitive price indicator published by Pakistan Bureau of Statistics on a weekly basis, represents a weighted index comprising of prices of different agricultural commodities (corn, wheat, maize, rice etc), energy (petrol, diesel & electricity costs) and labor (wages of workers in the primary/secondary sector). All other things equal, an increase in production costs at the beginning of the production cycle (proxied by an increase in SPI) would result in a decrease in planned production and hence higher prices at the end of the production cycle assuming competitive markets. Therefore, we expect the coefficient on SPI to be positive and statistically significant in the difference equations of both chick and broiler prices.

<sup>28</sup> Parameter estimates from a model specified in levels and in first differences have identical interpretation in a linear model.

<sup>29</sup> The SPI series was non-stationary in levels but stationary in first differences, thus we used the first difference of SPI in our econometric specification.



The summer season is particularly harsh in Punjab, the production hub of poultry in Pakistan, with temperatures ranging from of 100-120 F (38-50 C). Moreover, chronic energy shortages, amplified by the high demand for the air-conditioning in the summer season, lead to long hours of energy outage. Poultry farmers are adversely affected by the planned load shedding, since broilers and chicks raised in control environments are sensitive to changes in temperature. Due to electricity outages, the temperature in controlled sheds cannot be controlled properly (or controlled at a higher cost via privately generated energy) and excessive heat leads to high rates of mortality. Likewise, due to the rudimentary and often crude transportation methods, mortality rates during delivery of chicks and broilers increase substantially in the summer season. Both factors have an adverse impact on marketed production, leading to higher prices in the summer season. In order to capture this effect, we use a dummy variable, spanning from the beginning of May to the end of July (the hottest months in Punjab), to control for the effect of summer season on the prices of poultry products<sup>30</sup>.

In a country like Pakistan where the diet is deficient in proteins and other sources of animal protein (beef/lamb/fish) are relatively expensive, demand for poultry products is high. Moreover, given that poultry products are a food based commodity, all other things equal, the demand for poultry products remains fairly constant over the year. Nevertheless, Pakistan is also a low income country (GDP per capita of less than \$1,000), therefore, large increase in prices of poultry products result in a significant reduction in demand for chicken. Religious festivities also have an impact on the demand for chicken, especially in predominantly Muslim countries like Pakistan<sup>31</sup>. During Eid-ul-Azha the price of chicken declines due to a reduction in demand, as meat from cattle/lamb etc slaughtered on these days is stored and consumed for several weeks thereafter.<sup>32</sup> Although our model is primarily geared towards capturing the supply side dynamics, but in order to capture this key demand side effect in our model, we use a contemporaneous dummy variable to capture the effect of the festive season of Eid-ul-Azha on broiler and chick prices. However, unlike festivities in the USA & Europe (Thanksgiving, Christmas, Easter etc), festive seasons in Muslim countries like Pakistan are based on the lunar calendar as opposed to the Gregorian calendar. Therefore, we converted the Gregorian calendar into the lunar calendar to capture the effect of Eid-ul-Azha.<sup>33</sup> The Eid-ul-Azha dummy variable in our model corresponds to a 7-week period in a given lunar year, starting 2 weeks prior to Eid-ul-Azha and continuing thereafter for another 5 weeks. Based on survey responses, the reduction in demand

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<sup>30</sup> Extremely cold weather also has an adverse impact on the supply of poultry products. However, the winter season in Pakistan is both short and mild. More importantly, over the last 8 years the winter season has overlapped with Eid-ul-Azha, a period of low demand (hence lower prices) potentially confounding the effect of winter season (low supply and high prices). Therefore, we do not include the winter dummy in the model.

<sup>31</sup> Ramzan is another important month in Islamic societies, which includes fasting for 30 days, followed by feasting for 3 days. The effect of Ramzan on consumer behavior vis-à-vis poultry prices is less clear, on one hand fasting results in lower consumption. But on the other hand an increase in charitable giving/ feeding of the poor during the month, followed by feasting in the final days leads to higher consumption. At the same time over the past decade, Ramzan has overlapped with the summer season; therefore it is difficult to identify the effect of Ramzan on poultry prices due to the supply effect described earlier.

<sup>32</sup> Eid-ul-Azha is celebrated from 10<sup>th</sup>-12<sup>th</sup> of Zul-Hijjah, the 12<sup>th</sup> month of the Islamic Calendar. Muslims slaughter large animals (cattle, camel, lamb and sheep etc.) to commemorate the sacrifices of Prophet Abraham on this occasion.

<sup>33</sup> Since, the lunar calendar is based on the moon; the length of a given month is not fixed but depends on moon sighting. Consequently, the months of the lunar calendar shift by approximately 11 days vis-à-vis the Gregorian calendar in a given year, thus lunar months do not correspond to seasons. All lunar month dates were indentified based on the Ummul Qura, Saudia Arabia lunar calendar.

of chicken due to an increased consumption of lamb/beef is adequately captured in this 7-week period.

Taking the abovementioned exogenous factors into account and adding controls for any time trends in the difference equations for broiler and chick prices, gives us model-I<sup>34</sup>:

$$\Delta p_t^B = \alpha^B + \gamma^B t + \beta_{p,7}^B \Delta p_{t-7}^B + \beta_{\omega,7}^B \Delta \omega_{t-7}^C + \gamma_{t-7}^B \Delta SPI_{t-7} + D_S^B \text{Sum}_t + D_{\text{Eid}}^B \text{EidAzha}_t + \epsilon_t^B$$

$$\Delta \omega_t^C = \alpha^C + \gamma^C t + \beta_{p,3}^C \Delta p_{t-3}^B + \beta_{\omega,3}^C \Delta \omega_{t-3}^C + \gamma_{t-3}^C \Delta SPI_{t-3} + D_S^C \text{Sum}_t + D_{\text{Eid}}^C \text{EidAzha}_t + \epsilon_t^C$$

Given that both chick and broiler prices are endogenously linked, we call model-1 a restricted VAR model in first differences since intermediate lags are not included. Inclusion of intermediate lags results in a near VAR model in first differences<sup>35</sup>. For the sake of robustness we estimate both versions, to rule out the possibility that estimates from the restricted model are merely statistical artifacts.

However recall that unit root tests revealed that broiler and chick prices are I(1) variables, while the Johansen cointegration showed that both variables are cointegrated. Although model-1 fully captures the short run dynamics derived from our theoretical model, a failure to control for cointegration in model-I; throws away useful information about the long run behavior of prices. In order to remove this bias, we employ the Engle & Granger (1987) two-step procedure. Engle & Granger (1987) showed that if two variables are non-stationary in levels but their linear combination is stationary, then this implies that a common stochastic trend is driving these variables. Thus, although the variables may drift apart in the short run, they will tend to converge towards the equilibrium relationship in the long run.

In the first stage of the Engle & Granger (1987) two-step procedure, the long-run (equilibrium) relationship is estimated as a simple OLS regression of the I(1) variables. If the residuals from this regression are stationary, then this implies that the underlying variables are cointegrated. In the second stage, Engle & Granger (1987) showed that if two variables are cointegrated (residuals from stage-1 regression are stationary) then the underlying data generating process can be represented by an error correction model and vice versa. Following this approach, we first estimate the cointegrating (long-run) relationship between broiler and chick prices. Note that given that chicks are a key input for broiler production, it is straightforward to derive a long run relationship between broiler and chick prices i.e.  $p^B = f(\omega^B)$ <sup>36</sup>. Taking a linear approximation of this relationship, we can estimate a simple OLS

<sup>34</sup> Note that SPI was non-stationary in levels but stationary in first differences.

<sup>35</sup> A VAR model with different lag length in the underlying equations is called a near VAR model. The same definition applies for a near VECM model.

<sup>36</sup> This involves removing time subscripts (and hence expectations) from the profit maximization problem of broiler farmers in section-3, including long run costs into the model and taking first order conditions. In addition to the first order optimality conditions, the (long-run) zero profit condition will determine the number of farmers and hence total production in a long run equilibrium.

model, given by  $p_t^B = \alpha + \beta\omega_t^C + \mu_t$ <sup>37</sup>. The resulting residuals:  $\hat{\mu}_t = p_t^B - \hat{\alpha} - \hat{\beta}\omega_t^C$ , are tested for stationarity. The results are summarized below:

**Table 4-Engle & Granger Cointegration Test**

<i>A-Estimate the Cointegrating Relationship: <math>p_t^B = \alpha + \beta\omega_t^C + \mu_t</math></i>		
	$\alpha$	$\beta$
OLS Estimates	91.51***	0.52***
<i>B-Test for Cointegration: <math>\hat{\mu}_t = p_t^B - \hat{\alpha} - \hat{\beta}\omega_t^C</math> Stationarity Tests</i>		
	ADF	DF-GLS
$\hat{\mu}_t$	-4.23***	-5.08***

All statistical tests are based on average weekly prices from June 2008 to June 2015. The data was sourced from Pakistan Poultry Association (North). All prices are in nominal units of the local currency. Note that the null hypothesis in unit root tests is the presence of a unit root. Therefore, accepting the null corresponds to non-stationarity while rejecting the null corresponds to stationarity. \*\*\*, \*\*, \* represent significance at 1%, 5% and 10% respectively.

Unit root tests clearly indicate that the predicted residuals are stationary, confirming the earlier conclusion from the Johansen cointegration test. In the second step, Engle & Granger (1987) showed that if the underlying series is cointegrated, OLS estimates from an error correction model are super consistent. An error correction model captures how the underlying endogenous variables behave in the short run consistent with the long-run (cointegrating) equilibrium relationship. To this end, lagged residuals from the cointegrating equation are included in the model to capture the contemporaneous effect of deviations from long run equilibrium on the current dynamics of the endogenous variables. However, note that for the error correction model to be valid, the adjustment parameters ( $\rho$ ) should be negative, signifying that short run deviations from the long-run relationship are corrected. The error correction model applied to our settings can be represented by model-II:

$$\Delta p_t^B = \alpha^B + \gamma^B t + \beta_{p,7}^B \Delta p_{t-7}^B + \beta_{\omega,7}^B \Delta \omega_{t-7}^C + \gamma_{t-7}^B \Delta SPI_{t-7} + D_S^B Sum_t + D_{Eid}^B EidAzha_t + \rho^B \mu_{t-1} + \epsilon_t^B$$

$$\Delta \omega_t^C = \alpha^C + \gamma^C t + \beta_{p,3}^C \Delta p_{t-3}^B + \beta_{\omega,3}^C \Delta \omega_{t-3}^C + \gamma_{t-3}^C \Delta SPI_{t-3} + D_S^C Sum_t + D_{Eid}^C EidAzha_t + \rho^C \mu_{t-1} + \epsilon_t^C$$

Model-II does not include intermediate lags, doing so results in a near vector error correction model (VECM) or an unrestricted VECM. As before, for sake of robustness we will estimate both the restricted and unrestricted versions of model-II.

#### 4.4 Discussion of Estimation Results from Restricted Model

Note that all the variables in model-I and model-II are I(0), therefore we can use OLS estimation and employ standard hypothesis testing methods (t-statistics, F-statistics etc) and diagnostics to evaluate the estimation results. Since chick and broiler prices are endogenous, we estimate the difference equations in each model simultaneously to capture any contemporaneous

<sup>37</sup> Traditional diagnostics, like t-statistics from the cointegrating (long-run) equation are not easily interpretable, because the distribution of the t-ratio is not known due to the presence of I(1) variables. The only purpose of estimating the cointegrating equation is to test whether the residuals are stationary or non-stationary.

correlation between error terms in the chick and broiler price equations. Parameter estimates from the restricted VAR and VECM model are reported in Table-5.

**Table-5 Results from Restricted Model**

	<b>Model-I: Restricted VAR</b>		<b>Model-II: Restricted VECM</b>	
	Chick Price Equation ( $\Delta\omega_t^C$ )	Broiler Price Equation ( $\Delta p_t^B$ )	Chick Price Equation ( $\Delta\omega_t^C$ )	Broiler Price Equation ( $\Delta p_t^B$ )
$\alpha$	0.205 (0.635)	-0.252 (0.918)	-0.439 (0.799)	-3.669*** (1.103)
$\gamma$	0.00007 (0.003)	0.0010 (0.004)	0.003 (0.004)	0.019*** (0.005)
$\Delta\omega_{t-3}^C$	-0.092* (0.052)		-0.092* (0.052)	
$\Delta p_{t-3}^B$	0.096*** (0.037)		0.093** (0.038)	
$\Delta\omega_{t-7}^C$		0.145** (0.074)		0.176** (0.072)
$\Delta p_{t-7}^B$		-0.076 (0.054)		-0.040 (0.053)
$\Delta SPI_{t-3}^C$	0.500* (0.271)		0.484* (0.272)	
$\Delta SPI_{t-7}^B$		0.717* (0.388)		0.700* (0.375)
<i>Summer</i> <sub>t</sub>	0.642 (0.691)	1.553 (0.989)	0.798 (0.696)	2.419** (0.969)
<i>EidAzha</i> <sub>t</sub>	-3.019*** (0.923)	-2.107 (1.322)	-3.282*** (0.941)	-3.26*** (1.29)
$\mu_{t-1}$			-0.026 (0.019)	-0.136*** (0.026)
R – Square	5.73%	3.36%	6.28%	9.93%
Portmanteau Test	8.227	33.377***	7.946	38.351***

Estimates based on weekly prices from June 2008 to June 2015 for a total of 367 observations. The data was sourced from Pakistan Poultry Association (North). All prices are in nominal units of the local currency. Parameter estimates reported adjacent to the corresponding row variables while standard errors provided in the parenthesis below the parameter estimates. Summer denotes a dummy variable representing the period May-July in a given year. EidAzha denotes a dummy variable corresponding to the period 2 weeks before and 5 weeks after the 10<sup>th</sup> of Zul-Hijjah (12<sup>th</sup> month in the Islamic calendar). The underlying equations were estimated simultaneously using SUR. The portmanteau (or Q) test for white noise is based on 7 lags of the residuals, where the maximum lag length in the underlying model is used as the criterion for the selection of the number of residual lags to test for zero autocorrelation. Serially uncorrelated errors is the null hypothesis of the portmanteau test. Asterisks indicate statistical significance, where \*\*\*, \*\*, \* represent significance at 1%, 5% and 10% respectively.

For sake of brevity we focus on the dissecting the empirical estimates rather than delving into the economic intuition behind Results-I-IV, which has been discussed in section-3. The first column in table-5 reports the results from the Model-1 i.e. restricted VAR model. Estimates from the chick price equation lend strong support to our theory of endogenous price fluctuations in the poultry sector i.e.  $\beta_{p,3}^C > 0$  (Result-IV:  $\frac{\partial \omega_t^C}{\partial p_{t-3}^B} > 0$ ) with a p-value of 0.01 and  $\beta_{\omega,3}^C < 0$  (Result-III:  $\frac{\partial \omega_t^C}{\partial \omega_{t-3}^C} < 0$ ) with a p-value of 0.07. The estimates from the broiler price equation are also broadly consistent with the theory of endogenous price fluctuations. For instance,  $\beta_{\omega,7}^B > 0$

(Result-II:  $\frac{\partial p_t^B}{\partial \omega_{t-7}^C} > 0$ ) with a p-value of 0.05 and  $\beta_{p,7}^B < 0$  (Result-I:  $\frac{\partial p_t^B}{\partial p_{t-2}^B} < 0$ ), although, the latter coefficient is marginally insignificant at conventional levels of significance (p-value of 0.16). Column-2 reports the results from model-II, the restricted VECM model which accounts for the long run equilibrium relationship between chick prices and broiler prices whilst modeling the short-run price dynamics. The empirical estimates are similar to the results from model-I and broadly consistent with results I-IV.

Perhaps, one explanation for the insignificance of  $\beta_{p,7}^B$  is the dominance of the effect of actual chick prices on broiler farmers' production decisions (hence actual broiler prices in future) compared to the effect of (naïve) expected future prices of broilers on broiler farmers' production decisions. For example, if a broiler farmer is credit constrained, he may not be able to purchase additional chicks in lieu of high chick prices, even though higher expected prices of broilers in future call for additional procurement of chicks. Other possible explanations for the insignificance of  $\beta_{p,7}^B$  include noisy data, factors not captured in our simplified model (market power, bargaining, farmer heterogeneity etc) or the restrictive functional form assumptions used to identify the comparative static results for the broiler price equation. Nevertheless, it is very unlikely that cobweb cycles in the upstream product do not translate into cobweb cycles in the upstream products, given the fact that market for intermediate goods (chicks) clear and all chicks are eventually converted into broilers. Therefore, we conclude that taken together, the overall results presented in table-5 lend support to the theory of endogenous price fluctuations i.e. naïve expectations in the given institutional environment leads to cobweb cycles in the prices of poultry products in Pakistan.

Table-5 also shows that all else equal, an increase in price levels (proxy for feed prices) at the beginning of the production cycle leads to a reduction in planned production and hence higher prices (for both broilers and chicks) at the end of the production cycle. As explained before, this result is intuitive, since farmers facing increasing feed costs at the beginning of the production cycle curtail planned production resulting in higher prices at the end of the production cycle. Similarly, Eid-ul-Azha, a period characterized by increased consumption of beef/lamb, has a negative effect on the prices of both chicks and broilers, due to a reduction in the demand of chicken during this period. The summer season does not have a statistically significant (adverse) impact on the production of chicks. Although, broiler prices increase significantly during the summer season in lieu of supply-side reasons, described in section 3.1. Lastly, broiler prices on average experienced a positive trend over the past decade while chick prices remained stagnant, a finding substantiated by line plots in figure-2. Also, note that the signs of the error correction terms ( $\mu_{t-1}$ ) in column-II are negative for both price equations, indicating that the VECM is a valid representation of the underlying data generating process.

As far as model specification is concerned, controlling for the long run-relationship between chicks and broilers leads to a significant improvement in model fit, especially in the broiler price equation. Bearing in mind that a relatively low R-square is not uncommon in econometric models specified in first differences, due to the information lost as a result of first differencing the data. For model diagnostics, we use the portmanteau (or Q) test to check for

serially correlated residuals<sup>38</sup>. In order to check whether residuals from the underlying empirical model are white noise, the portmanteau test determines whether autocorrelations between residuals at multiple lags are statistically different from zero. Whereby, serially uncorrelated residuals (null hypothesis of the portmanteau test) suggest that the model is well specified while serially correlated residuals (alternative hypothesis of the portmanteau test) are an indication of model specification errors. Based on portmanteau test for white noise we cannot reject the null hypothesis of serially uncorrelated errors in the chick price equation (in both models), suggesting that the chick price equation is properly specified and hence parameter estimates are valid<sup>39</sup>. However, we reject the null hypothesis of no residual correlation for broiler equation (in both models), signaling towards possible model specification errors in the broiler price equation.

For the sake of robustness, estimates from the more conventional econometric models i.e. near VAR model (unrestricted model-I) and the near VECM model (unrestricted model-II) are presented in table-6. The purpose of these results is to determine whether or not parameter estimates obtained from the restricted models are statistical artifacts driven by the exclusion of intermediate lags of explanatory variables. Moreover, we also want to see if (possible) specification errors in the broiler equation are mitigated in the unrestricted models.

**Table-6 Results from Unrestricted Model**

	<b>Model-I: Near VAR</b>		<b>Model-II: Near VECM</b>	
	Chick Price Equation ( $\Delta\omega_t^C$ )	Broiler Price Equation ( $\Delta p_t^B$ )	Chick Price Equation ( $\Delta\omega_t^C$ )	Broiler Price Equation ( $\Delta p_t^B$ )
$\alpha$	0.278	-0.056	-0.905	-5.41***
$\gamma$	-0.0002	0.0003	0.006	0.027***
$\Delta\omega_{t-1}^C$	-0.037	0.120	-0.054	0.040
$\Delta\omega_{t-2}^C$	-0.057	0.015	-0.066	-0.043
$\Delta\omega_{t-3}^C$	-0.096*	0.100	-0.105*	0.042
$\Delta\omega_{t-4}^C$		0.127*		0.082
$\Delta\omega_{t-5}^C$		0.062		0.046
$\Delta\omega_{t-6}^C$		0.050		0.050
$\Delta\omega_{t-7}^C$		0.181***		0.187***
$\Delta p_{t-1}^B$	0.070*	0.256***	0.088**	0.347***
$\Delta p_{t-2}^B$	0.028	-0.155***	0.050	-0.047
$\Delta p_{t-3}^B$	0.092**	-0.012	0.115***	0.082
$\Delta p_{t-4}^B$		-0.087		-0.007
$\Delta p_{t-5}^B$		-0.018		0.055

<sup>38</sup> It is well known that the Durbin Watson Test is not applicable if lagged values of the dependent variables appear on the right hand side of the regression equation. Moreover, the portmanteau test is able to check for serially correlated errors at multiple lags compared to just one lag in the Durbin Watson test.

<sup>39</sup> Note that both comparative static results related to chick prices vis-à-vis cobweb cycles are validated by empirical estimates from the chick price equation.

$\Delta p_{t-6}^B$		-0.098*		-0.039
$\Delta p_{t-7}^B$		-0.067		0.004
$\Delta SPI_{t-1}$	0.209	-0.025	0.258	0.261
$\Delta SPI_{t-2}$	-0.334	-0.179	-0.299	0.040
$\Delta SPI_{t-3}$	0.497*	-0.400	0.558**	0.252
$\Delta SPI_{t-4}$		-0.383		-0.194
$\Delta SPI_{t-5}$		-0.590		-0.468
$\Delta SPI_{t-6}$		0.207		0.239
$\Delta SPI_{t-7}$		0.669*		0.770**
$Summer_t$	0.567	1.86**	0.732	2.54***
$EidulAzha_t$	-3.039***	-1.229	-3.527***	-2.56**
$\mu_{t-1}$			-0.046**	-0.191***
R – Square	7.51%	16.18%	8.77%	23.58%
Portmanteau Test	9.399	0.496	8.992	1.29

Estimates based on weekly prices from June 2008 to June 2015 for a total of 367 observations. The data was sourced from Pakistan Poultry Association (North). All prices are in nominal units of the local currency. Parameter estimates reported adjacent to the corresponding row variables while standard errors provided in the parenthesis below the parameter estimates. Summer denotes a dummy variable representing the period May-July in a given year. EidAzha denotes a dummy variable corresponding to the period 2 weeks before and 5 weeks after the 10<sup>th</sup> of Zul-Hijjah (12<sup>th</sup> month in the Islamic calendar). The underlying equations were estimated simultaneously using SUR. The portmanteau (or Q) test for white noise is based on 7 lags of the residuals, where the maximum lag length in the underlying model is used as the criterion for the selection of the number of residual lags to test for zero autocorrelation. Serially uncorrelated errors is the null hypothesis of the portmanteau test. Asterisks indicate statistical significance, where \*\*\*, \*\*, \* represent significance at 1%, 5% and 10% respectively.

## I. Estimates of the Chick Price Difference Equation-Near VAR and Near VECM models

The results from the near VAR and the near VECM model provide further support to the theory of endogenous price fluctuations. All lags of chick prices are negative in the chick price equation in the near VAR model. This implies that periods of increasing prices witnessed during the chick production cycle are followed by periods of decreasing prices (high production) and vice versa, this is a clear manifestation of the classic cobweb effect. Furthermore, as predicted by our model (Result-III:  $\frac{\partial \omega_t^C}{\partial \omega_{t-3}^C} < 0$ ), only the third lag i.e.  $\beta_{\omega,3}^C$  is statistically significant, suggesting that farmers use the current chick prices at the beginning of the production cycle to formulate expectation of future prices and hence production plans.

Moving onto the effect of broiler prices on chick farmer production decisions, consistent with Result-IV i.e.  $\frac{\partial \omega_t^C}{\partial p_{t-3}^B} > 0$ , we find that  $\beta_{p,3}^C$  is positive and statistically significant. This represents the cobweb effect in a vertically linked market, whereby (naïve) expectation of higher demand in lieu of high current broiler prices induces chick farmers to increase production and vice versa. Note that although, the first broiler price lag ( $\beta_{p,1}^C$ ) is also positive, its effect is both smaller in terms of magnitude and statistical significance (p-value of 0.09) compared to the magnitude and significance of  $\beta_{p,3}^C$  (p-value of 0.03). The significance of  $\beta_{p,1}^C$  is perhaps driven by the following phenomenon. First, from the organization of production in poultry sector we know that broiler farmers buying chicks in a given week are the ones who sold broilers in the last

week. Therefore, if broiler prices were high in last week, broiler farmers experienced a positive cash inflow and are now willing to pay a higher price for chicks and vice versa. Another intuitive explanation is that  $\beta_{p,1}^C$  denotes the positive effect of last week's broiler price on the bargaining over chick prices between chick and broiler farmers. For example, if broiler prices witnessed an increase in the last week, broiler farmers are willing to pay chick farmers a higher price in (naïve) expectation of the continuation of current increasing trend. These interpretations are consistent with survey responses, where participants noted that broiler prices served as reference price for transactions between chick/broiler farmers and broiler farmers /retailers. The effect of feed costs reaffirms findings from the restricted VAR model i.e. positive and statistically significant  $\gamma_{t-3}^C$ . Whereby, high feed prices at the beginning of the production cycle lead to lower production and hence higher prices at the end of production cycle. Interestingly, all intermediate lags of feed price changes are insignificant; suggesting that production plans are made in view of current feed prices only, consistent with the responses of farmers documented in section 3.1.

Estimates from the chick price equation in the near VECM model present an identical picture albeit at markedly higher significance levels compared to the results from the near VAR model. More specifically,  $\beta_{\omega,3}^C$  (Result-III:  $\frac{\partial \omega_t^C}{\partial \omega_{t-3}^C} < 0$ ), is negative and statistically significant (p-value 0.06),  $\beta_{p,3}^C$  (Result-IV:  $\frac{\partial \omega_t^C}{\partial p_{t-3}^B} > 0$ ) is positive and statistically significant (p-value 0.01) and  $\gamma_{t-3}^C$  is positive and statistically significant (p-value 0.05). Likewise, coefficients on intermediate lags of all explanatory variables are statistically insignificant except  $\beta_{p,1}^C$ , which reflects the role of last week's broiler prices as the reference price during bargaining between chick and broiler farmers over chick prices. The estimates presented in table-6 clearly show that empirical evidence in favor of cobweb cycles is not merely a statistical artifact, driven by the exclusion of intermediate lags in the restricted model.

Lastly, the negative effect of EidAzha on chick prices remains regardless of model specification, while summer season does not have statistically significant impact on chick prices (via reduced chick production channel) in either model. Note that compared to the restricted VECM model, the error correction term is negative and statistically significant (p-value 0.02) in the near VECM or unrestricted model, an indication that the near VECM is a valid approximation of the underlying data generating process. Also, the higher R-square in the near VECM model, along with smaller standard errors makes it our preferred specification. Although, based on portmanteau test statistic, we cannot reject the null of uncorrelated residuals in both the near VAR and near VECM models, suggesting that both models are well specified. Overall, empirical estimates presented in table-6 not only corroborate estimates reported in table-5 but in fact further strengthen the argument in favor of endogenous price fluctuations (cobweb cycles) in chick prices.

## II. Estimates of the Broiler Price Difference Equation-Near VAR and Near VECM models

We now focus our attention towards the broiler price equations. In the near VECM model, coefficients on all lags of chick prices are insignificant except  $\beta_{\omega,7}^B$ , which is not only significant at the 1% level but has a relatively large positive magnitude. Again, this is consistent with result-



II ( $\frac{\partial p_t^B}{\partial \omega_{t-7}^C} > 0$ ) derived from the stylized model of profit maximization by upstream and downstream farmers in a vertically linked agricultural value chain under the naïve expectations hypothesis. Similar, results are obtained from estimates of the broiler price equation in the near VAR model, although  $\beta_{\omega,4}^B$  is also marginally significant in this model. In both the near VAR and near VECM model,  $\beta_{p,1}^B$  is positive and highly significant. Similar to chick prices, this effect may represent the fact that last week's broiler prices serve as reference price during negotiations between broiler farmers and retailers over broiler prices. Therefore, high prices last week translate into higher prices today and vice versa. Another explanation, drawn from the price transmission literature, is that  $\beta_{p,1}^B$  merely represents the statistical effect of stickiness or inertia in final good (in this case broiler) prices. Since, final good (broiler) prices don't change abruptly due to the presence menu or transaction costs of relabeling etc. The effect of feed costs on broiler prices mirrors the results from the restricted models i.e. only  $\gamma_{t-7}^B$  is (positive) statistically significant amongst all the lags of  $\gamma^B$ .

In the case of result-I ( $\frac{\partial p_t^B}{\partial p_{t-7}^B} < 0$ ), empirical evidence is ambiguous at best. The estimates from the near VAR model offer some support to the cobweb effect. For instance other than  $\beta_{p,1}^B$ , all lags of broiler prices are negative, in the near VAR model, consistent with the standard cobweb effect, although only  $\beta_{p,2}^B$  and  $\beta_{p,6}^B$  are statistically significant. However, all lags of broiler prices are statistically insignificant apart from  $\beta_{p,1}^B$  in the near VECM model. In light of the strong empirical evidence in favor of cobweb cycles in chick production (hence prices), it is unlikely that cobweb cycles in chick prices do not translate into cobweb cycles in the broiler production (hence prices), given the fact that chicks are eventually converted into broilers. Nevertheless, we present three explanations to resolve this conundrum.

Perhaps, as argued earlier, the effect of chick prices on broiler farmers' production decision dominates the effect of future expectation of broiler prices. Second, recall that the production cycle of broilers varies from 6-7 weeks. At the same time, in contrast to chick farmers, broiler farmers have a certain degree of flexibility vis-à-vis timing of the sale of broilers. Therefore, it may be that at times broiler farmers sell their output in the 5<sup>th</sup> week in lieu of favorable prices while at other times they wait until the seventh week. As a result, the relevant price that is used to form (naïve) expectation about future prices changes from one cycle to another, diluting the predictive power of our empirical model that relies on the identifying assumption of a fixed production cycle of approximately 7 weeks. Consequently, ignoring intermediate lags, as is done in the restricted models, mitigates this bias to an extent. Since, the cobweb effect of excluded lags is partially captured by the 7<sup>th</sup> lag. Recall, that in the restricted model  $\beta_{p,7}^B$  was negative regardless of model specification and although p-values were high, but nonetheless within a neighborhood of acceptability. Third, recall that the identification of the comparative static results of the broiler price difference equation was not straightforward but relied on additional assumptions on the functional forms<sup>40</sup>. Perhaps, these restrictive assumptions weaken the effect of broiler price expectations on broiler production (and hence realized prices).

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<sup>40</sup> It is straightforward to note that identification of the comparative static results of the chick price difference equation is independent of any assumptions on functional forms, due to the separability of chick and broiler prices. However the comparative static results for the broiler price equations cannot be identified without additional assumptions on the nature of the broiler cost function and the broiler retail demand function due to the fact that broiler and chick price lags are not separable, and their difference is the argument in the RHS.

As observed in the restricted models, summer has a positive effect on broiler prices, an effect that seems to be based on the reduction of supply due to higher broiler mortality in hot weather. EidAzha, a period of increased lamb/beef consumption, has a negative effect on broiler prices due to a reduction in the demand of chicken. Likewise, consistent with previous empirical evidence, results in table-6 show that broiler prices have witnessed an upward trend over the past decade while chick prices have remained largely stagnant. Consistent with the prior literature of agricultural production, we also find a higher effect of feed costs on the prices of downstream product compared to the prices of the upstream sector i.e. ( $\gamma_{t-7}^B > \gamma_{t-3}^B$  with p-value of 0.03).

Lastly, regression diagnostics reveal that R-square increases significantly in the unrestricted model, especially for the broiler price equations. Most importantly in contrast to the restricted models, we cannot reject the null hypothesis of serially uncorrelated errors in the chick and broiler price equations in both unrestricted model (near VAR or VECM model) based on portmanteau test, suggesting that the unrestricted models are well specified. We arrive at similar conclusions vis-à-vis model specification based on the autocorrelation (ACF) and partial autocorrelation (PACF) functions.

### Summary

A simple dynamic model of profit maximization by downstream and upstream farmers in a vertically linked agricultural value chain provides us with a parsimonious theoretical framework to empirically test the theory of endogenous price fluctuations in the Pakistan poultry sector. In general, the empirical estimates based on a unique dataset comprising of weekly farm-gate prices of chicks and broilers from June 2008 to June 2015 lend support to the naïve expectation hypothesis and hence cobweb web cycles. Our findings are robust to different econometric specifications and estimation methodologies. More specifically, estimates from the restricted VAR and the restricted VECM models, derived directly from the theoretical model under the assumption of a quadratic cost function and a linear retail demand curve, are corroborated by estimates from the more conventional near VAR and near VECM models.

In addition to the major results, we also find that higher feed costs (proxied by SPI) at the beginning of the production cycle lead to lower production and hence higher prices at the end of the production cycle for both chick and broiler prices. Second, as predicted by theory, there is a strong long run relationship between prices of intermediate (chicks) and final (broiler) agricultural products, evidenced by the negative and statistically significant error correction terms in the estimated VECM models. Whereby, deviations from the long-run equilibrium are periodically corrected in the short run. Lastly, following the burgeoning literature on the affect of Islamic festivals on economic behavior (Gavriilidis et al. 2015 and Seyyed et al. 2005), we convert the Gregorian calendar into the lunar calendar to isolate the demand side effect of EidulAzha, a festive period characterized by increased consumption of beef/lamb, on chicken prices. Unsurprisingly, we document a negative effect of EidulAzha on broiler and chick prices due to the lower demand for chicken during this season.

Nevertheless, the abovementioned findings are not without important caveats. First, empirical tests of expectation regimes based on aggregate data are indirect by nature and hence inherently weak. Second, in the absence of aggregate output data, empirical tests of cobweb

cycles based on price dynamics, derived from an underlying model of vertically linked upstream and downstream farmers, assume that demand behavior for broiler chicken is given. Although, this does not seem like an unreasonable assumption because broiler chicken is a food based commodity with fairly stable demand. At the same time, given the constraint that aggregate price data is usually available at reasonable frequencies as opposed to quantity data, there seems to be no other viable alternatives. Third, the empirical tests are based on theoretical results derived from a stylized model with several simplifying assumptions. However, the validity of a model is not judged by its assumptions but by its ability to explain reality. Therefore in the next section, we use simulations to examine whether the stylized facts of the actual data can be reproduced by the price dynamics implied generated our theoretical model.

The low explanatory power of empirical models, in particular the restricted models, vis-à-vis poultry prices is another potential source of concern. Although, a relatively low R-square is not uncommon in models specified in first differences compared to models specified in levels. However, in our context, explanatory power is less of an issue given the fact that we are primarily interested in the verification or falsification of comparative static results as opposed to point estimates. Besides, regression diagnostics based on portmanteau test for white noise reveal that the empirical models are sufficiently well specified. Nevertheless, given the fact that a large body of theoretical literature has shown that, in the presence of non-linearities, chaotic dynamics can arise in simple non-stochastic cobweb markets. Where, chaotic systems are defined as deterministic dynamical systems that essentially generate random data, characterized by excessive variability and unpredictability. Therefore, we cannot rule out chaotic dynamics in the underlying system of difference equations as the underlying factor behind the low explanatory power of our empirical models. We examine some of the key issues related to non-linearities and the associated chaotic dynamics in the next section.

In summary, an econometrician cannot observe the underlying data generating process driving the prices of chicks and broilers in Pakistan. But, instead, only aim to find the best approximation of systematic components in the trajectory of poultry prices by capturing empirical regularities in a given dataset. In this regard, few would argue that the overall empirical estimates presented in this section are not broadly consistent with the naïve expectation hypothesis and hence the existence of cobweb cycles in the Pakistan poultry sector. At the same time, by imposing restrictions on the estimated parameters, our theoretical framework allows us to circumvent a common critique of empirical research on price dynamics based on standard autoregressive time-series models i.e. difficulty in “interpreting” parameter estimates due to the inherently atheoretical and often arbitrary structure of autoregressive time-series models. While, robustness checks ensure that our findings are not driven by mere statistical artifacts.

## **5. A simple model of endogenous price fluctuations: Numerical Analysis**

The evidence presented in the previous sections clearly illustrates the relevance of the theory of endogenous price fluctuations in explaining price dynamics in the Pakistan poultry sector. However, models and theories are judged upon their predictive ability in the framework of neoclassical economics (Lucas 1980). Therefore, in this section we employ numerical analysis to examine whether the stylized model of price fluctuations proposed in section-3 can reproduce the patterns observed in the actual data. But before delving into the simulations, we draw

attention towards some interesting features of the system of difference equations derived from the underlying theoretical model. This understanding is essential to grasp the intuition behind the forthcoming simulation results.

In a section-3 we showed that the system of coupled, time-delay difference equations for chick and broiler prices, derived from a simple model of profit maximization by upstream and downstream farmers in a vertically interlinked competitive cobweb agricultural market, is given by:

$$\begin{cases} p_t^B = \frac{a}{b} - \frac{N_2 k}{N_3 b} \left( \frac{k p_{t-6}^B}{\beta} - \frac{\omega_{t-6}^C}{\beta} \right)^{\frac{1}{\beta-1}} \\ \omega_t^C = k p_{t-3}^B - \left( \frac{N_1 \beta}{N_2} \right)^{\beta-1} \left( \frac{\omega_{t-3}^C}{\alpha} \right)^{\frac{\beta-1}{\alpha-1}} \end{cases}$$

Recall that  $\alpha$  and  $\beta$  represent the curvature of the cost functions for chick farmers ( $C_1(q_t^C) = (q_t^C)^\alpha$ ) and broiler farmers ( $C_2(q_t^C) = (q_t^C)^\beta$ ) respectively, where  $\alpha > 1$  and  $\beta > 1$  guarantees convexity. The curvature of the cost curves determine whether the underlying model behaves linearly or non-linearly, whereby the model is linear as long as  $\alpha = \beta = 2$  and in all other cases it is non-linear. The demand for broiler by retailers is given by  $Q_t^{B,D} = a - b p_t^B$  where  $a > 0$  denotes the extent (or maximum capacity) of a given retailer and  $b > 0$  represents the sensitivity of demand to broiler prices. A fixed proportions production technology, i.e. the conversion rate of chicks into broilers, is represented by  $k$  with  $k \in [0 - 1]$ . While, one week represents the discrete interval (or step) in the abovementioned differences equations.

$N_1, N_2$  and  $N_3$  represent the number of chick farmers, broiler farmers and retailers in the market respectively. Although, we documented that markets for poultry products were competitive, farmer surveys revealed significant bargaining over prices between counter parties in spot markets. Therefore, we use the ratio of the relative size (within reasonable bounds) of counter parties (chick farmers, broiler farmers and retailers) in a given transaction (chicks or broilers) to capture the effect of bargaining power on poultry price dynamics, under the assumption that relative size, which is inversely related to the number of farmers in a given category, is a valid proxy for bargaining power<sup>41</sup>. Our stylized model is simple yet powerful as it captures several key aspects of agricultural markets. For example, the effects of cost structures, technological advancements, broiler chicken demand and relative bargaining powers of counterparties on poultry price dynamics are embedded into the system of equations.

### 5.1 A Brief Overview-Effect of Time delays on the Behavior of Dynamic Models

Unlike static models, in dynamic models, we are not only interested in the existence of an equilibrium state but also in its local stability and the global behavior of orbits generated by the underlying model. By and large, the literature on chaotic agricultural cobweb markets is limited to 1-dimensional maps. This simplifies analysis, since standard analytical results on the stability

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<sup>41</sup> For example, broiler farmers and chick farmers bargain over chick prices while broiler farmers and retailers bargain over broiler prices at any given time. In the simulations, we keep relative bargaining power low in order to ensure that considerations for imperfect competition don't arise.

of one-dimensional systems are well known and thus characterization of system dynamics into the relevant categories is fairly straightforward. However, many standard results derived for one-dimensional chaotic maps cannot be easily extended for two-dimensional systems (Sedaghat 2003). Research by Dieci and Westerhoff (2009 and 2012) is one of the few attempts in the agricultural economics literature to study chaotic dynamics in two dimensional systems. But their work deals with a “standard” system of difference equations i.e. the future state of the system is completely determined by the current state of the system. This is no longer true in difference equations with time delays because the evolution of the system is dependent upon both the current and the past state of the system. Consequently, conventional tools used to analyze the behavior of “standard” dynamical systems are not directly applicable to the analysis of systems with time-delays<sup>42</sup>.

But in reality time delays in feedback mechanisms are frequently encountered in both the natural and social phenomenon. For example, the effect of time delays on the behavior of physical models is extensively studied in engineering sciences, particularly in the discipline of control systems (Zavaeri & Jamshidi 1987). Likewise, time delays are ubiquitous in biological models of cellular automaton, epidemics and population dynamics (Campbell 2007). Although compared to other disciplines, the study of time delays in the economics literature has been somewhat neglected over past decades. Nevertheless, analysis of the effects of lagged investment on business cycles and economic growth in the macroeconomics literature, popularly known as the Kaldor-Kalecki model due to Kaldor (1940) and Kalecki (1935), was one of the earliest attempts to study the impact of time delays on the behavior of dynamic models. While, the effects of time delays vis-à-vis transmission of information related to the competitor’s output on the stability of Nash equilibrium in the Cournot model continues to be an active research area in the industrial organization literature (Howroyd & Russell 1984; Chiarella & Khomin 1996; Yassen & Agiza 2003; Hassan 2004 and Elsadany 2010). Surprisingly, however to the best of our knowledge, the effect of time delays on the dynamics of agricultural prices has not been carefully examined in the agricultural economics literature.

So, how does the presence of time-delays affect the global behavior of dynamic models? It turns out that the answer is not straightforward<sup>43</sup>. Initially, time delays were generally considered to be a cause of instability, however later research has shown that this is not always true (Campbell 2007). For example Yassen and Agiza (2003) and Elsadany (2010) prove that presence of delays increase the probability of convergence towards the Nash equilibrium in a Cournot duopoly game. Likewise, Hassan (2004) shows that delays increase the region of stability in a Cournot duopoly game. However, Howroyd & Russell (1984) find that decreasing delays increases the likelihood of stability in a Cournot oligopoly game. In fact, Huang (2008) argues that the relationship between the system stability and delays is not monotonic but varies from one case to case.

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<sup>42</sup> Asymptotic stability of linear systems can be analyzed by computing the eigen-values of the system transition matrix. While eigen values of the Jacobian, used to linearize nonlinear systems around the steady state, are employed to study issues related to local stability of nonlinear systems. As will become obvious later, both strategies need to be significantly modified to examine the behavior of dynamical system with time delays.

<sup>43</sup> Although, the continuous time analogue of time-delay difference equations i.e. time-delay differential equations are used in the vast majority of these applications. Nonetheless, qualitative results related to the effects of time delays on the dynamics of continuous time problems are generalizable to discrete time problems as well.

Nevertheless, Campbell (2007) has identified several qualitative attributes commonly associated with the underlying dynamics of time-delay models. She finds that time delays often lead to delay induced oscillatory behavior created by Hopf bifurcations<sup>44</sup>. Whereas, other complicated dynamics associated with time-delay models include the existence of solutions with multiple frequencies (quasi-periodicity), attractor switching and multi-stability i.e. coexistence of more than one stable solution (Guckenheimer & Holmes 1983) and switching from one type of behavior to another as some parameter is varied (Kuznetsov 1995). In a nutshell, time delays significantly impact the behavior of dynamic models and thus, failing to account for time-delays in a model may lead to incorrect conclusions.

Many of the abovementioned features commonly observed in time-delay models are also typically found in chaotic systems. This is not surprising, since the continuous time analogue of time-delay differences equations i.e. time delay differential equations belong to the class of functional differential equations which are well known to be inherently infinite dimensional problems. Whilst, although finite dimensional in the strictest sense, the presence of time delays in difference equations also leads to an increase in the dimension of the state-space of the underlying model.<sup>45</sup> The increase in the dimension of the state space of an underlying model comes at the additional cost of both analytical difficulty and complicated dynamics. In fact, current research on linear chaotic operators has confronted the popular view that chaotic behavior can only arise in the presence of non-linearities. For example, Godefroy & Shapiro (1991) in their seminal work proved that a class of linear functions defined on an infinite dimensional state-space is chaotic. Likewise, Grosse-Erdmann et al (2011) document several linear operators on an infinite dimensional state-space that behave chaotically.

Even linear time-delay differential equations do not have closed form analytical solutions except under some special cases because continuous time-delay systems possess a spatial component in addition to a temporal component due to the effect of past states on the evolution of the dynamical system. Thus, unlike a system of ordinary differential equations, time delay systems belong to the class of systems with a functional state variable and hence an infinite dimensional state space. As a result, the characteristic equation of time-delay differential equations is a quasi-polynomial with infinite number of roots in the complex plane (Zavaeri & Jamshidi 1987). Consequently, characterization of eigen values on the real-imaginary axis, the workhorse of stability analysis, is no longer applicable. Just like the transition matrix, the Jacobian of the system also contains exponential terms, greatly complicating linearization of non-linear systems around the steady state<sup>46</sup>. Therefore, numerical methods and simulations are usually employed to study the local and global behavior of the continuous time delay systems. Of

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<sup>44</sup> In simple words, Hopf bifurcation is used to describe the change in the behavior of an underlying system from a stable equilibrium state to periodic trajectory as a given bifurcating parameter crosses some critical threshold. It is normally associated with purely imaginary eigen values. Of course, as mentioned before, adding delays to the model can also lead to the opposite effect i.e. transition from a periodic solution to a stable equilibrium point.

<sup>45</sup> A system of discrete time-delay equations can be recast into a system of higher order difference equations and order reductions methods used convert a system of higher order difference equations into a system of first order difference equations in order to perform stability analysis leads to an increase in the dimension of the state-space.

<sup>46</sup> The Lyapunov-Krasovski functionals, Razumikhin techniques and Padé approximations are some of the commonly used analytical tools to study the stability of systems with a functional state variable, each method with its own pros and cons.

course, numerical solutions rely on discretization of the infinite dimensional state space of the underlying model into a finite dimensional state-space.

Although, the state-space of time-delay difference equations is finite dimensional<sup>47</sup>, nonetheless many difficulties encountered in the analysis of time-delay differential equations remain whilst studying the dynamics of time-delay system in discrete time. For example, while, the eigen values of the characteristic equation of a system of time delay difference equations are finite. However, due to the presence of power terms in addition to polynomials in the characteristic equation, often eigen values cannot be analytically factorized as a function of model parameters (Zavaeri & Jamshidi 1987). Likewise, standard linearization methods, based on the Jacobian, used to study non-linear systems are no longer applicable. Moreover, the number of eigen values is directly proportional to the size or dimension of the state vector corresponding to an arbitrary initial condition i.e. length of the delay. Therefore, as the length of the delay increases the analytical study of a system's eigen values becomes cumbersome. Therefore, numerical analysis is often employed to study time-delay difference equations as well. However, due to the discrete-time nature of these problems, a simple forward looking loop can be used to simulate the underlying model in contrast to the complicated algorithms needed to numerically solve a system of time delay differential equations.

In summary, time delays in the feedback mechanisms limit the efficacy of standard analytical tools used in the study of dynamical systems. At the same time even in linear systems, time delays can result in complicated trajectories including delay induced oscillations, quasi-periodicity, attractor switching and multi-stability. In fact, in the case of time-delay differential equations even linear models can behave chaotically due to the existence of an infinite dimensional state-space. These behavioral features of time-delay systems are particularly relevant for agricultural economists interested in the theory of endogenous price fluctuations, an area where delays in feedback mechanisms are (in reality) a rule rather than an exception. Therefore, it will be interesting to see the insights that can be gained from incorporating time delays into cobweb-type models of agricultural markets, an endeavor that we pursue in the following pages.

## 5.2 Equilibrium & Stability Analysis

In light of the properties of time-delay systems documented in the previous subsection, we now direct our attention towards examining the equilibrium states and stability of the underlying system of time-delay difference equations. We begin our analysis with the special case of a linear system i.e.  $\alpha = \beta = 2$ . Note that in an equilibrium or steady state of the system  $p_t^B = p_{t-3}^B = p_{t-6}^B = p^B$  and similarly  $\omega_t^C = \omega_{t-3}^C = \omega_{t-6}^C = \omega^C$ . Substituting into the system of linear equations and after some algebraic manipulation we get the following equilibrium state:

$$p^B = \frac{2aN_3(N_2 + N_1)}{2bN_3(N_2 + N_1) + N_2N_1k^2}$$

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<sup>47</sup> The state spaces of discrete-time time delay systems is finite-dimensional because the state vector of past values at each instant has a finite number of elements, compared to delay differential equations where the past values have to be defined over a continuous interval with infinite number of sampling points.

$$\omega^c = \frac{2aN_3N_2k}{2bN_3(N_2 + N_1) + N_2N_1k^2}$$

In appendix-2, we use the single crossing property to prove the uniqueness of the equilibrium state of the underlying model in the more general case i.e. without imposing the restriction that  $\alpha = \beta = 2$ .

**Proposition-1: The equilibrium of the coupled time-delay system on the real axis is unique, non-zero and positive.**<sup>48</sup>

The steady state of possesses several interesting features. First of all note that both equilibrium prices are non-zero and strictly positive given the definition of the parameters in our model. Secondly, as expected, we have  $p^B > \omega^c$  in equilibrium<sup>49</sup>. Thirdly, the comparative static results are intuitive e.g.  $\frac{\partial p^B}{\partial a} > 0$  and  $\frac{\partial \omega^c}{\partial a} > 0$  i.e. equilibrium price of broiler and chicks increases if the retail chicken demand curve shifts upwards.  $\frac{\partial p^B}{\partial b} < 0$  and  $\frac{\partial \omega^c}{\partial b} < 0$  i.e. equilibrium price of broiler and chicks decreases if the retail chicken demand curve becomes steeper (consumers become more price sensitive). And lastly, an increase in number of farmers at a given level in the value chain leads to a decrease in equilibrium prices of products at that level, due to the decline in relative size vis-à-vis the counter party in the given transaction (the bargaining power effect) i.e.  $\frac{\partial p^B}{\partial N_2} < 0$  and  $\frac{\partial \omega^c}{\partial N_1} < 0$ . These findings lend support to the validity of the underlying model and also highlight that the equilibrium state is economically “relevant”. The abovementioned comparative static results also hold for the more general, non-linear system<sup>50</sup>.

In a dynamic economic model, in addition to the existence of an equilibrium state, researchers are also interested in the stability of the equilibrium. For example, is the equilibrium asymptotically stable? And how does the system respond to small perturbations from the equilibrium state etc.? However, as mentioned before, the tools used to study the stability of “standard” dynamical systems are not directly applicable to time-delay systems due to delayed feedback mechanisms. Nevertheless, economists have developed methods and techniques to analytically study the properties of time-delay difference equations.

A well-known method to examine the stability of time-delay models in the literature is based on the conversion of a system of time-delay difference equations into a sequence of first-order difference equations (Yassen & Agiza 2003; Hassan 2004 and Elsadany 2010). This approach is based upon the fact that a time-delay difference equation is equivalent to a higher order difference equation<sup>51</sup>. And given an arbitrary  $n^{\text{th}}$  order difference equation, it is fairly

<sup>48</sup> From a stability perspective if  $\alpha < \beta$ , the system blows up and the equilibrium is no longer unique. Nevertheless even if  $\alpha < \beta$ , the economically relevant equilibrium state i.e. on the positive real line is still unique.

<sup>49</sup> Since the denominators of the fixed points are equal, comparing the numerators for  $p^B = 2aN_3(N_2 + N_1)$  and  $\omega^c = 2aN_3N_2k$ , it is easy to see that  $p^B > \omega^c$  since  $N_2 + N_1 > N_2k$  because  $k < 1$  and  $N_i$  is a positive integer.

<sup>50</sup> The comparative statics of the nonlinear model are similar to the results derived from the linear model but require additional and at times considerably more involved algebra. Therefore, given the scope of this paper and for sake of brevity, we choose not to show these calculations here, but are available upon request.

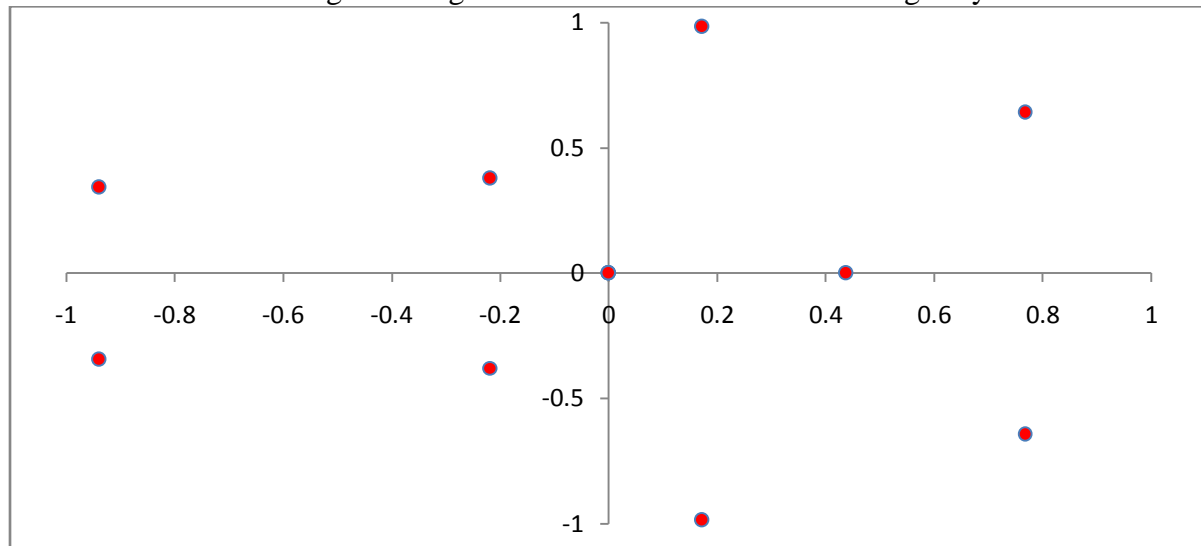
<sup>51</sup> The order of a system of difference equation refers to the maximal difference between the highest and lowest time indexes for a given variable in any system of difference equations.



straightforward to produce a system of  $n$  first order difference equations by defining  $n-1$  new variables for each higher order difference term (Neusser 2015). Thereafter, standard tools can be employed to study the stability of the resulting first order system of difference equations. In particular, it has been shown that the original time-delay system is asymptotically stable if and only if, all the eigen values of the augmented systems lie within the unit circle in the real-imaginary axis space and vice-versa. More importantly, well known rules pertaining to the eigen values of the augmented first order system can be used to understand the qualitative behavior of the original time-delay model i.e. convergence to equilibrium, diversion from equilibrium or oscillations around the equilibrium.

In appendix-3, we apply this method to study the stability in particular and the qualitative behavior in general of the system of linear coupled- time delay difference equations derived from our theoretical model under the naïve expectations hypothesis. For sake of brevity, the details of the computations are relegated to appendix-3, and eigen values under suitable calibrations of the model parameters are plotted below in figure-3<sup>52</sup>.

Figure-3-Eigen Values Plotted on the Real-Imaginary Axis



The horizontal axis represents the real component of the eigen values and the vertical axis represents the imaginary components of the eigen values, computed at the baseline parameter values given in appendix-3.

Figure-3 shows that the original system of linear time-delay difference equations is asymptotically stable because the eigen values are within the unit circle in the real-imaginary axis space. This is an important finding since an unstable system i.e. trajectories that explode, is not meaningful and in most cases a clear indication that the underlying economic model is an invalid depiction of reality because in reality prices don't explode. Moreover, it is well known that complex eigen values are associated with oscillatory behavior and it is interesting to note that 10 out of the 12 eigen values are complex conjugates, with large imaginary components. Qualitatively similar eigen values were obtained using different sets of parameter values. Whereby, a larger imaginary component relative to the real component is an indication of long-

<sup>52</sup> Due to the nature of the underlying time-delay system, the roots of the characteristic equation corresponding to the system transition matrix cannot be expressed in terms of the model parameters due to the presence of higher order terms; for details see appendix-3. Therefore we compute the eigen values numerically under reasonable calibrations. The details of the calibration under the baseline scenario are provided in the subsection on numerical simulations.

lasting oscillations around the equilibrium<sup>53</sup>. The persistent cycles in the underlying time-delay system are clearly highlighted during numerical simulations.

Mathematically, it is important to note that the channel of price cyclicity in the underlying coupled time-delay system is different from the source of cyclicity in a standard one dimensional cobweb market. Whereby, in the case of the latter, the border line case of an eigen value of negative one is driving the cyclical behavior, severely limiting the range of parameter values at which long-term price cycles arise. However, in higher dimensional systems, complex eigen values and hence persistent price cycles are associated with a large range of parameter values. The bifurcation diagrams presented in the next subsection clearly illustrate the changes in the behavior of the underlying system from a unique steady state ( $|\text{real eigen values}| < 1$ ) to persistent oscillations (complex eigen values) in our settings.

Lastly, the abovementioned analysis reveals that the dynamics of a two-dimensional linear time-delay system is equivalent to the dynamics in a 12 dimensional linear first-order system. Naturally, the complexity of system dynamics increases manifold with an increase in the state space of the model. And although, finite dimensional linear systems are never chaotic in theory, however, in practice high-dimensional linear systems can generate complicated behavior, in particular a markedly high sensitivity to initial conditions. Because, high dimensional systems are known to possess several complex eigen values, increasing the likelihood that different initial conditions correspond to the manifolds of different eigen vectors and hence produce different trajectories. Moreover, given that initial cannot be known with certainty, different sets of initial conditions result in different types of behavior which may lead an external observer to incorrectly conclude that the underlying data generating process is nonlinearly chaotic, unstable or changing over time.

Even for linear systems, direct delay-dependent stability criterion cannot be analyzed analytically (barring some special cases) due to the presence of matrix power terms in the characteristic equation (Zavaeri & Jamshidi 1987). Nevertheless, as shown above, indirect methods to study the stability of linear time-delay systems reveal that our underlying model is asymptotically stable and characterized by cyclical behavior. But, stability criterion independent of delay can be derived analytically in order to determine whether delays have a stabilizing or destabilizing effect on the underlying model.

Hale et. al (1985) have provided necessary and sufficient conditions of asymptotic stability independent of time delays for a given time-delay system. In appendix-4, we follow the approach of Hale et. al (1985) to prove that independent of time delays, the underlying model is asymptotically unstable since the absolute value of the dominant eigen value is greater than one.

**Proposition-2: The underlying linear time-delay system is asymptotically unstable independent of time-delays because the absolute value of the dominant eigen value is greater than 1.**

Proposition-2 and results from the stability analysis of the original linear time-delay systems using an indirect state-space augmentation approach lead us to the conclusion that

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<sup>53</sup> In the extreme case of non-zero imaginary-component but a real component of zero, the equilibrium is called a centre point i.e. perpetual oscillations around the steady state.

delays in feedback mechanisms are a source of stability in this case. These findings have important ramifications vis-à-vis the literature on agricultural cobweb markets which have by and large failed to incorporate time-delays into the price dynamics.

The objective of this subsection was to employ relevant analytical tools to study the fixed points and stability of the underlying system of time-delay difference equations. This analysis has revealed several important features of the model. First, the fixed point or equilibrium of the original unrestricted model is unique. Second, in the special case of a linear time delay system, the original model can be rewritten as set of twelve first order difference equations. The eigen values of the resulting system reveal that the original time-delay system is asymptotically stable and characterized by cyclical behavior due to the presence of complex eigen values with large imaginary components. Lastly, delays have a stabilizing effect on the dynamics of the model or in other words make an inherently unstable model stable by generating oscillatory behavior around the equilibrium state. We now turn to numerical analysis to further crystallize the conclusions derived from the analytical analysis in this subsection.

### 5.3 Numerical Analysis & Simulations

Numerical simulations are commonly employed to study the global dynamics of time-delay models. However, in contrast to the complicated algorithms needed to solve time-delay differential equations, a simple forward looking loop with appropriate model calibrations can be used to trace the trajectories or orbits of time-delay difference equations.

#### 5.3.1 Model Calibrations

We utilize the findings from fieldwork in Pakistan (refer to section-1 and section-3 for details), to calibrate the parameters of the model. The production technology of broiler farmers i.e. the conversion rate of chicks into broiler ( $k$ ) equals one minus the mortality rate of chicks in a given production cycle. Industry reports (Poultry Research Institute 2012, 2013 and 2014) and interviews with broiler farmers revealed that approximately 5% of the chicks die during the broiler production cycle on average. Therefore, in the baseline model we use  $k = 0.95$ . Contrary to conventional wisdom, it is well known that the upstream farm sector (chick farmers) is relatively more concentrated compared to the downstream farm sector (broiler farmers) in the Pakistan poultry industry. Likewise the downstream farm sector is relatively more concentrated compared to the retail sector. Given that industry concentration is inversely related to the number of firms in a sector we use  $N_1 = 10$ ,  $N_2 = 11$  and  $N_3 = 12$  in the baseline model. Whereby, as argued before, the relative size vis-à-vis the relevant counter party is employed as a proxy for the effect of bargaining power on poultry prices.

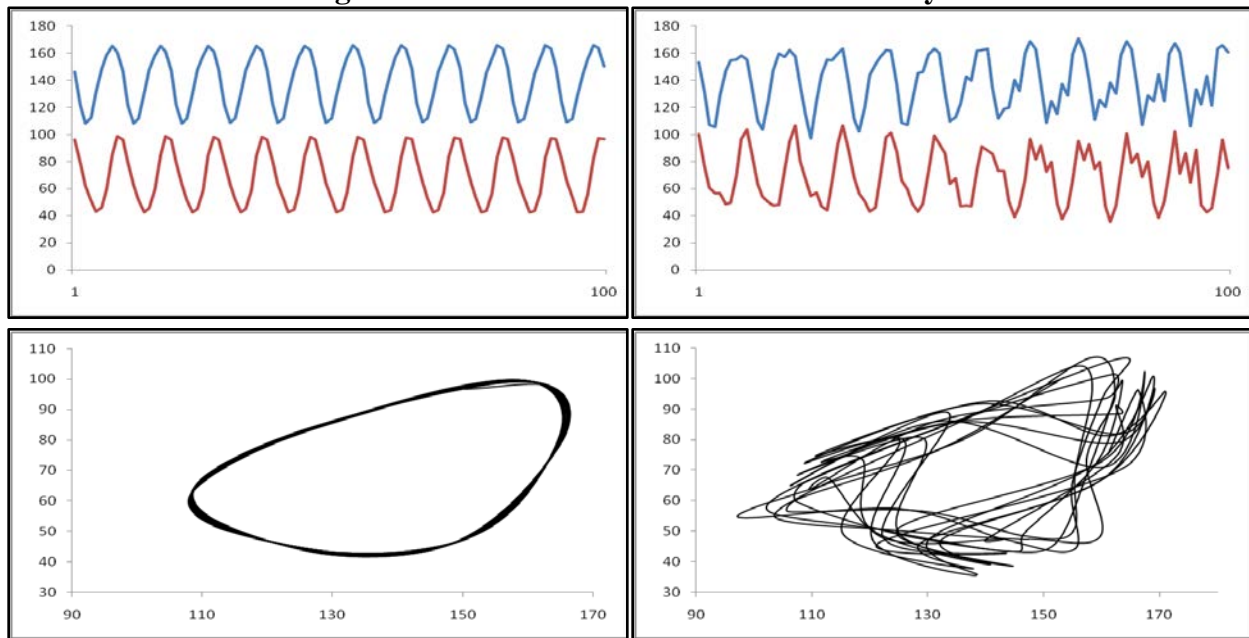
Lastly, after taking into account the overall economic environment in less developed countries like Pakistan, we assume a relatively elastic demand for broiler chicken. Because budget constrained consumers are sensitive to changes in the prices of chicken and curtail consumption if prices increase beyond certain thresholds (and vice versa). But at the same time, broiler chicken is a food based commodity and demand for food based commodities is relatively inelastic. In light of these observations we assume  $a = 90$  and  $b = 0.45$  in the retail demand curve for broilers in the baseline model. The simulation results showed that these calibrations yield point-price elasticity of demand between  $-1\%$  and  $-4\%$  and the elasticity of demand

equals  $-1.85\%$  at the mean broiler price. Anecdotal evidence and responses of consumers during fieldwork support these hypothesized estimates of the price-elasticity of demand.

### 5.3.2 Chaotic Dynamics

The top-left panel in Figure-4 illustrates the simulated trajectories of chick and broiler prices over time under the linear time-delay system i.e.  $\alpha = \beta = 2$  without an exogenous production shocks. Simulated prices with a production shock of  $\mu \sim (0, 0.75)$  each period related to the mortality of chicks i.e.  $\tilde{k} = k + \mu$  are shown in the top-right panel. The lower panel shows the corresponding phase-space plots.

**Figure-4: Simulated Prices-Linear Time Delay Models**

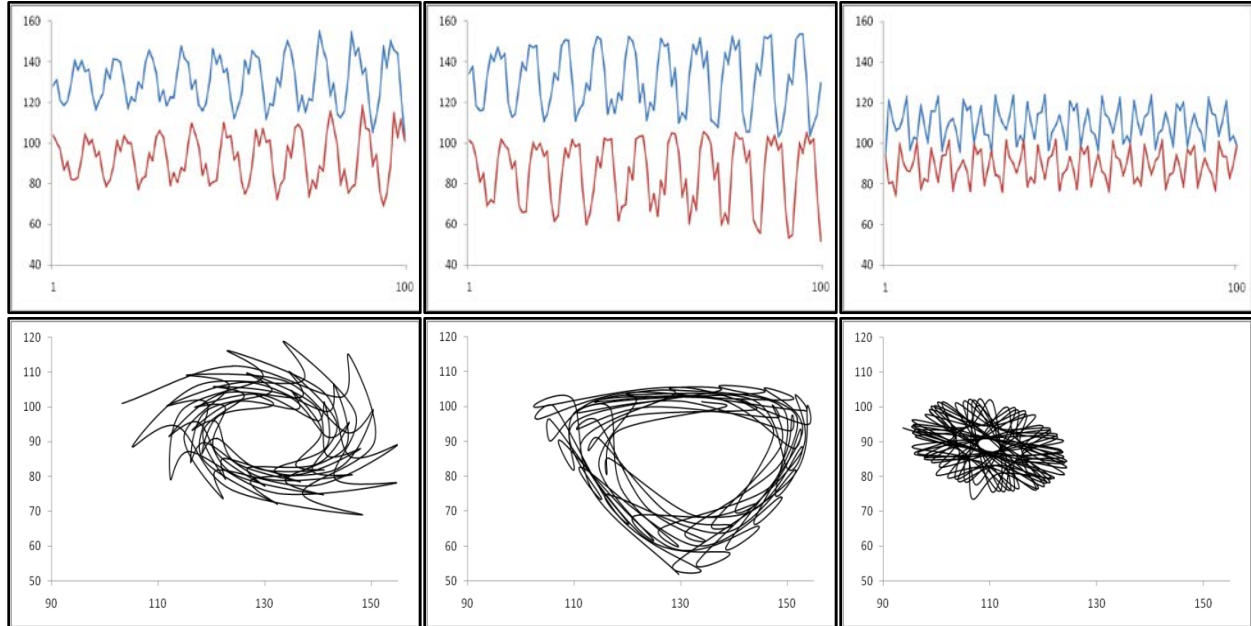


From left to right, the top panel shows the line plots of chick and broiler prices from the linear deterministic time-delay model and the linear stochastic time-delay model, respectively. The baseline model calibrations are  $\alpha = 2, \beta = 2, k = 0.95, a = 90, b = 0.45, N_1 = 10, N_2 = 11$  &  $N_3 = 12$ . Where, the red and blue lines represent weekly chick and broiler prices respectively, simulated over a period of 100 weeks after dropping the transient phase associated with initial conditions. The initial conditions for both simulations are identical. The bottom panel shows the corresponding phase-space plots i.e. plot of  $\omega_t^C$  (y-axis) on  $p_t^B$  (x-axis.)

Simulated prices from the deterministic linear time-delay model are characterized by oscillations around an equilibrium state with a fixed amplitude and period. This type of dynamical behavior is consistent with complex eigen values with large imaginary components relative to the real components. Moreover, in contrast to the price trajectories derived from standard cobweb models, the simulated prices from the linear time-delay model are smooth and positively autocorrelated. Introducing production shocks with a zero mean into the linear time-delay model results in more realistic random, non-periodic (the amplitude and period is non-constant) yet cyclical dynamic around an equilibrium state. The simulations demonstrate that a simple model of endogenous price fluctuations with exogenous production shocks can generate the stylized features commonly associated with commodity prices i.e. positive autocorrelation, cyclicity and random variation.

As expected, the corresponding phase-space plots show that the deterministic linear time-delay model possesses a smooth, stable limit cycle and is thus non-chaotic in strictest sense, while the stochastic linear time-delay model does not have a stable limit cycle due to production shocks in each period. However, it is well known that non-linearities in difference equations can lead to complicated and often chaotic dynamics even in purely deterministic systems. In order to examine the effect of non-linearities on the dynamics of the underlying system of time-delay difference equations we introduce different types of non-linearities into the system by varying the cost function parameters i.e.  $\alpha$  and  $\beta$ . The resulting price trajectories are shown in figure-5.

**Figure-5: Simulated Prices-Non-Linear Time Delay Model**



From left to right, the top panel shows the line plots of chick and broiler prices from different nonlinear deterministic time-delay models, parameter values for each simulation are given by: (1)  $\alpha = 2.5, \beta = 2.1, k = 0.95, a = 70, b = 0.45, N_1 = 10, N_2 = 11$  &  $N_3 = 12$ . (2)  $\alpha = 1.75, \beta = 1.65, k = 0.95, a = 1000, b = 6.5, N_1 = 10, N_2 = 10$  &  $N_3 = 10$  and (3)  $\alpha = 3, \beta = 2, k = 0.95, a = 70, b = 0.6, N_1 = 10, N_2 = 11$  &  $N_3 = 12$ . The initial conditions for all simulations are identical. The red and blue lines represent weekly chick and broiler prices respectively, simulated over a period of 100 weeks after dropping the transient phase associated with initial conditions. The bottom panel shows the corresponding phase-space plots i.e. plot of  $\omega_t^C$  (y-axis) on  $p_t^B$  (x-axis.)

The time series plots in the top panel depict bounded, non-periodic orbits which randomly oscillate around an equilibrium state. Although, approximately quasi-cyclical (periods of increasing prices followed by periods of decreasing prices), however, the orbits do not possess a constant period or amplitude. Moreover, the orbits do not show any repeating patterns. This is highlighted in the phase-space plots whereby, the price trajectories comprise of dense orbits encircling the equilibrium state without converging to a stable limit cycle or the equilibrium itself. It is well known that the aforementioned features typify chaotic systems i.e. deterministic non-linear maps exhibiting highly complicated, random and unpredictable behavior (Brock 1986). It is also interesting to note that even the slightest non-linearities e.g. scenario at the top-left of figure-5, leads to chaos. Likewise, different types of nonlinearities generate completely different price trajectories.

If our model accurately captures the price dynamics in the Pakistan poultry sector, it is more likely that the true data generating process is non-linear and hence chaotic because the

linear system of time-delay equations is only a special case of the underlying model. Note that if prices are chaotic and information acquisition costly, then naïve expectations about future prices are perfectly rational, as explained in detail in section-2. But is the actual price data chaotic?

Unfortunately, methods to detect chaos in actual data are rudimentary at best. Consequently, most applications of chaos in economics lack an empirical content (Brock 1999). In practice, identification of chaos in real world data employs a battery of statistical tests but the conclusions are seldom definitive. In fact, Sprott (2003) forcefully argues that due to the presence of environmental shocks, it is extremely difficult to disentangle deterministic randomness or chaos from noise in real world data. For example, Chatrath et al. (2002) use numerous empirical tests to determine whether daily futures prices of wheat, corn, soybean and cotton are chaotic. Although, they find evidence of non-linear dependence in the price data but nevertheless fail to conclusively detect chaotic dynamics. Finkenstadt and Kuhibier (1992) arrive at similar conclusions using weekly price data of pigs and potatoes in Germany from 1955 to 1989.

Nonetheless, data generated from chaotic systems often has a certain degree of structure or determinism relative to pure white noise. The BDS test statistic, due to Brock et al. (1987) is commonly employed to detect determinism in a dataset and thus serves as an ad-hoc test for deterministic chaos in the literature. The BDS test uses the concept of spatial correlation from chaos theory to compute the correlation integral for a given embedding dimension in the actual data and serves as a powerful statistical test for nonlinearities and deterministic chaos (Chatrath et al 2002 & Finkenstadt and Kuhibier 1992). Whereby, the rejection of the null hypothesis is construed as evidence in favor of deterministic chaos. Details of the related computations and the properties of the test statistic can be found in Brock et al. (1996).

We use, MATLAB code written by Kanzler (1998) to compute the BDS test statistic for the actual chick and broiler price series along with the companion program (Kanzler 1999), to adjust the corresponding p-values for small sample bias. The BDS test statistic for chick and broiler prices is 58.63 and 56.18, respectively and statistically significant at 1% in both cases. The rejection of the null-hypothesis suggests that the actual price data is generated by a chaotic system. However, as mentioned before, the BDS test is not a conclusive test of chaos but merely suggestive of determinism in the underlying price series. Moreover, as pointed out by Sprott (2003), the confounding effect of environmental noise in real world data makes clean identification of deterministic chaos virtually impossible. Nonetheless, from an econometrician's perspective, chaotic dynamics severely limit the efficacy of statistical models vis-à-vis long range price forecasts (Chatrath et al 2002). In fact, if the underlying data is chaotic then the current price is the "best" prediction of future price, very much in tune with the naïve expectation hypothesis. This may be one reason for the low explanatory power of the econometric models estimated in section-4 of this paper.

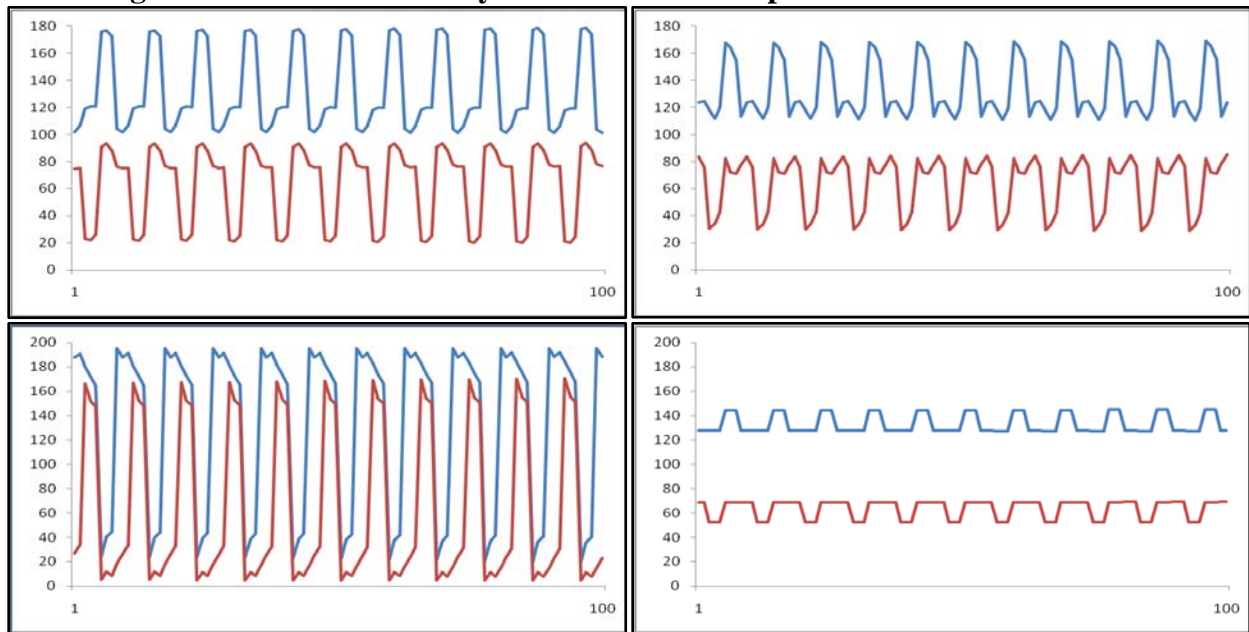
It is well known that nonlinearities can generate chaotic dynamics i.e. bounded, non-periodic and dense orbits characterized by sensitive dependence to initial conditions and small changes in parameters. Interestingly, as highlighted in the beginning of this section, many of these properties are also commonly observed in time-delay models. For example, delay induced oscillatory behavior created by Hopf bifurcations, existence of solutions with multiple frequencies (quasi-periodicity), multi-stability i.e. coexistence of more than one stable solution

(Guckenheimer & Holmes 1983) and attractor switching i.e. switching from one type of behavior to another as some parameter is varied (Kuznetsov 1995).

Research has also shown that many linear operators on an infinite dimensional state space can generate chaotic data e.g. Godefroy & Shapiro (1991) and Grosse-Erdmann et al (2011). It is important to note that time delay *differential* equations belong to a class of functional differential equations known to be inherently infinite dimensional problems. Whilst, although finite dimensional in the strictest sense, in practice the presence of time delays in linear difference equations leads to an increase in the dimension of the state-space of the underlying model. And it is well known that increases in the dimension of the state space lead to complicated dynamics (as shown in appendix-3).

Although, we do not find dense, non-periodic orbits without a stable limit cycle in simulations of the linear model. However, we document a markedly high sensitivity to initial conditions and small changes in parameter values in the linear time delay model. Figure-6 shows the simulated price trajectories of the baseline linear time-delay model with different initial conditions. The initial condition vectors for each scenario are sampled from a reasonable range within the domain of chick and broiler prices.

**Figure-6: Linear Time Delay Model-Sensitive Dependence on Initial Conditions**



The figure shows line plots of chick and broiler prices from the linear deterministic time-delay model with calibrations of parameter identical to the baseline model shown in the top left corner of figure-4 but different initial conditions. The red and blue lines represent weekly chick and broiler prices respectively, simulated over a period of 100 weeks after dropping the transient phase associated with initial conditions. The initial conditions for the baseline model shown in figure 4 are  $\{[p^B], [\omega^c]\} = \{[100 \ 85 \ 79 \ 70 \ 72 \ 80][20 \ 25 \ 30 \ 30 \ 27 \ 20]\}$ . Clockwise from left the initial condition vector for is given by  $\{[149 \ 141 \ 134 \ 117 \ 105 \ 101][51 \ 41 \ 39 \ 35 \ 25 \ 21]\}$ ,  $\{[121 \ 132 \ 135 \ 135 \ 133 \ 127][33 \ 53 \ 57 \ 51 \ 43 \ 45]\}$ ,  $\{[110 \ 110 \ 110 \ 110 \ 110][36 \ 36 \ 36 \ 36 \ 36 \ 36]\}$  and  $\{[40 \ 46 \ 44 \ 52 \ 58 \ 56][33 \ 32 \ 33 \ 30 \ 28 \ 19]\}$ .

Note that although the price trajectories for different initial conditions are qualitatively similar i.e. oscillations around an equilibrium state, nevertheless, they look remarkably different despite the fact that model calibrations are identical to the baseline model. This type of behavior is uncharacteristic of asymptotically stable linear systems. However, as mentioned before time-delays in the feedback mechanisms add significant complexity to the behavior of otherwise

simple linear dynamical systems. Although, the underlying linear time-delay system is not chaotic in the strictest sense, due to the presence of a clearly observable stable limit cycle<sup>54</sup>. Yet, a remarkably high sensitivity to initial conditions points towards “thin” chaos or complicated behavior in the absence of dense, non-periodic orbits. Thin chaos poses major problems to the discipline of statistical modeling for long-range price forecasts since it is often not possible to precisely identify the “true” initial conditions. And as highlighted in figure-5, small measurement errors in initial conditions can lead to very different price trajectories even in “thinly” chaotic systems.

Intuitively, the abovementioned sensitive dependency to initial conditions is perhaps driven by two factors. First, recall that the dynamics of the underlying 2-dimensional system of linear time-delay difference equations is equivalent to a 12-dimensional system of first order difference equations (see Appendix-3 for details). Therefore, different initial conditions may be associated with different eigen values as they lie on the stable manifold of different eigen vectors, resulting in different, albeit qualitatively similar behavior. Secondly, retarded differential equations, the continuous time analogue of time-delay difference equations are infinite dimensional problems, and, as mentioned before, chaotic behavior of linear functions on an infinite dimensional state-space is well documented in the literature (Grosse-Erdmann et al 2011). Although, time-delay difference equations are not infinite dimensional because of a fixed step-size, nevertheless, as the length of the delay increases the behavior of a system of time-delay difference equations approaches that of a retarded differential equation. Since, a large number of past values have to be determined at each state of the system in order to determine the future state of the system.

Moreover, given that in practice, numerical methods to solve time-delay differential equations rely on discretization algorithms, which effectively means that a time-delay differential equation is equivalent to a system of first order difference equations with a very large but finite state-space dimension. And it is well known that the complexity of system dynamics and in some cases even emergence of chaotic behavior is directly related to the dimension of the state-space. Therefore, if retarded linear differential equations can behave chaotically in principle due to a high dimensional state-space, the same can be expected of linear time delay difference equations with long time-delays in general. In summary, although technically finite dimensional, the presence of time-delays in difference equations results in an increase in the dimension of the underlying state-space, which may potentially lead to “thin” chaos, as documented in the simulations in figure-6.

The sensitive dependence of chaotic systems to small changes in parameters is studied with the help of bifurcation diagrams. Bifurcation diagrams illustrate the transitions in the nature of the limiting behavior of a dynamical system as a parameter of interest is systematically varied. And a bifurcation is said to have occurred if the limiting behavior of the underlying system is qualitatively different for parameter values on both sides of a given critical threshold. For example, transition from steady state to periodic state or transition from a periodic state to an unstable state etc and vice versa. This type of “switching” behavior is well documented in time-delay systems (Kuznetsov 1995). Given the plethora of research on bifurcations in non-linear

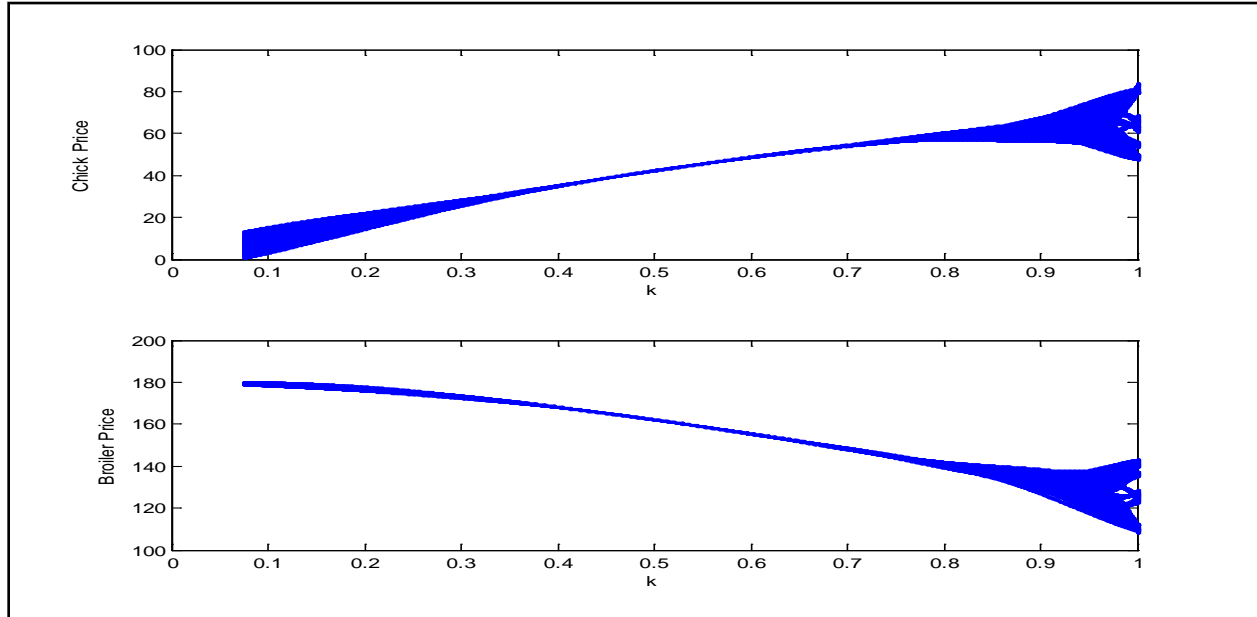
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<sup>54</sup> However, note that lack of a stable limit cycle or in other words existence of solutions with multiple frequencies (quasi-periodicity) is not in time-delay systems (Campbell 2007).



systems, we again choose to study the bifurcating behavior of the linear time-delay system. We use  $k$  as the bifurcating parameter, since it is conveniently bounded between 0 – 1 by definition and at the same time it possess a meaningful interpretation i.e. the efficiency of chick to broiler conversion technology. The bifurcation diagram is presented in figure-7.

**Figure-7: Bifurcation Diagram of Linear Time-Delay System**



The bifurcation diagram is based on following parameter values  $\alpha = 2, \beta = 2, a = 90, b = 0.5, N_1 = 10, N_2 = 11$  &  $N_3 = 12$ . To ensure non-negative chick prices values of  $k$  were restricted between 0.075 and 1, with an incremental step size of 0.001. The transient phase comprising of 50 periods was omitted and the states variables were simulated over an additional 200 periods to sketch out the limiting behavior of the system. The associated MATLAB code is given in Exhibit-2

The bifurcation diagram reaffirms earlier conjectures about the supposedly chaotic behavior of the underlying system of linear time-delay equations. For instance, the limiting behavior of chick and broiler price trajectories transitions or “switches” from a unique stable equilibrium state into a periodic oscillatory cycle as  $k$  crosses the (approximate) threshold of 0.8. This type of behavior is characteristic of Hopf bifurcations, usually associated with a change from real to imaginary eigen values in low dimensional non-linear systems. At the same time it is well known that delay induced oscillations, a commonly encountered feature of time-delay systems; often arise out of Hopf bifurcations (Campbell 2007). The thick lower tail of the bifurcation diagram, corresponding to values of  $k$  (approximately) below 0.3, represents the opposite phenomenon i.e. transition from an oscillatory periodic state to a stable equilibrium state as  $k$  increases beyond the (approximate) threshold of 0.3, albeit at a lesser scale.

The above mentioned numerical analysis highlighted several interesting features of the price trajectories generated by the underlying system of time-delay difference equations derived from a simple model of profit maximization by farmers in a vertically linked cobweb market. In a nutshell, the simulated prices exhibit complicated cyclical behavior with and without any nonlinearity in the system. For example, the behavior of the system is clearly chaotic in the presence of any type of nonlinearity as shown by an absence of stable limit cycles. While, although, the special case of linear system possesses a stable limit cycle, however its dynamical behavior is characterized by sensitive dependence to initial conditions and small changes to

parameters. These findings limit the efficacy of long range statistical price forecasting methods and lend credence to the argument that naïve expectations about future prices are rational in chaotic markets given information acquisition is costly.

#### 5.4 Comparison of statistical properties of simulated and actual price data

As mentioned at the beginning of this section, the primary objective of numerical simulations is to determine whether the stylized patterns observed in the actual price data can be reproduced by a proposed model. An aspect of economic modeling that has been largely ignored in the theoretical research on chaotic cobweb models. To this end, we compare and contrast the statistical properties of the simulated price data (table-7) with the actual data (table-2) to better understand the strengths and limitations of the underlying model of endogenous price fluctuations. The parsimonious structure of the model allows us to look at several scenarios including exogenous shocks to mortality rate i.e. chick to broiler conversion technology, effect of different types of nonlinearities on price trajectories etc.

**Table-7: Relevant Statistical Measures of Simulated Price Data**

Scenario		Coefficient of Variation	Skewness	Kurtosis	Autocorrelation
1	Broiler-linear deterministic	0.15	-0.23	-1.47	0.75
	Chick-linear deterministic	0.29	0.14	-1.50	0.75
2	Broiler-linear stochastic	0.15	-0.24	-1.20	0.60
	Chick-linear stochastic	0.28	0.18	-1.17	0.59
3	Broiler-nonlinear1	0.09	-0.03	-0.86	0.60
	Chick- nonlinear1	0.12	0.06	-0.96	0.61
4	Broiler-nonlinear2	0.12	-0.12	-1.32	0.60
	Chick- nonlinear2	0.19	-0.46	-1.18	0.59
5	Broiler-nonlinear3	0.08	0.00	-1.09	-0.15
	Chick- nonlinear3	0.09	-0.03	-1.09	-0.12

Firstly, simple visual inspection of line plots of simulated prices reported in figure 4 and 5 reveal a non-explosive cyclical/quasi-cyclical behavior broadly similar to the strongly cyclical behavior of actual prices depicted in figure-2. Secondly, the average (normalized) variation in chick prices is twice that of broiler prices in both the simulated data and the actual data. The simulated price series also possess sufficiently high, positive first order autocorrelations, comparable to the autocorrelations (approximately 0.9) in the actual price data. This, in particular, is an important finding because negative autocorrelation in standard cobweb type models is a common criticism of the theory of endogenous price fluctuations. Likewise, the kurtosis is negative in both the price series, although rather high in the simulated data. But simulated broiler prices show negative skewness in contrast to the positive skewness observed in the actual broiler prices, although, simulated chick prices show positive skewness similar to the actual chick price data. It is evident that scenarios 1-3 are the best depiction of the underlying data generating process<sup>55</sup>.

<sup>55</sup>Note that it has already been shown that even a linear-time delay system can be characterized by “thin” chaos i.e. sensitive dependence to initial conditions and small changes in parameters despite possessing a stable limit cycle. In this case, a linear model is not inconsistent with rationality of naïve expectations.

#### 5.4.1 Model Appraisal-Limitations and Extensions

Taken together, the results from table-7 suggest that, apart from scenario-5, the underlying model generates, to a reasonable extent, the stylized features observed in the actual price data. At the same time, it is clear that the underlying model cannot reproduce the actual price series to a level of satisfactory precision. However, economic models are not expected to reproduce the exact data generating process behind actual commodity prices in the first place. Models of commodity prices are created to shed light on fundamental mechanisms behind price fluctuations whilst capturing the decisions making processes of agents in a specific economic environment. Obviously, this process entails (intentionally) overlooking potentially important factors like capacity constraints, market power, adjustment costs and risk aversion etc.

These weaknesses withstanding, nevertheless, from a theoretical perspective the major objective of this paper was to examine the effects of vertical linkages and asymmetric production lags (time-delays) in agricultural value chains on commodity price dynamics. Given the data limitations, a deliberate attempt was made to keep the structure of the model as simple as possible in order to ensure that the analytical results derived from the underlying model can be evaluated statistically using real world data. Of course, this simplicity could not be achieved without ignoring some importation factors pertinent to agricultural markets. Therefore, we take some time out here to point out important weaknesses of the underlying model with the intention of pursuing them in future work and hopefully succeeding in addressing the shortcomings of the existing model vis-à-vis recreating the original price series.

Firstly, in reality, farmers cannot simply increase/decrease planned production as a best response to favorable/unfavorable changes in prices due to short-run adjustment costs and long-run capacity constraints. Especially in the case of livestock business, planned production is constrained in the short-run by the size of the breeding herd (parent stock in the case of poultry industry). The resulting inertia leads to higher first order autocorrelation in the actual data compared to the simulated data. In a model with nonlinear supply, Onozaki et al. (2000) incorporate the short run effect of adjustment costs by allowing only partial adjustment towards optimal production in response to a given price change. They show that faster adjustment towards optimal production in face of a given price change increases the likelihood of chaotic behavior and vice versa. Incorporating heterogeneous expectations seems like another valuable modification to the baseline model. For instance, Chavas (1999a) and Chavas (2000) find strong empirical evidence in support of heterogeneous expectation regimes in US broiler and beef sectors, respectively. In presence of heterogeneous expectations about future prices, different farmers respond differently to price changes, resulting in comparatively stable production (hence stable prices) and higher first order autocorrelation in actual prices.

Another limitation of the model is the failure to incorporate long run dynamics into the price trajectories<sup>56</sup>. For instance, a cursory look at figure-2 clearly reveals several vertical shifts

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<sup>56</sup>Consequently, in order to ensure minimal influence of long-run dynamics on prices, we limit the time-span of simulations to approximately 100-120 periods in the numerical section. This period is approximately equal to two years and in all likelihood long-run dynamics can be assumed to be fixed over such a short duration in practice. Furthermore, we do not compare the statistical properties related to the levels of accrual data due to the aforementioned shifts in the actual prices.

(regime switching) in the actual price data over time, consistent with entry and exit of farmers to preserve the long-run zero profit condition. Such type of vertical shifts are by and large absent from the simulated price trajectories. One way to incorporate long run dynamics into the model is to endogenize the number of chick farmers, broiler farmers and retailers via the free-entry equilibrium condition. Of course, this stratagem entails the inclusion of additional state variables making the analysis much more complicated but at the same time more realistic. Dieci & Weshterhoff (2010) make an attempt to pursue this line of inquiry by using the profit differentials between two horizontally linked cobweb markets to determine the equilibrium number of participating farmers in each market, although the total number of farmers is held to be fixed in their paper. Market power, especially in vertically linked agricultural markets, is another pertinent issue that was not considered in our paper. Interestingly, to the best of our knowledge, the effect of market power on the price dynamics in cobweb type agricultural markets has not been fully examined in the literature. Although, the effect of market power on the chaotic dynamics of prices in Cournot-type markets is an active area of research in the industrial organization literature (Yassen & Agiza 2003; Hassan 2004 and Elsadany 2010).

It may also be worthwhile to explore a scenario in which planned production responses to a price increase differs from changes in planned production in response to a price decrease. Because additions to breeding herd in lieu of favorable price changes will bear fruition vis-à-vis increased production several periods later. On the contrary, breeding herd may be immediately culled in lieu of unfavorable expected prices, resulting in relatively faster decline in future production. This effect can be clearly seen in the actual price data depicted in figure-2, whereby prices do not rise and fall at the same “speed”. Anecdotal evidence also suggests that risk averse farmers rapidly decrease production in response to adverse price movements by culling breeding stock or leaving crop land fallow. Interestingly, Boussard (1996) shows that in the presence of risk averse producers, prices depict chaotic behavior and hedging facilities like future markets fail to reduce the magnitude of price fluctuations. Undoubtedly, pursuing this line of enquiry will lead to the introduction of further complexity into the model due to the discontinuities in the production function.

Some important and seemingly important extensions can be easily incorporated into the original model e.g. adaptive expectations instead of naïve expectations and predetermined percentage of planned production produced at a pre-contracted fixed rate. We choose not to pursue these extensions for sake of brevity with the reasoned belief that incorporating these factors will not have a qualitatively meaningful impact on the existing price dynamics.

## **6-Conclusion**

The behavior of prices of agricultural products is potentially interesting and at the same time complicated due to the interaction of myriad factors e.g. production delays, expectation regimes, random supply shocks and seasonality of demand etc. Although, commodity prices depict quasi-cyclical behavior, however, identification of the period and amplitude of cycles is often impossible due to the presence of several unobserved systematic components in agricultural prices e.g. seasonality of demand and endogenously changing production patterns. Moreover, interactions of continuous random supply shocks and “cobweb” responses of farmers to price changes may lead to periods of relative stability (quasi-periodic cycles) and instability

(explosive cycles). Thus, researchers have to be very careful given the numerous pitfalls associated with examining issues related to agricultural commodity prices.

In this paper we integrated research on theoretical models of chaotic cobweb markets with standard econometric tools for analysis of time-series data and insights from extensive fieldwork to examine the underlying reasons behind the price fluctuations in the Pakistan poultry sector. In doing so we have tried to address the well known shortcomings associated with purely theoretical research, empirical work and descriptive studies. For example, though illuminating, theoretical work often fails to adequately consider whether key assumptions of cobweb models hold in practice. While, statistical relationships derived from empirical work devoid of an economic framework are difficult to interpret and are often meaningless. Lastly, farmer surveys and interviews lend support to the conclusions of the paper and provide an interesting context for the research questions. Although, we do not claim that the aforementioned pieces of evidences i.e. qualitative study, empirical estimation, theoretical modeling and numerical simulation is individually conclusive but their accumulation does present a coherent picture. In summary, we conclude that numerical simulations and empirical analysis lend support to anecdotal evidence in favor of the relevance of the theory of endogenous price fluctuations in the Pakistan poultry industry.

Methodologically, in the first stage of this project, structured interviews were carried out with different stakeholders in the poultry value chain during field work in Pakistan. The primary purpose of this endeavor was to understand the production and price discovery process, in particular the structure of the poultry supply chain, information flows and the economics of poultry farms in Pakistan. Unsurprisingly, we documented significant organizational differences between the supply chain and institutional environment of the poultry sector in Pakistan and USA. Given an understanding of the domestic price formation mechanisms and a selective review of the literature on price fluctuations, we developed an economic model to derive relationships between chick and broiler prices under a naïve expectations regime, given the constraint that price data is generally available at reasonable frequencies as opposed to quantity data. In contrast to the previous literature on cobweb markets, we specifically incorporated vertical linkages in poultry supply chain and asymmetric production time-delays into our model of price fluctuations. Thereafter, standard time-series econometric tools are employed to determine whether the actual broiler and chick prices conform to the predictions made from the underlying model. Lastly, numerical analyses were used to highlight the “strange” behavior of the underlying system of time-delay difference equations. Under reasonable calibrations, simulations reproduced the stylized patterns observed in the real world data i.e. quasi-cyclical behavior, positive first-order autocorrelation, high variability and negative kurtosis.

The paper makes several key contributions to the literature. Firstly, fieldwork in Pakistan sheds light on the often poorly understood mechanics of agricultural markets in less developed countries. From an economic theory perspective, we extend the literature on chaotic cobweb markets by incorporating two important aspects, which have been largely overlooked by previous research i.e. vertical linkages in agricultural value chains and the associated asymmetric production delays. We also add an empirical dimension to the otherwise largely theoretical literature on cobweb markets by employing standard time-series econometrics methods to statistically evaluate the comparative static results derived from the underlying model using real-world data. Lastly, our numerical analysis highlights that incorporating linkages between

vertically linked farmers and asymmetric production-delays helps overcome commonly cited critiques of cobweb models i.e. negatively autocorrelated prices and simple, predictable dynamics<sup>57</sup>. These are non-trivial findings because backward looking expectations are boundedly rational if prices are unpredictable i.e. chaotic and information acquisition is costly.

### **6.1-Policy Implications**

In light of the abovementioned findings, we conclude the paper by offering some policy recommendations to mitigate price volatility in the Pakistan poultry sector. The development of future markets on poultry products in Pakistan is highly improbable given the prevalent institutional environment. Therefore, in the short-run some sort of market intervention is needed to mitigate endogenous price fluctuations. For example, government may fix broiler and chick price over regular periods to ensure that farmers are aware of the expected price and hence future profitability of production plans. This will lead to stabilization of production and hence prices, if done over a significantly long period time, such that both upstream and downstream farmers are able to benefit from predetermined prices. Stabilization the upstream sector, seldom ignored in policy debates of price fluctuations, is key to mitigating price fluctuations, since a production glut or shortage originates from the upstream sector and feeds into the downstream sector. Increases in the market power of the upstream sector is one way to achieve stabilization of production because a monopolist sets production (and hence prices) such that marginal revenue equals marginal cost. In fact simulations (not reported) revealed that increasing the bargaining power of the chick farmers mitigated the price cyclicalities to a large extent. Of course, both of these policies are merely stop-gap solutions and it is well known that market interventions and imperfections leads to significant deadweight loss.

In the long run, policies that promote vertical integration of the entire poultry value chain are perhaps the best solution to the underlying problem of endogenous price fluctuations. Even basic level integration between hatcheries and broiler farmers can also lead to significant reduction in the observed price volatility. By stabilizing short run supply, storage leads to a significant reduction in price volatility (Mitra & Boussard 2012), especially given the strong seasonal patterns in the demand for broiler chicken in Pakistan. Whereby, seasonal swings in demand can have a large impact on current prices, and hence future prices given current prices are used as a proxy of future price. However, this is not possible without engendering a smooth transition from a live-bird market to frozen bird market. Of course, this requires significant investment in cold storage facilities across the poultry value chain along with relevant extension work to educate farmers and consumers about the benefits of frozen chicken over live-chicken. The Pakistan government was fairly successful in converting customers from consumption of untreated open-milk to packaged milk through extensive advertising campaigns in the 1990s.

At the poultry association level, investments in information systems to collect, process and disseminate relevant poultry data e.g. placement of parent stock, chicks in hatcheries and broilers sold in a given week etc. will go a long way in helping farmers formulate production plans efficiently. However, the success of such systems relies on voluntary information sharing by participating farmers but in the context of developing countries, the incentives to accurately share production information are usually absent.

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<sup>57</sup> For example, we find evidence of chaotic behavior, even in the absence of any explicit nonlinearity

## Appendix-1

In order to calibrate the theoretical model presented in section-3, we use a power function to represent the convex cost functions of poultry farmers and a linear function to capture the retail demand for broilers<sup>58</sup>. More specifically we employ:

- A convex cost function of chick farmers at time t:  $C_1(q_t^C) = (q_t^C)^\alpha$  where  $\alpha > 1$
- A convex cost function of broiler farmers at time t:  $C_2(q_t^C) = (q_t^C)^\beta$  where  $\beta > 1$
- A linear retail demand function for broilers at time t:  $Q_t^{B,D} = a - bp_t^B$  where  $a > 0$  represents the extent (or maximum capacity) of a given retail demand and  $b > 0$  represents the sensitivity of demand to broiler prices.

Given these primers, recall that the derivation of broiler price dynamics under the naïve expectations hypothesis yielded the following difference equation:

$$p_t^B = F^{-1} \left( \frac{kN_3}{N_3} (C_2')^{-1} (kp_{t-2}^B - \omega_{t-2}^C) \right)$$

Substituting in the above-mentioned functional forms and performing some algebraic manipulations yields:

$$p_t^B = \frac{a}{b} - \frac{N_2 k}{N_3 b} \left( \frac{kp_{t-2}^B}{\beta} - \frac{\omega_{t-2}^C}{\beta} \right)^{\frac{1}{\beta-1}}$$

Similarly the difference equation for broiler prices was given by:

$$\omega_t^C = kp_{t-1}^B - C_2' \left( \frac{N_1}{N_2} (C_1')^{-1} (\omega_{t-1}^C) \right)$$

Substituting in the relevant functional forms, and performing some algebraic manipulations gives us the following price dynamic for chick prices:

$$\omega_t^C = kp_{t-1}^B - \left( \frac{N_1 \beta}{N_2} \right)^{\beta-1} \left( \frac{\omega_{t-1}^C}{\alpha} \right)^{\frac{\beta-1}{\alpha-1}}$$

In the special case of quadratic cost functions ( $\alpha = \beta = 2$ ), the difference equations of broiler and chick prices can be reduced to the following system of linear equations:

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<sup>58</sup> Since the domain of cost functions is strictly positive by definition, therefore all power functions are convex functions. Likewise, both power functions and linear functions have a well defined inverse, note that since prices are always positive, the range of the inverse function is also the real line.

$$p_t^B = \frac{a}{b} - \frac{N_2 k^2}{2bN_3} p_{t-2}^B + \frac{N_2 k}{2bN_3} \omega_{t-2}^C$$

$$\omega_t^C = k p_{t-1}^B - \frac{N_1}{N_2} \omega_{t-1}^C$$

### Appendix-2

Again in an equilibrium or steady state we have  $p_t^B = p_{t-3}^B = p_{t-6}^B = p^B$  and  $\omega_t^C = \omega_{t-3}^C = \omega_{t-6}^C = \omega^C$ . Substituting into the system of equations we get:

$$p^B = \frac{a}{b} - \frac{N_2 k}{N_3 b} \left( \frac{k p^B}{\beta} - \frac{\omega^C}{\beta} \right)^{\frac{1}{\beta-1}}$$

$$\omega^C = k p^B - \left( \frac{N_1 \beta}{N_2} \right)^{\beta-1} \left( \frac{\omega^C}{\alpha} \right)^{\frac{\beta-1}{\alpha-1}}$$

From the chick price equation we get:

$$p^B = \frac{1}{k} \left( \omega^C + \left( \frac{N_1 \beta}{N_2} \right)^{\beta-1} \left( \frac{\omega^C}{\alpha} \right)^{\frac{\beta-1}{\alpha-1}} \right)$$

Substituting the above expression into the broiler price equation and simplifying yields:

$$\omega^C + \left( \frac{N_1 \beta}{N_2} \right)^{\beta-1} \left( \frac{\omega^C}{\alpha} \right)^{\frac{\beta-1}{\alpha-1}} = \frac{ka}{b} - \frac{N_2 k^2}{N_3 b} \left( \omega^C + \left( \frac{N_1 \beta}{N_2} \right)^{\beta-1} \left( \frac{\omega^C}{\alpha} \right)^{\frac{\beta-1}{\alpha-1}} - \omega^C \right) \left( \frac{1}{\beta} \right)^{\frac{1}{\beta-1}}$$

$$\omega^C + \left( \frac{N_1 \beta}{N_2} \right)^{\beta-1} \left( \frac{\omega^C}{\alpha} \right)^{\frac{\beta-1}{\alpha-1}} + \frac{N_1 k^2 \beta}{N_3 b} \left( \frac{\omega^C}{\alpha} \right)^{\frac{1}{\alpha-1}} \left( \frac{1}{\beta} \right)^{\frac{1}{\beta-1}} - \frac{ka}{b} = 0$$

This equation is non-linear as long as  $(\alpha = \beta \neq 2)$ . In order to study the characteristics of the fixed point of this equation we can decompose it into a function  $f(\omega^C) = \omega^C + \left( \frac{N_1 \beta}{N_2} \right)^{\beta-1} \left( \frac{\omega^C}{\alpha} \right)^{\frac{\beta-1}{\alpha-1}} + \frac{N_1 k^2 \beta}{N_3 b} \left( \frac{\omega^C}{\alpha} \right)^{\frac{1}{\alpha-1}} \left( \frac{1}{\beta} \right)^{\frac{1}{\beta-1}}$ , where the domain of  $\omega^C$  is the positive real line and a constant  $C = \frac{ka}{b}$ , where  $0 < C < \infty$  by definition of the parameters in the model. Obviously at the fixed point  $f(\omega^C) = C$ .

Clearly,  $f(0) = 0$  and  $f(\infty) = +\infty$ . Note that  $f'(\omega^C) = 1 + \frac{\beta-1}{\alpha-1} \left( \frac{N_1 \beta}{N_2} \right)^{\beta-1} \left( \frac{\omega^C}{\alpha} \right)^{\frac{\beta-1}{\alpha-1}-1} + \frac{N_1 k^2 \beta}{(\alpha-1)N_3 b} \left( \frac{\omega^C}{\alpha} \right)^{\frac{1}{\alpha-1}-1} \left( \frac{1}{\beta} \right)^{\frac{1}{\beta-1}} > 0$ , since  $\beta > 1$  and  $\alpha > 1$  by definition and all other model parameters are strictly positive. Therefore, given  $f(\omega^C)$  is a strictly increasing function on its domain, we can use the single crossing property to conclude that  $f(\omega^C)$  intersects  $C$  at a single



point. Given the unique steady state  $\omega^c$ , we can compute the corresponding equilibrium broiler price  $p^B$ . Note, that we do not consider economically irrelevant steady states i.e. when  $\omega^c$  and/or  $p^B$  are less than zero, since prices are positive by definition. The actual steady states are only implicitly defined and cannot be expressed as a function of the model parameters. Nevertheless, actual values can be computed via numerical methods given suitable model calibrations.

### Appendix-3

The special case of a system of linear time-delay difference equations is given by:

$$p_t^B = \frac{a}{b} - \frac{N_2 k^2}{2bN_3} p_{t-6}^B + \frac{N_2 k}{2bN_3} \omega_{t-6}^C$$

$$\omega_t^C = k p_{t-3}^B - \frac{N_1}{N_2} \omega_{t-3}^C$$

It is straightforward to notice that this is a system with sixth-order difference equations and two state variables. In order to study the stability and qualitative behavior of the system we can rewrite it as a system of first order difference equations with a dimension of 2x6. Whereby, n-1 new variables are defined to represent each higher order difference term along with a new system constant  $C$ . Therefore, let  $x_t^i = p_{t-i}^B$  and  $y_t = \omega_{t-1}^C$  for  $i = 1$  to 6.

$$\begin{bmatrix} p_t^B \\ \omega_t^C \\ x_t^1 \\ y_t^1 \\ x_t^2 \\ y_t^2 \\ x_t^3 \\ y_t^3 \\ x_t^4 \\ y_t^4 \\ x_t^5 \\ y_t^5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{N_2 k^2}{2bN_3} & \frac{N_2 k}{2bN_3} \\ 0 & 0 & 0 & 0 & k & \frac{N_1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_t^1 \\ y_t^1 \\ x_t^2 \\ y_t^2 \\ x_t^3 \\ y_t^3 \\ x_t^4 \\ y_t^4 \\ x_t^5 \\ y_t^5 \\ x_t^6 \\ y_t^6 \end{bmatrix} + C$$

It is easy to notice that the augmented system comprises of first order difference equations i.e. after making the appropriate substitutions we get the relevant price equations and 10 identities:

$$\begin{bmatrix} p_t^B \\ \omega_t^C \\ p_{t-1}^B \\ \omega_{t-1}^C \\ p_{t-2}^B \\ \omega_{t-2}^C \\ p_{t-3}^B \\ \omega_{t-3}^C \\ p_{t-4}^B \\ \omega_{t-4}^C \\ p_{t-5}^B \\ \omega_{t-5}^C \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{N_2 k^2}{2bN_3} & \frac{N_2 k}{2bN_3} \\ 0 & 0 & 0 & 0 & k & \frac{N_1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{t-1}^B \\ \omega_{t-1}^C \\ p_{t-2}^B \\ \omega_{t-2}^C \\ p_{t-3}^B \\ \omega_{t-3}^C \\ p_{t-4}^B \\ \omega_{t-4}^C \\ p_{t-5}^B \\ \omega_{t-5}^C \\ p_{t-6}^B \\ \omega_{t-6}^C \end{bmatrix} + C$$

For notational simplicity, let  $X_t$  denote the LHS vector of variables and  $A$  represent the transition matrix, then the model can be succinctly written as a first order system:  $X_t = AX_{t-1} + C$ . It is well known, that eigen values of the matrix  $A$  will determine the behavior of the original time-delay system. Note that in equilibrium  $X_t = X_{t-1} = X$ , substituting into the augmented first order system and simplifying we get:  $X = (I - A)^{-1}C$ . It is easy to show that  $(I - A)^{-1}$  is always invertible and thus the augmented system has a unique fixed point just like the original time-delay system.

Due to the large dimension of the system matrix  $A$ , formulating and factorizing the associated characteristic equation is not feasible. Therefore, we use MATLAB to calculate eigen values under suitable model calibrations, these calibrations are identical to ones used for numerical simulations of the baseline model in section 5.3 i.e.  $k = 0.95, b = 0.45, a = 90, N_1 = 10, N_2 = 11$  and  $N_3 = 12$ . The eigen values are:  $\lambda_{1,2} = -0.9392 \pm 0.3434i$ ,  $\lambda_{3,4} = 0.1722 \pm 0.9851i$ ,  $\lambda_{5,6} = 0.7670 \pm 0.6417i$ ,  $\lambda_{7,8} = -0.2184 \pm 0.3784i$ ,  $\lambda_{9,10} = -1.4733e - 06 \pm 2.5519e - 06i$ ,  $\lambda_{11} = 0.4367$  and  $\lambda_{12} = 2.9467e - 06$  respectively. Qualitatively, similar eigen values are obtained for within the domain of  $k$ .

Although, the above-mentioned analysis has been performed for the special case of our time-delay model i.e. a linear system when  $\alpha = \beta = 2$ . This method can be followed to study the local stability of the non-linear systems as well i.e. when  $\alpha = \beta \neq 2$ , by replacing the coefficient or system transition matrix by the Jacobian of the resulting first order system so as to linearize the non-linear system around the fixed point.<sup>59</sup> Again, for sake of brevity, we choose not to pursue this line of inquiry further since it entails numerical computation of the fixed point of the non-linear system (please refer to appendix-2) along with other detailed computations.

<sup>59</sup> Note that for linear systems, local stability and asymptotic stability are equivalent notions. While for non-linear system we can only study the local behavior of the system analytically and need to use numerical simulations to investigate the global behavior.

#### Appendix-4

Derivation of delay-dependent stability criterion for time-delay systems is possible in very few cases, usually for very simple systems. However, stability criterion independent of delay can be easily derived for most time-delay systems. Hale et al. (1989) have provided necessary and sufficient conditions for the asymptotic stability of time-delay systems independent of delay:

Let  $x$  represent a vector of endogenous state variables with a fixed time-delay  $d$  and related coefficient matrix  $A$  and  $A_d$ , such that the resulting system of retarded differential equations is given by:

$$\dot{x} = Ax + A_d x(t-d). \text{ This system is asymptotically stable, independent of delay if } \text{Re } |\lambda(A + A_d)| < 0 \text{ under standard normality conditions.}$$

It is well known that time-delay difference equations with a fixed delay are just the discrete time analogue of retarded differential equations, and can be readily converted into the latter, the resulting theorem for difference equations is given by:

$$\Delta x_t = Ax_{t-1} + A_d x(t-d). \text{ This system is asymptotically stable, independent of delay if } \text{Re } |\lambda(A + A_d)| < 1 \text{ under standard normality conditions.}^{60}$$

Applying this theorem to time-delay system under study in this paper, we get:

$$\begin{bmatrix} \Delta p_t^B \\ \Delta \omega_t^C \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} p_{t-1}^B \\ \omega_{t-1}^C \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ k & -\frac{N_1}{N_2} \end{bmatrix} \begin{bmatrix} p_{t-3}^B \\ \omega_{t-3}^C \end{bmatrix} + \begin{bmatrix} -\frac{N_2 k^2}{2bN_3} & \frac{N_2 k}{2bN_3} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_{t-6}^B \\ \omega_{t-6}^C \end{bmatrix} + \begin{bmatrix} a \\ b \\ 0 \end{bmatrix}$$

Following Hale et al. (1989), we need to look at the eigen values of the following matrix in order to determine the asymptotic stability of the abovementioned system independent of time delay:

$$\lambda(A + A_d) = \begin{bmatrix} -1 - \frac{N_2 k^2}{2bN_3} & \frac{N_2 k}{2bN_3} \\ k & -1 - \frac{N_1}{N_2} \end{bmatrix}$$

The corresponding characteristic equation is given by:

$$\left(1 + \frac{N_2 k^2}{2bN_3} + \lambda\right) \left(1 + \frac{N_1}{N_2} + \lambda\right) - \frac{N_2 k^2}{2bN_3}$$

Multiplying out and collecting terms we get:

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<sup>60</sup> Simply subtract  $x_{t-1}$  from both sides, such that we have the change in  $x$  on the left hand side, see Vas (2016) for details. Note that in contrast to continuous time whereby the eigen values are the exponent in the symbolic dynamic of the system, in discrete time the eigen values are the base of the exponent in the symbolic dynamics. Therefore, asymptotic stability in discrete time systems is achieved if the absolute value of the eigen value is less than 1 i.e. all eigen values are within the unit circle.

$$\lambda^2 + \lambda \left( 2 + \frac{N_1}{N_2} + \frac{N_2 k^2}{2bN_3} \right) + 1 + \frac{N_1}{N_2} + \frac{N_1 k^2}{2bN_3}$$

Now, using the quadratic root formula:  $\frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$ , where  $b = 2 + \frac{N_1}{N_2} + \frac{N_2 k^2}{2bN_3}$ ,  $c = 1 + \frac{N_1}{N_2} + \frac{N_1 k^2}{2bN_3}$  and  $a = 1$ . It is straightforward to notice that  $\left| \frac{-b}{2a} \right| > 1$ , since  $\frac{N_1}{N_2}$  and  $\frac{N_2 k^2}{2bN_3}$  are non-zero positive numbers by definition of the parameters. Similarly, simplifying the expression  $(b^2 - 4ac)$  we get:  $\frac{N_1^2}{N_2^2} + \frac{N_2^2 k^4}{4b^2 N_3^2} + \frac{k^2(2N_2 - N_1)}{bN_3}$ , this expression is also strictly positive by definition of the parameters<sup>61</sup>.

We know with certainty that that absolute value of  $\lambda_1 = -\frac{b}{2a} - \frac{\sqrt{(b^2 - 4ac)}}{2a}$  is greater than 1, since both  $\frac{b}{2a}$  and  $\frac{\sqrt{(b^2 - 4ac)}}{2a}$  are positive, while  $\left| \frac{b}{2a} \right| > 1$  by definition. For our purposes it suffices to note that  $|\lambda_1| > |\lambda_2|$  where  $\lambda_2 = -\frac{b}{2a} + \frac{\sqrt{(b^2 - 4ac)}}{2a}$ . Because the asymptotic behavior of the system is determined by the dominant or largest eigen value and it is well known that the underlying system is asymptotically unstable if the absolute value of the dominant eigen value is greater than 1. Therefore, we conclude that the underlying system is asymptotically unstable independent of time-delays.

Note that in addition to  $|\lambda_1| > 1$ ,  $\lambda_1$  is also negative, consistent with oscillatory diverging behavior, while the sign of  $\lambda_2$  depends on the parameter values. Likewise, depending on the parameter values either  $|\lambda_1| > 1 > |\lambda_2|$  which corresponds to a saddle point or an unstable explosive system  $|\lambda_1| > |\lambda_2| > 1$ . In order to get a better idea of the behavior of the underlying model without the effects of time-delays on system dynamics, we compute the eigen values using model calibrations for the baseline case i.e.  $k = 0.95$ ,  $b = 0.45$ ,  $a = 90$ ,  $N_1 = 10$ ,  $N_2 = 11$  and  $N_3 = 12$ . The resulting eigen values are  $\lambda_1 = -2.70$  and  $\lambda_2 = -0.96$ , consistent with a saddle-point, whereby the system is best characterized by oscillatory divergence away from the equilibrium or explosion in the direction of  $\lambda_1$  and oscillatory convergence towards equilibrium in the direction of  $\lambda_2$ . Also, note that  $\lambda_2$  is approximately equal to -1 in the baseline case, indicative of the borderline case of perpetual oscillations around the equilibrium. Of course, the baseline model independent of time-delay is also asymptotically unstable due to the effect of the dominant eigen value.

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<sup>61</sup> Recall that the  $N_i$  was used to measure relative bargaining power of the  $i$ th counterparty and assumed to be such that the ratio of any two  $N_i$  is within reasonable bounds such that considerations of imperfect competition do not arise. Therefore  $2N_2 < N_1$  is not feasible given our model. Also note that based on surveys data assumed  $N_1 < N_2$ .

## **Exhibit-1**

A stacked survey methodology was adopted to gain maximum information about the poultry industry at minimum cost. Approximately 50 subjects, belonging to different types of agents in the poultry value chain, were interviewed by a professional marketing research company called The Learning Organization (TLO) under my supervision between August-October 2015. The questions for structured interviews, usually lasting for approximately 1-2 hours, are provided below along with the relevant subject headings.

### **1-Business Model and the Poultry Value Chain**

- Briefly describe your role in the poultry value chain?
- Who are the most powerful players in the chick and broiler value chain?
- What is the reason for engaging intermediary in the value chain instead of direct dealing with hatchery/retailer?
- What is the commission structure of middlemen in the value chain?

### **2-Demand/Supply Trends and Determinants**

- What are the major trends of demand for chicken in Pakistan?
- What are the major determinants of supply (production) of chicks and broiler in Pakistan?
- How do individual farmers make their decisions about when/how much to supply in a given time period?
- What are the reasons for overproduction?
- Are the market dynamics for chick and broiler different in other regions of Pakistan? Yes/No? Why?

### **3-Poultry Data related to Demand and Supply**

- What are the main cost components in the production of chick and broiler?
- Do you think that cost components have shown considerable volatility over the past five years? If yes then why and which components?
- What are the main cost components vis-à-vis the selling and marketing expenses of chick and broiler? Do you think that selling/marketing costs have changed considerably over the past 5 years? If yes then why?

### **4-Price determination and Communication**

- How are farm gate prices set in the chick and broiler industry?
- How are the prices communicated to various stakeholders?
- Is there any mechanism to ensure that everyone is selling chick and broiler at the circulated prices?

### **5-Price Volatility**

- What do you think are the possible reasons behind the volatility in prices of chick and broiler?
- What is the impact of price volatility on your business?
- What is the possible impact of price volatility on other participants in the value chain (e.g. feed mills, breeding farms, broiler farms, retailers, final consumer)?

- Who do you think gains the most in the value chain due to these price fluctuations?
- Who do you think loses the most in the value chain due to these price fluctuations?
- What measures can be taken to stabilize the prices of poultry products?

### Exhibit-2

```

clear all
close all
clc
A=2;%alpha in appendix-1%;
B=2;%beta in appendix-1%
n1=10;
n2=11;
n3=12;
a=90;
b=0.5;
Npre = 50; Nplot = 200;
chickprice = zeros(Nplot,1);
broilerprice=zeros(Nplot,1);
for k = 0.075:0.0001:1,
    chickprice(1) = 20;
    chickprice(2) = 25;
    chickprice(3) = 30;
    chickprice(4) = 30;
    chickprice(5) = 27;
    chickprice(6) = 20;
    broilerprice(1) = 100;
    broilerprice(2) = 85;
    broilerprice(3) = 79;
    broilerprice(4) = 70;
    broilerprice(5) = 72;
    broilerprice(6) = 80;
    for t = 7:Nplot,
        chickprice(t)= k*broilerprice(t-3) - ((n1*B/n2)^(B-1))*(chickprice(t-
3)/A)^((B-1)/(A-1));
        broilerprice(t)= a/b-((n2*k)/(n3*b))*(k*broilerprice(t-6)/B-chickprice(t-
6)/B)^(1/(B-1));
    end,
subplot(211)
    plot(k*ones(Nplot-Npre,1),real(chickprice(Npre:Nplot-1)), '.',
'markersize', 3);
    hold on
subplot(212)
    plot(k*ones(Nplot-Npre,1),real(broilerprice(Npre:Nplot-1)), '.',
'markersize', 3);
    hold on;
end,
xlabel('k'); ylabel('Price');
set(gca, 'xlim', [0 1]);
hold off;

```

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