Measuring price discovery in agricultural markets

Evgenia Pavlova, Georg-August-University Göttingen, Germany, epavlov@gwdg.de

Stephan von Cramon-Taubadel, Georg-August-University Göttingen, Germany, scramon@gwdg.de

Selected Paper prepared for presentation at the 2016 Agricultural & Applied Economics Association Annual Meeting, Boston, Massachusetts, July 31-August 2

Measuring price discovery in agricultural markets

Abstract

The present paper is devoted to the issue of price discovery in agricultural markets. Price discovery is considered one of the central functions of financial markets and is usually analyzed empirically for highly homogeneous assets traded on these markets. We investigate whether this approach can provide useful insights into the pricing of agricultural commodities. We follow classical approaches for the measurement of price discovery: the permanent-transitory (PT) approach proposed by Gonzalo and Granger (1995); the information shares (IS) approach proposed by Hasbrouck (1995); and the information leadership share (ILS) proposed by Putnins (2013) that is a combination of both. These approaches are based on the vector error-correction model (VECM) that separates long-run price movements from short-run market microstructure effects. We analyse the differencies between argicultural commodities and financial assets that are relevant for deriving and calculating the above measures and propose respective modifications. We apply our methodology to the analysis of weekly pork meat prices in four EU countries (Germany, Netherlands, Belgium and France) for the period 1987-2013 and perform pairwise price discovery analysis using PT, IS and ILS measures.

1 Introduction

Price discovery is considered one of the central functions of financial markets and is usuallyanalyzed empirically for the assets traded on these markets. Though the trading venues are different, they deal with identical (or closely related) assets, therefore, intermarket arbitrage keeps the prices from drifting too far apart from each other. Econometrically speaking, in this case prices are $I(1)$ cointegrated variables, sharing one (or, more rarely, several) common stochastic trends or factors. In the bivariate case of two prices it means that their linear combination is stationary, and the components of cointegrating vector are the coefficients in this linear combination. This common stochastic factor is referred to as the unobservable efficient price, and price discovery can be viewed as the process of uncovering this fundamental value.

The vast majority of existing studies on the subject, both theoretical and empirical, analyse highly homogeneous financial assets. Our aim is to investigate whether the price price discovery approach can provide useful insights into pricing on agricultural markets. Agricultural commodities differ from financial instruments in a number of ways, most importantly in less frequent observations available for the analysis: one trading of day stock exchange provides 21,600 observations, whereas much agricultural price data is only weekly or monthly. Another difference is that the same agricultural commodities traded at different venues are less homogeneous than identical financial assets. For latter is is usually assumed that the cointegrating vector is (1,-1), which is not necessarily the case in agricultural commodity settings.

There are two established approaches for the measurement of price discovery: the permanent-transitory (PT) approach proposed by Gonzalo and Granger (1995), and the information shares (IS) approach proposed by Hasbrouck (1995). Both approaches are based on the vector error-correction model (VECM) that separates long-run price movements from short-run market microstructure effects.

In the PT approach, a special linear combination of the variables is constructed that forms a common factor of the system. Coefficients of this common factor are orthogonal to the speed of adjustment (error-correction) vector and under certain conditions can be normalized to represent the percentage share of each market's contribution to the total price discovery process.

The IS approach defines price discovery as the variance of the innovations to the common factor and measures each market's contribution to this variance as its information share. This approach uses the vector moving average (VMA) representation, however it can be shown that in the bivariate case the final IS measure can be derived directly from the VECM.

In several studies it is argued that the above measures may provide incorrect information about price discovery if the markets under consideration differ in their speed (responce to changes) and noise level (how effectively these changes are recognised). To account for this, [Yan and Zivot, 2010] have proposed an information leadership (IL) measure that combines elements of the PT and IS approaches, that was later developed into the information leadership share (ILS) by **??**.

This paper is organized as follows: in Section 2 the notion of price discovery and its different metrics are analysed. Section 3 describes major differences between agricultural commodities and financial assets and how these impact the price discovery measures. Section 4 contains the results of applying the measures to the pork meat prices in Germany, Netherlands, Belgium and France in 1987-2015. All relevant tables and technical calculations are carried out in Appendix 5.

2 The subject of price discovery and its measures

2.1 Definition of price discovery

For market participants it is important to understand where price information is being produced. Suppose we have a set of informationally-linked markets trading a homogeneous commodity at prices that are compatible with the Law of One Price (LOP).

If upon arrival of new information each of the markets sets a different new price, subsequent arbitrage will force the prices to return to values that are compatible with the LOP. Some market (or markets) will play a leading role in this process, i.e. vary their price only a little bit and thus dominate price discovery, whereas remaining markets will be mostly adjusting to the new price level.

Let P_t^* denote an unobservable permanent price that reflects the *fundamental value* of a commodity. The observable market price P_t is distinct from it and can be decomposed into two components

$$P_t = P_t^* + \varepsilon_t, \tag{1}$$

where ε_t stands for various transitory effects, market noise etc. Price discovery can be viewed as the movement of a market towards the new level of P_t^*.

There is no unambiguous definition of price discovery. For example, the above definition is consistent with Harris' view of price discovery as "the process by which security markets attempt to identify permanent changes in equilibrium transaction prices" ([Harris et al., 2002, p.2]), understanding it in broad terms as prices reacting to new information.

At the same time, Hasbrouck in [Hasbrouck, 1995] defines price discovery as " "who moves first" in the process of price adjustment", focusing attention on the *speed* component of the process. Some other studies ([Cao et al., 2009]) interpret price discovery by how informative prices are in depicting the true permanent value.

In a speciall issue of *Journal of Financial Markets* price discovery was defined as "efficient and timely incorporation of the information ... into market prices", highlighting both *speed* and *efficiency* characteristing of the process. There are other studies that accept this definition, eg. [Yan and Zivot, 2010] and [Putnins, 2013].

2.2 Measures of price discovery

The way researches define price discovery plays a crucial role in how they measure it.

Historically, one of the first price discovery models belongs to [Garbade and Silber, 1983] where futures and cash prices of financial instruments were analyzed in order to find out which one plays a dominant role. However, since the present study focuses on a spatial price discovery across markets (same or similar commodity traded across countries, regions of the same country etc.) we outline those price discovery measures that are applicable to this case.

There are two classical price discovery metrics:

1. Information shares in [Hasbrouck, 1995], hereafter referred to as IS, and
2. Permanent-transitory decomposition in [Gonzalo and Granger, 1995], referred to as PT,

that are explained in greater detail below.

Consider two $I(1)$ time series $X_{1,t}$ and $X_{2,t} := X_t \in \mathbb{R}_{2\times T}$ that are cointegrated with vector $\beta = (\beta_1, \beta_2)$, meaning that $Z_t = \beta_1 X_{1,t} + \beta_2 X_{2,t}$ is stationary, $I(0)$.

[Stock and Watson, 1988] show that if the series are cointegrated, there must be a common factor representation of the form

$$\begin{pmatrix} X_{1,t} \\ X_{2,t} \end{pmatrix} = \begin{pmatrix} -\beta_2 \\ \beta_1 \end{pmatrix} f_t + \begin{pmatrix} \tilde{X}_{1,t} \\ \tilde{X}_{2,t} \end{pmatrix}, \tag{2}$$

where f_t (common factor) depicts the long-run dynamics of the system. The above equation allows to decompose the time series into a permanent component f_t and a cyclical or transitory component $(\tilde{X}_1, \tilde{X}_2)'$.

Estimation of f_t is based on two conditions:

1. f_t is a linear combination of $(X_{1,t}, X_{2,t})$,

2. The transitory component $(\tilde{X}_{1,t}, \tilde{X}_{2,t})'$ has no permament effect on $(X_{1,t}, X_{2,t})'$, s.t. f_t solely represents long-run behavior of the variables.

The starting point for computation of f_t is the vector error-correction model (VECM) of the form

$$\Delta X_t = \alpha\beta' X_{t-1} + \sum_{j=1}^{k} A_j \Delta X_{t-j} + e_t, \quad t = 1, \dots T \tag{3}$$

where $\beta \in \mathbb{R}_{2\times 1}$ is a cointegrating vector, $\alpha \in \mathbb{R}_{2\times 1}$ is an error-correction vector and $e_t = (e_{1,t}, e_{2,t})' \in \mathbb{R}_{2\times T}$ are innovations with zero mean and covariance matrix Ω

$$\Omega = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}. \tag{4}$$

It can be shown (cf. [Gonzalo and Granger, 1995, Proposition 2]) that the only linear combination of $(X_{1,t}, X_{2,t})$ satisfying the above conditions is

$$f_t = \gamma X_t, \tag{5}$$

where $\gamma = (\gamma_1, \gamma_2)$ is a vector orthogonal to error-correction vector $\alpha = (\alpha_1, \alpha_2)$.

Since γ represents the weights of both prices in the common factor, it can also be viewed as the price discovery metric, meaning that the price that moves closer to the common factor must be dominating the process of price discovery. The price that shows greater adjustment to the common factor is the one following the leader.

Estimation of γ in the bivariate case is trivial, and by imposing additional normalisation conditionof $\gamma_1 + \gamma_2 = 1$ we get that

$$PT_1 = \gamma_1 = \frac{\alpha_2}{\alpha_2 - \alpha_1}, \quad PT_2 = \gamma_2 = \frac{\alpha_1}{\alpha_1 - \alpha_2}. \tag{6}$$

The fact that the common factor weights γ is orthogonal to α is intuitive, cf. for example [Gonzalo and Granger, 1995] for a consumption-GNP example. According to the model, income shows adjustment to the error-correction term, but consumption doesn't, i.e. $\alpha = (0,1)'$ and $\alpha_\perp = (1,0)'$. The whole weight in the common factor goes to consumption, and more "dependent" income that shows 100% adjustment does not play any role in price discovery.

Unlike the above, Hasbrouck uses the vector moving average (VMA) representation of the equation (3)

$$\Delta X_t = \Psi(L)e_t, \tag{7}$$

where $\Psi(L)$ is a matrix polynomial in the lag operator L. Metric is derived with the integrated form of VMA, given as

$$X_t = \Psi(1)\sum_{s=1}^{t} e_s + \Psi^*(L)e_t. \tag{8}$$

Here the matrix $\Psi(1)e_t \in \mathbb{R}_{2\times 2}$ represents permanent impact of the market innovations on both prices.

Hasbrouck's model was developed for the financial markets where the following assumptions are usually made:

1. Cointegrating vector $\beta = (1,-1)$, and

2. The effect of innovation is the same for both prices, hence the rows of the matrix $\Psi(1)$ are identical, i.e.
$$\Psi(1) = \begin{pmatrix} \psi_1 & \psi_2 \\ \psi_1 & \psi_2 \end{pmatrix} = \begin{pmatrix} \psi \\ \psi \end{pmatrix}. \tag{9}$$

Both models (3) and (8) are connected through the following relation (cf. [Johansen, 1991, Theorem 4.1]):

$$\Psi(1) = \beta_\perp \Pi \alpha_\perp^{'}, \tag{10}$$

where
$$\Pi = \left(\alpha_\perp^{'}\left(I - \sum_{j=1}^{k} A_j\right)\beta_\perp\right)^{-1}. \tag{11}$$

It is easy to see that in the bivariate case matrix Π is a scalar. Furthermore, from the assumption of $\beta = (1, -1)$ it follows that $\beta_\perp = (1, 1)$. Then from (9) and (10)

$$\Psi(1) = \Pi \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_1 & \gamma_2 \end{pmatrix}, \tag{12}$$

and

$$\frac{\psi_1}{\psi_2} = \frac{\gamma_1}{\gamma_2}. \tag{13}$$

IS price discovery measure is then defined as a share of the total variance of innovations, given by $var(\psi e_t)$, that is explained by each of the prices. Since the matrix $\Psi(1)$ has identical rows, the total variance

$$var(\psi e_t) = var\left(\psi_1 e_{1,t} + \psi_2 e_{2,t}\right) = \psi_1^2\sigma_1^2 + 2\psi_1\psi_2\rho\sigma_1\sigma_2 + \psi_2^2\sigma_2^2.$$

If the innovations across markets are uncorrelated, i.e. $\rho = 0$, then $var(\psi e_t) = \psi_1^2\sigma_1^2 + \psi_2^2\sigma_2^2$, and the information share of market i becomes

$$IS_i = \frac{\left(\psi_i^2\sigma_i^2\right)^2}{\psi_1^2\sigma_1^2 + \psi_2^2\sigma_2^2} = \frac{\left(\gamma_i^2\sigma_i^2\right)^2}{\gamma_1^2\sigma_1^2 + \gamma_2^2\sigma_2^2}, \quad i = 1, 2, \tag{14}$$

meaning that IS measure can also be calculated by using solely the VECM representation in (3).

It schould be noted that the attribution of information in Cholesky decomposition depends on the ordering of the variables preliminary to the decomposition (i.e. whether X_1 or X_2 is considered to be "first" or "second" market price). Each ordering yields a different information share, so it is common to compute IS measure for both orderings and then take a mean value as a final estimate.

The same holds for the case of correlated innovations, i.e. $\rho \neq 0$. To eliminate contemporaneous correlation, Hasbrouck uses the Cholesky decomposition of matrix

$\Omega = MM'$, where M is a lower triangular matrix

$$M = \begin{pmatrix} m_{11} & 0 \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sigma_2\sqrt{1-\rho^2} \end{pmatrix}. \tag{15}$$

In this case,

$$IS_i = \frac{([\psi M]_i)^2}{\psi\Omega\psi'}, \tag{16}$$

or more specifically, for a bivariate case,

$$IS_1 = \frac{(\gamma_1 m_{11} + \gamma_2 m_{21})^2}{\gamma_1^2\sigma_1^2 + 2\gamma_1\gamma_2\rho\sigma_1\sigma_2 + \gamma_2^2\sigma_2^2}, \quad IS_2 = \frac{(\gamma_2 m_{22})^2}{\gamma_1^2\sigma_1^2 + 2\gamma_1\gamma_2\rho\sigma_1\sigma_2 + \gamma_2^2\sigma_2^2}. \tag{17}$$

As noted above, different price discovery measure comes from different defintions of this process. Some studies (cf. [Yan and Zivot, 2010]) argue that in a number of cases PT and IS measure only capture the speed or efficiency component of price discovery and thus do not measure it precisely. As an alternative, the information leadership shares (ILS) metric was proposed by [Putnins, 2013]:

$$ILS_1 = \frac{IL_1}{IL_1 + IL_2}, \quad ILS_2 = \frac{IL_2}{IL_1 + IL_2}, \tag{18}$$

where

$$IL_1 = \left|\frac{IS_1 CS_2}{IS_2 CS_1}\right|, \quad IL_2 = \left|\frac{IS_2 CS_1}{IS_1 CS_2}\right|. \tag{19}$$

3 Special characteristics of agricultural commodities

Price discovery was historically viewed as one of the major functions of financial markets[1], and the vast majority of existing literature is devoted to analysing financial data (stock exchange tradings), with few exceptions such as GDP and consumption (cf. [Gonzalo and Granger, 1995]).

3.1 Lower data frequency

Agricultural commodities differ from financial instruments in a number of ways, most importantly in being less homogeneous. Also, the observations are often weekly (or monthly) aggregated data unlike highly frequent data for financial assets. For example, studies calculating price discovery measures on stock exchanges often use 1-second intervals within a trading day, which generates 21,600 observations. Given weekly data, the same number of observation require roughly 415 years.

Since the number of observations n and its frequency are considerably lower than in the existing empirical studies, applying the same model (3) or (8) to the whole period may lead to false or incorrect results, simply because the underlying economic situation might have changed during this time. This may also be important for the stationarity analysis, as shown below.

In the present study we analyse weekly pork prices in four European countires: Germany, Netherlands, Belgium and France for the period 1987 to 2015. The total amount of observations is 1439.

Technically, cointegration technique and VECM are based on the assumption that the time series in question are integrated of order one, $I(1)$, cf. Section 2.2. To test this assumption, we apply the following unit root tests:

1. Augmented Dickey-Fuller (ADF) test,
2. Phillips-Perron (PP) test,

[1] See [Figuerola-Ferreti and Gonzalo, 2010]

3. Elliot-Rothenberg-Stock (ERS) test,

as well as the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) stationarity test. Unit root tests and stationarity tests differ in their null and alternative hypotheses. In the unit root tests, H_1, an alternative, is that time series is stationary, whereas in stationariy tests $I(0)$ is a null hypothesis H_0.

The testing results are given in Tables 10 to 13 in Appendix 5. Here, different tests provide different results. ADF and PP tests are often critisized for having a low power if the process is stationary but with a root close to the non-stationary boundary (cf. [DeJong et al., 1992]). This drawback, however, should be corrected in the ERS and KPSS tests, and since those two also show contradictory results, we do not have enough evidence to assume that time series are integrated $I(1)$.

This ambiguity has been noted in numerous other studies (e.g. [Hjalmarsson and ́Osterholm, 2007]). There is little theoretical reason to expect a strict unit root in economic time series, and our full pork price series is perhaps long enough to reveal that it should be considered a near-integrated process. When we divide our series into three sub-samples of 480 (479) observations each, unit root tests uniformly indicate that all prices in all sub-samples are $I(1)$. In the following we proceed with the separate analysis of these three sub-sample periods.

3.2 Cointegrating relation

When introducing the IS measure in Section 2.2 it was mentioned that one of the assumptions for the model is that the cointegrating vector $\beta = (1, -1)$, meaning that in the long rung equlibrium prices on two markets are identical. However, this does not necessarily hold for less homogeneous agricultural commodities.

Consider $\beta = (\beta_1, \beta_2)$ with $\frac{\beta_1}{\beta_2} \neq -1$. Further, let $\beta_\perp = (\beta_{1\perp}, \beta_{2\perp})$ be a vector orthogonal to β. In this case, Π in (10) would still be a scalar.

Now consider

$$\Psi(1) = \begin{pmatrix} \psi_{11} & \psi_{12} \\ \psi 21 & \psi_{22} \end{pmatrix} = \Pi \begin{pmatrix} \beta_{1\perp}\gamma_1 & \beta_{1\perp}\gamma_2 \\ \beta_{2\perp}\gamma_1 & \beta_{2\perp}\gamma_2 \end{pmatrix}. \tag{20}$$

In order for the rows of this matrix to be identical (as in financial market case) it must hold that

$$\begin{cases} \beta_{1\perp}\gamma_1 = \beta_{2\perp}\gamma_1 \\ \beta_{1\perp}\gamma_2 = \beta_{2\perp}\gamma_2 \end{cases} \Leftrightarrow \quad \beta_{1\perp} = \beta_{2\perp},$$

meaning that for the cointegrating vector $\beta = (\beta_1, \beta_2)$

$$\beta_1 + \beta_2 = 0,$$

and $\beta \propto (1, -1)$, which contradicts with the assumtion above. Hence, matrix $\Psi(1)$does not have identical rows and the relation (13) does not hold. Therefore, VECM (3) does not contain all the information required for the computation of the IS measure, s.t. $\Psi(1)$ must be estimated from (12).

However, in a bivariate case it can still be shown that for cointegrating vector $\beta \neq (1, -1)$ IS measure can still be computed by (17) (cf. Appendix 5 for detailed proof).

3.3 Error-correction relation

PT measure (6) only depends on the error-correction vector, or, more precisely, the vector orthogonal to it.

In order to be interpretable, the measures must be bounded by $[0, 1]$ interval, which is only the case if coefficients of the vector $\alpha = (\alpha_1, \alpha_2)$ are of different signs. This restriction is usually satisfied for financial models (cf. [Campbell and Hendry,]) but is not always the case for agricultural commodities.

The reasons for α_1 and α_2 in $\alpha = (\alpha_1, \alpha_2)$ being of different sign may include:

1. Small number of observations in agricultural compated to financial one.
2. Structural break, meaning that there are different cointegrating/error-correction relations before and after a break point.
3. Bias connected with the analysis of the complex multivariate system by means of the simplified bivariate model.

In the present work we calculate three price discovery measures: PT, IS, ILS, for agricultural commodities, using (6), (17) and (18), respectively.

4 Results

Below we demonstrate application results of the price discovery measures to weakly pork meat prices in four European countries: Germany, Natherlands, Belgium and France in 1987-2015. The data set was divided into three subsets containing 480 (479) observations, as mentioned in Section 3.1:

1. Period 1: March 1987 - July 1996,

2. Period 2: July 1996 - January 2006,

3. Period 3: January 2006 - March 2015.

PT price discovery measures calculated for all three periods are given in Table 1 to Table 3, representing the pairwise relations between countries. Apart from the Germany-Netherlands cell in the first table, all PT values are interpretable. It can also be assumed that Germany is 100% dominating Netherlands in this case, as it is done in many empirical studies (cf. [Putnins, 2013]).

The causality relations in all three periods differ, for example, in the first period Germany dominates all other countries and Belgium always is the follower. However, in the second period this relation changes and Belgium becomes a dominant market to Germany. The third period demonstates a shift in the dominating role from Belgium to France.

Tables 4 - 6 depict IS price discovery measure computed for the same countries, pairwise. In this case, all the values are interpretable, i.e. in the $[0, 1]$ range, and support causality. Here, France is always in the follower role, dominated by Belgium, then Netherlands and finally, Germany is the leading market in all three periods.

Finally, the ILS measure is computed for the data, cf. Tables 7-9. Again, the the results are transitive and interpretable, however, causality scheme is different, eg. in the first period Germany is dominated by all three markets.

	Germany	Netherlands	Belgium	France
Germany		1.07 Germany -0.07 Netherlands	0.89 Germany 0.11 Belgium	0.91 Germany 0.09 France
Netherlands			0.54 Netherlands 0.46 Belgium	0.53 Netherlands 0.47 France
Belgium				0.43 Belgium 0.57 France
France				

Table 1: PT measure for period 1, 1987-August 1996

	Germany	Netherlands	Belgium	France
Germany		0.51 Germany 0.49 Netherlands	0.48 Germany 0.52 Belgium	0.80 Germany 0.20 France
Netherlands			0.23 Netherlands 0.77 Belgium	0.62 Netherlands 0.38 France
Belgium				0.79 Belgium 0.21 France
France				

Table 2: PT measure for period 2, August 1996-January 2006

	Germany	Netherlands	Belgium	France
Germany		0.59 Germany 0.41 Netherlands	0.29 Germany 0.71 Belgium	0.50 Germany 0.50 France
Netherlands			0.19 Netherlands 0.81 Belgium	0.38 Netherlands 0.62 France
Belgium				0.30 Belgium 0.70 France
France				

Table 3: PT measure for period 3, January 2006 - March 2015

	Germany	Netherlands	Belgium	France
Germany		0.65 Germany 0.35 Netherlands	0.58 Germany 0.42 Belgium	0.60 Germany 0.40 France
Netherlands			0.70 Netherlands 0.30 Belgium	0.80 Netherlands 0.20 France
Belgium				0.70 Belgium 0.30 France
France				

Table 4: IS measure for period 1, 1987-August 1996

	Germany	Netherlands	Belgium	France
Germany		0.85 Germany 0.15 Netherlands	0.83 Germany 0.17 Belgium	0.60 Germany 0.40 France
Netherlands			0.80 Netherlands 0.20 Belgium	0.60 Netherlands 0.40 France
Belgium				0.62 Belgium 0.38 France
France				

Table 5: IS measure for period 2, August 1996-January 2006

	Germany	Netherlands	Belgium	France
Germany		0.82 Germany 0.18 Netherlands	0.73 Germany 0.27 Belgium	0.60 Germany 0.40 France
Netherlands			0.88 Netherlands 0.12 Belgium	0.70 Netherlands 0.30 France
Belgium				0.68 Belgium 0.32 France
France				

Table 6: IS measure for period 3, January 2006 - March 2015

	Germany	Netherlands	Belgium	France
Germany		0.01 Germany 0.99 Netherlands	0.17 Germany 0.83 Belgium	0.02 Germany 0.98 France
Netherlands			0.80 Netherlands 0.20 Belgium	0.92 Netherlands 0.08 France
Belgium				0.91 Belgium 0.09 France
France				

Table 7: ILS measure for period 1,1987-August 1996

	Germany	Netherlands	Belgium	France
Germany		0.97 Germany 0.03 Netherlands	0.96 Germany 0.04 Belgium	0.13 Germany 0.87 France
Netherlands			0.99 Netherlands 0.01 Belgium	0.45 Netherlands 0.55 France
Belgium				0.16 Belgium 0.84 France
France				

Table 8: ILS measure for period 2, August 1996-January 2006

	Germany	Netherlands	Belgium	France
Germany		0.91 Germany 0.09 Netherlands	0.98 Germany 0.02 Belgium	0.70 Germany 0.30 France
Netherlands			0.99 Netherlands 0.01 Belgium	0.93 Netherlands 0.07 France
Belgium				0.96 Belgium 0.04 France
France				

Table 9: ILS measure for period 3, January 2006-March 2015

References

[Campbell and Hendry,] Campbell, B. and Hendry, S. Price discovery in canadian and u.s. 10-year government bond markets. Working Paper 2007-43, Bank of Canada, August 2007.

[Cao et al., 2009] Cao, C., Hansch, O., and Wang, X. (2009). The information content of an open limit-order book. *Journal of Futures Markets*, (29):16–41.

[DeJong et al., 1992] DeJong, D., Nankervis, J., Savin, N., and Whiteman, C. (1992). The power problems of unit root test in time series with autoregressive errors. *Journal of Econometrics*, 53:323–343.

[Figuerola-Ferreti and Gonzalo, 2010] Figuerola-Ferreti, I. and Gonzalo, J. (2010). Modelling and measuring price discovery in commodity markets. *Journal of Econometrics*, 158:95–107.

[Garbade and Silber, 1983] Garbade, K. and Silber, W. (1983). Price movements and price discovery in futures and cash markets. *The Review of Economics and Statistics*, 65:289–297.

[Gonzalo and Granger, 1995] Gonzalo, J. and Granger, C. (1995). Estimation of common long-memory components in cointegrated systems. *Journal of Business and Economic Statistics*, 13:27–36.

[Harris et al., 2002] Harris, F., McInish, T., and Wood, R. (2002). The dynamics price adjustment across exchanges: an investigation of common factor components for dow stocks.

[Hasbrouck, 1995] Hasbrouck, J. (1995). One security, many markets: Determining the contributions to price discovery. *Journal of Finance*, 50:1175–1199.

[Hjalmarsson and ´Osterholm, 2007] Hjalmarsson, E. and ´Osterholm, P. (2007). Testing for cointegration using the johansen methodology when variables are near-integrated. *IMF Working Paper*.

[Johansen, 1991] Johansen, S. (1991). Estimation and hypothesis testing of cointegration vectors in gaussian vector autoregressive models. *Econometrica*, 59:1551–1580.

[Putnins, 2013] Putnins, T. (2013). What do price discovery metrics really measure? *Journal of Empirical Finance*, 23:68–83.

[Stock and Watson, 1988] Stock, J. and Watson, M. (1988). Testing for common trends. *Journal of the American Statistical Association*, 831:1097–1107.

[Yan and Zivot, 2010] Yan, B. and Zivot, E. (2010). A structural analysis of price discovery measures. *Journal of Financial Markets*, 13:1–19.

5 Appendix

5.1 Appendix A

Let $(X_{1,t}, X_{2,t}) \coloneqq X_t \in \mathbb{R}_{2\times T}$ $I(1)$ time series cointegrated with $\beta = (\beta_1, \beta_2)$, s.t. that $Z_t = \beta_1 X_{1,t} + \beta_2 X_{2,t} \sim I(0)$. Further, let $\beta \neq (1, -1)$.

The IS price discovery measure can still be computed with (17).

Proof:

According to (8), matrix $\Psi(1)e_t \in \mathbb{R}_{2\times 2}$ represents long-run impact of the market innovations impounded in price i, where $i = 1, 2$.

$$\Psi(1)e_t = \begin{pmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{pmatrix} \begin{pmatrix} e_{1,t} \\ e_{2,t}. \end{pmatrix}$$

Further,

$$var\left(\psi_{11}e_{1,t} + \psi_{12}e_{2,t}\right) = \psi_{11}^2\sigma_1^2 + 2\psi_{11}\psi_{12}\rho\sigma_1\sigma_2 + \psi_{12}^2\sigma_2^2,$$
$$var\left(\psi_{21}e_{1,t} + \psi_{22}e_{2,t}\right) = \psi_{21}^2\sigma_1^2 + 2\psi_{21}\psi_{22}\rho\sigma_1\sigma_2 + \psi_{22}^2\sigma_2^2$$

are total innovation variances for each of the markets. The patrial impact of an innovation in price j on price i $(i, j = 1, 2)$ is therefore given by $\phi_{ij}e_{j,t}$.

In case of uncorrelated innovations, i.e. $\rho = 0$,

$$IS_{ij} = \frac{\left(\psi_{ij}^2\sigma_j^2\right)^2}{\psi_{i1}^2\sigma_1^2 + \psi_{i2}^2\sigma_2^2} \tag{21}$$

is IS price discovery metric of price j with respect to price i, showing the relative effect of market j on price on market i. Normalisation assures that the effects for each market sum to one, allowing for interpretation.

Analogously, if the innovations are correlated,

$$IS_{ij} = \frac{\left([\Psi M]_{ij}\right)^2}{\left(\Psi\Omega\Psi'\right)_{ii}}, \tag{22}$$

where M is a lower triangular matrix from the Cholesky decomposition of the matrix Ω, cf. (15).

Since

$$\Psi M = \begin{pmatrix} \psi_{11}m_{11} + \psi_{12}m_{21} & \psi_{12}m_{22} \\ \psi_{21}m_{11} + \psi_{22}m_{21} & \psi_{22}m_{22} \end{pmatrix},$$

we get that the pair of IS metrics for the first market is

$$IS_{11} = \frac{(\psi_{11}m_{11} + \psi_{12}m_{21})^2}{\psi_{11}^2\sigma_1^2 + 2\psi_{11}\psi_{12}\rho\sigma_1\sigma_2 + \psi_{12}^2\sigma_2^2}, \tag{23}$$

$$IS_{12} = \frac{(\psi_{12}m_{22})^2}{\psi_{11}^2\sigma_1^2 + 2\psi_{11}\psi_{12}\rho\sigma_1\sigma_2 + \psi_{12}^2\sigma_2^2}, \tag{24}$$

and the second pair, representing how price on the second market is influenced by the its own innovation and those from the first market,

$$IS_{21} = \frac{(\psi_{21}m_{11} + \psi_{22}m_{21})^2}{\psi_{21}^2\sigma_1^2 + 2\psi_{21}\psi_{22}\rho\sigma_1\sigma_2 + \psi_{22}^2\sigma_2^2}, \tag{25}$$

$$IS_{22} = \frac{(\psi_{22}m_{22})^2}{\psi_{21}^2\sigma_1^2 + 2\psi_{21}\psi_{22}\rho\sigma_1\sigma_2 + \psi_{22}^2\sigma_2^2}. \tag{26}$$

However, in a bivariate case (20) holds, where the rows of $\Psi(1)$ have a common coefficient $\Pi\beta_{i\perp}$, $i = 1, 2$. More specifically,

$$\psi_{11} = \Pi\beta_{1\perp}\gamma_1, \quad \psi_{12} = \Pi\beta_{1\perp}\gamma_2$$

for the first row of matrix $\Psi(1)$ and

$$\psi_{21} = \Pi\beta_{2\perp}\gamma_1, \quad \psi_{22} = \Pi\beta_{2\perp}\gamma_2$$

fot the second row.

By subsituting this into (23)-(26), the common coefficient will be eliminated, and hence

$$IS_{11} = IS_{21} \qquad \text{and} \qquad IS_{12} = IS_{22},$$

meaning that information share of price j on market i and j are identical. Therefore, relation (17) can be used for the computation of the measure in this case.

5.2 Appendix B

Test	Details (in R)	Test statistic	Decision	Result
ADF Test	Model with "drift", BIC	-4.5871	Reject H_0, accept H_1	Stationary, $I(0)$
PP Test	Z-alpha, short lags	-40.9579	Reject H_0, accept H_1	Stationary, $I(0)$
ERS Test	Model with "constant"	-4.7729	Reject H_0, accept H_1	Stationary,$I(0)$
KPSS Test	Model "mu", short lags	0.6843	Reject H_0, accept H_1	Unit root, $I(1)$

Table 10: Testing for unit roots, Germany, full data set

Test	Details (in R)	Test statistic	Decision	Result
ADF Test	Model with "drift", BIC	-5.0456	Reject H_0, accept H_1	Stationary, $I(0)$
PP Test	Z-alpha, short lags	-45.7075	Reject H_0, accept H_1	Stationary, $I(0)$
ERS Test	Model with "constant"	-3.7165	Reject H_0, accept H_1	Stationary,$I(0)$
KPSS Test	Model "mu", short lags	0.9415	Reject H_0, accept H_1	Unit root, $I(1)$

Table 11: Testing for unit roots, Netherlands, full data set.

Test	Details (in R)	Test statistic	Decision	Result
ADF Test	Model with "drift", BIC	-4.2295	Reject H_0, accept H_1	Stationary, $I(0)$
PP Test	Z-alpha, short lags	-38.6677	Reject H_0, accept H_1	Stationary, $I(0)$
ERS Test	Model with "constant"	-3.1673	Reject H_0, accept H_1	Stationary,$I(0)$
KPSS Test	Model "mu", short lags	2.9266	Reject H_0, accept H_1	Unit root, $I(1)$

Table 12: Testing for unit roots, Belgium, full data set.

Test	Details (in R)	Test statistic	Decision	Result
ADF Test	Model with "drift", BIC	-5.0491	Reject H_0, accept H_1	Stationary, $I(0)$
PP Test	Z-alpha, short lags	-40.4179	Reject H_0, accept H_1	Stationary, $I(0)$
ERS Test	Model with "constant"	-3.8854	Reject H_0, accept H_1	Stationary,$I(0)$
KPSS Test	Model "mu", short lags	1.7152	Reject H_0, accept H_1	Unit root, $I(1)$

Table 13: Testing for unit roots, France, full data set.

	Test	Test statistic	Decision	Result
Period 1	ADF	-2.0825	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.5326	Cannot reject H_0	Unit root, $I(1)$
	ERS	-2.3705	Under 5% significance reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	0.6499	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$
Period 2	ADF	-2.8024	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.2436	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.0662	Cannot reject H_0	Unit root, $I(1)$
	KPSS	0.5114	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$
Period 3	ADF	-2.8169	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.0935	Cannot reject H_0	Unit root, $I(1)$
	ERS	-2.1564	Under 5% significance reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	1.752	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$

Table 14: Testing for unit roots, Germany, three periods.

	Test	Test statistic	Decision	Result
Period 1	ADF	-2.5352	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.2109	Cannot reject H_0	Unit root, $I(1)$
	ERS	-2.0988	Under 1% significance reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	1.7979	Reject H_0, accept H_1	Unit root, $I(1)$
Period 2	ADF	-3.4966	Reject H_0, accept H_1	Stationary, $I(0)$
	PP	p-value = 0.0824	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.497	Cannot reject H_0	Unit root, $I(1)$
	KPSS	0.4802	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$
Period 3	ADF	-3.0038	Reject H_0, accept H_1	Stationary, $I(0)$
	PP	p-value = 0.1155	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.8558	Under 5% significance cannot reject H_0	Unit root, $I(1)$
	KPSS	1.5617	Reject H_0, accept H_1	Unit root, $I(1)$

Table 15: Testing for unit roots, Netherlands, three periods.

	Test	Test statistic	Decision	Result
Period 1	ADF	-2.4956	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.2589	Cannot reject H_0	Unit root, $I(1)$
	ERS	-2.3059	Under 5% significance reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	1.6635	Reject H_0, accept H_1	Unit root, $I(1)$
Period 2	ADF	-2.9294	Reject H_0, accept H_1	Stationary, $I(0)$
	PP	p-value = 0.32	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.1315	Cannot reject H_0	Unit root, $I(1)$
	KPSS	0.635	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$
Period 3	ADF	-2.8624	Under 5% significance cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.1703	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.8636	Under 5% significance cannot reject H_0	Unit root, $I(1)$
	KPSS	1.2754	Reject H_0, accept H_1	Unit root, $I(1)$

Table 16: Testing for unit roots, Belgium, three periods.

	Test	Test statistic	Decision	Result
Period 1	ADF	-2.1794	Cannot reject H_0	Unit root, $I(1)$
	PP	p-value = 0.5638	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.9765	Under 5% significance reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	1.9458	Reject H_0, accept H_1	Unit root, $I(1)$
Period 2	ADF	-3.4149	Under 1% significance reject H_0, accept H_1	Stationary, $I(0)$
	PP	p-value = 0.0767	Cannot reject H_0	Unit root, $I(1)$
	ERS	-1.2051	Cannot reject H_0	Unit root, $I(1)$
	KPSS	0.5989	Under 5% significance reject H_0, accept H_1	Unit root, $I(1)$
Period 3	ADF	-3.4054	Under 1% significance reject H_0, accept H_1	Stationary, $I(0)$
	PP	p-value = 0.089	Cannot reject H_0	Unit root, $I(1)$
	ERS	-2.8658	Reject H_0, accept H_1	Stationary, $I(0)$
	KPSS	2.1757	Reject H_0, accept H_1	Unit root, $I(1)$

Table 17: Testing for unit roots, France, three periods.

	critical values		
Test	**1%**	**5%**	**10%**
ADF	-3.43	-2.86	-2.57
ERS	-2.57	-1.94	-1.62
KPSS	0.739	0.463	0.347

Table 18: Critical values for ADF, ERS and KPSS tests.