Buy Locally?
Big-Box Stores and Time-Inconsistent Preferences

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Abstract

The widespread closure of Mom-and-Pop shops, or small, locally owned stores, in the United States and elsewhere is a well-recognized phenomenon. These closures are generally met with dissatisfaction from the local community. These stores, however, often close due to a lack of patronage on the part of the same consumers who are upset by their closure. This behavior and subsequent disappointment are puzzling. I show that this phenomenon can be explained using a hyperbolic discounting, or time-inconsistency, framework. Using a theoretical model, I show that time-inconsistent consumers are less likely to patronize local businesses than time-consistent consumers. The sophistication effect, however, makes consumers more likely to shop at the local store than if they were naïve. Further, I show that local stores are more likely to close in a community with a larger proportion of naïve time-inconsistent consumers and that these closures cause a loss in welfare.

JEL Classification Codes: D03, D11, R22
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1 Introduction

Between 1998 and 2011 there was an 8% decline in the number of retail businesses with fewer than 10 employees in the United States (US Census Bureau 2014). Many consumers blame the introduction of big businesses, or "big-box stores," as the largest culprit of the closure of small, locally-owned businesses (Brooklyn Bureau 2014, AlterNet 2014). Wal-Mart and Target, in particular, have been blamed for the closure of local grocery stores, Starbucks for the closure of local coffee shops, and Barnes & Noble and Amazon.com for the closure of local bookstores.

Consumers often bemoan the closing of local Mom-and-Pop shops. This dissatisfaction has prompted "buy local" campaigns in many cities across the US. For example, the "Keep Austin Weird" campaign was launched to encourage Austin, Texas residents to support local businesses. These campaigns support not only shopping at local businesses, but also buying local foods and other types of local goods (PennLive 2014).

However, local shops often close due to a lack of patronage by the same consumers who are distressed by their closures. At first glance, this behavior is entirely irrational for forward-looking consumers. I explain the phenomenon via a "time-inconsistency" (or hyperbolic discounting) framework from the behavioral economics literature. The contribution of this paper is to suggest a testable hypothesis to explain the closure of local businesses and the subsequent disappointment.

Using a theoretical model I show that (1) time-inconsistent consumers are less likely to patronize local businesses than time-consistent consumers, (2) local stores are more likely to close in a community with a larger proportion of naive time-inconsistent consumers, (3) there is a loss in welfare due to the closure of local stores.

Section 2 discusses the literature relevant to addressing this question. Sections 3 and 4 present a theoretical model of consumer behavior. Section 5 discusses the outcome of the consumer behavior shown in Section 4 and shows the implications of the model. Section 6 discusses these implications and draws conclusions.

2 Literature Review

Mom-and-Pop Stores & Big-Box Stores

Big-box stores are able to sell items at lower prices (Hausman & Leibtag 2007), and consumers often favor them for this reason. Small, locally-owned stores close when a big-box store opens in close proximity, if the big-box store sells the same types of items as the local store (Haltiwanger, Jarmin & Krizan 2010, Boyd 2014). When the big-box store sells different types of items, there may be an increase in business for the nearby local store.

The presence of Wal-Mart that results in the closure of local stores depresses social capital stocks in local communities. These externalities represent real costs for communities in the form of reduced economic growth (Oetz & Upasingha 2006). This suggests that while big-box stores may be good for consumers, they could have negative consequences for society. Consumers may not be internalizing the costs associated with shopping at a big-box store in lieu of supporting local businesses. Individuals are both consumers and social beings, and often a sales price doesn't reflect the full price that they pay as workers and citizens (Reich 2005).

Time-Inconsistency

Neoclassical economic theory assumes that a consumer exponentially discounts future utility when solving a multi-period utility maximization problem. That is, a consumer chooses $\{x_t\}_{t=\tau}^{\infty}$ to maximize the present discounted value of her utility at time τ:

$$U_\tau = \max_{\{x_t\}_{t=\tau}^{\infty}} [u(x_\tau) + \sum_{t=\tau+1}^{\infty} \delta^{t-\tau} u(x_t)]$$

where $0 < \delta \leq 1$.

A consumer who is an exponential discounter experiences a constant rate of decline of the value of consumption in future periods. I refer to this consumer as "time-consistent" because her relative preference for consumption in an earlier period compared to a later period is independent of the period in which she is asked about her preferences.

More recent literature in the field of behavioral economics finds evidence that consumers discount the immediate future sharply and discount later periods at an exponential rate (Angeletos, Laibson, Repetto, Tobacman & Weinberg 2001, Shapiro 2005, Soman 2004, Ashraf, Karlan & Yin 2006). This is often referred to as "hyperbolic discounting" or "(β, δ) preferences." I refer to a consumer with this type of preference for consumption over time as "time-inconsistent" because her relative preference for consumption in an earlier period compared to a later period is dependent on *when* she is asked about her preferences.

Phelps & Pollak (1968) proposes an elegant simplification of the hyperbolic discount function to model a consumer's preference for consumption over time. The authors model this intertemporal time preference as the consumer choosing $\{x_t\}_{t=\tau}^{\infty}$, to solve:

$$U_\tau = \max_{\{x_t\}_{t=\tau}^{\infty}} [u(x_\tau) + \beta \sum_{t=\tau+1}^{\infty} \delta^{\tau-t} u(x_t)]$$

where $0 < \delta \leq 1$, and $0 < \beta \leq 1$.

Notice that δ is present in both the time-consistent model and the time-inconsistent model. In both models, δ represents the time-consistent portion of preferences. Notice, however, that β is only present in the time-inconsistent model, and β represents the consumer's preference for the present over the future. If $\beta = 1$, then the consumer is simply an exponential, or time-consistent, consumer. If $\beta < 1$, then the consumer is time-inconsistent.

Figure 1 displays the preference for utility over time for time-consistent and time-inconsistent consumers. The reader can see that the time-inconsistent consumer perceives a steep drop in the value of future period utility. As $\beta \to 0$, the consumer derives less utility from future periods to the point that she only values the current period.

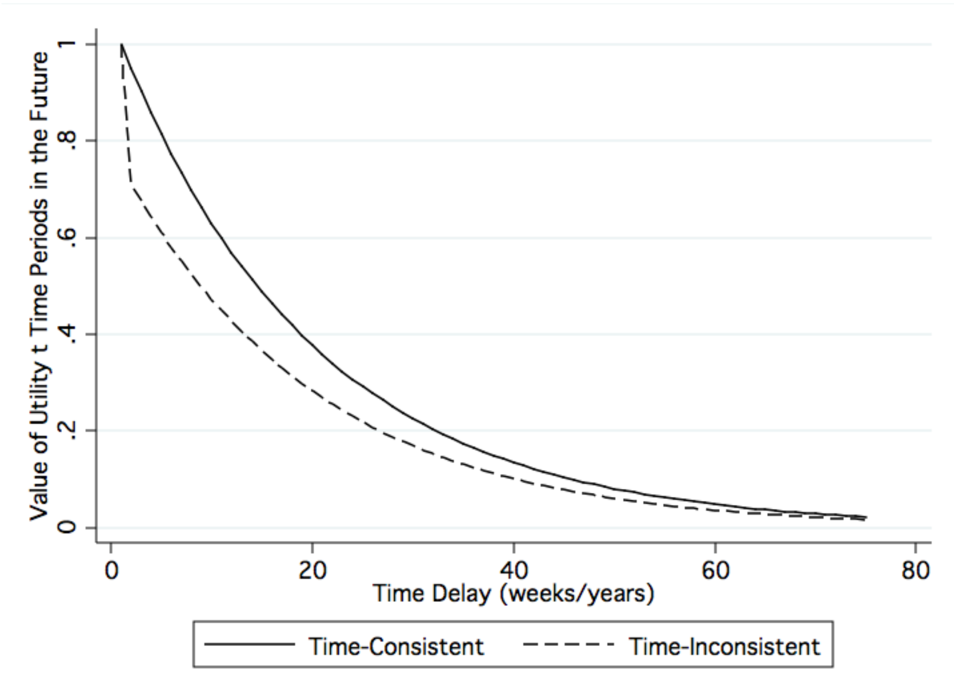

Figure 1: $\delta = 0.95$ and $\beta = 0.7$

O'Donoghue & Rabin (1999) refers to time-inconsistent preference as "present-biased preferences" because a consumer with this type of preference for consumption over time gives an even higher relative weight to an earlier period as the earlier period becomes closer. Present bias is often interpreted or manifested as a lack of self-control or impatience. Researchers have identified and measured these self-control problems in situations such as savings and debt (Angeletos et al. 2001, Laibson 1997, Laibson, Repetto, Tobacman, Hall, Gale & Akerlof 1998), food and exercise (DellaVigna & Malmendier 2006, Scharff 2009), and purchasing decisions (Soman 2004, Shapiro 2005). These studies show that the behavior of individuals follows the predictions of a time-inconsistent model much more closely than the predictions of a time-consistent model.

The economics literature has not addressed the closure of local stores and the subsequent disappointment using a time-inconsistency framework, or any other model. This paper attempts to fill this gap.

Sophisticates and Naïves

In both the time-consistent and time-inconsistent models, we can view the consumer as a separate agent in each period. That is, she chooses the choice variable to maximize the present discounted value of utility in period τ, and then her future self chooses again in period $\tau + 1$ to maximize her future self's present discounted value of utility, etc. That is, the consumer can only choose her behavior in the current period and is unable to constrain her choice in future periods. With this in mind, it is important to consider what we think about the consumer's beliefs about the behavior of her future selves.

There are two types of time-inconsistent consumers: "naïves" and "sophisticates" (O'Donoghue & Rabin 1999, Laibson et al. 1998, Angeletos et al. 2001). A naïve time-inconsistent consumer is one who is unaware of her time-inconsistent preferences. She believes that her future self will behave according to her current self's long-run preferences. Conversely, a sophisticated time-inconsistent consumer is fully aware of her present-biased tendencies and that her future self will experience the same present bias, or self-control problem.

Undoubtedly, most consumers have some amount of sophistication and some amount of naïveté. Evidence in favor of sophistication is the existence of commitment devices such as

Alcoholics Anonymous and Weight Watchers. An individual tends to join these groups when she is aware that her future selves' preferences are different than her long-run preferences. Ausubel (1991) includes a story of consumers that immerse their credit cards in trays of water and place them in a freezer in order to commit in advance of temptation to not make impulse purchases. In contrast, there is also evidence of naïveté in that an individual will continually have a lack of will power or give in to impulses that are in opposition to long-term goals. Nevertheless, rather than considering the entire spectrum of sophistication and naïveté possible, it is useful to dichotomize this awareness of future selves' preferences to model resulting behavior.

3 Theoretical Model

For the purposes of this paper, I model a stylized decision-making process of a consumer selecting to purchase groceries at either a local grocery store or a big-box store such as Wal-Mart or Target. Note that this analysis of decision-making could be easily applied to other types of local and big-box retail stores.

Assume that the consumer can choose to purchase food at either the local store or the big-box store. The consumer must buy non-food items at the big-box store. Most local grocery stores sell food only and not other types of goods.

Let q_τ be the quantity of food purchased in period τ, where q_τ is a function of $S_\tau \in \{l_\tau, b_\tau\}$. $S_\tau = l_\tau$ indicates that the consumer shopped at the local store in period τ, and $S_\tau = b_\tau$ indicates that the consumer shopped at the big-box store in period τ. Let g_τ represent the non-food composite good purchased in period τ. Let L_τ represent the existence of the local store in period τ.

Then, single-period utility takes the following form:

$$u_\tau = u(q_\tau(S_\tau), L_\tau, g_\tau). \tag{1}$$

Notice that L_τ enters explicitly into the utility function. This is the utility that the consumer gains from knowing that the local store is in business even if she does not shop at the local store in period τ. This is in line with Lancaster (1975), which shows that consumers have increasing utility in product variety, particularly in a case such as this in which the number of stores is already somewhat limited.

Assumption 1. *The consumer's preferences over consumption and the existence of the local store are the same over time, i.e.,* $u_\tau(\cdot) = u(\cdot)$.

Assumption 2. *Utility is non-decreasing in quantity of food,* q_τ*, and the non-food composite good,* g_τ.

Assumption 3. *The consumer's utility is non-decreasing in the existence of the local store,* $u(L_\tau = 1, \cdot) \geq u(L_\tau = 0, \cdot)$. *This implies that the consumer is not worse off if the store exists, and may have higher utility if the store does exist.*

Next, consider the consumer's budget constraint. Let $p_{l,\tau}$ be the price of food at the local store and $p_{b,\tau}$ be the price of food at the big-box store in period τ. Recall that $l_\tau = 1$ if the consumer shops at the local store in period τ and 0 otherwise. Let a_τ be the price of the non-food composite good, and let y_τ bet the income of the consumer in period τ.

Assumption 4. *There are no credit markets and Walras' Law holds. Walras' Law implies that consumer expends all her wealth. Together these imply that the consumer must spend all of her income in each period.*

Assumption 4 is a simplifying assumption that allows us to isolate the effects of time-inconsistency rather than complicate the model with a consumer's ability to borrow and save.

If Assumption 4 holds, the consumer's single period budget constraint is:

$$l_\tau p_{l,\tau} q_\tau + (1 - l_\tau) p_{b,\tau} q_\tau + a_\tau g_\tau = y_\tau, \tag{2}$$

where $p_{l,\tau}, p_{b,\tau}, a_\tau > 0$, and $q_\tau, g_\tau, y_\tau \geq 0$.

Assumption 5. *Assume that the price paid for food at the local store is always higher than the price paid for food at the big-box store, that is, $p_{l,\tau} > p_{b,\tau}$.*

Assumption 5 is often the case in reality because small local businesses do not benefit from the economies of scale that large chain stores do (Hausman & Leibtag 2007, Jones & Doucet 2000, Barber & Tietje 2004).

Proposition 1. *If Assumptions 4 and 5 hold, then either the quantity of food purchased, q_τ, will always be larger if the consumer shops at the big-box store, i.e., if $l_\tau = 0$.*

Assumption 6. *The existence of the local store is entirely dependent upon a sufficient number of consumers shopping at the store in period $\tau - 1$.*

Assumption 7. *If the store closes in period τ it can never be reopened.*

Proposition 2. *If Proposition 1 is satisfied and Assumptions 2 and 6 hold, single-period utility is higher $\forall\ \tau$ if the consumer shops at the big-box store.*

Proof. Under Assumption 2, utility is non-decreasing in q_τ and g_τ. Under Assumption 6, L_τ is dependent upon $S_{\tau-1}$, not S_τ, which implies that the consumer's behavior in period τ cannot affect whether or not the store is open in period τ. This together with Proposition

1, which states that the consumer can buy more q_τ if she shops at the big-box store, implies that single-period utility is always higher if the consumer shops at the big-box store in the current period. □

When choosing where to shop in period τ, the consumer considers not only her single-period utility, but also her infinite horizion, intertemporal utility function in which she is maximizing the present discounted value of her utility:

$$U_\tau = \max_{\{S_t\}_{t=\tau}^{\infty}} u(q_\tau(S_\tau), L_\tau, g_\tau) + \beta \sum_{t=\tau+1}^{\infty} [\delta^{t-\tau} u(q_t(S_t), L_t, g_t)]$$

$$\text{s.t. } l_t p_{l,t} q_t + (1 - l_t) p_{b,t} q_t + a_t g_t = y_t, \ \forall \ t \geq \tau$$

where $0 < \beta \leq 1$, $0 < \delta \leq 1$, $p_{l,\tau}, p_{b,\tau}, a_\tau > 0$, and $q_\tau, g_\tau, y_\tau \geq 0$.

Recall that in period τ, the consumer can only choose her behavior in the current period and cannot commit to a certain behavior for future periods. That is, she can choose only which store she will shop at in period τ and which store she *intends* to shop at in future periods. Let this be defined as her strategy set $\mathbf{s}_\tau \equiv (s_\tau^\tau, s_\tau^{\tau+1}, s_\tau^{\tau+2}, \ldots)$ where $s_\tau^{\tau+t} \in \{l_\tau, b_\tau\}, \forall \ t \geq 0$. However, the consumer does not have to adhere this strategy in the next period. In period $\tau + 1$, she defines a new strategy set $\mathbf{s}_{\tau+1}$, where $s_{\tau+1}^{\tau+1}$ as defined in $\mathbf{s}_{\tau+1}$ is not necessarily the same as $s_\tau^{\tau+1}$ as defined in $\mathbf{s}_\tau$.

While it is possible that utility is not dependent upon the existence of the local store or that a consumer derives negative utility from the existence of the local store, in this paper I only consider the case in which the consumer derives positive utility from the option to shop at the local store, i.e. I maintain Assumption 3 throughout the paper. It is also possible that a consumer's single-period utility is higher if she shops at the local store in that period. In order to account for this possibility, it would be necessary to allow for the consumer to derive utility from other aspects of the local store such as convenience or ambiance. Throughout this paper, I maintain the assumptions that lead to Proposition 2, i.e., that the consumer's single-period utility is largest if she shops at the big-box store.

Assume that the consumer has subjective probabilities that the local store will remain open. Let μ_l be the subjective probability that the local store will be in business in period $\tau + 1$ if she shops at the local store in period τ, and let μ_b be the consumer's subjective probability that the local store will be open in period $\tau + 1$ if she does *not* shop at the local store in period τ. Let $0 < \mu_l < 1$ and $0 < \mu_b < 1$. Note that even if the consumer shops at the local store in period τ, there is a non-zero probability, $1 - \mu_l$, that the store will be closed in period $\tau + 1$ and the consumer will be forced to shop at the big-box store.

Assumption 8. *Let $\mu_l > \mu_b$, that is, the consumer believes that the local store is more likely to remain open if she shops at the local store than if she shops at the big-box store.*

Assumption 9. *Utility from consumption and from the existence of the local store are additively separable.*

Assumption 9 generates the following functional form of the utility function:

$$u_\tau = \hat{c_\tau} + \hat{L}_\tau, \tag{3}$$

where $\hat{c_\tau}$ represents the utility from consumption of q_τ and g_τ, and $\hat{L_\tau}$ represents the gain in utility from the existence of the local store. I assume that the consumer chooses q_τ and g_τ to maximize her utility from consumption and I remain agnostic toward the functional form of this process. This means that, for example, the consumer does not gain a different level of utility from consuming an apple while the local store is open as compared to when the store is closed. Let $\hat{c_\tau} \in \{\hat{b}_\tau, \hat{l}_\tau\}$, where $\hat{b}_\tau$ and $\hat{l}_\tau$ represent the utility that the consumer receives from the consumption of q_τ and g_τ if she shops at the big-box store or local store, respectively.

Assumption 10. *Let $y_\tau = y$, $p_{l,\tau} = p_l$, $p_{b,\tau} = p_b$, and $a_\tau = a$ such that income and prices are the same in each period.*

Assumption 10 is made in order to isolate the effects of present bias rather than unnecessarily complicate the model with price and income uncertainty (Bellemare, Barrett & Just 2013, Sandmo 1970).

Proposition 3. *If Assumptions 1 and 10 hold, then $\hat{b_\tau} = \hat{b}$ and $\hat{l_\tau} = \hat{l}$. That is, the consumer will buy the same bundle of goods every time she shops at the big-box store, and will buy the same bundle of goods every time she shops at the local store.*

Proof. If the consumer's preferences over consumption are stable over time (Assumption 1) and prices and income are stable over time (Assumption 10), the consumer will buy the same bundle of goods in each period, differing only by in which store she shops. □

Assumption 11. *The utility derived from the existence of the local store is the same in each period such that $\hat{L}_\tau = \hat{L}$.*

Assumption 12. *Let $\hat{L} > \hat{b} - \hat{l} > 0$, that is, the gain in utility from the existence of the local store is greater than the difference in utility gained from consumption when shopping at the big-box store, relative to the utility from consumption when shopping at the local store. Proposition 2 is maintained in this assumption.*

If Assumption 12 does not hold, the consumer would never opt to shop at the local store.

4 Behavior

I consider three types of consumers: (1) a time-consistent consumer ($\beta = 1$), (2) a time-inconsistent consumer ($\beta < 1$) who is sophisticated, and (3) a time-inconsistent consumer ($\beta < 1$) who is naïve. Comparing a time-consistent consumer to a naïve consumer shows how the strategy set for a consumer with present-biased preferences differs from the strategy set she believes she will have in the long run. A comparison of naïves and sophisticates indicates how the consumer's perceptions of future preferences change behavior. This is known as the "sophistication effect" (O'Donoghue & Rabin 1999).

Time-Consistent

Although single-period utility is always highest if the consumer shops at the big-box store, when the consumer defines her strategy set, she is taking into account the present discounted value of the utility she could gain from various strategy sets.

Proposition 4. *If Assumptions 1 and 10 hold, then the time-consistent consumer will choose between the two strategy sets* $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$ *and* $\mathbf{s}_\tau = (b_\tau, b_{\tau+1}, b_{\tau+2}, \ldots)$*.*

Proof. Because preferences over consumption and the existence of the local store are the same across time (Assumption 1) and prices and income are the same across time (Assumption10), the consumer will make the same choice in every period. Further, because the consumer is time-consistent, she is aware that she will face the exact same problem in each period. She will never have plans to change her strategy. Therefore the only two viable strategies are shopping at the local store in every period or shopping at the big-box store in every period, i.e., $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$ and $\mathbf{s}_\tau = (b_\tau, b_{\tau+1}, b_{\tau+2}, \ldots)$. □

Let $U_\tau(l)$ and $U_\tau(b)$ denote the present discounted value of utility associated with the strategy sets $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$ and $\mathbf{s}_\tau = (b_\tau, b_{\tau+1}, b_{\tau+2}, \ldots)$, respectively. At any point in time τ, the consumer compares the following utility possibilities:

$$U_\tau(l) = \hat{l}_\tau + \hat{L}_\tau + \delta[\mu_l(\hat{l}_{\tau+1} + \hat{L}_{\tau+1}) + (1-\mu_l)\hat{b}_{\tau+1}] \\ + \delta^2[\mu_l^2(\hat{l}_{\tau+2} + \hat{L}_{\tau+2}) + (1-\mu_l^2)\hat{b}_{\tau+2}] + \ldots \tag{4}$$

$$U_\tau(b) = \hat{b}_\tau + \hat{L}_\tau + \delta[\hat{b}_{\tau+1} + \mu_b\hat{L}_{\tau+1}] + \delta^2[\hat{b}_{\tau+2} + \mu_b^2\hat{L}_{\tau+2}] + \ldots \tag{5}$$

Proposition 5. *If Assumptions 1, 9, 10, and 11 hold, then the time-consistent consumer will compare the following utility possibilities:*

$$U_\tau(l) = (\hat{l} + \hat{L} - \hat{b}\delta\mu_l)\left(\frac{1}{1-\delta\mu_l}\right) + \hat{b}\left(\frac{\delta}{1-\delta}\right) \tag{6}$$

$$U_\tau(b) = \hat{b}\left(\frac{1}{1-\delta}\right) + \hat{L}\left(\frac{1}{1-\delta\mu_b}\right) \tag{7}$$

Proof. Assumption 9 generates the structure of the utility function. Recall from Proposition 3 that if Assumptions 1 and 10 hold, the consumer will buy the same amount of q and g in each period such that, $\hat{l}_\tau = \hat{l}$ and $\hat{b}_\tau = \hat{b}$ in each period. Further, if Assumption 11 holds then $\hat{L}_\tau = \hat{L}$. Then, begin with the utility the time-consistent consumer derives from strategy set: $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$, which is equation 4:

$$U_\tau(l) = \hat{l}_\tau + \hat{L}_\tau + \delta[\mu_l(\hat{l}_{\tau+1} + \hat{L}_{\tau+1}) + (1-\mu_l)\hat{b}_{\tau+1}] + \delta^2[\mu_l^2(\hat{l}_{\tau+2} + \hat{L}_{\tau+2}) + (1-\mu_l^2)\hat{b}_{\tau+2}] + \ldots$$

With the assumptions given above, this becomes:

$$U_\tau(l) = \hat{l} + \hat{L} + \delta[\mu_l(\hat{l} + \hat{L}) + (1 - \mu_l)\hat{b}] + \delta^2[\mu_l^2(\hat{l} + \hat{L}) + (1 - \mu_l^2)\hat{b}] + \ldots$$

Collecting terms, this becomes:

$$U_\tau(l) = (\hat{l} + \hat{L})\left(\frac{1}{1 - \delta\mu_l}\right) + \delta(1 - \mu_l)\hat{b} + \delta^2(1 - \mu_l^2)\hat{b} + \delta^3(1 - \mu_l^3)\hat{b} + \ldots$$

Applying the laws of geometric series, this becomes:

$$U_\tau(l) = (\hat{l} + \hat{L})\left(\frac{1}{1 - \delta\mu_l}\right) + \hat{b}\left(\frac{\delta}{1 - \delta}\right) - \hat{b}\left(\frac{\delta\mu_l}{1 - \delta\mu_l}\right)$$

Collecting terms, we arrive at equation 6:

$$U_\tau(l) = (\hat{l} + \hat{L} - \hat{b}\delta\mu_l)\left(\frac{1}{1 - \delta\mu_l}\right) + \hat{b}\left(\frac{\delta}{1 - \delta}\right)$$

I now turn to the utility the time-consistent consumer derives from strategy set: $\mathbf{s}_\tau = (b_\tau, b_{\tau+1}, b_{\tau+2}, \ldots)$, which is equation 5:

$$U_\tau(b) = \hat{b}_\tau + \hat{L}_\tau + \delta[\hat{b}_{\tau+1} + \mu_b\hat{L}_{\tau+1}] + \delta^2[\hat{b}_{\tau+2} + \mu_b^2\hat{L}_{\tau+2}] + \ldots$$

With the assumptions given above, this equation becomes:

$$U_\tau(b) = \hat{b} + \hat{L} + \delta(\hat{b} + \mu_b\hat{L}) + \delta^2(\hat{b} + \mu_b^2\hat{L}) + \delta^3(\hat{b} + \mu_b^3\hat{L}) + \ldots$$

Applying the laws of geometric series and collecting terms, this becomes:

$$U_\tau(b) = \hat{b}\left(\frac{1}{1 - \delta}\right) + \hat{L}\left(\frac{1}{1 - \delta\mu_b}\right)$$

□

Under what circumstances would a consumer's lifetime utility be higher if she shops at the local store in each period even when her single-period utility is higher from shopping at the big-box store?

Proposition 6. *If Assumptions 1, 9, 10, 11, and 12 hold, then long-run utility is higher from shopping at the local store, $U(l) > U(b)$, even while single-period utility is highest from shopping at the big-box store, only if $\frac{1-\delta\mu_b}{1-\delta\mu_l} > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$.*

Proof.

$$U(l) > U(b) \implies (\hat{l} + \hat{L} - \hat{b}\delta\mu_l)\left(\frac{1}{1-\delta\mu_l}\right) + \hat{b}\left(\frac{\delta}{1-\delta}\right) > \hat{b}\left(\frac{1}{1-\delta}\right) + \hat{L}\left(\frac{1}{1-\delta\mu_b}\right)$$

Rearranging terms,

$$\implies (\hat{l} + \hat{L})\left(\frac{1}{1-\delta\mu_l}\right) - \hat{b}\left(1 + \frac{\delta\mu_l}{1-\delta\mu_l}\right) > \hat{L}\left(\frac{1}{1-\delta\mu_b}\right)$$

$$\implies (\hat{l} + \hat{L})\left(\frac{1}{1-\delta\mu_l}\right) - \hat{b}\left(\frac{1}{1-\delta\mu_l}\right) > \hat{L}\left(\frac{1}{1-\delta\mu_b}\right)$$

Under Assumption 12, $\hat{l} - \hat{b} + \hat{L} > 0$

$$\implies \frac{1-\delta\mu_b}{1-\delta\mu_l} > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$$

□

That is, if $\frac{1-\delta\mu_b}{1-\delta\mu_l} > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$, the time-consistent consumer's lifetime utility is higher if she shops at the local store, even though her single-period utility is higher if she shops at the big-box store. The time-consistent consumer with these preferences will shop at the local store in each period.

This condition is most likely to hold if:

- The subjective probability of the local store remaining open is much larger if the consumer shops at the local store than if she doesn't: $\mu_l >> \mu_b$

- The difference between the utility she gains from shopping at the big-box store and shopping at the local store is small.

Time-Inconsistent

As discussed in Section 2, one can think of the time-inconsistent consumer as a separate agent in each period. Thus, it is important to consider what the consumer believes about her future selves' preferences. Assume that sophisticated and naïve time-inconsistent consumers have the same preferences (including the same β); they only differ in their beliefs about their future selves' preferences.

Sophisticated Time-Inconsistent

Let us first consider the sophisticated time-inconsistent consumer.

Proposition 7. *If Assumptions 1, 9, and 10 hold, then the sophisticated time-inconsistent consumer will choose between strategy sets* $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$ *and* $\mathbf{s}_\tau = (b_\tau, b_{\tau+1}, b_{\tau+2}, \ldots)$*.*

Proof. Recall from Proposition 3 that if Assumptions 1 and 10 hold, the consumer will buy the same amount of q and g in each period such that, $\hat{l}_\tau = \hat{l}$ and $\hat{b}_\tau = \hat{b}$ in each period. Further, if Assumption 1 holds then the consumers preference for consumption and the existence of the local store are stable over time. Recall that the sophisticated time-inconsistent consumer is aware of her present-bias. She knows that she will face the exact same problem in each period. If $U_\tau(l) > U_\tau(b)$, then $U_{\tau+t}(l) > U_{\tau+t}(b) \ \forall \, t \geq 1$. Similarly if $U_\tau(b) > U_\tau(l)$, then $U_{\tau+t}(b) > U_{\tau+t}(l) \ \forall \, t \geq 1$. The sophisticated time-inconsistent consumer will never have a strategy set in which she plans to shop at different stores. She will either shop at the local store in each period or shop at the big-box store in each period, i.e., either $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$ or $\mathbf{s}_\tau = (b_\tau, b_{\tau+1}, b_{\tau+2}, \ldots)$. □

In each period, τ, the sophisticated time-inconsistent consumer compares the utility possibilities for strategy sets $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$ and $\mathbf{s}_\tau = (b_\tau, b_{\tau+1}, b_{\tau+2}, \ldots)$:

$$U_\tau(l) = \hat{l}_\tau + \hat{L}_\tau + \beta[\,\delta\,[\mu_l(\hat{l}_{\tau+1} + \hat{L}_{\tau+1}) + (1-\mu_l)\hat{b}_{\tau+1}] + \delta^2[\mu_l^2(\hat{l}_{\tau+2} + \hat{L}_{\tau+2}) + (1-\mu_l^2)\hat{b}_{\tau+2}] + \ldots] \tag{8}$$

$$U_\tau(b) = \hat{b}_\tau + \hat{L}_\tau + \beta[\,\delta\,[\hat{b}_{\tau+1} + \mu_b\hat{L}_{\tau+1}] + \delta^2[\hat{b}_{\tau+2} + \mu_b^2\hat{L}_{\tau+2}] + \ldots] \tag{9}$$

Proposition 8. *If Assumptions 1, 9, 10, and 11 hold, the consumer compares:*

$$U_\tau(l) = (\hat{l} + \hat{L})\left(1 + \frac{\beta\delta\mu_l}{1-\delta\mu_l}\right) + \hat{b}\left(\frac{\beta\delta}{1-\delta}\right) - \hat{b}\left(\frac{\beta\delta\mu_l}{1-\delta\mu_l}\right) \tag{10}$$

$$U_\tau(b) = \hat{b}\left(1 + \frac{\beta\delta}{1-\delta}\right) + \hat{L}\left(1 + \frac{\beta\delta\mu_b}{1-\delta\mu_b}\right) \tag{11}$$

Proof. Begin with the utility the sophisticated time-inconsistent consumer derives from strategy set: $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$, which is equation 8. Assumption 9 establishes the functional form of the utility function:

$$U_\tau(l) = \hat{l}_\tau + \hat{L}_\tau + \beta[\,\delta\,[\mu_l(\hat{l}_{\tau+1} + \hat{L}_{\tau+1}) + (1-\mu_l)\hat{b}_{\tau+1}] + \delta^2[\mu_l^2(\hat{l}_{\tau+2} + \hat{L}_{\tau+2}) + (1-\mu_l^2)\hat{b}_{\tau+2}] + \ldots]$$

If Assumptions 1, 10, and 11 hold, this equation becomes:

$$U_\tau(l) = \hat{l} + \hat{L} + \beta[\,\delta\,[\mu_l(\hat{l}+\hat{L}) + (1-\mu_l)\hat{b}] + \delta^2[\mu_l^2(\hat{l}+\hat{L}) + (1-\mu_l^2)\hat{b}] + \ldots]$$

Collecting terms, this becomes:

$$U_\tau(l) = \hat{l} + \hat{L} + \beta\left[(\hat{l}+\hat{L})\frac{\delta\mu_l}{1-\delta\mu_l} + \hat{b}\frac{\delta}{1-\delta} - \hat{b}\frac{\delta\mu_l}{1-\delta\mu_l}\right]$$

Using the properties of geometric series, this becomes equation 6:

$$U_\tau(l) = (\hat{l}+\hat{L})\left(1 + \frac{\beta\delta\mu_l}{1-\delta\mu_l}\right) + \hat{b}\left(\frac{\beta\delta}{1-\delta}\right) - \hat{b}\left(\frac{\beta\delta\mu_l}{1-\delta\mu_l}\right)$$

Now turning to the utility from shopping at the big-box store, I begin with the utility the sophisticated time-inconsistent consumer derives from strategy set: $\mathbf{s}_\tau = (b_\tau, b_{\tau+1}, b_{\tau+2}, \ldots)$, which is equation 9:

$$U_\tau(b) = \hat{b}_\tau + \hat{L}_\tau + \beta[\,\delta\,[\hat{b}_{\tau+1} + \mu_b\hat{L}_{\tau+1}] + \delta^2[\hat{b}_{\tau+2} + \mu_b^2\hat{L}_{\tau+2}] + \ldots]$$

Given the assumptions listed above, this equation becomes:

$$U_\tau(b) = \hat{b} + \hat{L} + \beta[\,\delta\,[\hat{b} + \mu_b\hat{L}] + \delta^2[\hat{b} + \mu_b^2\hat{L}] + \ldots]$$

Using the properties of geometric series and rearranging, this becomes

$$\Longrightarrow\; U_\tau(b) = \hat{b} + \hat{L} + \beta\left[\hat{b}\frac{\delta}{1-\delta} + \hat{L}\frac{\delta\mu_b}{1-\delta\mu_b}\right]$$

Rearranging, this becomes equation 11

$$\Longrightarrow\; U_\tau(b) = \hat{b}\left(1 + \frac{\beta\delta}{1-\delta}\right) + \hat{L}\left(1 + \frac{\beta\delta\mu_b}{1-\delta\mu_b}\right)$$

□

Proposition 9. *If Assumptions 1, 9, 10, 11, and 12 hold, then the sophisticated time-*

inconsistent consumer has a higher long-run utility from shopping at the local store only if $\left(\frac{1-\delta\mu_b}{1-\delta\mu_l}\right)\left(\frac{1-\delta\mu_l+\beta\delta\mu_l}{1-\delta\mu_b+\beta\delta\mu_b}\right) > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$.

Proof.

$$U_\tau(l) > U_\tau(b) \implies (\hat{l}+\hat{L})\left(1+\frac{\beta\delta\mu_l}{1-\delta\mu_l}\right) + \hat{b}\left(\frac{\beta\delta}{1-\delta}\right) - \hat{b}\left(\frac{\beta\delta\mu_l}{1-\delta\mu_l}\right) >$$
$$\hat{b}\left(1+\frac{\beta\delta}{1-\delta}\right) + \hat{L}\left(1+\frac{\beta\delta\mu_b}{1-\delta\mu_b}\right)$$

Rearranging terms,

$$\implies (\hat{l}+\hat{L}-\hat{b})\left(\frac{1-\delta\mu_l+\beta\delta\mu_l}{1-\delta\mu_l}\right) > \hat{L}\left(\frac{1-\delta\mu_b+\beta\delta\mu_b}{1-\delta\mu_b}\right)$$

Under Assumption 12, $\hat{l}-\hat{b}+\hat{L} > 0$

$$\implies \left(\frac{1-\delta\mu_b}{1-\delta\mu_l}\right)\left(\frac{1-\delta\mu_l+\beta\delta\mu_l}{1-\delta\mu_b+\beta\delta\mu_b}\right) > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$$

□

If this condition holds, then the sophisticated time-inconsistent consumer will shop at the local store in each period, even while single-period utility is higher from shopping at the big-box store.

Proposition 10. *If Assumptions 1, 8, 9, 10, 11, and 12 hold, and if* $\left(\frac{1-\delta\mu_b}{1-\delta\mu_l}\right)\left(\frac{1-\delta\mu_l+\beta\delta\mu_l}{1-\delta\mu_b+\beta\delta\mu_b}\right) > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$, *then* $\frac{1-\delta\mu_b}{1-\delta\mu_l} > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$. *That is, if the sophisticated consumer chooses to shop at the local store, then the time-consistent consumer also opts to shop at the local store.*

Proof. If Assumption 8 holds, i.e., $\mu_l > \mu_b$, then $\left(\frac{1-\delta\mu_l+\beta\delta\mu_l}{1-\delta\mu_b+\beta\delta\mu_b}\right) < 1$. This implies that a stronger condition is necessary for the sophisticated time-inconsistent consumer to shop at the local store than the condition necessary for the time-consistent consumer to shop at the local store in each period. Thus, if the sophisticated consumer chooses to shop at the local store, then the time-consistent consumer definitely opts to shop at the local store. This is the case if $\frac{1-\delta\mu_b}{1-\delta\mu_l} > \left(\frac{1-\delta\mu_b}{1-\delta\mu_l}\right)\left(\frac{1-\delta\mu_l+\beta\delta\mu_l}{1-\delta\mu_b+\beta\delta\mu_b}\right) > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$. $\square$

Naïve Time-Inconsistent

While neither the time-consistent nor the sophisticated time-inconsistent consumer would ever plan to switch strategies, it is possible that the naïve time-inconsistent consumer would plan to change strategies.

Accordingly, the naïve time-inconsistent consumer compares the utility possibilities from strategy sets $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$ and $\mathbf{s}_\tau = (b_\tau, b_{\tau+1}, b_{\tau+2}, \ldots)$ as well as a variety of other strategies such as $\mathbf{s}_\tau = (l_\tau, b_{\tau+1}, l_{\tau+2}, b_{\tau+3} \ldots)$ or $\mathbf{s}_\tau = (b_\tau, l_{\tau+1}, b_{\tau+2}, l_{\tau+3} \ldots)$. That is, a naïve's possible strategy sets span the whole (l, b) space.

Proposition 11. *If Assumptions 1, 9, 10, and 11 hold, and* $\frac{1-\delta\mu_b}{1-\delta\mu_l} > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$*, then the naïve time-inconsistent consumer will only compare utilities from the following two strategy sets:* $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$ *and* $\mathbf{s}_\tau = (b_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$.

Proof. First, I can eliminate any strategies that change store choice after period $\tau + 1$. This is because the naïve consumer believes that beginning in period $\tau + 1$ she will behave as a time-consistent consumer. Thus, I narrow the possible strategy sets down to:

1. $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$

2. $\mathbf{s}_\tau = (b_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$

3. $\mathbf{s}_\tau = (b_\tau, b_{\tau+1}, b_{\tau+2}, \ldots)$

4. $\mathbf{s}_\tau = (l_\tau, b_{\tau+1}, b_{\tau+2}, \ldots)$

Recall that $\frac{1-\delta\mu_b}{1-\delta\mu_l} > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$ implies that we are in the state of the world in which the time-consistent consumer chooses $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, \ldots)$. The naïve consumer believes that beginning in period $\tau + 1$, she will behave as a time-consistent consumer, therefore she believes that she will also shop at the local store in each period beginning in period $\tau + 1$. We can eliminate any strategies that include the naïve time-inconsistent consumer shopping at the big-box store after period τ. Thus, we can eliminate the strategies 3 and 4 shown above and compare only strategies 1 and 2. □

Let $U_\tau(b, l)$ denote the present discounted value associated with the strategy set $\mathbf{s}_\tau = (b_\tau, l_{\tau+1}, l_{\tau+2}, l_{\tau+3}, \ldots)$. The consumer compares the life-time utility possibilities from Equations 8 and 12.

$$\begin{aligned} U_\tau(b,l) = \hat{b}_\tau + \hat{L}_\tau + \beta[\, \delta\, [\mu_b(\hat{l}_{\tau+1} + \hat{L}_{\tau+1}) + (1-\mu_b)\hat{b}_{\tau+1}] \\ + \delta^2[\mu_l\mu_b(\hat{l}_{\tau+2} + \hat{L}_{\tau+2}) + (1-\mu_l\mu_b)\hat{b}_{\tau+2}] \\ + \delta^3[\mu_l^2\mu_b(\hat{l}_{\tau+3} + \hat{L}_{\tau+3}) + (1-\mu_l^2\mu_b)\hat{b}_{\tau+3}] + \ldots] \end{aligned} \tag{12}$$

Proposition 12. *If Assumptions 1, 9, 10, and 11 hold, then the naïve time-inconsistent consumer compares the following utility possibilities:*

$$\begin{aligned} U_\tau(l) &= (\hat{l} + \hat{L})\left(1 + \frac{\beta\delta\mu_l}{1-\delta\mu_l}\right) + \hat{b}\left(\frac{\beta\delta}{1-\delta}\right) - \hat{b}\left(\frac{\beta\delta\mu_l}{1-\delta\mu_l}\right) \\ U_\tau(b,l) &= \hat{b} + \hat{L} + \mu_b\left(\frac{\beta\delta}{1-\delta\mu_l}\right)(\hat{l} + \hat{L} - \hat{b}) + \hat{b}\left(\frac{\beta\delta}{1-\delta}\right) \end{aligned} \tag{13}$$

Proof. The first equation is explicated in the proof of Proposition 8. To show the second is true, I begin with equation 12.

$$U_\tau(b,l) = \hat{b}_\tau + \hat{L}_\tau + \beta[\ \delta\ [\mu_b(\hat{l}_{\tau+1} + \hat{L}_{\tau+1}) + (1-\mu_b)\hat{b}_{\tau+1}] + \delta^2[\mu_l\mu_b(\hat{l}_{\tau+2} + \hat{L}_{\tau+2}) + (1-\mu_l\mu_b)\hat{b}_{\tau+2}]$$
$$+\delta^3[\mu_l^2\mu_b(\hat{l}_{\tau+3} + \hat{L}_{\tau+3}) + (1-\mu_l^2\mu_b)\hat{b}_{\tau+3}] + \ldots]$$

If Assumptions 1, 9, 10, and 11 , this equation becomes:

$$U_\tau(b,l) = \hat{b} + \hat{L} + \beta[\ \delta\ [\mu_b(\hat{l} + \hat{L}) + (1-\mu_b)\hat{b}] + \delta^2[\mu_l\mu_b(\hat{l} + \hat{L}) + (1-\mu_l\mu_b)\hat{b}]$$
$$+\ \delta^3[\mu_l^2\mu_b(\hat{l} + \hat{L}) + (1-\mu_l^2\mu_b)\hat{b}] + \ldots]$$

Using the properties of geometric series and rearranging, this becomes:

$$= \hat{b} + \hat{L} + (\hat{l} + \hat{L})\left(\frac{\beta\delta\mu_b}{1-\delta\mu_l}\right) + \hat{b}\left(\frac{\beta\delta}{1-\delta}\right) - \hat{b}\left(\frac{\beta\delta\mu_b}{1-\delta\mu_l}\right)$$

Rearranging, this becomes equation 13

$$= \hat{b} + \hat{L} + \mu_b\left(\frac{\beta\delta}{1-\delta\mu_l}\right)(\hat{l} + \hat{L} - \hat{b}) + \hat{b}\left(\frac{\beta\delta}{1-\delta}\right)$$

□

Let us compare the strategy sets $\mathbf{s}_\tau = (b_\tau, l_{\tau+1}, l_{\tau+2}, l_{\tau+3}, \ldots)$ and $\mathbf{s}_\tau = (l_\tau, l_{\tau+1}, l_{\tau+2}, l_{\tau+3}, \ldots)$.

Proposition 13. *If Assumptions 1, 9, 10, 11, and 12 hold, then $U(b,l) > U(l)$ only if* $\frac{\hat{b}-\hat{l}}{\hat{l}+\hat{L}-\hat{b}} > \frac{\beta\delta}{1-\delta\mu_l}(\mu_l - \mu_b)$.

Proof.

$$U_\tau(b,l) > U(l) \implies \hat{b} + \hat{L} + \mu_b \left(\frac{\beta\delta}{1-\delta\mu_l}\right)(\hat{l}+\hat{L}-\hat{b}) + \hat{b}\left(\frac{\beta\delta}{1-\delta}\right) >$$
$$(\hat{l}+\hat{L})\left(1+\frac{\beta\delta\mu_l}{1-\delta\mu_l}\right) + \hat{b}\left(\frac{\beta\delta}{1-\delta}\right) - \hat{b}\left(\frac{\beta\delta\mu_l}{1-\delta\mu_l}\right)$$

Rearranging and collecting terms,

$$\implies \hat{b} + \hat{L} + \frac{\beta\delta\mu_b}{1-\delta\mu_l}(\hat{l}+\hat{L}-\hat{b}) > \hat{l} + \hat{L} + \frac{\beta\delta\mu_l}{1-\delta\mu_l}(\hat{l}+\hat{L}-\hat{b})$$
$$\implies \hat{b} - \hat{l} > (\hat{l}+\hat{L}-\hat{b})\left(\frac{\beta\delta\mu_l}{1-\delta\mu_l} - \frac{\beta\delta\mu_b}{1-\delta\mu_l}\right)$$

Under Assumption 12, $\hat{l} - \hat{b} + \hat{L} > 0$

$$\implies \frac{\hat{b}-\hat{l}}{\hat{l}+\hat{L}-\hat{b}} > \frac{\beta\delta}{1-\delta\mu_l}(\mu_l - \mu_b)$$

□

If this condition holds, then the naïve time-inconsistent consumer will shop at the big-box store in the current period, and plan to shop at the local store in all future periods. However, when the next period arrives, she will again shop at the big-box store while planning to shop at the local store in all future periods. This will continue *ad infinitum*. Thus, the naïve time-inconsistent consumer will shop at the big-box store in each period.

Let us further interpret the meaning of this expression.

Proposition 14. *Let Assumptions 1, 9, 10, 11, and 12 hold, then* $\frac{\hat{b}-\hat{l}}{\hat{l}-\hat{b}+\hat{L}} > \frac{\beta\delta}{1-\delta\mu_l}(\mu_l - \mu_b)$ *is more likely to hold if:*

1. *$\hat{b}$ is much larger than $\hat{l}$,*

2. *β is very small, and*

3. *$\mu_l - \mu_b$ is small.*

These imply that the naïve time-inconsistent consumer is most likely to shop at the big-box store in the current period, with the intention of shopping at the local store in future periods if:

1. the utility from shopping at the big-box is much larger than the utility from shopping at the local store,
2. she has a very low valuation of the future, and
3. she perceives that her importance in keeping the local store open is small, i.e., the difference between the probability that the store remains open if she shops at the local store and the probability the store will remain open if she shops at the big-box store is small.

All of these conditions are what one may intuitively assume about a present-biased consumer who shops at the big-box store with the intention of shopping at the local store in future periods.

Is it possible for the sophisticated time-inconsistent consumer to shop at the local store in each period while the naïve time-inconsistent consumer shops at the big-box store in each period with the intention of shopping at the local store in later periods? That is, can $\left(\frac{1-\delta\mu_b}{1-\delta\mu_l}\right)\left(\frac{1-\delta\mu_l+\beta\delta\mu_l}{1-\delta\mu_b+\beta\delta\mu_b}\right) > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$ and $\frac{\hat{b}-\hat{l}}{\hat{l}-\hat{b}+\hat{L}} > \frac{\beta\delta}{1-\delta\mu_l}(\mu_l - \mu_b)$ hold simultaneously?

Proposition 15. *If Assumptions 1, 9, 10, 11, and 12 hold, then* $\left(\frac{1-\delta\mu_b}{1-\delta\mu_l}\right)\left(\frac{1-\delta\mu_l+\beta\delta\mu_l}{1-\delta\mu_b+\beta\delta\mu_b}\right) > \frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}}$ *and* $\frac{\hat{b}-\hat{l}}{\hat{l}-\hat{b}+\hat{L}} > \frac{\beta\delta}{1-\delta\mu_l}(\mu_l - \mu_b)$ *must hold simultaneously.*

Proof. Recall Assumption 12, i.e., $\hat{L} > \hat{b} - \hat{l}$. This assumption implies that $\frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}} > \frac{\hat{b}-\hat{l}}{\hat{l}-\hat{b}+\hat{L}}$. This implies that these two equations necessarily hold simultaneously. □

Proposition 15 implies that if we are in the case in which the sophisticated time-inconsistent consumer shops at the local store in each period (and thereby implying that time-consistent consumers also shop at the local store in each period, see Proposition 10), then the naïve consumer will shop at the big-box store in each period with the intention of shopping at the local store in all future periods.

Discussion of Results

If the consumer shops at the local store, she experiences immediate costs through the higher prices at the local store, as well as a delayed reward through a higher probability that the local store remains open in the next period. Conversely, if she shops at the big-box store, she experiences an immediate reward through lower prices, as well as a delayed cost through the higher probability that the local store will close in the next period.

The results derived above are consistent with the findings in the hyperbolic discounting, or time-inconsistency, literature that a naïve time-inconsistent consumer will "misbehave" relative to her long-run self's preferences in the face of delayed gratification (O'Donoghue & Rabin 1999). Behaviors that result in delayed gratification are hard to induce in the presence of present bias. The behavior of sophisticated time-inconsistent consumers is harder to predict. In this case, we have found a condition in which the sophisticated time-inconsistent consumer will shop at the local store. There are clearly also cases in which the sophisticated time-inconsistent consumer would shop at the big-box store, even though her long-run self would be better off if she shopped at the local store. In the model in this paper, this would be the case if $\frac{\hat{L}}{\hat{l}-\hat{b}+\hat{L}} > \left(\frac{1-\delta\mu_b}{1-\delta\mu_l}\right)\left(\frac{1-\delta\mu_l+\beta\delta\mu_l}{1-\delta\mu_b+\beta\delta\mu_b}\right)$.

As O'Donoghue & Rabin (1999) note, "... sophistication helps you when knowing about future misbehavior increases your perceived cost of current misbehavior, thereby encouraging you to behave yourself now." The result from the case emphasized in this paper corroborates O'Donoghue & Rabin (1999)'s claim.

5 Outcome and Welfare Loss of Time-Inconsistency

Thus far, I have analyzed the preferences and behaviors of individual consumers. I now turn to the consequences of time-inconsistency for a local economy.

Let us consider the case in which the sophisticated consumer shops at the local store in each period, and the naïve time-inconsistent consumer shops at the big-box store in each period. Assume, for simplicity, that all consumers are time-inconsistent consumers; some are naïve and some are sophisticated. This is an innocuous assumption because the sophisticated time-inconsistent consumer behaves like the time-consistent consumer in this case.

Recall that $L_\tau = 1$ if the local store is open in period τ, and $L_\tau = 0$ if the store is closed in period τ. Then, let$l_{\tau,i} = 1$ if individual i shops at the local store in period τ, and zero otherwise. Let I be the total number of consumers in the local economy, and $\bar{L}$ is some critical level of patronage needed for the local store to remain in business. Then, let

$$L_\tau = \begin{cases} 1, & \text{if } \sum_{i=1}^{I} l_{\tau-1,i} \geq \bar{L} \\ 0, & \text{if } \sum_{i=1}^{I} l_{\tau-1,i} < \bar{L} \end{cases}$$

This simplification implicitly assumes that the ability of the store to remain in business is entirely dependent on the number of store visits and not on what is purchased during the store visit. This is inline with Assumption 6. A more general way to think of this assumption is to let $l_{\tau,i}$ represent the average amount spent in each transaction minus the average cost incurred by the store. Then, $\bar{L}$ is the critical value of total revenue minus the costs necessary to keep the store in business. Notice also that if the store is closed in period τ, there is no possibility for the store to reopen in a later period. This is inline with Assumption 7.

Now, let $\lambda \in [0, 1]$ be the proportion of the sophisticated time-inconsistent consumers.

Then,

$$L_\tau = \begin{cases} 1, & \text{if } \lambda I \geq \bar{L} \\ 0, & \text{if } \lambda I < \bar{L} \end{cases}$$

Thus, if the total proportion of sophisticated time-inconsistent consumers drops below a certain level, the local store will close and never reopen.

Welfare Loss

As discussed in Section 4, because the three types of consumers have the same long-run utility, comparing the strategy set of a time-consistent consumer to the strategy set of a naïve consumer depicts how the behavior of a consumer with present-biased preferences differs from her "optimal" long-run behavior, given her long-run preferences. Comparing naïves and sophisticates indicates how the consumer's perception of future preferences changes behavior (O'Donoghue & Rabin 1999). Further, making these comparisons allows us to measure the welfare loss experienced due to present bias as well as the effect of sophistication. To measure welfare losses I measure the differences is long-run utility.

Again, let us consider the case in which the time-consistent consumer will shop at the local store and her long-run utility will be $U_{tc}(l) = (\hat{l} + \hat{L} - \hat{b}\delta\mu_l)\left(\frac{1}{1-\delta\mu_l}\right) + b\left(\frac{\delta}{1-\delta}\right)$. The naïve time-inconsistent consumer will shop at the big-box store in each period, and her long-run utility will be $U_n(b) = \hat{b}\left(\frac{1}{1-\delta}\right) + \hat{L}\left(\frac{1}{1-\delta\mu_b}\right)$. Note that to compare welfare, we consider the utility that the time-inconsistent consumer *actually* experiences rather than her perceived lifetime utility. That is, while the consumer believes that she will shop at the local store in future periods, she does not. Further, she never actually experiences the β decrease in future utility, because once the future arrives, she fully experiences the outcome. Then, the loss in utility for the naïve time-inconsistent consumer is $U_{tc}(l) - U_n(b)$, which is:

$$U_{tc}(l) - U_n(b) = \frac{\hat{l} + \hat{L} - \hat{b}}{1 - \delta\mu_l} - \frac{\hat{L}}{1 - \delta\mu_b} \tag{14}$$

Because $\mu_l > \mu_b$ (Assumption 8) and because $\hat{l} + \hat{L} - \hat{b} > 0$ (Assumption 12), we can conclude that $U_{tc}(l) - U_n(b)$ is positive. Thus, there is a loss in welfare for the naïve time-inconsistent consumer.

We now consider the gain in welfare due to sophistication in the case in which the sophisticated consumer shops at the local store in each period. Again, we measure the loss in welfare by considering the utility that the consumer *actually* experiences; she never experiences the drop in utility from β. Then, $U_s(l) - U_n(b) = -[U_{tc}(l) - U_n(b)]$, so the gain in welfare due to sophistication is the negative of the welfare loss due to present bias. In this particular case (it is not the case in all situations), the sophistication effect exactly off-sets the time-inconsistency effect.

6 Discussion

This paper analyzed how time-inconsistency (or hyperbolic discounting) can affect consumers' behavior and perception of future behavior. A misperception of future behavior can lead a consumer to incorrectly believe that she will shop at the local store in future periods but actually shop at the big-box store in every period. Failure to shop at the local store increases the probability that the local store will close. Dissatisfaction arises from the closing of the local store because the consumer's lifetime utility is higher if the store is open, even if the same consumer never patronized the local store.

Undoubtedly, there are attributes of local and big-box stores that affect choice of store that are not characterized in this analysis (e.g., convenience of the store, organization of the store, variety of food items, pleasantness of the staff, etc.). Preferences for these attributes of shopping can be added to the model, but the core result will remain. As such, this paper provides a useful framework for thinking about a mechanism by which consumers become upset when local stores close.

While this paper models and analyzes a stylized decision-making process for a consumer's grocery shopping choice, the analysis is readily transferable to other types of local and big-box stores, such as bookstores, restaurants, pubs, and coffee shops.

Future research should empirically test if the ideas in this paper hold. This could be tested using consumer data on shopping behavior along with proxy variables for time-inconsistency.

There are other possible explanations for the closure of local stores and subsequent disappointment. An analysis that models the local store as a public good is the most salient alternative. Future research should model this phenomenon using a public goods framework. The competing models should then be tested to determine which explanation is closer to reality.

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