Examining the effects of uncertainty on second-generation biofuel investment by using a two stochastic process approach

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1. Introduction

Investment in the production of advanced biofuel has been encouraged by the federal regulation which enacted the Renewable Fuel Standard (RFS). However, investments and production of advanced biofuel has been lower than expected in recent years, leading regulators to substantially reduce required volumes of advanced biofuel (U.S. Environmental Protection Agency, 2015).

Real option analysis is used to help explain why uncertainty has reduced investment in advanced biofuel and eroded the requirements put forth by USEPA. As Dixit (1989) showed, hysteresis can have a significant effect on a firm's decision to enter the market. Uncertainty and irreversibility are key ingredients to hysteresis, leading to prices which trigger entry (exit) that is greater (Brigo, Dalessandro, Neugebauer, & Triki) than would be expected using traditional discounted cash flow methods. Schmit, Luo, and Conrad (2011), solve for optimal entry and exit trigger prices for a firm producing ethanol from corn, a first-generation biofuel. First-generation biofuels are produced from sugars such as those found in corn or sugar cane. The authors extend previous research, which relied on a single stochastic variable, by assuming both revenues and costs follow a stochastic process. When originally conceived, the RFS targeted second-generation biofuels to account for 16 billion gallons of biofuel production by year 2022. Second generation biofuels are produced from non-food feedstocks, and thus do not face the same volatilities in feedstock cost, as is the case with first-generation biofuels. Relying on a single stochastic variable, McCarty and Sesmero (2014), have conducted an analysis on the effect of hysteresis in the investment of second-generation biofuel production.

This study examines the effect of uncertainty and irreversibility on a second-generation biofuel investment while considering the volatility of Renewable Identification Number (RIN) price, conventional fuel price and their correlations by applying a two stochastic variable approach.

Previous studies have not included both conventional fuel price and RIN price as two sources of uncertainty faced by the firm. Producers of renewable fuel face an output price which can be decomposed into the price of conventional fuel, plus the price of applicable RINs. Therefore the firm is subject to uncertainty in both markets. Results have important policy implications, as uncertainties in the RIN market, created to encourage biofuel production, may be counteracting intended policy design.

2. Renewable Identification Numbers

The Energy Policy Act of 2005 (EPAct) established the Renewable Fuel Standard (RFS), requiring the use of biofuels (such as ethanol) in the U.S. automotive fuel supply. The RFS mandated that ethanol production increase from the 4 billion gallons of ethanol produced in 2006 to 7.5 billion gallons per year by 2012. Two years later the Energy Independence and Security Act of 2007 (EISA) amended the RFS to include:

- Blending requirements for diesel as well as gasoline;
- An increase in the volume of renewable fuel required to be blended into transportation fuel from the original RFS requirement of 7.5 billion gallons per year by 2012 to 36 billion gallons per year by 2022;
- New categories of renewable fuel with separate minimum volume requirements for each category; and
- Lifecycle greenhouse gas performance threshold standards requiring the renewable fuel used to satisfy the RFS emit fewer greenhouse gases (GHG) than the petroleum-based fuel it replaces (Yacobucci, 2012).

The RFS is implemented by requiring *Obligated Parties*, or refiners that produce gasoline or diesel fuel in the United States and anyone who imports gasoline or diesel fuel into the United States *(U.S. Environmental Protection Agency, 2014)*, to meet four different *Renewable Volume Requirements* (RVOs): (1) Total Renewable Fuel, (2) Advanced Biofuel, (3) Biomass-based Diesel, and (4) Advanced Cellulosic Biofuel. The RVOs are based on RFS percentages, and may be amended from year-to-year. The RFS percentages are the ratio of renewable fuels to all non-renewable gasoline and diesel fuels. For example, USEPA set the 2013 total renewable fuel requirement at 16.55 billion gallons, the advanced biofuel requirement at 2.75 billion gallons, the biomass-based diesel requirement at 1.28 billion gallons and the cellulosic biofuel requirement at 6 million gallons. The 6 million gallon requirement for cellulosic fuels is less than half of the initial proposed minimum volume of 14 million gallons. These minimum requirements translate to a total renewable fuel percentage standard for 2013 of 9.74% (since the 16.55 billion gallons of renewable fuel represented 9.74% of all fuel volume), an advanced biofuel percentage standard of 1.52%, a biomass-based diesel percentage standard of 1.13% and a cellulosic biofuel percentage standard of 0.004% (U.S. Environmental Protection Agency, 2013).

Obligated Parties show compliance with RVOs using a tracking system developed by USEPA. In this system, every gallon qualifying as a renewable fuel produced or imported is assigned a renewable identification number (RIN). A RIN is a 38-character numeric code which biofuel producers self-generate every time a gallon of qualifying renewable fuel is produced or imported. To qualify as a renewable fuel, the fuel must meet the requirements above. The RIN system was designed to provide a flexible way of meeting annual RVOs by obligated parties.

Once it is determined a qualifying feedstock has been acquired, the feedstock is then tracked through its processing at the biofuel plant and a RIN is attached to that batch of biofuel. The RIN must not be separated from the biofuel as it moves through the distribution system, and is transferred along with the biofuel as ownership changes. This process is similar for importers of renewable fuel as well. Once the renewable fuel is blended into a transportation fuel, the RIN is separated from the renewable fuel and used for compliance by Obligated Parties, or it may be traded as a stand-alone commodity (McPhail et al 2011).

The type of RIN generated depends on the type of renewable fuel being produced, feedstock and production process. EISA requires that for a fuel produced from a Renewable Biomass to qualify as renewable fuel under the RFS2, the fuel's lifecycle GHG emissions must be less than the lifecycle GHG emissions of the 2005 baseline average gasoline or diesel fuel that it replaces. How much less, depends upon the category of biofuel.

Table 1: RIN type by D-code and production pathway

Fuel (D Code)	**Fuel Type**
Cellulosic Biofuel (D3)***	Cellulosic ethanol, Renewable CNG/LNG, Naphtha, Renewable Gasoline
Cellulosic Diesel (D7)***	Cellulosic diesel, Renewable Jet Fuel
Biomass-Based Diesel (D4)**	Biodiesel, Renewable Diesel, Jet Fuels, etc.
Advanced Biofuel (D5)**	Sugarcane ethanol, Renewable Heating oil, etc.
Renewable Fuel (D6)*	Corn ethanol, etc.

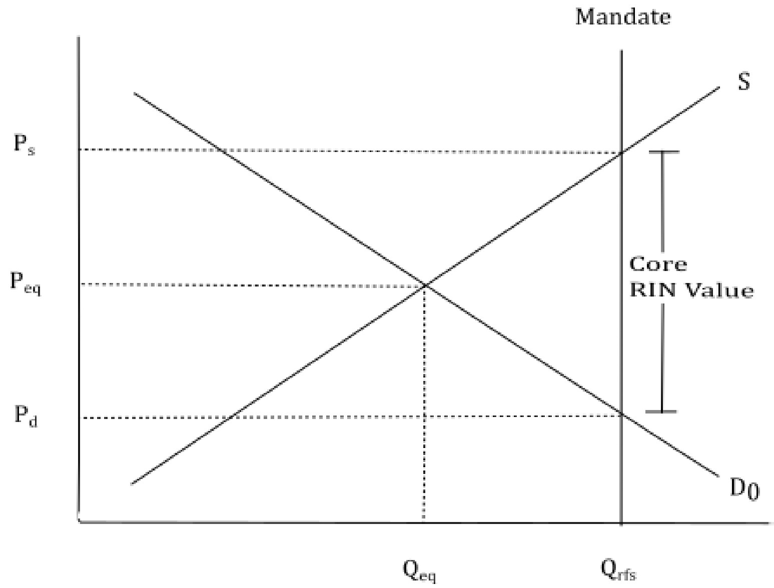
*20% reduction in GHG emissions **50% reduction in GHG emissions ***60% reduction in GHG emissions

Table 1 summarizes RIN types and the primary fuel types which qualify under the corresponding categories. U.S. Environmental Protection Agency (2016), provides a full listing of approved pathways and detailed fuel types.

2.1 RIN Price

RIN prices are determined on a secondary market where they are traded amongst obligated parties after the renewable fuel has been blended with conventional fuels. To form a basic idea of how a RIN price is determined suppose, the market for biofuel is represented by the following diagram (McPhail, Westcott, & Lutman, 2011).

Figure 1: Core Value of the RIN



When the market is in equilibrium without any mandates, equilibrium price is given p_e , and equilibrium quantity is given by Q_e . Under the Renewable Fuel Standard (RFS) mandated quantities of biofuel are required to be blended into the nation's fuel supply. When the mandated quantity is greater than equilibrium quantity demanded the mandate is said to be binding and will lead to a supply price (p_s) which is greater than demand price (p_d).   This creates a gap between the willingness to pay for biofuel at the mandated quantities and a price of supplying biofuel at the mandated quantities. RIN prices bridge this gap and ensure the mandate is met, therefore the core RIN price is determined by the size of this gap. RIN prices also have a time value. When a RIN is generated in year (t) it can be held and submitted for future compliance for up to two years. This is also known as RIN banking and provides a time value to RINs. However, the core value of the RIN is the largest component of a RINs value.

Figure 2: Historical biodiesel RIN prices and biodiesel blend margin

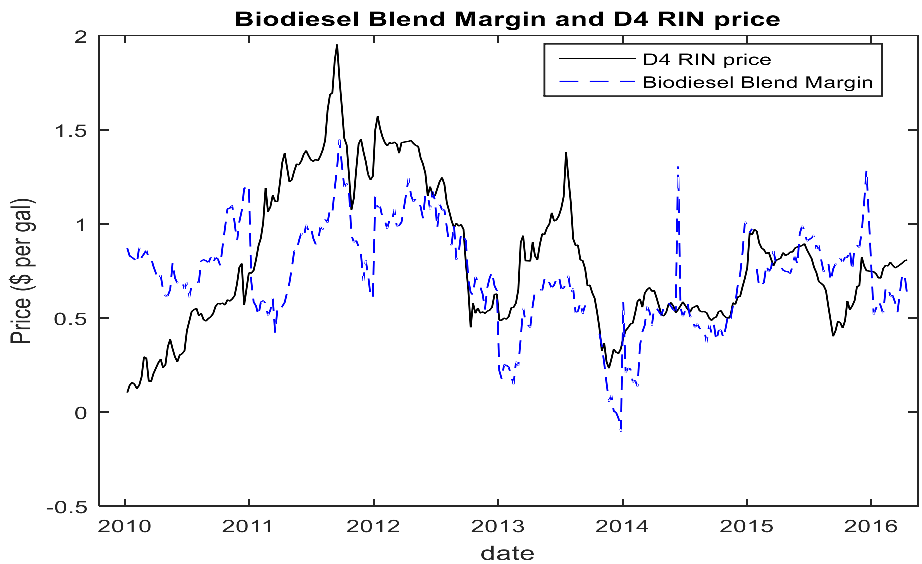


Figure 2 depicts the biodiesel blend margin and biomass-based D4 RIN prices over time. The biodiesel blend margin is calculated as the difference between conventional diesel prices and biodiesel prices, plus applicable tax credits and adjusted for the ethanol equivalent value (Irwin, 2014). Since the core value of any RIN is determined by the gap between the supply price and demand price, it comes as no surprise that the biodiesel blend margin and D4 RINs follow quite similar paths. A simple ordinary least squares regression of D4 RIN price against the biodiesel blend margin, while suppressing the constant results in an R-squared of 0.8645, indicating that blend margins explain a large portion of D4 variation.

For this study, the focus is on producers of second-generation biofuels which generate biodiesel D4 RINs. Recall that the four different *Renewable Volume Requirements*

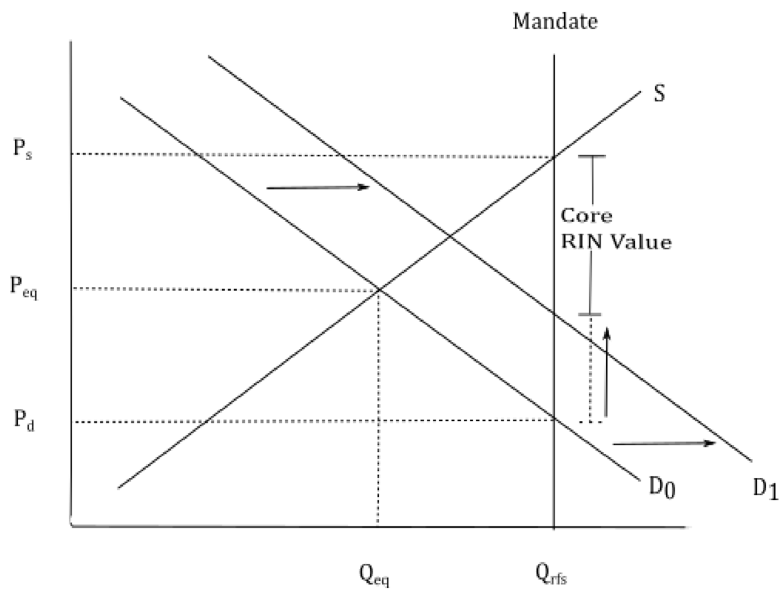
(RVOs) are: (1) Total Renewable Fuel, (2) Advanced Biofuel, (3) Biomass-based Diesel, and (4) Advanced Cellulosic Biofuel. These four renewable fuel standards are nested within each other so that fuels with higher GHG reductions can be used to meet the standards for a lower GHG reduction. For example, cellulosic biofuels with a lifecycle greenhouse gas emission that results in a 60% reduction from the baseline can be submitted for the cellulosic biofuel category, the biomass-based diesel category, the advanced category or the conventional renewable fuel category. Therefore the cellulosic D7 and D3 RINs are worth at least as much as D4, D5, and D6 RINs. Similarly, fuels in the biomass-based diesel D4 category are worth at least as much as D5 and D6 RINs. However, production of cellulosic biofuel has not been in significant quantity and as of March 10, 2016 the total RIN count was 2,842,575,051 and only 14,755,743 D3 RINs and zero D7 RINs were generated. Therefore historical data for D3 and D7 RIN price is lacking and thus D4 RIN prices are evaluated in this study.

3. Motivation for the Two-Variable Model

A two-variable model allows the conventional fuel price and RIN price to be modeled separately and account for the relationship between these two variables. Figure 1 illustrates the relationship between conventional fuel price and RIN price. Recall that (p_d) of Figure 1 represents the price at which biofuel is demanded. Therefore this implies that (p_d) also represents the price of conventional fuel, since producers and importers of conventional fuel have no incentive to purchase a biofuel which is more costly than conventional fuel. Furthermore, biofuel and conventional fuels are meant to be substitutes. One could argue that the two may well be complement goods under the RFS, but this is not borne out in the data, as shown in section 7.

Regardless whether the two goods are substitutes or complements, a change in conventional fuel price will lead to a demand shift in biofuel, and thus a change in the core value of the RIN.

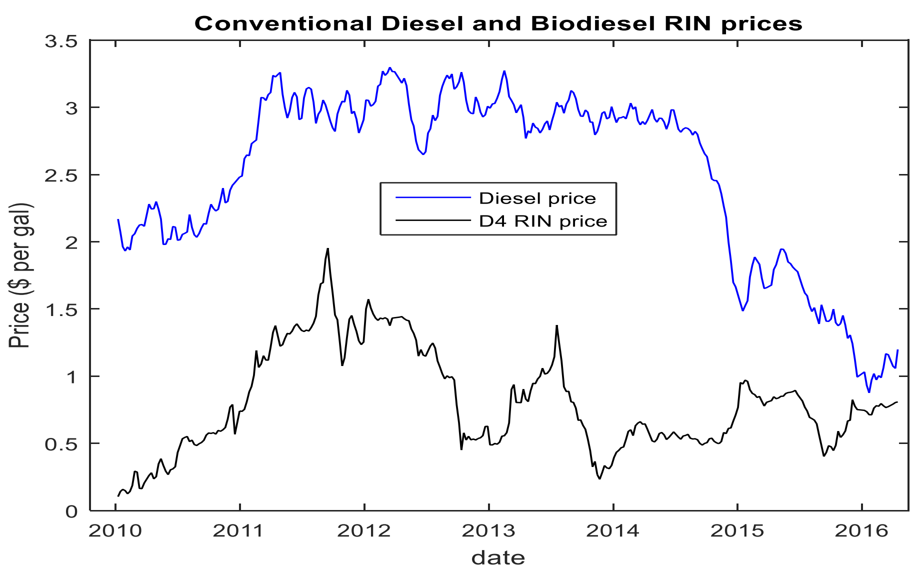
Figure 3: Outward demand shift causes decrease in RIN value



If the two goods are indeed substitutes, an increase in the price of conventional fuel, will cause an outward shift in the demand for biofuel, which would decrease the core value of RINs. This negative relationship between RIN price and conventional fuel price may affect a firm's investment decisions where the effect is dependent on a variety of factors. For example, if the price demanded for biofuel increases by $1 per gallon, the core value of RINs would decline and thus the price of RINs would decline. In reality however, it is not clear how large of a decline in RIN price would occur. Thus it is also not clear what the overall effect on total price paid to suppliers would be. Presumably this overall effect would be an increase in the price paid to suppliers of $\$0.00 < \Delta P < \$1.00$ where ΔP represents the change in total price paid to biofuel suppliers.

Potential entrants to the market for second-generation biofuels are influenced by the observed price of both conventional fuel and RIN price. The price of the RIN will be reflected in the price of the biofuel. However, second-generation biofuels are not yet produced in large quantities and the market is in a nascent stage. There is no common exchange or market for cellulosic biofuel, green-diesel or renewable jet fuels. And as such, there are no well-known prevailing prices or historical data for which reasonable estimates for future price and future price behavior can be made. However, the price of second-generation biofuels is modeled as the price of conventional fuel, plus the applicable RIN price. A firm interested in generating economic rents through the investment of second-generation biofuel production is likely to observe the market behavior of conventional fuels and applicable RINs when deciding to enter the market or remain idle. Seemingly the RFS and the RIN market would appear to lower the risk of investing in second-generation biofuels. However, the RIN market itself is very volatile.

Figure 4



In Figure 4, RIN prices appear to be even more volatile than price for conventional diesel fuel.

One possible reason for highly volatile RIN prices is due to the uncertain nature of annual renewable fuel standards. The RFS is evaluated and potentially revised each year. U.S. EPA has the authority to revise the annual standards upwards or downwards based on projected demand and production capacities, and there is precedent for these revisions. There is also potential for the standards to be challenged judiciously. For example, the District of Columbia Circuit Courts vacated 2012 cellulosic standards.

Investigating how the price volatility of both conventional fuel price, RIN price and their correlations effect the firm's decision to enter the market, remain idle or to abandon the market if already active, will provide important insights into an industry which may be critical to long term energy independence and national security. Furthermore, results could have important policy implications. If high levels of price volatility are show to depress investment in second-generation biofuels, then perhaps a more prudent policy effort would be to reduce uncertainties in the market for advanced biofuels, as opposed to setting production floors.

A real options analysis provides the framework for the problem of the firm, which is when to invest if inactive, and when to abandon if already active. Real option models are well suited to model the adaptive decision making of a firm while accounting for the irreversibility and uncertainty inherent in the firm's decision.

4. Literature

In 1977, the term "real option" was coined by Stewart Myers in "Determinants of Corporate Borrowing." Myers observes that firms are valued as an ongoing operation, a value which reflects the expectation of continued investment. Investment decisions however, are discretionary and dependent on the future net present values. Future net present values are a function of the state of the world, and future states which present unfavorable conditions will lead the firm to invest nothing. Thus a portion of the firm's value is accounted for by the present value of the *option* to make future investments. A firm's value can be segmented into two distinct asset types; real assets which have market value independent of strategy, and real options or "growth options" which are opportunities for discretionary investment (Myers, 1977). Before Myers introduced us to growth opportunities viewed as an option, Black and Scholes (1973) and Merton (1973) provided us with a concise mathematical valuation of financial options. This valuation, known as the Black-Scholes formula, is a function of known variables and estimated volatility of returns, and sets the foundation for quantitative modeling of options (Merton, 1998). Derivation of the Black-Scholes formula relies on an equilibrium condition such that the expected return of a portfolio of hedged positions is equal to the return on a riskless asset (Black & Scholes, 1973) . Hedged positions are created by combining investments to reduce risk. If the portfolio of hedged positions provided an expected return different from the return on a riskless asset, the market would be in violation of the No-Arbitrage Condition and would not be in equilibrium. The importance of arbitrage conditions or the absence of arbitrage was first recognized in the work of Modigliani and Miller (1958). The authors show that arbitrage cannot exist in equilibrium, as any opportunities for arbitrage will be exploited by investors causing the value of overpriced shares to fall, underpriced shares to rise and thereby eliminating arbitrage opportunities.

Dixit (1989), develops an entry and exit model with general implications while explicitly taking into account the effect of hysteresis on a firms entry and exit decisions. Hysteresis in this setting can be best explained as a lasting effect from a reversible cause. So even in the case of reversible decisions, hysteresis can act as a partial irreversibility. In the presence of sunk costs hysteresis has a large effect on a firm's decision to enter and exit. Given an output of unity a firm's revenues can be represented by output price alone. Under Marshallian theory entrance should occur when output price is greater than variable plus fixed costs, this is the trigger price of entry. Conversely when the output price is less than variable cost, the firm should shut down, and this price is the trigger price of exit. Due to uncertainty and the value of options, Dixit (1989) shows that hysteresis causes the gap between the trigger price of entry and exit to be substantially larger than the Marshallian gap between full and variable costs. For example the author finds the trigger price for entry to be 33 percent above the variable plus fixed costs, while the trigger price for exit is found to be 24 percent below variable costs.

Irreversibility and uncertainty are two key characteristics needed in determining to apply a real options analysis approach. This was made clear by Dixit and Pindyck (1994) in their landmark text which provides a thorough and accessible treatment of real options analysis. Later, Trigeorgis (1996) provided another highly accessible resource and together the two illuminating texts further propelled the application of real option analysis. One area that has recently become of interest is the application of real option analysis to renewable energy and biofuel investment problems.

Schmit et al. (2011), solve for optimal entry and exit trigger prices for a firm producing ethanol from corn. The authors extend previous research, which relied on a single stochastic variable, by incorporating two stochastic variables to model revenues and costs separately. By separating the revenues and costs, the effect of financial incentives and policy measures can be identified and analyzed. Trigger prices for entry were found to be approximately 85% and 25% higher than NPV and single-variable solutions respectively. Trigger prices for exit were found to be approximately 30% and 18.5% lower than NPV and single-variable solutions, suggesting a significant effect of hysteresis in the investment of first-generation biofuels. Policy measures supporting the use of ethanol have lowered entry trigger prices, increased exit trigger prices, and thus increased the overall volatility of investment (Schmit et al., 2011).

Relying on a single stochastic variable, McCarty and Sesmero (2014), have conducted an analysis on the effect of hysteresis in the investment of second-generation biofuel production. The authors find a significant effect on the trigger prices of entry and exit in the cellulosic ethanol market. In their model, the firm has the option to switch between idle to active, active to mothballed, mothballed to exit, and mothballed to active. Each of these four options to switch has a trigger price which would induce the firm into each decision. The authors find a real options trigger price for entry to be 72% higher than a traditional break-even price, while the real option trigger price for exit was about the same as the Marshallian exit price.

The literature on real option theory and it's applications to biofuel investments is growing. As energy demands rise there is an increasing need for alternative energy sources. The importance of understanding the economic challenges to bringing alternative energy to market is of increasing need as well. There are many areas of challenge, but one in particular is the area of private investment. Applying the theory of real options to biofuel investment problems will contribute to the understanding of the challenges in this area, and the impacts of policy instruments. This paper contributes to the literature by examining the uncertainty effects of renewable identification numbers on biofuels investment. To do so, a two-variable real options model is employed while allowing RIN price and conventional fuel price to follow stochastic processes.

5. The Model

Producers of renewable fuel face an output price which is composed of the price of conventional fuel, plus the price of applicable RINs. Both prices are determined exogenously. The firm producing renewable fuel decides to enter the market based on conditions observed in both the conventional fuel markets and the RIN market. Therefore the firm is subject to uncertainty in both markets.

Should an inactive firm decide to enter the market for renewable fuel the firm will incur an initial cost of investment which is sunk and irreversible. An active firm has the option to either continue operation or exit operations by incurring a sunk cost which is less than or equal to the initial investment cost.

Given the option to delay, the firm's decision to enter or exit is impacted by the irreversibility and uncertain nature of the investment. The plant can be in two different states, idle or active. In an idle state the firm is not paying either fixed or capital costs since the investment has not yet been initiated and therefore the building of the plant has not begun. When the firm decides to become active it incurs the investment cost, k, to enter the market and pays operating costs w, and earns revenue $P + R$. The revenue is a decomposition of the biofuel producer's output price which is determined by the price of conventional fuel, P, plus the RIN price, R (Figure 1). The conventional fuel could be gasoline, diesel or jet fuel, depending on the advanced biofuel produced. Similarly the particular RIN type is dictated by the type of biofuel being produced, production pathways, and feedstocks. In this case, the conventional fuel is ultra-low sulfur number two diesel and the applicable RINs are biomass-based diesel D4 RINs. Operating costs w, consist of the feedstock costs, plus transportation of that feedstock to the conversion facility.

An active plant has the option to exit the market and recover a salvage value s . The trigger prices which induce each of the decisions under the real options approach are P_h , P_l , R_h and R_l respectively. Output price (trigger price) which induces entry and exit under conventional break-even analysis are denoted by W_h and W_l respectively.

Price per gallon P is the price of a gallon of conventional fuel. Price is assumed to follow a Geometric Brownian Motion such that the change in price

$$dP = \alpha_p P dt + \sigma_p P dz_p \tag{1}$$

Where α_p represents a drift parameter and σ_p represents the standard deviation. Drift changes over the time increment dt . The change in z , dz_p , follows a Wiener process such that $dz_p = \varepsilon_t \sqrt{dt}$ where, ε_t is a normally distributed random variable with a mean of zero and a standard deviation of unity. Furthermore ε_t is serially uncorrelated so that $E(\varepsilon_t, \varepsilon_s) = 0 \ \forall \ t \neq s$ and thus the values of dz_p for any two different intervals of time are independent, following a Markov process with independent increments. It is also assumed that the discount rate δ is greater than the drift rate α_p, which indeed must hold otherwise investment would never be optimal as the growth rate would outpace the discount rate. Hence it would always be possible to do better by waiting longer.

The price at which RINs are traded is determined on a secondary market and prices have exhibited high degrees of volatility throughout RIN history. One primary cause of RIN price volatility is uncertainty in annual RFS obligations. This uncertainty is captured by assuming

RINs follow a stochastic process. In this case, RIN prices are assumed to follow a GBM just as is the case for the price of conventional gasoline.

$$dR = \alpha_r R dt + \sigma_r R dz \qquad (2)$$

Where α_r represents a drift parameter, σ_r represents the standard deviation and the same assumptions of equation (1) apply.

Feedstock production costs and feedstock transportation costs are influenced by the price of conventional fuel. Therefore, these costs can be expressed as a function of the conventional fuel price multiplied by some scale factor, η.

$$w = \eta P \qquad (3)$$

As the conventional fuel price increases, the cost of a given feedstock changes according to this relationship. This assumption represents an extreme case where feedstock costs are linearly related to fuel price over all ranges of fuel price. Modeling the operating costs in this manner achieves two things. First, it captures the influence of oil price on the production of alternative fuels. Crude oil and conventional fuels are used throughout the input stream of producing alternative fuels. The cost of fertilizers, machinery, harvesting, processing and transportation are all directly influenced by the cost of conventional fuels. Indirect effects are also captured. As the price of oil and conventional fuel increases, the attractiveness of biofuels and biodiesel increase, this in turn puts upward pressure on the prices of biofuel and biodiesel feedstocks. Second, modeling operating costs as linearly related to the price of conventional fuel allows for an analytical solution. Later this assumption is relaxed under a numerical analysis and operating costs are modeled as being non-linearly related to conventional fuel price.

In a 2013 study, the International Energy Agency examined the influence of oil price on production costs of biofuels. To examine oil price effect on the feedstock procurement and transportation costs, the authors use two methods. The first method is to use a petroleum intensity method where they estimate the quantity of petroleum used in the production and transportation of each feedstock. As oil price increases, feedstocks with the highest petroleum intensity incur the highest rises in cost. The second method is the historical trend method, which relies on the historical relationship between the price of oil and global average price of a given feedstock. In both methods, the authors found cellulosic ethanol and biodiesel to have large changes in production cost as oil price fluctuates (Cazzola et al., 2013). An advantage of the historical trend method is that it captures indirect effects, whereas the petroleum intensity method only accounts for direct effects.

6. Firm Decisions

6.1 The Firm’s Decision to Enter

The idle project’s expected net present value is denoted as $V_0(P, R)$. Over the range of prices P and R where it is optimal for an idle firm to remain idle, the asset of the investment opportunity must be willingly held. Since there are no operating profits being generated, the only return is the expected capital appreciation $E_t[dV_0(P, R)]dt^{-1}$.

The discounted expected net present value on the investment opportunity is represented by the function $\delta V_0(P, R)$. The no-arbitrage condition of efficient markets sets these two returns equal, resulting in the Bellman equation.

$$\delta V_0(P, R) = E_t[dV_0(P, R)]dt^{-1} \tag{4}$$

Under the efficient markets theorem, equation (4) must hold and implicitly defines the entry trigger prices for jet fuel and RINs. Equation (4) essentially says that over an infinitesimal period dt, the total expected return on the investment opportunity is equal to its expected rate of capital appreciation and facilitates the evaluation of the affect that changes in output price have on the value of an inactive firm.

Using Ito's Lemma $dV_0(P,R)$ is expanded using a Taylor series expansion. It is assumed that P and R are both continuous-time stochastic processes as represented by equation (1) and (2). Consider that the function $V_0(P,R)$ is at least twice differentiable in P and R. Total differentiation of $V_0(P,R)$ to higher-order terms, and making the necessary substitutions yields

$$dV_0(P,R) = \frac{\partial V_0}{\partial P}\alpha P dt + \frac{\partial V_0}{\partial P}\sigma P dz + \frac{\partial V_0}{\partial R}\alpha_r R dt + \frac{\partial V_0}{\partial R}\sigma_r R dz + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}(\sigma P)^2 dt + \frac{1}{2}\frac{\partial^2 V}{\partial R^2}(\sigma_r R)^2 dt + \frac{\partial^2 V}{\partial P \partial R}(\sigma P \sigma_r R)\rho_{pr} dt \tag{5}$$

Now, substituting equation (5) into equation (4) the following is obtained

$$\delta V_0(P,R) = E_t\left[\frac{\partial V_0}{\partial P}\alpha P dt + \frac{\partial V_0}{\partial P}\sigma_p P dz_p + \frac{\partial V_0}{\partial R}\alpha_r R dt + \frac{\partial V_0}{\partial R}\sigma_r R dz_p + \frac{1}{2}\frac{\partial^2 V}{\partial P^2}(\sigma P)^2 dt + \frac{1}{2}\frac{\partial^2 V}{\partial R^2}(\sigma_r R)^2 dt + \frac{\partial^2 V}{\partial P \partial R}(\sigma P \sigma_r R) dt\right] dt^{-1}. \tag{6}$$

Given that $E(dz) = 0$ the middle terms $\frac{\partial V_0}{\partial P}\sigma_p P dz_p = \frac{\partial V_0}{\partial R}\sigma_r R dz_r = 0$ and equation becomes the second-order homogenous differential equation

$$\frac{\sigma_p \sigma_r \partial^2 V_0}{\partial P \partial R}(\rho_{pr} PR) + \frac{{\sigma_p}^2}{2}\frac{\partial^2 V_0}{\partial P^2}(P)^2 + \frac{{\sigma_r}^2}{2}\frac{\partial^2 V_0}{\partial R^2}(R)^2 + \frac{\partial V_0}{\partial P}\alpha_p P + \frac{\partial V_0}{\partial R}\alpha_r R - \delta V_0(P,R) = 0 \tag{7}$$

6.2. The Firm's Decision to Exit

The value of the active firm is denoted as $V_1(P,R)$. Over the range of prices P and R, where it is optimal for an active firm to remain active, the firm is earning $(P+R-w)$. Similar to the Bellman equation (4) the active firm has an option value and a value of ongoing operations $(P+R-w)$ and efficiency in markets requires

$$\delta V_1(P,R) = (P+R-w) + E_t[dV_1(P,R)]dt^{-1}. \tag{8}$$

Applying Ito's Lemma to expand $dV_1(P,R)$ and following similar procedures as in equations (5)-(7) results in the second-order non-homogenous differential equation

$$\frac{\sigma_p \sigma_r \partial^2 V_1}{\partial P \partial R}\left(\rho_{pr} PR\right) + \frac{{\sigma_p}^2}{2}\frac{\partial^2 V_1}{\partial P^2}(P)^2 + \frac{{\sigma_r}^2}{2}\frac{\partial^2 V_1}{\partial R^2}(R)^2 + \frac{\partial V_1}{\partial P}\alpha_p P + \frac{\partial V_1}{\partial R}\alpha_r R \\ - \delta V_1(P,R) = w - P - R. \tag{9}$$

Firms will face a single composite price of conventional fuel price plus the price of the RIN. Therefore the value function can be written as $V(P,R) = f(P+R)$. Because this value function is homogenous of degree one in prices, the price of conventional fuel is normalized and the value function becomes a function of a numeriare plus the relative price of RINs over conventional fuel price. Therefore the value function is written as $V(P,R) = f(P+R) = Pf\left(1+\frac{R}{P}\right)$. Using this form, the partial derivatives for $V(P,R)$ are found and substituted into equations (7) and (9). Making this substitution results in a differential equation for the function $f(r)$.

$$-\frac{r}{P}f''(r)(\sigma_p\sigma_r\rho_{pr}PR) + \frac{\sigma^2}{2}\frac{r^2}{P}f''(r)(P)^2 + \frac{{\sigma_r}^2}{2}\frac{1}{P}f''(r)(R)^2 + f(r) \\ - rf'(r)\alpha_p P + f'(r)\alpha_r R - \delta P f(r) = 0 \tag{10}$$

Rearranging terms and dividing through by P results in:

$$\left(\frac{1}{2}\sigma_p^2 - \sigma_p\sigma_r\rho_{pr} + \frac{1}{2}\sigma_r^2\right)r^2 f''(r) + (\alpha_r - \alpha_p)rf'(r) - (\delta - \alpha_p)f(r) = 0 \tag{11}$$

Equation (11) can now be solved using the same general solution as in the single variable case.

The solution $f(\mathrm{r}) = r^\beta$ is tried and substituted into equation (11).

$$\left(\frac{1}{2}\sigma_p^2 - \sigma_p\sigma_r\rho_{pr} + \frac{1}{2}\sigma_r^2\right)r^2\beta(\beta-1)r^{\beta-2} + (\alpha_r - \alpha_p)r\beta r^{\beta-1} - (\delta - \alpha_p)r^\beta = 0 \tag{12}$$

This becomes:

$$\left(\frac{1}{2}\sigma_p^2 - \sigma_p\sigma_r\rho_{pr} + \frac{1}{2}\sigma_r^2\right)r^\beta(\beta^2-\beta) + (\alpha_r - \alpha_p)\beta r^\beta - (\delta - \alpha_p)r^\beta = 0 \tag{13}$$

Dividing both sides of the equation by r^β results in the characteristic equation:

$$\left(\frac{1}{2}\sigma_p^2 - \sigma_p\sigma_r\rho_{pr} + \frac{1}{2}\sigma_r^2\right)(\beta^2-\beta) + (\alpha_r - \alpha_p)\beta - (\delta - \alpha_p) = 0 \tag{14}$$

To solve for the roots of the characteristic equation the terms are rearranged to isolate β^2.

$$\left(\frac{1}{2}\sigma_p^2 - \sigma_p\sigma_r\rho_{pr} + \frac{1}{2}\sigma_r^2\right)\beta^2 + \left[(\alpha_r - \alpha_p) - \left(\frac{1}{2}\sigma_p^2 - \sigma_p\sigma_r\rho_{pr} + \frac{1}{2}\sigma_r^2\right)\right]\beta = (\delta - \alpha_p) \tag{15}$$

Let $\left(\frac{1}{2}\sigma_p^2 - \sigma_p\sigma_r\rho_{pr} + \frac{1}{2}\sigma_r^2\right) = u$, $(\alpha_r - \alpha_p) = \mu$ and $(\delta - \alpha_p) = z$ and divide both sides by u.

$$\beta^2 + \frac{\mu}{u}\beta - \frac{u}{u}\beta = \frac{z}{u} \tag{16}$$

Let $\frac{\mu}{u} = \gamma$ and $\frac{z}{u} = \tau$.

$$\beta^2 + \gamma\beta - \beta = \tau \tag{17}$$

$$\beta^2 - (1-\gamma)\beta - \tau = 0 \tag{18}$$

To solve for β complete the square by adding $\frac{(1-\gamma)^2}{4}$ to both sides resulting in two solutions, the positive root and negative root.

$$\beta = \frac{(1-\gamma)}{2} \pm \frac{\sqrt{4\tau + (1-\gamma)^2}}{2} \tag{19}$$

Dixit and Pindyck (1994), show that the negative root β_1 can be any negative number, but the positive root β_2 must be any number greater than one. To show why let equation (18) be denoted as $\emptyset(\beta)$ and note that

$$\emptyset(0) = -\tau < 0 \tag{20}$$

and $\emptyset(1) = \gamma - \tau < 0$ which must be the case since it must be that $\alpha_r < \delta$ (21)

Equation (20) must hold because as noted above, if the discount rate δ is not greater than the drift rate (a.k.a. growth rate) α_r, it would never be optimal to invest, and the firm could always do better by waiting longer. The same condition and corresponding intuition holds true for the growth rate of the conventional fuel price α_p so that $\alpha_p < \delta$. Since $\emptyset(\beta = 1) < 0$, then it must be true that the positive root $\beta_2 > 1$. Therefore the two roots of the characteristic equation (18) are:

$$\beta_1 = \frac{(1-\gamma)}{2} - \frac{\sqrt{4\tau + (1-\gamma)^2}}{2} < 0 \tag{22}$$

$$\beta_2 = \frac{(1-\gamma)}{2} + \frac{\sqrt{4\tau + (1-\gamma)^2}}{2} > 1 \tag{23}$$

Under the conditions $\alpha_r, \alpha_p < \delta$ and $\beta_1 < 0$ and $\beta_2 > 1$ the general solution to equation (11) is found to be

$$f_0(r) = A_0 r^{\beta_1} + B_0 r^{\beta_2}. \tag{24}$$

The constants A_0 and B_0 are yet to be determined, $\beta_1$and β_2 are known constants whose values depend on the parameters $\sigma_p, \sigma_r, \rho_{pr}, \alpha_p, \alpha_r$ and δ. The term $A_0 r^{\beta_1}$ represents the value of the option to switch states if r is very small and $B_0 r^{B_2}$ represents the value of the option to switch states when r increases. When the firm is inactive and r is very small, there is no value in the option to enter the market. In order for the solution to hold on the whole domain

$$f_0(r) = 0 \;\; as\, r \to 0. \tag{25}$$

To comply with the condition in (25) A_0 is set to equal zero. So the value of an idle firm in equation (24) can be rewritten as

$$f_0(r) = B_0 r^{\beta_2}. \tag{26}$$

Applying the same methods to the active firm, the ratio r is substituted into the equation (9) to obtain

$$\begin{aligned}\left(\frac{1}{2}\sigma_p^2 - \sigma_p\sigma_r\rho_{pr} + \frac{1}{2}\sigma_r^2\right) r^2 f''(r) + (\alpha_r - \alpha_p) r f'(r) - (\delta - \alpha_p) f(r) \\ = \frac{w}{P} - (1 + r)\end{aligned} \tag{27}$$

Recall from (3) that $w = \eta P$, where η is the influence of conventional fuel price on operating costs. Making this substitution will further simplify (27) to result in

$$\begin{aligned}\left(\frac{1}{2}\sigma_p^2 - \sigma_p\sigma_r\rho_{pr} + \frac{1}{2}\sigma_r^2\right) r^2 f''(r) + (\alpha_r - \alpha_p) r f'(r) - (\delta - \alpha_p) f(r) \\ = \eta - (1 + r)\end{aligned} \tag{28}$$

Equation (28) is a second-order *non*-homogenous differential equation. Differential equations of the non-homogenous form can be generally solved in two parts by trying a particular solution for the homogenous part, and a specific solution to the non-homogenous part. It has already been shown that the general solution to the homogenous part of equation (28) is equal to equation (24). To find a particular solution to the non-homogenous part of equation (28) the method of undetermined coefficients is used and a particular solution of the form $f(r) = \psi - \xi - \omega r$ is substituted in. This substitution results in (29) and (30).

$$\left(\frac{1}{2}\sigma_p - \sigma_p\sigma_r\rho_{pr} + \frac{1}{2}\sigma_r\right)(0) - (\alpha_r - \alpha_p)r\omega - (\delta - \alpha_p)(\psi - \xi - \omega r) = \eta - (1 + r) \tag{29}$$

$$-(\alpha_r - \alpha_p)r\omega - (\delta - \alpha_p)(\psi - \xi - \omega r) = \eta - 1 - r \tag{30}$$

Equating the coefficients set $(\delta - \alpha_r)\omega = -1$; $(\delta - \alpha_p)\psi = -1$; and $(\delta - \alpha_p)\xi = -1$; to solve for $\psi, \xi,$ and ω.

Therefore the solution to the non-homogenous second-order differential equation in (28) is

$$f_1(r) = A_1 r^{\beta_1} + B_1 r^{\beta_2} + \frac{r}{\delta - \alpha_r} + \frac{1}{\delta - \alpha_p} - \frac{\eta}{(\delta - \alpha_p)} \tag{31}$$

Similarly to equation (24), the term $A_1 r^{\beta_1}$ represents the value of the option to switch states if the ratio decreases and $B_1 r^{\beta_2}$ represents the value of the option to switch states when the ratio increases. In this case, the firm is currently active and has the option to exit ongoing operations if the ratio becomes very small and the option to exit operations if the ratio becomes very large. As the ratio becomes very large, the value of the option to exit approaches zero. For the solution to hold on the whole domain, B_1 is set to equal zero. So equation (31) can be simplified as

$$f_1(r) = A_1 r^{\beta_1} + \left\{\frac{r}{\delta - \alpha_r} + \frac{1}{\delta - \alpha_p} - \frac{\eta}{\delta - \alpha_p}\right\} \tag{32}$$

The expression in brackets of equation (32) represents the expected value of ongoing operations if the firm stays active. So equation (32) is the expected present value of the firm when active, composed of an option value to exit the market and the expected present value of operations.

7. Deriving the Trigger Prices

The firm can be in two different states, idle or active, giving decision makers two options to either enter, or exit. The ratio of prices which induce each of the decisions under the real options approach are r_H, and r_L respectively. The conditions which characterize each of the trigger ratios are known as value matching and smooth pasting conditions. Smooth pasting conditions require the tangency of the firm's value in each state. If the slopes of the two value functions were to differ, the firm could exploit the kink formed by the two and depart from the supposedly optimal policy to obtain a better payoff (Dixit 1989). The value matching condition requires that switching from one state to another occurs when the value of the current state becomes less than the value of the next most desirable state minus the cost of switching states. The firm incurs a cost to switch to active and to exit. Given that r_H triggers entry to an active state then based on value matching and smooth pasting conditions r_H must satisfy

$$V_0(r_H) = V_1(r_H) - k \tag{33}$$

$$V_0'(r_H) = V_1'(r_H) \tag{34}$$

Similarly, the ratio of prices r_L which triggers the firm to exit the market must satisfy

$$V_1(r_L) = V_0(r_L) + s \tag{35}$$

$$V_1'(r_L) = V_0'(r_L) \tag{36}$$

The next step is to substitute equations (33) and (35) into the value matching and smooth pasting conditions. This will generate a system of four equations with four unknowns which can be solved simultaneously to derive the unknown constants of A_1, B_0, r_H and r_L.

$$B_0 {r_H}^{\beta_2} = A_1 {r_H}^{\beta_1} + \left\{\frac{r_H}{\delta - \alpha_r} + \frac{1}{\delta - \alpha_p} - \frac{\eta}{\delta - \alpha_p}\right\} - k \tag{37}$$

$$\beta_2 B_0 {r_H}^{\beta_2 - 1} = \beta_1 A_1 {r_H}^{\beta_1 - 1} + \frac{1}{\delta - \alpha_r} \tag{38}$$

$$A_1 {r_L}^{\beta_1} + \left\{\frac{r_L}{\delta - \alpha_r} + \frac{1}{\delta - \alpha_p} - \frac{\eta}{\delta - \alpha_p}\right\} = B_0 {r_L}^{\beta_2} + s \tag{39}$$

$$\beta_1 A_1 {r_L}^{\beta_1 - 1} + \frac{1}{\delta - \alpha_r} = \beta_2 B_0 {r_L}^{\beta_2 - 1} \tag{40}$$

This system can now be solved for unique solutions for each of the four unknown values. The system is solved using MATLAB, and what is left is to estimate the unknown parameters which determine β_1 and β_2. These parameters are the variance and covariance parameters, growth rates and the discount rate denoted as $\sigma_r, \sigma_p, \rho_{pr}$, α_r, α_p and δ respectively. The growth rates, covariance and variance parameters are characteristic of the stochastic nature of output price in equation (1) where $dP = \alpha P dt + \sigma P dz$, RIN price in equation (2) $dR = \alpha_r R dt + \sigma_r R dz$ and their covariance.

8. Data Analysis

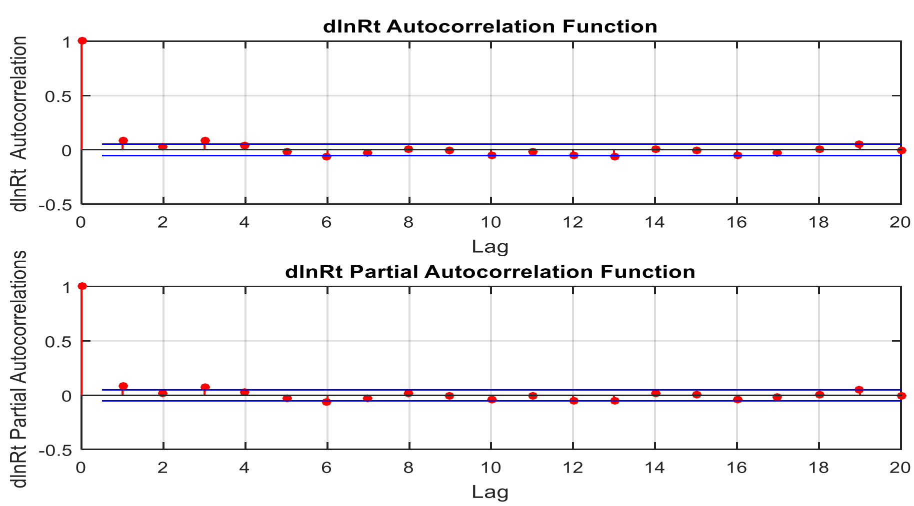
Two parameters are estimated for each stochastic process, the drift rate α, and the standard deviation σ. Next the covariance of the two stochastic processes ρ_{pr} is estimated. One potential method to estimate the parameters is maximum likelihood estimation (MLE).

Solving for the first-order conditions, MLE estimates for $\hat{\mu}$ and $\hat{v}$ yield

$$\hat{\mu} = \sum_{k=1}^{n} \frac{X(t_k)}{n} \quad and \quad \hat{v} = \sum_{k=1}^{n} \frac{(X(t_k) - \hat{\mu})^2}{n}.$$

The above expressions are simply the sample mean and sample variance of the normally distributed ΔlnP_t . This result requires ΔlnP_tbeing independent of autoregressive lags and normally distributed and is therefore a naïve approach to the GBM parameter estimation. To examine the independence or potential lack thereof, the autocorrelation and partial autocorrelation are plotted for both the transformed series of conventional fuel price ΔlnP_t and RIN price ΔlnR_t.

Figure 5: ACF and PACF of ΔlnR_t



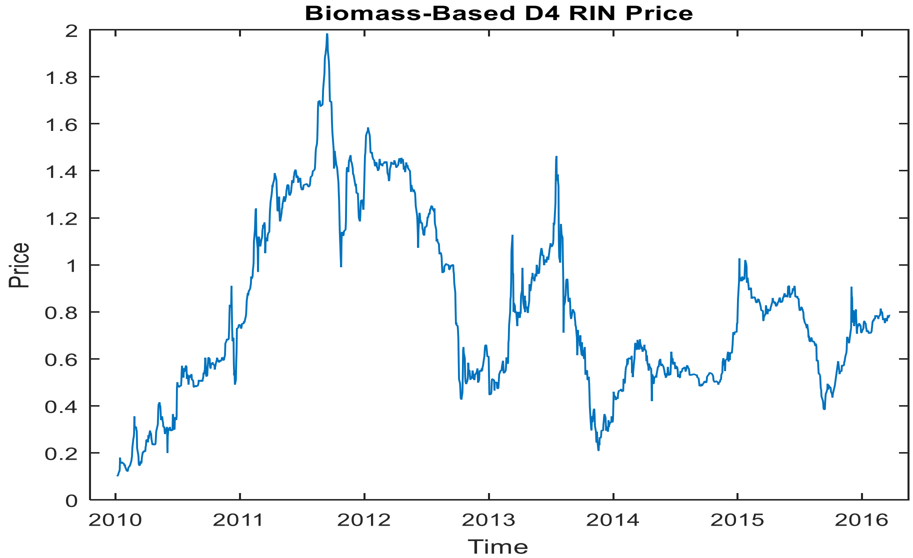
Based on Figure 5, it does not appear that independence is tenable for ΔlnR_t. This implies $\in_{R_t}$ is not white noise in $\Delta lnR_t = \varphi_1 \Delta lnR_{t-1} + \in_{R_t}$. Therefore, additional lags are added to the specification until $\in_{R_t}$ becomes white noise in $\Delta lnR_t = \sum_{i=1}^{p-1} \varphi_i \Delta lnR_{t-i} + \in_{R_t}$. A Portmanteau test for white noise is used to determine that the third lag produces white noise in the residual term. Given that $\in_{R_t}$ is white noise, I specify the Augmented Dickey Fuller (ADF) test to verify the process is a unit root and thus satisfies GBM properties. By simply including an additional term of lnR_{t-1} and a constant term, I can test for a unit root using Case II of the Augmented Dickey Fuller such that

$$\Delta lnR_t = \gamma_0 + \gamma_1 lnR_{t-1} + \sum_{i=1}^{p-1} \varphi_i \Delta lnR_{t-i} + \in_{R_t}.$$

The null hypothesis in Case II is that $\gamma_1 = 0$. After conducting the ADF test under Case II a p-value of 0.5763 is obtained, signaling a failure to reject the null hypothesis and thus reaching a conclusion that the series is indeed unit root. This implies the estimated autoregression includes a constant term, but the true process follows a unit root with no drift. The result is consistent with GBM, but the sensitivity of such tests to null specification motivates a second test.

Based on an examination of D4 RIN price against time, the data appear to have a non-zero intercept and follow a time trend.

Figure 6: D4 RIN price in levels



Therefore, Case IV of the ADF test is specified such that

$$\Delta lnR_t = \gamma_0 + \gamma_1 lnR_{t-1} + \gamma_2 t + \sum_{i=1}^{p-1} \varphi_i \Delta lnR_{t-i} + \epsilon_{R_t}.$$

In Case IV the null hypothesis is that $\gamma_1 = \gamma_2 = 0$ and γ_0 is allowed to be any value. In this case, test results are even stronger in failing to reject the null with a p-value of 0.9957. D4 RIN prices are thus appropriately modeled as a GBM. Therefore drift and variance parameters can be deduced from the regression of

$$\Delta lnR_t = c + \sum_{i=1}^{p-1} \varphi_i \Delta lnR_{t-i} + \epsilon_{R_t}$$

.

Recall that $\mu_r = (\alpha_r - \frac{1}{2}\sigma_r^2)$ such that, $\Delta lnR(t) = \mu_r dt + \sigma_r dz$ and $\Delta lnR(t) = (\alpha_r - \frac{1}{2}\sigma_r^2)dt + \sigma_r dz$. If $dt = 1$ then $E[\Delta lnR(t)] = \mu_r$ and taking the expectation of both sides of the above regression

$$E[\Delta lnR_t] = E\left[c + \sum_{i=1}^{p-1} \varphi_i \Delta lnR_{t-i} + \epsilon_{R_t}\right]$$

results in $\mu_r = c + \mu_r \sum_{i=1}^{p-1} \varphi_i$. Therefore, $\mu_r = \frac{c}{1-\sum_{i=1}^{p-1} \varphi_i} = (\alpha_r - \frac{1}{2}\sigma_r^2)$ and the standard deviation can be read directly from the regression results root mean squared error (Schmit et al., 2011).

Table 2

Ordinary Least Squares result for the equation: $\Delta lnR_t = c + \sum_{i=1}^{p-1} \varphi_i \Delta lnR_{t-i} + \epsilon_{R_t}$

Dependent ΔlnR_t	Coef.	Std. Err.	t	P > t	[95% Conf. Interval	
Independent						
ΔlnR_{t-1}	0.08366	0.0254	3.300	0.001	0.033927	0.1333873
ΔlnR_{t-2}	0.01104	0.0254	0.430	0.664	-0.03886	0.060936
ΔlnR_{t-3}	0.07701	0.0253	3.040	0.002	0.027295	0.1267253
c	0.00099	0.0012	0.810	0.417	-0.00141	0.0033868
Observations	1543					
F(3,1539)	7.16					
Prob > F	0.0001					
Root MSE	0.04793					

OLS results indicate the constant c is not statistically different from zero. Therefore $\left(\hat{\alpha}_r - \frac{1}{2}\hat{\sigma}_r^2\right)$ is set equal to zero and the drift parameter is solved for by plugging the root mean squared error in for $\hat{\sigma}_r$.

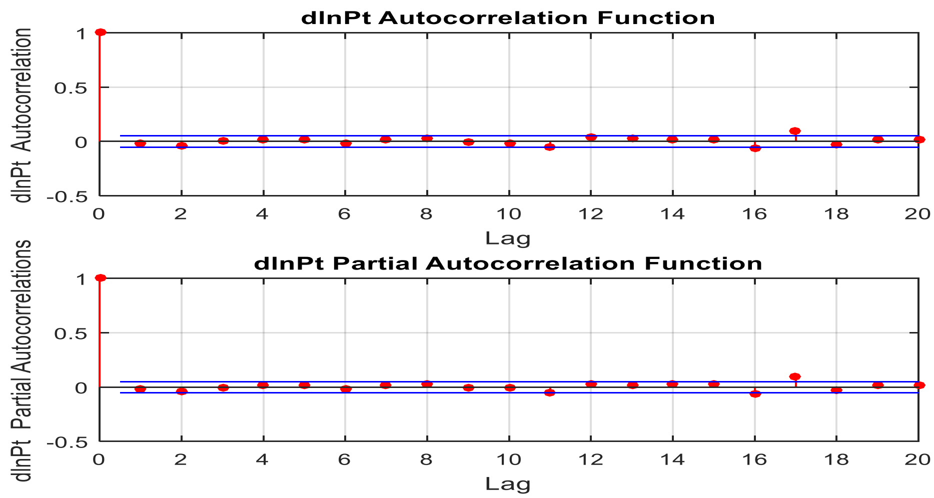
$$\left(\hat{\alpha}_r - \frac{1}{2}\hat{\sigma}_r^2\right) = 0$$

$$\hat{\alpha}_r = \frac{1}{2}\hat{\sigma}_r^2$$

$$\hat{\alpha}_r = \frac{1}{2}(0.04793)^2 = 0.0011486$$

Recall that dt was set to equal one, therefore the drift and variance parameters are annualized following the convention of converting 1-day parameters to h-day parameters. This requires scaling by a factor of $\sqrt{h}$ resulting in a drift rate of $\hat{\alpha}_r = 0.0011486 * \sqrt{252} = 0.0182$ and a standard deviation of $\hat{\sigma}_r = 0.04793 * \sqrt{252} = 0.7608$. This same process is carried out for the drift and standard deviation of conventional fuel price. By examining the correlogram of ΔlnP_t it appears that independence is tenable.

Figure 7: ACF and PACF of ΔlnP_t

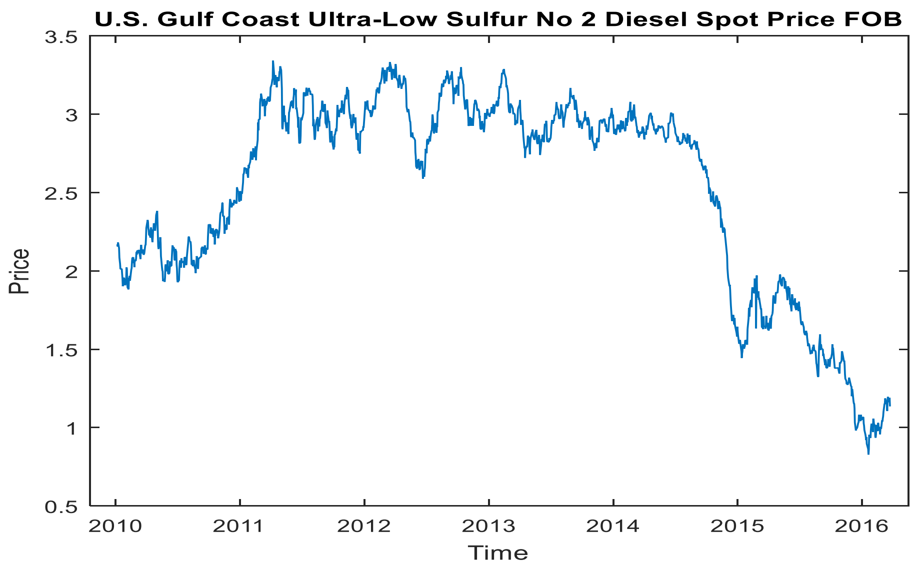


A Portmanteau white noise test is conducted with the null hypothesis that white noise is present in the residuals ϵ_{P_t} of $\Delta lnP_t = \varphi_1 \Delta lnP_{t-1} + \epsilon_{P_t}$. The Portmanteau Q-test statistic is 43.079, resulting in a p-value of 0.3411, signaling a failure to reject the null hypothesis. Based on an examination of diesel price against time, the data appear to have a non-zero intercept and time trend. So an ADF test of Case IV is specified such that

$$\Delta lnP_t = \gamma_0 + \gamma_1 lnP_{t-1} + \gamma_2 t + \epsilon_{P_t}.$$

The null hypothesis in Case IV is that $\gamma_1 = \gamma_2 = 0$ and γ_0 is allowed to take any value. The above ADF test with zero lags produces a p-value of 0.7915 signaling a failure to reject the null hypothesis of unit root. Variations of the Case IV ADF test with additional lags is also tested and results signal a failure to reject the null hypothesis in all cases.

Figure 8



To obtain the drift and variance parameters an OLS regression of $\Delta lnP_t = c + \varphi_1 \Delta lnP_{t-1} + \epsilon_{P_t}$ is conducted just as was the case for D4 RIN price. Regression results are presented in Table 2 below. Once again the constant term is not statistically different from zero. Reading the root mean squared error directly from the OLS results and plugging into the expression for the mean, the drift rate is found to be

$$\hat{\alpha}_p = \frac{1}{2}(0.01844)^2 = 0.00017.$$

With a 1-day drift rate of 0.00017 the annual drift rate becomes $\hat{\alpha}_p = 0.00017 * \sqrt{252} =$ .0026986 and the annual standard deviation becomes$\hat{\sigma}_p = 0.01844 * \sqrt{252} = 0.2927$.

Table 3:

Ordinary Least Squares result for the equation: $\Delta lnP_t = c + \varphi_1 \Delta lnP_{t-1} + \epsilon_{P_t}$

Dependent					
ΔlnP_t	Coef.	Std. Err.	t	P > t	[95% Conf. Interval
Independent					
ΔlnP_{t-1}	-0.0209821	0.025505	-0.82	0.411	-0.07101 0.0290455
c	-0.0004234	0.000469	-0.90	0.367	-0.00134 0.0004969
Observations	1,545				
F(1, 1543)	0.680				
Prob > F	0.41080				
Root MSE	0.01844				

Next, the correlation coefficient of conventional fuel price and D4 RIN price is obtained by estimating the correlation between the residuals of the fitted OLS regressions on ΔlnR_t and ΔlnP_t. Results are presented below in Table 4.

Table 4: Parameter Estimation Results

Parameter	Definition	Value	Scale
α_p	Drift Rate Conventional Fuel	0.0027	Per year
α_r	Drift Rate D4 RINs	0.0182	Per year
σ_p	Standard Deviation Conventional Fuel	0.2927	Per year
σ_r	Standard Deviation RINs	0.7609	Per year
ρ_{pr}	Correlation of Fuel and RINs	-0.0129	Per year

9. Cost Parameters

Production of renewable fuel can occur through a number of technological processes. One such technological process which shows considerable promise is a Green Fuel Technology capable of producing three different types of alternative fuel through the conversion of eight or more different feedstocks. A firm deciding to invest will employ this technology to produce the most profitable alternative fuel. Presumably this would imply that the firm produces the alternative fuel which generates the most valuable RIN at the lowest operating cost and is the alternative fuel with the greatest demand. Of the three types of RINs under consideration in this study, D4 Biodiesel RINs have the greatest value due to the nested structure of RINs discussed in section 2. Based on this nested structure cellulosic biofuel (D3) and cellulosic diesel (D7) RINs are more valuable than biodiesel D4 RINs. However, D3 and D7 RINs are not considered in this case due to a lack of significant historical production and thus lacking historical data. The first year D3 and D7 RINs were generated was 2012, but of the 15,306,658,432 total RINs generated in 2012, only 21,810 of those were D3 or D7 RINs. That translates to just 0.0001425% of total RINs attributed to D3 or D7 RINs. In 2015 the total RIN count, including D3, D4, D5, D6 and D7 was 17,908,121,504 RINs. Of the 17.9 billion 2015 RINs, D3 and D7 RINs made up just 141,557,292, or 0.7905%. As of March 10, 2016 the total RIN count was 2,842,575,051 and only 14,755,743 D3 RINs and zero D7 RINs were generated.

D4 RINs are generated through the production of biodiesel, green-diesel and renewable jet fuel. Firms employing the Green Fuel Technology are able to produce either of these fuels. Producers of conventional diesel are required to submit a specified number of RINs each year based on their annual production of conventional diesel. So the Renewable Fuel Standard supports demand for either biodiesel or green-diesel. On the other hand, producers and importers of conventional jet fuel are not obligated to submit RINs.

Green-diesel is a second-generation alternative fuel that can be blended at any proportion with conventional diesel. Chemically, green-diesel is identical to conventional petroleum based diesel, so there is no blend limit. Vehicles can operate on 100% green-diesel without any engine modifications. It requires no changes to infrastructure and it has a higher cetane rating the conventional diesel (UOP, 2016). A promising feedstock for this conversion technology is pennycress oil. Pennycress is a non-food, winter annual cover crop that produces high quality oil seeds. As a winter cover crop, it does not displace any land for food crops, and it provides all the soil benefits associated with other cover crops. Additionally the high quality oil seed has potential to generate significant farm revenue for land normally left fallow (Moser et al. 2009). Cost parameters are therefore based on the production of a green-diesel using purchased pennycress oil. Typically a techno-economic analysis is required to estimate cost parameters of a production pathway. Techno-economic analysis (TEA) is a method used to assess the technical and economic performance of a particular production pathway (Brown & Brown, 2013). TEA's represent a simplified version of commercial scale projects, that allows the biorefinery's production pathway to be evaluated for feasibility. The primary method of developing a TEA is through a detailed process model. Performance information is collected on the technologies under consideration. Appropriate production scenarios are identified and the process model is designed using process engineering software. Based on the process design, capital investments and determined and a discounted cash flow analysis is performed. This allows the investment and production cost of a biorefinery to be determined.

Due to the detailed and specialized information required to conduct a TEA, cost parameters are sourced from previous literature. Two primary cost parameters utilized in this study are operating costs and capital costs (initial investment cost). English, Menard, Yu Edward T., and Jensen (2016), provide a thorough review of operating and capital costs for a variety of production pathways. For green fuel production pathways, the authors examine four different conversion processes: 1) hydro-processing of purchased oils, 2) pyrolysis to hydro-processing, 3) gasification and 4) pyrolysis of biomass. Since pennycress is the chosen feedstock under consideration for this study, cost parameters represent the hydro-processing of purchased oils process. Based on a conversion facility which demands 96.3 million pounds of pennycress oil, with a production capacity of 13 million gallons, the capital costs are estimated to be \$3.93 per gallon. Operating costs are based on feedstock production and transportation costs and are estimated to be \$3.38 per gallon (English et al., 2016). Table 5 summarizes the cost parameters and their source.

Table 5: Cost parameters obtained from prior literature

Parameter	Definition	Value	Scale	Source
δ	Discount Rate	0.10	per year	(Brown & Brown, 2013)
η	Oil Price Influence	2.34	per gallon	(Cazzola et al., 2013)
w	Operating Cost	3.38	per gallon	(English et al., 2016)
k	Capital Cost	3.93	per gallon of total capacity	(English et al., 2016)
s	Salvage Value	2.48	per gallon of total capacity	(Schmit, Luo, & Tauer, 2009)

10. Results

In a single parameter model break-even entry price is defined as $\frac{W_h}{\delta-\alpha_p} \geq \frac{w}{\delta} + k$ where α_p represents the annual drift rate of conventional fuel price, implying the firm produces an alternative fuel whose price is on par with the conventional fuel. The price which triggers investment in this single variable break-even analysis is $\frac{W_h}{0.10-0.0027} \geq \frac{3.38}{0.10} + 3.93$ or that $W_h \geq \$3.67$ per gallon. Similarly the single variable break-even exit price is defined as $\frac{W_l}{\delta-\alpha_p} \leq \frac{w}{\delta} + s \rightarrow \frac{W_l}{0.10-0.0027} \leq \frac{3.38}{0.10} + 0.9825$ or that $W_l \leq \$3.38$ which is equivalent to operating costs for the production pathway being modeled.

In a two variable model, break-even entry price is defined as $\frac{W_h}{\delta-\alpha_r-\alpha_p} \geq \frac{w}{\delta} + k$ where the output price is now a composite of both the conventional fuel price and D4 RIN price, as is the case under a binding RIN market. Using the drift rate for both D4 RIN and conventional diesel price, the two-variable break-even prices which triggers investment is $\frac{W_h}{0.10-0.0027-0.0182} \geq \frac{3.38}{0.10} + 3.93$ or that $W_h \geq \$2.98$. Two-variable break-even exit price is defined as $\frac{W_l}{\delta-\alpha_r-\alpha_p} \leq \frac{w}{\delta} + s$ and is calculated to be \$2.75 per gallon. So when uncertainty and irreversibility are not considered, RIN prices reduce the entry price and reduce the gap between the entry and exit prices. From this perspective RIN markets are accomplishing the goals set forth by policy makers. However, RIN markets have been highly volatile since their inception. The volatility in RIN markets exacerbates the volatility already present in the market for transportation fuels, and entering into production of biofuels requires significant capital investment which is at least partially irreversible. The trigger price which induces investment is significantly larger when uncertainty and irreversibility is considered. Before examining the results of the real options approach using two stochastic variables, the trigger prices under a single variable real option analysis is examined.

In the absence of a RIN market, firms considering investment into the production of drop-in alternative fuel such as green-diesel would face a single output price. Because the firm is producing a drop-in alternative, the firm would observe the price of conventional diesel when deciding to enter or exit the market. Using this rational a real option analysis with a single stochastic variable is performed using the same parameters from conventional diesel, and the same cost parameters found for the production pathway discussed earlier. To summarize, the parameters used in the single stochastic variation are presented in Table 6.

Table 6: Parameters used for Single Variable ROA

Parameter	Definition	Value	Scale
α_p	Drift Rate Conventional Fuel	0.0027	per year
σ_p	Standard Deviation Conventional Fuel	0.2927	per year
δ	Discount Rate	0.10	per year
w	Operating Cost	3.38	per gallon
k	Capital Cost	3.93	per gallon
s	Salvage Value	0.9825	per gallon

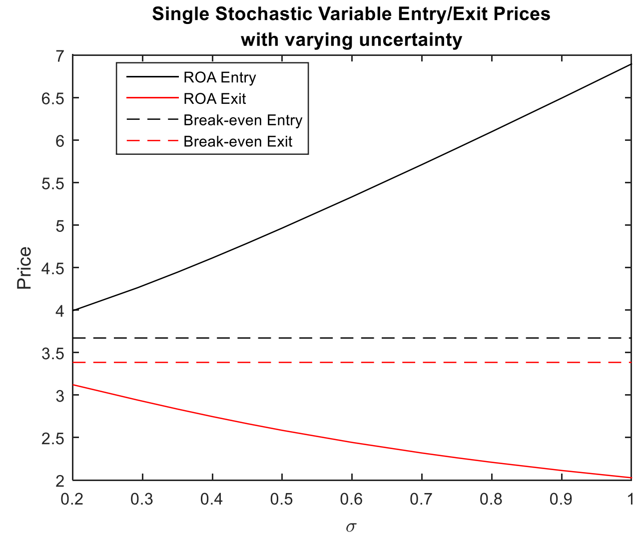
10.1 Results of the Single Stochastic Variable Case

Assuming the above parameters, the single stochastic variable case shows that uncertainty increases the price required for a firm to enter the market. Furthermore the price with induces the firm to exit the market is decreased under uncertainty, creating a gap between the entry and exit price that is larger than under the break-even analysis. This gap represents hysteresis in the market which is a zone of inaction for the firm. The greater the uncertainty the firm is subject to, the larger this hysteresis effect becomes. In this case, the entry price is found to be $4.26 per gallon and the exit price is found to be $2.94 per gallon. Recall that the break-even entry (exit) price was found to be $3.67 ($3.38) per gallon. Table 7 and Figure 9 illustrate the uncertainty effect on entry and exit prices for the single variable case. The level of hysteresis increases with increasing levels of uncertainty as can be seen in Figure 5.

Table 7: Entry and Exit Prices with varying uncertainty

σ_p	P_h	P_l
0.2000	3.992	3.122
0.2927	4.262	2.941
0.3500	4.445	2.835
0.4000	4.613	2.747
0.4500	4.786	2.663
0.5000	4.965	2.585
0.6000	5.332	2.443
0.7000	5.711	2.318
0.7500	5.904	2.262
0.8000	6.099	2.209
0.9000	6.493	2.113
1.0000	6.895	2.027
α_p	0.00269	
δ	0.10	
w	3.38	
k	3.93	
s	-0.9825	

Figure 9: Entry and Exit Prices with varying uncertainty

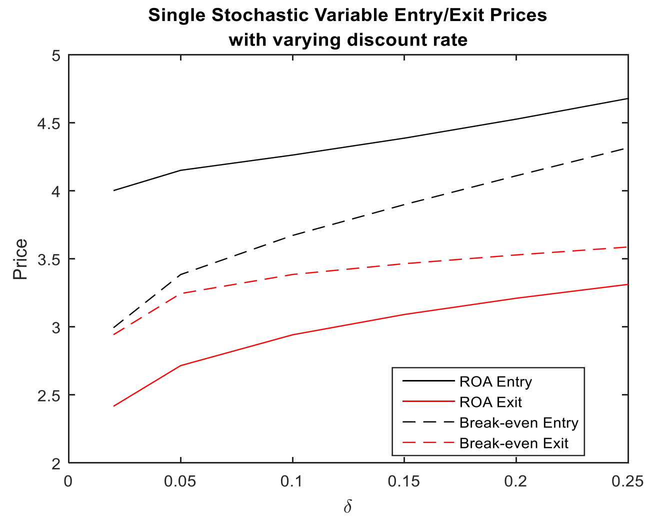


Varying the standard deviation parameter from 0.20 up to 1 demonstrates the effect uncertainty has on the firm’s investment decision. Results are consistent with theoretical expectations. The larger the price uncertainty becomes, the more valuable is the option to invest. It is this option value that induces the firm to delay until the discounted value of observed price outweighs the value of holding the option. This implies that the chosen discount rate also has a significant effect on the firms decision and indeed is does. Figure 10 and Table 8 demonstrate the effect of a varying discount rate on both the real option trigger prices and the break-even entry and exit prices. In this case, the gap between the real option entry and exit price remains constant over the range of discount rates. However, a varying discount rate creates a widening gap for the break-even entry and exit price.

Table 8: Entry and Exit prices with varying discount rate

δ	P_h	P_l
0.0200	4.0011	2.4165
0.0500	4.1499	2.7153
0.1000	4.2620	2.9415
0.1500	4.3862	3.0910
0.2000	4.5259	3.2102
0.2500	4.6777	3.3121
α_p	0.00269	
σ_p	0.2927	
w	3.38	
k	3.93	
s	-0.9825	

Figure 10: Entry and Exit with varying discount rate



10.2 Results of the Two-Stochastic Variable Case

In the two-variable case, the firm is subject to uncertainty in both conventional fuel price and RIN price. The interesting question becomes how these two uncertainties interact with each other and how that interaction effects the firm’s investment decision. Theoretical RIN prices are thought to be negatively correlated with conventional fuel price. As the price of conventional fuel increases, substitution effects take hold and shift the demand for alternative fuels which in turn causes a decrease in the price of RINs. This negative relationship between RIN price and conventional fuel price is borne out in the data as evidenced by the estimated correlation of -0.0129.

When solving the two-variable case, the variables were transformed into a ratio to facilitate a derived analytical solution to the value functions. Therefore the solutions in the two-variable case are in the form of a ratio. However, our primary interest is not in the ratio itself, rather in the composite price of conventional fuel plus RIN price. To convert ratios back into a composite price compatible with model assumptions, the implied conventional fuel price in operating costs is employed. Recall that operating cost is defined as $w = \eta P$ where η is the scale factor as determined by historical fuel price influence on feedstock and transportation costs.

Since η and w are specified, they together imply a conventional fuel price of $\frac{w}{\eta} = P$. In the case of this model the implied conventional fuel price becomes $\frac{3.38}{2.34} = P = 1.4\bar{4}$. From the variable transformation $V(P, R) = f(P + R) = Pf\left(1 + \frac{R}{P}\right)$ and therefore ratios are converted back into trigger prices by $1.4\bar{4} * (1 + r) = (P_T + R_T)$ where P_T and R_T represent trigger prices and $T \in [h, l]$.

Using the parameters specified in Table 3 and Table 4, entry and exit ratios are found to be 2.9915 and 0.7182 respectively. To convert these ratios into entry and exit prices compatible with the model, the entry price of the two-variables becomes $1.4\bar{4} * (1 + 2.9915) = P_h = \$5.765$. This is the price per gallon of biofuel that would be required to trigger investment in the two-variable case. The price per gallon of biofuel that would trigger the firm to exit the market is found to be$1.4\bar{4} * (1 + 0.7182) = P_l = \$2.4819$.

Solving the optimal switching problem through a system of equations and unknowns, of equal dimension, results in a unique solution for the assumed parameters. Any combination of RIN price and conventional fuel price with a combined price of $5.765 is predicted to trigger the firm to enter the market. Conversely any combination of RIN price and conventional fuel price with a combined price of $2.4819 is predicted to trigger the firm to abandon the project and exit the market.

Figure 11

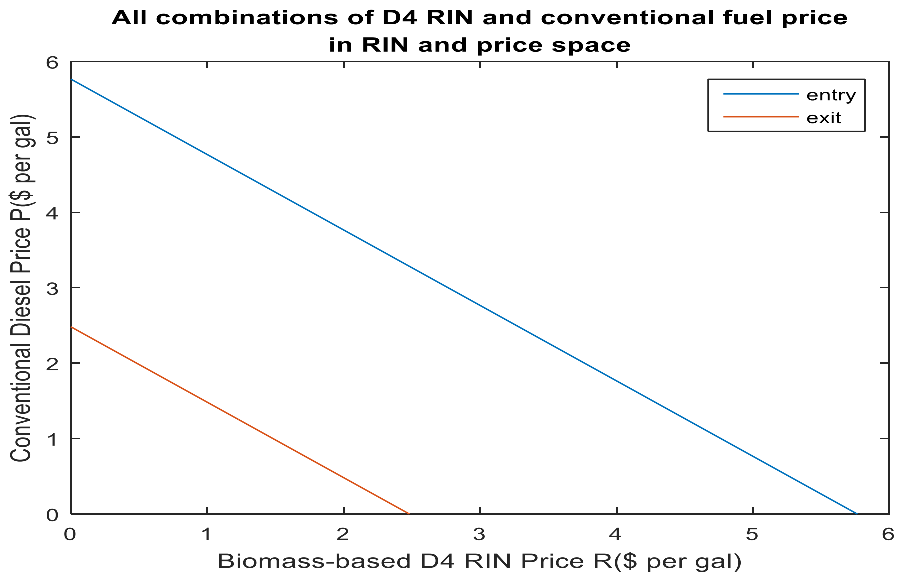


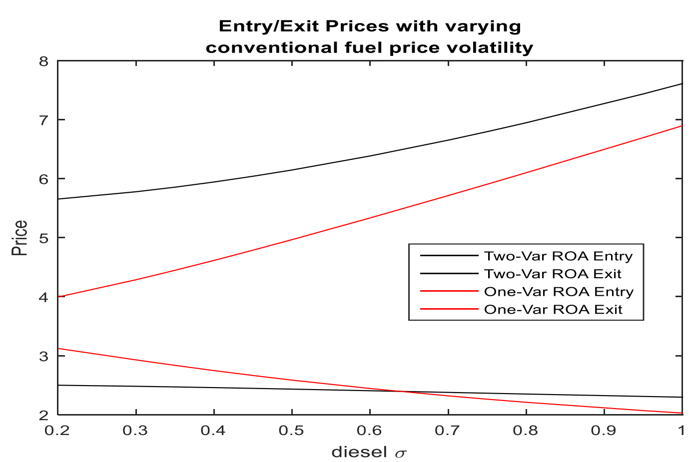
Figure 11 traces out all combinations of entry and exit prices found to trigger the firm's action to switch states. The resulting trigger prices of the two-variable model are significantly higher than results found in the one-variable model. For the two-variable model, entry trigger prices are approximately $1.50 per gallon higher than in the one-variable model. The higher trigger price which induces the firm to enter the market under the two-variable model is thought to be the result of high volatility in RIN price and its negative correlation to conventional fuel price. Increasing levels of uncertainty have been shown to increase the price required by the firm to enter the market, and also results in increasing strength of hysteresis effect. The additional uncertainty of RIN price leads to an exit trigger price of $2.4819, which is nearly $0.50 per gallon less than that was found in the one-variable case. Compared to the two-variable break-even entry price, the real option trigger price under the two-variable model is nearly twice as large with a difference of $2.7811 per gallon. However, the two-variable real option model predicts an exit price which is approximately $0.27 per gallon less than the two-variable break-even exit price.

Table 9: Comparison of Entry and Exit prices

Method	Entry	Exit
Single Variable Break-Even	\$3.6711	\$3.3843
Two Variable Break-Even	\$2.9844	\$2.7513
Single Variable Real Option	\$4.2620	\$2.9415
Two Variable Real Option	\$5.7655	\$2.4819

Table 9 demonstrates the entry and exit prices found under each case. As discussed previously, the addition of the RIN market would appear to accomplish the goal of encouraging investment if viewed in the light of a two-variable break-even approach. However, when accounting for the uncertainty in conventional fuel price, RIN price and their correlations, the entry and exit prices required for investment are higher than the single variable case. In the one-variable case, the uncertainty level of conventional fuel price was varied from a standard deviation of 0.20 up to one to examine the effect of a changing volatility. To draw comparisons between the two-variable case and the one-variable case, RIN price volatility is held constant at the estimated level of 0.7608 while allowing conventional fuel price to vary again from 0.20 up to one. Doing so results in an increasingly wide gap between entry and exit price that is substantially greater than the one-variable case.

Figure 12: RIN volatility constant at the estimated 0.7608



When the standard deviation of RIN price is held constant at the estimated value of 0.7608 and the standard deviation of conventional diesel is varied from 0.20 up to one, the difference between the two-variable case and the one-variable case is significant, particularly for the entry prices. Figure 12 indicates that additional uncertainty brought forth by the RIN market has a significant effect on the firm's investment decision. If the standard deviation of conventional diesel is held constant at the estimated rate of 0.2927 and the standard deviation of RIN price is varied from 0.20 up to one, the path of the entry and exit prices are quite similar between the two-variable case and one-variable case.

Figure 13: Conventional diesel volatility constant at the estimated 0.2927

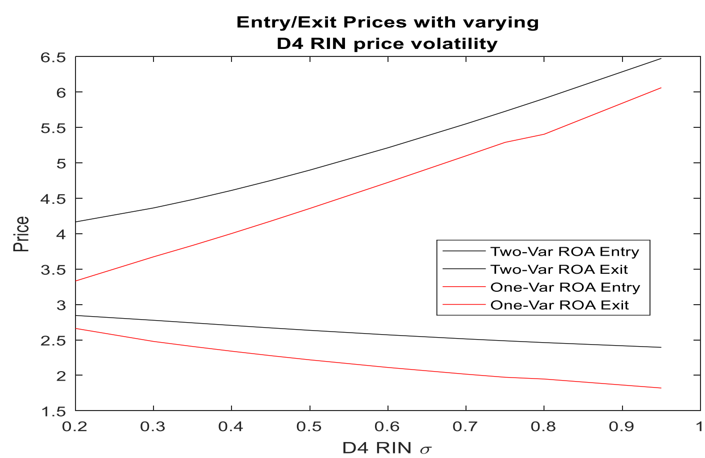


Figure 13 depicts the two-variable model against the one-variable model over varying levels of RIN price volatility, while holding diesel price volatility constant. As is the case in Figure 7, the gap between entry and exit trigger prices is wider under the two-variable model, at lower levels of volatility. As volatility nears unity, the zone of inaction widens to a comparable area for both the one-variable and two-variable models. The one-variable model predicts entry and exit trigger prices which are clearly lower than the two-variable model, across the range of RIN price volatility. Intuitively, this is likely the result of increased volatility in the two-variable model, in the form of diesel price volatility.

The choice of η is based on findings of Cazzola et al. (2013) where the historical influence of oil price on feedstock production and transportation costs ranged from 2.07 to 2.62. When operating costs consist of feedstock production and transportation, this implies that the operating cost would range from 2.07 to 2.62 times the historical price of conventional fuel. The median of this range is chosen in parameter estimations to facility analytical solutions. Using this historical relationship, the operating cost of \$3.38 per gal implies the price of conventional fuel is \$1.44 per gal. This is the price which is used to convert the entry and exit trigger ratios into entry and exit trigger prices, given the parameter specifications. Therefore, results are inspected for sensitivity to η, given that operating costs are known and held constant.

Figure 14: Examining the sensitivity of results to η

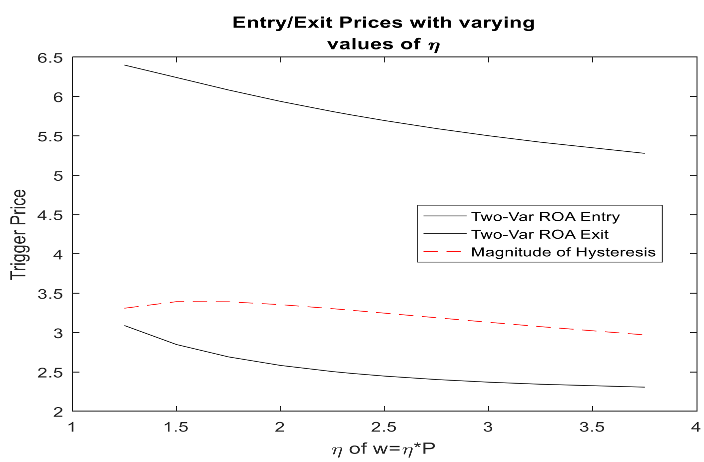


Figure 14 shows that the results are not highly sensitive over a wide range of values for η. The lack of sensitivity is clearer when the range of η is restricted to the historical range of 2.07 to 2.62 as found by (Cazzola et al., 2013).

Figure 15: Examining the sensitivity of results to η ranging from 2.07 to 2.62

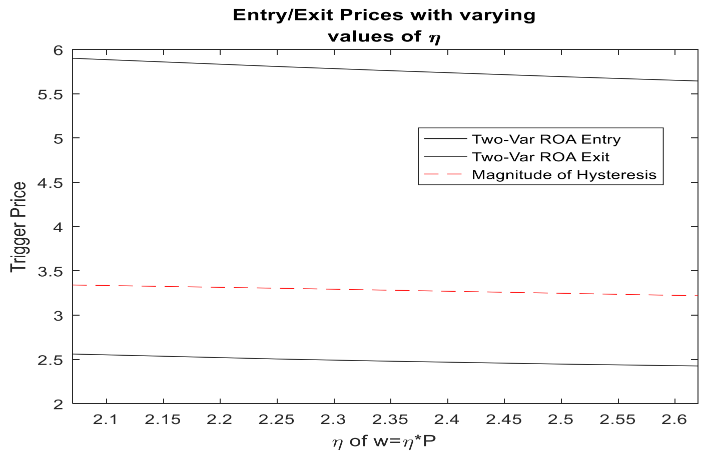


Figure 15 indicates that entry and exit trigger prices under the two-variable model are not highly sensitive over the range of historical values of η. By modelling the operating cost as linearly related to conventional fossil fuels and relying on prior literature to arrive at reasonable estimates of η, an analytical solution for the two-variable model was made possible. A relaxing of these assumptions requires the use of numerical methods to approximate solutions to the entry and exit thresholds. Solving the entry and exit problem without these assumptions will provide a sort of robustness to the analytical results by verifying similar results are found under alternative methods.

11. Numerical Approach

This section relies on a function approximation and collocation method to solve the firm's problem of optimal entry and exit timing. Fackler (2008), provides a general approach and MATLAB implementation which is applied to the optimal entry and exit problem of second-generation biofuel producers.

Recall from equation (7) the firm's decision to enter can be represented as

$$\frac{\sigma_p \sigma_r \partial^2 V_0}{\partial P \partial R}\left(\rho_{pr} P R\right)+\frac{\sigma_p{}^2}{2} \frac{\partial^2 V_0}{\partial P^2}(P)^2+\frac{\sigma_r{}^2}{2} \frac{\partial^2 V_0}{\partial R^2}(R)^2+\frac{\partial V_0}{\partial P} \alpha_p P+\frac{\partial V_0}{\partial R} \alpha_r R=\delta V_0(P, R).$$

Combining this with the value matching condition (33)

$$V_0(P, R)=V_1(P, R)-k,$$

the optimal value function thus satisfies the conditions,

$$\frac{\sigma_p \sigma_r \partial^2 V_0}{\partial P \partial R}\left(\rho_{pr} P R\right)+\frac{\sigma_p{}^2}{2} \frac{\partial^2 V_0}{\partial P^2}(P)^2+\frac{\sigma_r{}^2}{2} \frac{\partial^2 V_0}{\partial R^2}(R)^2+\frac{\partial V_0}{\partial P} \alpha_p P+\frac{\partial V_0}{\partial R} \alpha_r R \leq \delta V_0(P, R) \tag{41}$$

and

$$V_0(P, R) \geq V_1(P, R)-k. \tag{42}$$

Whichever of these two hold with equality determines the optimal firm decision. So if the value matching condition holds with strict equality such that $V_0(r_H)=V_1(r_H)-k$, then the optimal firm decision is to invest the sunk cost k and enter the market.

Allow

$$\begin{aligned}\beta_0(P, R)= & \ \delta V_0(P, R)-\alpha_r R \frac{\partial V_0}{\partial R}-\alpha_p P \frac{\partial V_0}{\partial P}-(R)^2 \frac{\sigma_r{}^2}{2} \frac{\partial^2 V_0}{\partial R^2}-(P)^2 \frac{\sigma_p{}^2}{2} \frac{\partial^2 V_0}{\partial P^2} \\ & -\left(\rho_{pr} P R\right) \frac{\sigma_p \sigma_r \partial^2 V_0}{\partial P \partial R}\end{aligned}$$

The optimality conditions (41) and (42) can then be written in equivalent form

$$0=\min [(\beta_0(P, R)-0), min(V_0(P, R)-V_1(P, R)+k)] \tag{43}$$

(Fackler, 2008).

Similarly the optimality conditions for the active firm can be expressed as

$$0=\min [(\beta_1(P, R)-\pi_1(P, R)), min(V_1(P, R)-V_0(P, R)-s)]. \tag{44}$$

where $\pi_1(P, R)$ is the flow of payments generated by the active firm.

To find the optimal value function in each state, suppose the firm's value function $V_i(P, R)$, where $i=0,1$, can be approximated by $\phi(P, R) \theta_i$ where $\phi(\cdot)$ represents a family of n-basis functions and θ_i is an n-vector of approximating coefficients. The approximating coefficients θ_i solve the optimality conditions at n-nodal state values and are found by collocation methods.

The set of n-basis functions for a family of approximating functions form the $n \times n$ matrix represented by Φ and B represents $n \times n$ matrix of $\beta_i(P, R)$ evaluated at n-nodal points. Similarly π_i is defined as the flow of payments in state (i), and C_{ij} is defined as the cost to switch from state $(i)$to state (j).

Now the problem is written as an extended vertical linear complementary problem (EVLCP) which solves for the value of $\theta_1$and $\theta_2$in

$$0 = \min(M_1\theta_1 + q_1\,, M_2\theta_2 + q_2). \tag{45}$$

$$M_1 = \begin{bmatrix} \mathrm{B}_1 & 0 & \ldots & 0 \\ -\Phi & \Phi & 0 & 0 \\ \vdots & 0 & \Phi & \vdots \\ -\Phi & 0 & \ldots & \Phi \end{bmatrix}$$

$$M_2 = \begin{bmatrix} \Phi & -\Phi & \ldots & 0 \\ 0 & \mathrm{B}_2 & 0 & 0 \\ \vdots & -\Phi & \Phi & \vdots \\ 0 & -\Phi & \ldots & \Phi \end{bmatrix}$$

$$q_1 = \begin{bmatrix} 0 \\ C_{2,1} \end{bmatrix}$$

$$q_2 = \begin{bmatrix} C_{1,2} \\ -\pi_1(P, R) \end{bmatrix}$$

Note that the switching costs $C_{2,1}$ and $C_{1,2}$ represent the exit cost and the entry cost respectively. So practically speaking, these are represented as the salvage cost (s) and the capital cost (k). Similarly, $\pi_1(P, R)$ represents the flow of payments when the firm is active which is specified as $P + R - w$.

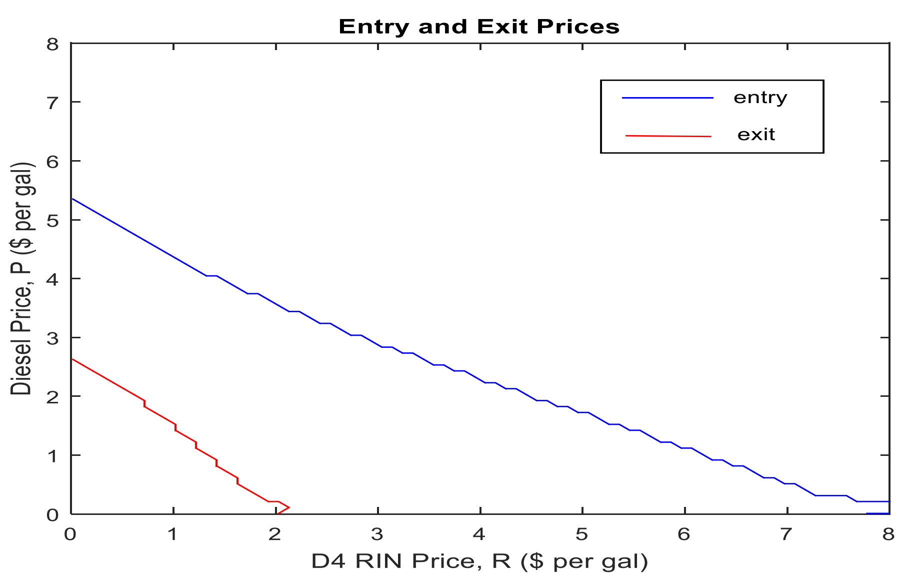
To solve the EVLCP, a smoothing Newton algorithm is employed while specifying the family of approximating solutions to be a piecewise linear function using upwind finite differences to approximate first and second order differentiation. Piecewise linear functions are most appropriate for entry and exit problems because of the inherent discontinuities in the second derivative of the value function where it becomes optimal for the firm to switch states from idle to active, or from active to idle.

An important choice in specifying the model is the choice of lower and upper limits on the approximation interval and the number of nodal points. If the interval is too wide, a large number of node points will be required to obtain accurate solutions. Conversely if the approximation interval is too narrow, such that the process starts near an optimal switching point, the solutions may be inaccurate (Fackler, 2008).

In this case, the approximation interval spans the possible prices of conventional fuel price and RIN price. Therefore the lower bound is set to \$0.01 and the upper bound is set to \$10 per gallon for both conventional fuel and RIN price. Between each lower and upper bound, 100 nodes are specified, so that the matrices of nodal points for both conventional fuel price and RIN price are both 100×100. Therefore M_1 and M_2 are each $10{,}000 \times 10{,}000$ matrices, while q_1 and q_2 are each $10{,}000 \times 1$ matrices.

Performing the numerical approximation results in entry and exit trigger prices which are very similar to those found under the analytical approach. Numerical approximation results in zone of inaction which widens with lower conventional fuel price and higher RIN price. In other words the ratio of relative prices is not constant over the entire range of prices. For example, if RIN prices are at the lower bound of the approximation interval of \$0.01, the entry and exit thresholds are found to be $5.3682 and $2.6436 respectively. This implies that if RIN prices are near zero, firms would require a conventional fuel price of $5.3582 per gallon to enter the market. Conversely, firms would exit the market if conventional fuel price fell to $2.6436 per gallon. These two entry and exit thresholds also represent the lower bound of entry trigger prices and the upper bound of exit trigger prices, so that the zone of inaction is most narrow when RIN prices are near zero at $0.01 per gallon. However, this minimum entry price threshold occurs for larger values of RIN price as well. For example if conventional fuel prices are $4.1473 per gal. the RIN price which would induce the firm to enter the market is found to be $1.2209 resulting in a combined output price of $5.3682 per gallon. Similarly, if conventional fuel price were to fall to $1.9273, a RIN price of $0.7164 would induce the firm to abandon operations and exit the market. Figure 16 traces out the combination of conventional fuel price and RIN price which triggers entry and exit actions.

Figure 16: Numerical Solution entry and exit price thresholds



Notice in Figure 16 that the slopes of the entry and exit thresholds are not constant. As lower conventional fuel prices prevail, higher RIN prices are required to trigger firm entry and lower

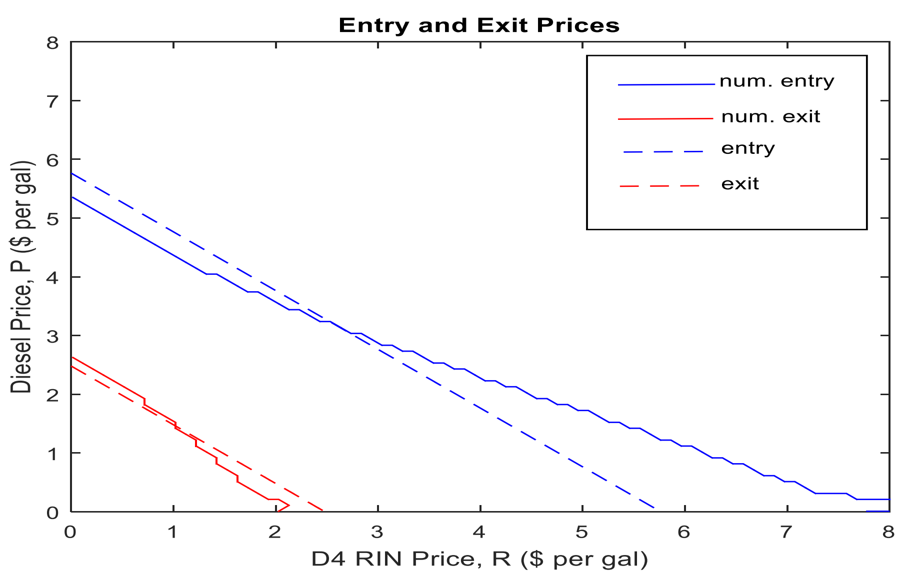
RIN prices are required to trigger firm exit. Intuitively this result seems logical. RIN prices are substantially more volatile than conventional fuel price, so with less price support in the form of conventional fuel price, and even higher RIN price is required to overcome the price uncertainty.

Table 10 reports the entry and exit thresholds under the various methods discussed. The entry and exit trigger prices reported for the two-variable numerical approximation are the lower bound of the entry trigger price and the upper bound of the exit trigger price.

Table 10: Comparison of entry and exit thresholds

Method	Entry	Exit
Single Variable Break-Even	\$3.6711	\$3.3843
Two Variable Break-Even	\$2.9844	\$2.7513
Single Variable Real Option	\$4.2620	\$2.9415
Two Variable Real Option	\$5.7655	\$2.4819
Two Var. Numerical Solution	\$5.3682	\$2.6436

Figure 17: Entry and exit thresholds under both the numerical and analytical approach



By plotting the entry and exit thresholds found under both the analytical approach and the numerical approach, similarities in the solutions are even clearer. For both the entry and exit thresholds, the two solutions intersect at \$5.7655 and \$2.4819, which is the value of the entry and exit thresholds along the entire solution curve for the analytical approach. So it seems that both analytical and numerical methods arrive at similar solutions. Under the two-variable model, the entry trigger price threshold is substantially larger than the threshold predicted under the one-variable real option model and the traditional break-even analysis models. Similarly the exit trigger price is lower than the exit price predicted in the one-variable and break-even models. It appears then, that while accounting for the volatility in both conventional fuel price and RIN price, the hysteresis effect, brought about by uncertainty and irreversibility, is stronger than would be predicted under a single-variable model.

12. Conclusion

The price which triggers the firm to invest and enter the market is significantly higher under the real options model with two stochastic variables. The underlying cause is price volatility and partially irreversible capital costs. In the market for second-generation biofuels, the price volatility faced by potential market entrants comes in the form of both conventional fuel price and RIN price. Conventional fuel prices have historically exhibited large volatilities, but RIN prices are found to be even more volatile, compounding the uncertainty faced by potential market entrants.

Analytical solutions are found under the two-variable approach by transforming the two-stochastic variables into a single ratio, following the technique pioneered by (Schmit et al., 2011). In this case however, the ratios of variables are both output prices, in the form of conventional fuel price and RIN price. The analytical model of a single stochastic ratio, results in entry and exit ratios which trigger the firm’s action. The ratios are then converted back into trigger prices, using an assumed relationship between operating costs and conventional fuel price. The resulting entry and exit thresholds are found to be in line with expectations. The entry and exit thresholds are significantly higher (lower) than the entry (exit) thresholds found in the single-variable real option model. The analytical model relies on the assumption of geometric Brownian data generating process for both stochastic variables. A second assumption needed to arrive at analytical solutions is that operating costs are linearly related to the price of conventional fossil fuels. Operating costs consist of feedstock production costs and feedstock transportation costs, so these costs are certainly dependent upon fossil fuel input costs. However, the linearity is a strong assumption. To avoid this assumption, numerical solutions are found through the use of a projection and collocation method.

Solving the two-variable real option model through the use of numerical methods has a number of advantages. First, the GBM data generating assumption can be relaxed, and a linearly related operating cost can be relaxed as well. Numerical solutions for the two-variable model are obtained while specifying a GBM process. Data analysis in section 8 support the notion of a GBM data generating process, however, the addition of a jump diffusion process has been argued for in previous studies. Plans are in place to specify a jump diffusion process in subsequent revisions of this paper.

Both numerical and analytical methods produce similar results and are in line with expectations. However, numerical methods rely on fewer assumptions and seem to more accurately account for differing volatilities between the two stochastic prices. For example, when diesel price is near zero, the firms would require very large RIN prices of $8 per gallon. Conversely if RIN prices are near zero, diesel prices are only about $5 per gallon. So the slopes of the thresholds vary over the range of prices under the numerical solution. Intuitively this would be explained by the larger uncertainties in RIN price compared to conventional fuel price. Using the analytical method, the slope of the thresholds remains constant over the range of prices.

Results indicate that high levels of uncertainty in RIN price, contribute to the widening of the range of inaction. Compared to the one-variable model, the entry (exit) trigger prices are higher (lower) in the two-variable model. This implies that uncertainties in annual volume standards and the future stability of the renewable fuel standard may be contributing to market volatility and actually impeding private investment. If this is indeed the case, it seems a more prudent policy effort would be to reduce uncertainties faced by producers of second-generation biofuels, rather than mandate minimum quantities of consumption.

This study could be strengthened by including additional policy incentives, such as USDA’s Advanced Biofuel Payment Program 9005. Enacted in the 2008 Farm Bill and reauthorized in the 2014 Farm Bill, program 9005 authorizes payment to producers of advanced biofuels (English et al., 2016). Directions for future research include the addition of these types of policy incentives, the use of alternative stochastic processes and the inclusion of investment lags in the firm’s optimal decision analysis.

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