Sensitivity of miscanthus supply: Application of Faustmann's rule in deterministic and stochastic cases

N. Ben Fradj\textsuperscript{1,2,*} and P.-A. Jayet\textsuperscript{1,†}

\textsuperscript{1}UMR Economie Publique, INRA, AgroParisTech, Université Paris-Saclay, 78850 Thiverval-Grignon, France.

\textsuperscript{2}UMR ECOSYS, INRA, AgroParisTech, Université Paris-Saclay, 78850 Thiverval-Grignon, France.

\*nosra.benfradj@gmail.com

\†jayet@grignon.inra.fr

Selected Paper prepared for presentation at the 2016 Agricultural & Applied Economics Association Annual Meeting, Boston, Massachusetts, July 31-August 2

Copyright 2016 by N. Ben Fradj and P.-A. Jayet. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provided that this copyright notice appears on all such copies.
Abstract

This paper aims to analyze the sensitivity of the supply of a perennial crop, i.e., miscanthus for which high interest arises when it is dedicated to second generation biofuels production. We develop a methodology based on the "Faustmann's rule" usually used in forest management fields. We first determine the yield growth function over time and the discounted present value of this crop in a deterministic case. Then, a stochastic process based on a beta distribution is introduced to manage the variability of miscanthus yield. A short-term agricultural model (AROPA) is used to highlight the large scale impact of annual yield randomization. This analysis details the impact assessment regarding optimal length cycle, land use, N input demand and nitrate losses. Ideally, miscanthus would be grown on marginal land. However, miscanthus profitability causes farmers to cultivate it on the most productive land generally devoted to food crops. An increase in yield potential leads to significant direct and indirect land re-allocation, favoring therefore the competition between food and biofuel production. This change in land use leads to a substantial decrease in N-input application and, consequently, in nitrate losses. Results significantly change when yields are affected by annual randomized variability. Throughout a sensitivity analysis, we notice that yields, renewal cycle costs and the discount rate may interact with yield randomization and significantly affect the future profitability of miscanthus.

Keywords: bioenergy perennial crop; Faustmann rule; stochastic process; Present Net Value; optimal rotation age; land use change;

1 Introduction

Rapid changes in the world's climate, as well as an increased interest in energy security, have triggered investigations into biomass production based on non-food crops. If we are to fuel our energy needs with biomass rather than petroleum, a large-scale production of biomass is required. Moreover, greenhouse gas emissions (GHG), one of the driving factors behind climate change, can be reduced by using plant-based biofuels because the useful biomass can fix atmospheric carbon (Coetto, 2008) and sequester carbon in the soil (Benbi & Brar, 2009). One of the benefits of second (2nd) generation biofuels, based on lignocellulosic biomass, is that they reduce GHG emissions up to 85% compared with conventional fuels (Wang et al., 2007) and can be produced from diverse raw materials such as wood, grasses, and crop residues. It is also known that 2nd generation feedstocks are more land-use efficient than first (1st) generation crops (Fischer et al., 2010). Producing high biomass yields per unit of land area, they can be used to ensure bioenergy demand because they require less land than low-yielding crops (Heaton et al., 2008). For instance, the high yielding perennial Miscanthus x giganteus (15 to 20 tonnes dry matter per hectare), one of the relevant industrial crops, could require 87% less land to produce the same amount of low-input biomass because the yield of Miscanthus x giganteus is nearly eight times greater (Heaton et al., 2008).

Miscanthus x Giganteus has other features that make it suitable as a source of biomass. It is a perennial grass which can be easily established and distributed under a wide range of European and North American climatic conditions (Lewandowski et al., 2000). Miscanthus is also environmentally friendly crop. Because of its roots which can reach 2 meters deep, it thrives on low N input and decreases risk of ground water pollution by pesticides and nitrates. However, the farmer who opts to cultivate miscanthus faces with a major question: the crop has a long rotation period and its last cutting should be at least in the 15th year. So, at which date will the crop provide the highest market value? In other words, In which year should the final cutting take place? In the field of natural resource economics, studies have sought to answer the question of timing by applying the Faustmann rule.

Faustmann's formula (Faustmann, 1849) is a commonly used method to address questions focusing on optimal resource management. The simplicity of this rule comes from the fact that the growth function and prices of miscanthus are assumed to be known over time. Over the years, Faustmann's formula under deterministic conditions has remained unchanged. However, methods have been developed in numerous ways in order to generalize the application of this rule. Focusing on optimal rotation, some studies have accounted for uncertainty. Resulting from random fluctuations in the productivity level or from random changes in
natural conditions, biomass uncertainty can play a central role in forest management. Different approaches have been developed to deal with this uncertainty. The tree size is modeled through a diffusion process (Miller & Voltaire, 1983) as well as a geometric Brownian motion (Clarke & Reed, 1989), with a view to solve the rotation problem. Willassen (1998) considers a general stochastic differential equation model for the growth process in continuous time. Buongiorno (2001) models the growth process in discrete time by employing Markov decision processes. Despite the fact that miscanthus raises a similar question of harvest timing under biomass uncertainty, to our knowledge, Faustmann’s formula has not been used as a means to analyze this issue. To manage the random yield of miscanthus, we develop a simple stochastic model in which the yield process is based on a popular distribution, the so-called beta-distribution. Because of its versatility, this distribution has been used by Nelson & Preckel (1989), among others, to model a variety of uncertainties. It is generally used for representing processes with lower and upper limits while it has the flexibility to model both positive and negative skewed data.

In this study, we address the question of biomass uncertainty in a continuous time for a perennial resource, in our case miscanthus. By applying two Faustmann models, we can derive harvesting rules to deal with the optimal rotation age and the value of miscanthus when its growth function is accounted for in a deterministic way as well as when it is governed by a stochastic process. Given the differences in results between these two cases, we undertake the use of the AROP Aj model to highlight them. The AROP Aj model is a one-year period mathematical programming model devoted to agricultural supply in Europe. It belongs to a class of models based on a micro-economic approach (Arni, 2001). The model covers the European Union by way of a large set of representative farm groups. It describes the annual supply choices of European farmers in terms of surface allocation, crop and animal production. The feasible production set is limited by several constraints: land endowment, animal demography, livestock limit, animal feeding, and Common Agricultural Policy (CAP) requisites (Galko & Jayet, 2011). Among the AROP Aj outputs, we find also the consumption in fertilizers, ammonia (NH$_3$) losses, as well as nitrous oxide (N$_2$O) and nitrate (NO$_3$) emissions. All these outputs are estimated by coupling AROP Aj with a crop model, namely STICS (Godard et al., 2008). In light of the above, several questions arise: how are the issues of timing and valuation altered when the farmer cannot foresee the future (the stochastic approach) compared to when we can foresee it (the deterministic approach)? More specifically, what are the differences between the deterministic and uncertain case, in terms of land use and N-losses? Moreover, in addition to yield, what other economic parameters affect the timing of harvest and the value of miscanthus?

2 Faustmann modeling

Here, we calculate the value and the optimal rotation age of miscanthus under deterministic and stochastic conditions. The value of miscanthus is determined by using the Faustmann rule, which is usually associated with forest which is cut at the end of the cycle. In our case, it is applied to miscanthus which is harvested each year.

2.1 Deterministic value expectation of miscanthus

The method we used to calculate the value and optimal rotation age of miscanthus in the deterministic case is explained in Bourgeois et al. (2014) and Ben Fradj et al. (2016). These papers detail the two-step procedure for determining the yield growth function over time and the discounted present value of this crop. Based on research conducted by Miguez et al. (2008), Clifton-Brown et al. (2007) and Christian et al. (2008), we first determine the miscanthus growth function, which represents the quantity of harvested biomass. We then adjust the average regional yield of miscanthus to the average regional yield of cereals in order to deal with a lack of data on miscanthus yields. The second step is based on a Faustmann dynamic approach aimed at estimating the average annual yield and discounted costs that optimize the value and rotation period of miscanthus (Bourgeois et al., 2014 Ben Fradj et al., 2016). Using the Faustmann rule, the farmer’s goal is to choose the rotation period T that maximizes the miscanthus value. The net cumulative revenue over one rotation of duration T, discounted at the beginning of the rotation, is as follows:

$$V_m(T) = -c_0 + \sum_{t=1}^{T} M(t) e^{-(\delta-\alpha)t}$$ (1)
Where $c_0$ is the rotation cost paid over each $T$ cycle duration, $\delta$ is the discount rate and $\alpha$ is the inflation rate. $M(t)$ is the annual gross margin. It is noted that first year growth is insufficient to be economically worth harvesting ($M(1) = 0$) and for $t \geq 2$, $M(t) = p_t y(t) - c_t$, $c_t$ are the annual production costs paid at any of the (T-1) years and $p_t$ is the price of a ton dry matter of the harvested miscanthus yield at $t$.

When the farmer opts for cultivating miscanthus, he is assumed to maximize the cumulative revenue over infinite time denoted by $W(T)$

$$W(T) = \sum_{n=1}^{\infty} V_m(T) e^{-\delta n T}$$

(2)

That leads to provide the annual equivalent discounted revenue which will be introduced in AROPAj, as well as the average yield.

### 2.2 Introduction of stochastic process in the Faustmann modeling

In this section, we suppose that only biomass quantity is random and all the other economic factors remain unchanged. The stochastic yield process is based on a beta distribution that represents how the harvest yield expectations change during the course of a rotation period.

Our model assumes $T + 1$ periods in a growing cycle. The first period begins at time $t = 0$, the planting date, and continues to $t = T$, the clear-cutting date. We are interested in yield expectations at each time $t$. We assume that yield realizations are positive and finite, and that the distribution is restricted to values between 0 and the value given by the potential function $y(t)$. Each year, the expected harvest yield is multiplied by $\epsilon_t = E[\tilde{y}(t)]$. Each random yield $\tilde{y}$ follows a beta distribution. The standard beta probability distribution function for a random variable $\tilde{y}$ is

$$f(\tilde{y}, \beta, \gamma) = \frac{\tilde{y}^{(\beta-1)}(1-\tilde{y})^{(\gamma-1)}}{B(\beta, \gamma)}$$

(3)

where $B(\beta, \gamma) = \int_{0}^{1} t^{\beta-1}(1-t)^{\gamma-1} dt$, $0 < \tilde{y} < 1$, and $\beta, \gamma > 0$.

The proposed technique consists of generating, at each time $t$, samples of yield according to the theoretical beta function given by equation 3. This randomized generation is renewed over a large number of succeeding cycles, when the cycle length is given. The sample beta distribution is fitted by an envelope represented by the potential yield function of miscanthus (Ben Fradj et al., 2016). An example of generated random sample and the deterministic distribution is presented by Figure 1.

![Figure 1: Miscanthus yield distribution: an example of generated yield sample](image)

Using the Faustmann rule, the farmer’s goal is to choose the rotation period $\bar{T}$ that maximizes the expected miscanthus value. The objective function is determined by maximizing the expected sum of the
annually discounted profits in an infinite sequence, at time $t = 1$. Therefore, the discounted expected value of the cumulative net income is as follows:

$$\tilde{W}_m(T) = E \left[ \sum_{n=1}^{\infty} \left( -c_0 + \sum_{t=1}^{T} \tilde{M}(t) e^{-\delta(n-1)T} \right) e^{-\delta(n-1)T} \right]$$

(4)

Where $\tilde{M}(t) = p_t f(\tilde{y}, \beta, \gamma) - c_t$.

2.3 Hypothesis and Scenarios

In our case, the AROPAj-model is run solely France which divided into 157 farm groups clustered into 21 regions. The introduction of miscanthus in the model requires estimates of its yield at the farm group level. However, miscanthus has been only recently introduced in France, and there is few, or no available data on yield for the full rotation period. We assume that miscanthus yield increases with the quality of the land, as does wheat which is a common crop presented in 80% of the French farm-groups in AROPAj. In order to study the sensitivity of land use change to modifications in miscanthus yield, we proceed to a homogeneous reduction of miscanthus yield from 0 up to 100% by 10% increments over all farm groups. Because of the heated "Food vs Fuel" debates, we assume that the part of the farm group’s UAA devoted to miscanthus does not exceed 20%.

As miscanthus nitrogen demand is low and nitrate leaching is potentially high in the first year after planting, zero nitrogen fertilisation during the first two years of cultivation is recommended, unless grown on poor soils. We therefore suppose that fertilizers are added only as of the the third year until the end of the rotation. Based on information provided by agricultural experts, miscanthus N-losses from the third year are near to zero because of its developed root system. Thus, we suppose that miscanthus engenders no N-losses.

3 Sensitivity analysis

This section provides an analysis of one factor, i.e. yield, which comes into play when the farmer is given a possibility of adopting miscanthus. We highlight the differences between the deterministic yield expectations and stochastic yield expectations of miscanthus in terms of profitability, land-use allocation and N-losses.

Based on the initial estimates of parameter values, the Cumulative Net Margin (CNM), is calculated by adding the potential net margins acquired over the rotation period. It indicates when the farmer will maximize his profit if he decide to cultivate miscanthus. However, planting this crop requires an important investment which will be returned as of the 4th year in the deterministic case and as of the 10th year in the uncertain case (Figure 2). The simulated CNMs for the potential and random cases are compared with the CNMs that would be calculated in the case of 50% reduction of the miscanthus potential. The higher the miscanthus yield, the earlier the investment is returned. Indeed, the investment return starts from the 9th for 50% reduction of yield. In forestry economics, the economically optimum rotation age is defined as the age when the harvest generates the maximum discounted margin. In other words, the optimal rotation of miscanthus corresponds to the maximum CNM. Table 1 shows the evolution over time of the cumulative net margin for the potential, 50% reduction of potential and uncertain cases. In the first two cases, the optimal rotation age is 16 years. The introduction of a random factor in the growth function delays the optimal rotation for 5 years, until to 21 years.

Regarding the distribution of the yield, the net margin and the rotation period over the farm groups, Figure 6 and 4 shows the frequency of these parameters in the potential and uncertain cases as well as in the case of a 50% reduction of the potential. For the first case, the yield among AROPAj farm-groups is typically about 25 tDM/ha. Yields between 20 and 30 tDM/ha are very frequent and the lowest yield is 4 tDM/ha. Regarding the Net Margin (NM), the highest value is about 1,300 €/ha and the lowest is 300 €/ha. The NMs between 1,000 and 1,600€/ha are very frequent. The optimal age is typically about 16 years and rotation periods between 15 and 17 years are frequent. In the case of a 50% reduction of miscanthus potential, the highest yield is about 12 tDM/ha. Yields between 11 and 15 tDM/ha are very frequent and the lowest yield
Figure 2: The cumulative net margin of miscanthus calculated for potential (blue), 50% of potential (yellow) and random cases (red).

<table>
<thead>
<tr>
<th>Year</th>
<th>Potential</th>
<th>50% of Potential</th>
<th>Random</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-28,0759</td>
<td>-28,0759</td>
<td>-62,4877</td>
</tr>
<tr>
<td>2</td>
<td>-14,3888</td>
<td>-14,3888</td>
<td>-32,0248</td>
</tr>
<tr>
<td>3</td>
<td>-6,90799</td>
<td>-9,45691</td>
<td>-20,3935</td>
</tr>
<tr>
<td>4</td>
<td>-1,88678</td>
<td>-6,36332</td>
<td>-14,0164</td>
</tr>
<tr>
<td>5</td>
<td>1,80241</td>
<td>-4,17902</td>
<td>-9,24406</td>
</tr>
<tr>
<td>6</td>
<td>4,52661</td>
<td>-2,59875</td>
<td>-6,26544</td>
</tr>
<tr>
<td>7</td>
<td>6,51571</td>
<td>-1,4554</td>
<td>-4,65318</td>
</tr>
<tr>
<td>8</td>
<td>7,95337</td>
<td>-0,63104</td>
<td>-1,97487</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>10,9317</td>
<td>1,1047</td>
<td>1,97345</td>
</tr>
<tr>
<td>15</td>
<td>10,9943</td>
<td>1,14751</td>
<td>1,92835</td>
</tr>
<tr>
<td>16</td>
<td>11,0033</td>
<td>1,15941</td>
<td>2,55291</td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>10,7138</td>
<td>1,01641</td>
<td>2,87434</td>
</tr>
<tr>
<td>21</td>
<td>10,594</td>
<td>0,952168</td>
<td>2,93158</td>
</tr>
<tr>
<td>22</td>
<td>10,4611</td>
<td>0,881539</td>
<td>1,85044</td>
</tr>
<tr>
<td>23</td>
<td>10,3268</td>
<td>0,806161</td>
<td>1,93928</td>
</tr>
</tbody>
</table>

Table 1: Evolution of the cumulative net margin of miscanthus (k€/ha) in potential, 50% of potential and random cases.

is 2 tDM/ha. Regarding the NM, the highest value is about 500€/ha and the lowest is 100€/ha. The NMs between 200 and 600 are frequent. The optimal age is typically about 16 years. Unusually rotation periods about 16 years are recorded in some groups. In the uncertain case, the typical yield is about 16 tDM/ha. Yields between 12 and 19 tDM/ha are frequent. The lowest yield is 4 tDM/ha. As the NM is concerned, the highest value is about 700 €/ha and the lowest is 100 €/ha. The NMs between 500 and 1,000 €/ha are very frequent. The optimal age is typically about 20 years and rotation periods between 16 and 24 years are frequent.

In addition to the long-term profitability problem, the farmer has to deal with questions related to land use change. Figure 5 shows that the miscanthus is more abundant in the French North than the South. Indeed, high yields are mostly recorded in the North where climatic conditions are more favorable and water availability is higher. Another main result is that the area devoted to miscanthus is sensitive to a change in its yield potential. In fact, introducing yield uncertainty delays the adoption of miscanthus until its potential is sufficiently high to motivate the farmer’s decision. More specifically, in the deterministic case, miscanthus is adopted when 30% of the yield potential is reached. In the stochastic case, this occurs when 60% of the yield potential is reached. Favorable yield potential helps the farmer make the decision to adopt miscanthus despite the high degree of uncertainty. Figure 6 shows a significant decrease in the food-crop areas when
miscanthus is progressively introduced with increasing yield potential in the certain and uncertain cases. Cereals and pasture areas are the most affected areas that undergo remarkable changes. More precisely, when the miscanthus yield reaches its full potential (as estimated by our method), cereals’ areas decrease by 25% in the certain case, and by 7% in the uncertain one. At the same time, grasslands decrease by 13% and 7.5%, in the certain and uncertain cases, respectively. Simulations show that changes in land use have a substantial effect on the N-input demand and N-losses. Even though the results in the deterministic and uncertain cases seem to be alike in terms of trend, they represent different magnitudes. Figure 7 shows that fertilizer consumption decreases when miscanthus becomes more profitable. Lower N input levels are reached by increasing the miscanthus yield level. For instance, we notice a decrease of fertilizer demand by 22% in the deterministic case and by 13% in the uncertain case when the miscanthus yield reaches its full potential. This decrease in N-demand leads to a 26% reduction in nitrate losses in the certain case and 13% in the uncertain one. The introduction of miscanthus decreases also the NH$_3$, N$_2$O emissions, but this decrease still be insignificant in comparison to NO$_3$ losses.

4 Discussion

Because of their high land-use efficiency, lignocellulosic crops are ideally cultivated on marginal areas in order to reduce the competition between food and biofuel production. However, we show in the stochastic case that these crops will be cultivated on the most fertile arable areas which are usually reserved for food production. Indeed, when the yield is governed by a stochastic process, the farmer is discouraged from planting miscanthus. The farmer has to be careful before making any decision to invest, so that if he decides to adopt miscanthus, he ensures its cultivation on the most productive areas on which the yield potential is the highest. This decision causes direct and indirect changes in land allocation due to the displacing of food activities and the conversion of uncultivated areas to biofuel production. Marginal areas and grasslands are the first areas that will be converted to produce biofuels. However, the progressive introduction of miscanthus with increasing yield potential reduces not only the grasslands and marginal areas, but also cereals’ areas. In all cases, the farm group’s UAA devoted to this crop does not exceed 13%.
Miscanthus x giganteus is an environmentally friendly crop which requires low input level. Introducing this crop into the farming system reduces the arable areas generally devoted to crops, which are characterized by a high N-input demand, i.e. cereals. This decrease greatly reduces nitrate losses. This result is the consequence of a hypothesis that stipulates that as nitrogen demand is low and nitrate leaching is potentially high in the two first years after planting, the addition of fertilizers starts only from the 3rd year when the root system is well developed. Nevertheless some soil scientists contest this advantage, arguing that the root depth and the perennial character of miscanthus would be damageable for the soil structure.

A positive present value of miscanthus accounts for a good argument in favor of the adoption of this crop, but it must be examined more rigorously. After showing the sensitivity of the value of miscanthus to a change in the yield, it seems primordial to test the variation of other parameters. Testing alternative assumptions that are more or less favorable in terms of rotation age and net value provides some indication of how certain unexpected parameters would have a beneficial or critical effect on crop profitability. These parameters are the establishment cost \( c_0 \), the annual cost \( c \) and the discount rate \( \delta \). The price is perceived as a certain variable since miscanthus is sold under contract to the renewable energy market at a fixed price and annual inflation. Figure 8 shows the sensitivity of both the optimal rotation age (ORA) and the net present value (NPV) to a change in the establishment cost \( c_0 \), the annual costs and the discount rate \( \delta \). If \( c_0 \) were 30% higher than expected, the ORA would increase, and the NPV would be almost 100 €/ha less. If \( c_0 \) were 30% lower than expected, the ORA would decrease, and the NPV would be almost 100 €/ha higher than

![Figure 5: Comparison of miscanthus land area between the certain and uncertain cases](image)

![Figure 6: Comparison of land-use allocation between the certain and uncertain cases](image)
expected. However, if $c_0$ were 80% lower than expected, the ORA would decrease down to 1 year, and the NPV would be almost 1000 €/ha higher than expected. In this case, the farmer would consider miscanthus as an annual crop and clear-cut it at the end of the 1st year to replant it again in the next year. Concerning the discount rate ($\delta$), a variation in this parameter would delay or move forward the ORA and decrease or increase the NPV. A remarkable change would occur only when $\delta$ is lower than 3%. In fact, if $\delta$ equaled 20%, the NPV would be almost 500 €/ha higher and the miscanthus would be clear-cut at the end of the 8th year instead of the 16th year. In the case of annual costs ($c$), an increase would move forward the ORP and decrease the NPV. In fact, if $c$ were 10% higher than expected, the NPV would be almost 100 €/ha less, but the ORA would move forward to 15 years. If $c$ were 10% lower than expected, the NPV would be almost 100 €/ha higher and the ORA would remain the same (16 year). A strong decrease in $c$ would have an impact on the NPV and the ORA. If $c$ were 80% lower than expected, the ORA would increase up to 18 years, and the NPV would be almost 200 €/ha higher than expected. Compared to $c_0$, the annual costs are less important. The farmer can incur them by delaying the rotation period, especially when the value of miscanthus is high enough.

5 Conclusion

We have assessed the sensitivity of the supply of a perennial bioenergy crop, i.e., miscanthus, to physical and economic factors. We show that the adoption of miscanthus depends on its present value that is calculated in certain and uncertain cases. An uncertainty in yield potential, establishment cost or discount rate alters the future value of this crop. An increasing bioethanol demand increases the miscanthus areas, which accentuate the direct and indirect land allocation. A detailed study is needed to analyze the interactions between agricultural activities inside a representative farm group.
Figure 8: Sensitivity analysis
References


Faustmann, M. (1849). *On the determination of the value which forest land and immature stands possesses for forestry*. In m. gane (ed) (1968) martin faustmann and the evolution of discounted cash flow, oxford forest institute paper 42 edition. (Commonwealth Forestry Institute, Oxford University).


