The Value of Being Socially Responsible. A DEA Approach for Analyzing Efficiency and Recovering Shadow Prices of CSR Activities

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Selected Paper prepared for presentation at the
2016 Agricultural & Applied Economics Association Annual Meeting,
Boston, Massachusetts, July 31 - August 2

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Abstract

Corporate social responsibility (CSR) has experienced a rather unique and difficult path towards becoming a well received subject of academic interest likely because there is a certain degree of ambiguity and disagreement with respect to its definition and the nature of the core principles that identify CSR. The existing research has analyzed mainly the business management and finance implications of CSR trying to justify its strategic role. This paper takes a very different perspective and attempts to formalize and explain the process through which CSR is created incorporating it into a production framework. To this extent, we develop a joint production model for characterizing the technology and representing the transformation process of multiple inputs into multiple (desirable/marketed and undesirable) outputs. CSR is one of these outputs and plays a mitigation role. The application focuses on the food and beverages manufacturing sector which is particularly interesting as it faces both very specific and more common CSR challenges. Because of data limitation, a non-parametric DEA approach is used to implement the empirical analysis. The general and flexible production model together with the parsimonious and computationally accessible empirical methodology adopted in this study constitute a powerful framework for characterizing the production technology for CSR, analyze technical efficiency and deriving a system of internal shadow values for CSR that allows for evaluating the overall value as well as the marginal impact of engaging in socially responsible activities for the firm. Our results indicate that in the sample of 175 firms included in the analysis efficiency levels are very high as approximately 75 percent of the firms are found to be technically efficient. For inefficient and just efficient firms the average shadow value of socially responsible activities is positive, implying that the cost of implementing these activities is compensated by their mitigating effect. For extreme efficient firms the average marginal value of increasing their socially responsible commitment is positive indicating that more CSR is considered beneficial for adding value to the firm. Conversely, the average marginal value of decreasing the CSR effort is negative indicating that lower levels of CSR are perceived as costly and damaging so that firms want to be compensated for reducing their socially responsible performance.

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1 Introduction

Corporate social responsibility (CSR) has experienced a rather unique and difficult path towards becoming a well received subject of academic interest likely because there is a certain degree of ambiguity and disagreement with respect to its definition and the nature of the core principles that identify CSR. According to the World Business Council for Sustainable Development (1998) ”Corporate Social Responsibility is the continuing commitment by business to contribute to economic development while improving the quality of life of the workforce and their families as well as of the community and society at large”. While CSR seeks to create long-term economic value as any conventional business activity, this definition highlights the fact that its scope is much broader as it calls for a more comprehensive commitment of the firm to society. The growing attention toward CSR among not only firms, but also civil society and government, has become a reality particularly since 1990. The advancement of CSR to a core management or board-level function together with the dramatic increase in CSR implementation and reporting has allowed scholars to have access to CSR data so that the focus of research could shift from a normative and theoretical framework to a more applied one. Nonetheless the literature on CSR, so far, has been mainly restricted to the business management and financial fields and only in the 2000s this literature has started to build a direct connection between CSR and the economic concept of profit maximization.

Baron (2001) and McWilliams and Siegel (2001) were the first to explicitly model ”strategic” and ”profit maximizing” CSR suggesting that firms undertake CSR activities expecting a net benefit from them. Both contributions emphasize the fact that CSR is a way for firms to compete for socially responsible consumers by either linking their social contribution to product sales or adding social attributes and features to their products. A key implication of this perspective on CSR is that it represents a product differentiation strategy to gain competitive advantage as argued by Bagnoli and Watts (2003) and Siegel and Vitaliano (2007). In addition, there is empirical evidence in the literature supporting the conjecture that CSR practices have an influence on consumers' purchasing intentions and willingness to pay (see Creyer (1997), Auger et al. (2003), Pelsmacker et al. (2005), or Ailawadi et al. (2011)).

CSR has been analyzed also in the theoretical finance literature with models assuming the existence of a class of investors who prefer to invest in CSR stocks, nonetheless the impact of their preferences on the value of the stocks is not always clear. Empirical evidence, on the other hand, seems to document a negative association between CSR and systemic risk and cost of equity capital, and a positive association between CSR and firm value and shareholder wealth. Thus, higher levels of CSR (or higher CSR scores) imply a lower systemic risk, a lower cost of capital, a higher firm value, and higher stocks evaluations (see Sharfman and Fernando (2008), Oikonomou et al. (2012), Margolis et al. (2009), Galema et al. (2008), or Dimson et al. (2012)).

All these contributions, in different ways and with different emphasis, try to identify and quantify to what extent the pursuit of a more advanced and comprehensive social agenda that may go beyond short-run profits or mandated minimum standards impacts the economic value of the firm. In fact, they provide formal and rigorous support to a conviction that has already reached a wide consensus among firms, consumers, and policy makers; i.e. that CSR should be a prominent business practice.

In spite of the efforts spent in corroborating the strategic role of CSR, practically no work has been done to formalize and explain the process through which CSR is created. Simply put, the existing literature justifies CSR practices with economic, managerial, or financial motivations trying to provide some insight on whether CSR is good for business in general and for what it might be good in particular. However, considerations regarding the way CSR is produced and what implications CSR production has on a firm's dynamics, such as the technology underlying CSR production, the impact of CSR on the other outputs produced, or the kind of inputs needed to produce CSR are completely ignored. This may be due to the fact that CSR is a relatively new and still unfolding concept for both firms and scholars, therefore it is not easy to precisely identify, define, model, measure, and quantify it. Even so, as existing research has been almost exclusively focused on *why* CSR is done, this study takes a very different perspective and attempts to shed light on *how* CSR is done by incorporating it into a formal production framework. This approach seems especially appropriate as a lot of CSR activity is directly related to production. The application focuses on the food and beverages manufacturing sector that is particularly interesting since it faces specific CSR challenges, such as food safety controversies, demand for healthier food products, responsible sourcing of raw materials, along with more common CSR issues such as water and energy efficiency, supply chain management, labor standards, and safety in the workplace.

We adopt a joint production model for characterizing the technology and representing the transformation process of multiple inputs into multiple outputs. The types of different inputs and outputs involved in the production process are a distinctive feature of the model. In particular, each firm is assumed to produce a desirable, marketed output but, because the production of this desirable output may require the use of some "socially irresponsible" input, an undesirable output can be generated along with the desirable one. Thus, the firm needs to engage in socially responsible activities, namely CSR, which is an additional output produced to mitigate the unwanted output. Common inputs as well as "socially responsible" inputs are used to produce CSR. The joint technology is obtained as a composition of two separate sub-technologies: one describing the desirable-output production and the other describing the generation of the undesirable output. CSR is the link between these two technologies as it simultaneously represents the opportunity cost of producing socially responsible activities in terms of desirable output and its mitigating effect with respect to undesirable output.

Empirically, the implementation of the analysis relies on a parsimonious non-parametric approach known as Data Envelopment Analysis (DEA) which allows for constructing the joint technology as the intersection of the desirable production and the undesirable production sub-technology. Once the technology is fully characterized, a set of internal, shadow values for inputs and outputs can be obtained that reveal how much the production of CSR is worth to the firm in terms of the other outputs produced. Moreover, even though DEA technologies are not amenable to standard differential calculus arguments, at least for the extreme efficient firms, as they display kinks in the primal (quantity) space that maps into flat portions in the dual (price) space, recent developments by Chambers and Färe (2008) are used to derive the shadow value of CSR as a measure of willingness to gain for producing one more unit of CSR and willingness to lose for relinquishing the production of one unit of CSR.

Our results indicate that in the sample of 175 firms included in the analysis efficiency levels are very high as approximately 75 percent of the firms are found to be technically efficient. For inefficient and just efficient firms the average shadow value of socially responsible activities is positive, implying that the cost of implementing these activities is compensated by their mitigating effect. For extreme efficient firms the average marginal value of increasing their socially responsible commitment is positive indicating that more CSR is considered beneficial for adding value to the firm. Conversely, the average marginal value of decreasing the CSR effort is negative indicating that lower levels of CSR are perceived as costly and damaging so that firms want to be compensated for reducing their socially responsible performance.

The reminder of the paper is organized as follows. Section 2 presents a concise discussion on the existing literature on multi-output production focusing on the issues related to modeling multiple outputs technologies, especially in the presence of bad outputs. This discussion provides the insight and motivation behind the theoretical framework that is also formalized in this section. Section 3 illustrates the empirical approach and explains how DEA methods can be used to characterize the technology, analyze technical efficiency and derive a set of internal values of inputs and outputs. This section also shows how to calculate shadow values, especially the shadow value of CSR, when the technology is not smooth and therefore non-differentiable. Section 4 describes the data used to carry out the empirical analysis. In Section 5 the results of the analysis are presented and discussed. Section 6 concludes.

2 Theoretical Framework

2.1 Issues and challenges of modeling multi-output and joint technologies

Multiple outputs are the rule rather than the exception at the micro level of production. This is because the same input, or set of inputs, can be employed to produce different outputs and because there are many instances of jointness in production that can reach the extreme form of different outputs needed to be produced in fixed proportions.

The first formalization of a multi-output process of production dates back to Klein (1947) who, in his study of U.S. railroads, pioneers the research on production functions by explicitly allowing multiple inputs to be used in the production of multiple outputs. However, as pointed out by Nerlove (1965) and Mundlack (1963 and 1964), modeling and estimating multi-output production relations presents several challenges mainly related to the choice of a proper aggregation procedure that keeps the measure of aggregate output constant at all points of a given transformation curve and guarantees convexity. In addition, Lau (1972) and Hasenkamp (1975) identify a set of desirable properties that a joint production function representing the technology of a M-output, N-input firm should have. These properties are known as free disposability of outputs and inputs and are mathematically expressed as $\frac{\partial f(y,x)}{\partial y_m} \geq 0 \quad \text{for } m = 1, ..., M$ and $\frac{\partial f(y,x)}{\partial x_n} \leq 0 \quad \text{for } n = 1, ..., N$, respectively with $f(y,x) \leq 0$ being a transformation function specifying the input-output combinations that are technically feasible. Free disposability of outputs implies that $\frac{dy_m}{dy_r} = -\frac{\partial f(y,x)}{\partial y_r} / \frac{\partial f(y,x)}{\partial y_m} < 0 \quad \text{for } m, r = 1, ..., M$, while free disposability of inputs entails that $\frac{dy_m}{dx_n} = -\frac{\partial f(y,x)}{\partial x_n} / \frac{\partial f(y,x)}{\partial y_m} > 0 \quad \text{for } m = 1, ..., M, \ n = 1, ..., N$. An economically meaningful interpretation of free disposability of outputs is that a technically efficient firm (for which $f(y,x) = 0$) should be producing a combination of outputs for which an increase in one output always generates the decrease in at least one other output. On the other hand, free disposability of inputs can be simply characterized as the requirement for input usage not to be expanded to yield negative marginal productivities.

A natural extension to the standard multi-output framework consists of recognizing that not all the outputs produced are of the same kind, i.e. joint production often occurs with the generation of undesirable, bad outputs, usually represented in the form of pollution/residuals. Baumol and Oates (1988) propose to extend the general representation of multi-output production possibilities to incorporate the generation of pollution as $f(y, z, x) = 0$ where z is a M'-dimensional vector of residuals. In this context the vector y represents desirable outputs (good outputs) that are the purpose of the production activity while the vector z represents undesirable outputs (bad outputs), i.e. residuals/pollution, that are an unavoidable by-product of the main production process. If pollution is treated as an output and assuming efficiency in production, the output free disposability condition requires that $\frac{\partial f(y,z,x)}{\partial z_k} > 0$ for $k = 1, ..., M'$ which implies that $\frac{dy_m}{dz_k} = -\frac{\partial f(y,z,x)}{\partial z_k} / \frac{\partial f(y,z,x)}{\partial y_m} < 0 \quad \text{for } m = 1, ..., M,\ k = 1, ..., M'$. That is, for a technically efficient firm the increase in one desirable output should always come at the expenses of a decrease in pollution. However, in the absence of explicitly specified abatement activities, this does not seem to be an assumption that reasonably captures the phenomenon of residual generation. In fact, if residuals are generated together with desirable outputs, it is not plausible to assume that it is always possible to increase those outputs while reducing residuals. On the other hand, treating pollution/residuals as an input implies that $\frac{dy_m}{dz_k} > 0$ because of free disposability of inputs but, for the same reason, also implies that $\frac{dz_k}{dx_n} < 0 \quad \text{for } k = 1, ..., M',\ n = 1, ..., N'$. Nonetheless, this is also problematic as it is certainly not appropriate to assume that it can be technologically feasible to indefinitely increase the usage of a bad input keeping the environmental resource required to absorb it fixed.

Since considering bads as outputs under standard output free disposability assumptions is questionable and considering bads as inputs is not appropriate because it misrepresents some relevant fatures pertaining to a technology that includes undesirable outputs, a new strand of research, inaugurated by Färe et al. (1986), proposes to assume weak disposability of bad outputs together with free disposability of good outputs and inputs instead. Weak disposability imposes that a reduction of bad outputs is feasible only if good outputs are simultaneously reduced, given a fixed level of inputs. Whereas free disposability guarantees maximum flexibilities because of the pure trade-off between outputs, weak disposability introduces a restriction on the trade-off between goods and bads capturing the opportunity cost of reducing bads. Furthermore, weak disposability of bads is usually paired with null jointness, a property that describes the inevitability of residuals generations, that is if a good output is produced in a positive amount, some bad output must also be produced.

Even if weak disposability and null jointness are helpful in rendering some relevant features of a multiple output technology with bads in a more realistic and appropriate way, the models adopting these assumptions appear still incomplete. This is because in this literature the generation of bad outputs remains in the background without being explicitly modeled, i.e. the inputs involved in the production of bads and the relations between goods and bads implied by the technology remain unspecified. Murty et al. (2012) are the first to introduce the generation of bad outputs directly in the modeling of a pollution-generating, or "by-production", technology. In particular, they argue that models which treat pollution as a freely disposable input, or as a weakly disposable and null joint output, may generate unreasonable implication for the trade-offs between inputs, good outputs, and pollution. Their proposed solution to these issues consists of specifying two distinct production possibility sets: one describing the technology for producing desirable (good) outputs and the other representing the technology behind the undesirable residuals production mechanism. The overall technology set is then the intersection of these two sets. In addition, they impose a peculiar condition, usually called costly disposability, on the residuals which imposes maximum inflexibility by enforcing a positive relationship between desirable outputs and residuals that should be interpreted as the inevitability of deploying productive resources for the disposal of pollution. Assuming costly disposability tacitly opens the door to the last piece of the puzzle, i.e. the express inclusion of abatement in a multi-output framework in the presence of bads. The option (or the requirement) to implement abatement activities represents a tool for mitigating the negative effects that such collateral outputs may cause.

2.2 A multi-output model of Corporate Social Responsibility

In light of these considerations on multi-output and joint production processes, we model CSR incorporating it into a multiple input multiple output technology framework and assuming that CSR is an additional output in the production process. This is because CSR activities are not freely available nor easily bought on the market like standard inputs, therefore their implementation requires firms to allocate resources (such as capital, labor, and materials) to the production of CSR effectively diverting them from the production of other outputs. The fact that CSR needs to be actually generated at an opportunity cost makes it more characteristic of an output rather than an input. In addition, because of the nature of socially responsible activities usually carried out by firms (i.e. environmental programs, sustainability programs, community programs), CSR appears to embody the notion of mitigation particularly well. Much like abatement is implemented to clean up pollution, CSR can be implemented to improve a dirty production processes, to support the use of sustainable inputs, or to establish a good reputation among consumers and community. Hence, building on Murty et al. (2012), the following model regards CSR as an output produced to mitigate the negative effects of another undesirable output, i.e. an output that is unwanted but inevitably generated within the production process and can potentially be detrimental to the firm.

Consider a joint production technology in which N^1 inputs x_1, x_2 and x_3 are utilized to produce M^2 outputs that can be categorized as desirable output(s) y_D, undesirable output(s) y_U, and the socially responsible output(s) y_R. More specifically, y_D is the primary, marketed output for which the production process is set up, y_U is a by-product generated during the production of the desirable output, and y_R consists of socially responsible activities implemented to reduce the undesirable output. The production process of the desirable output y_D requires inputs x_1, x_2, and x_3, where x_1 is a standard input used in the production of y_D, x_2 is a socially irresponsible input that leads to the generation of the by-product, i.e. the undesirable output y_U, and x_3 is a socially responsible input that, while used to produce y_D, specifically contributes to the production of the socially responsible output y_R as well. Because of the nature of this joint technology, a firm that aims to be socially responsible needs to engage in the production of y_R to mitigate the undesirable output. Nonetheless, producing y_R is costly, meaning that a firm has to divert resources away from the desirable output production to generate socially responsible activities.

The *by-production*[3], or joint technology T can be characterized as the intersection of two different technologies T_1 and T_2, so that

$$T = T_1 \cap T_2 \tag{2.1}$$

$$T_1 = \left\{ \langle y_D, y_U, y_R, x_1, x_2, x_3 \rangle \in \mathbb{R}_+^{M+N} \mid f(y_D, y_R, x_1, x_2, x_3) \leq 0 \right\} \tag{2.2}$$

$$T_2 = \left\{ \langle y_D, y_U, y_R, x_1, x_2, x_3 \rangle \in \mathbb{R}_+^{M+N} \mid y_U \geq q(y_R, x_2) \right\} \tag{2.3}$$

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[1] Where N is a $(N_1 + N_2 + N_3)$-dimensional vector of inputs.

[2] Where M is a $(M_D + M_U + M_R)$-dimensional vector of outputs.

[3] By-production is a term specifically coined by Murty et al. (2012) to describe a technology in which the desirable production process generates unwanted residuals, or by-products.

with f and q being continuously differentiable functions. The set T_1 is a conventional convex technology set representing the transformation process of the inputs into the desirable output and the socially responsible output. Assuming that f satisfies

$$f_{y_D}(y_D, y_R, x_1, x_2, x_3) \geq 0 \tag{2.4}$$

$$f_{y_R}(y_D, y_R, x_1, x_2, x_3) \geq 0 \tag{2.5}$$

$$f_{x_n}(y_D, y_R, x_1, x_2, x_3) \leq 0 \quad \text{for } n = 1, 2, 3 \tag{2.6}$$

then the technology T_1 displays the standard free disposability properties in desirable output, socially responsible output, and inputs, respectively, i.e.

$$\langle y_D, y_U, y_R, x_1, x_2, x_3 \rangle \in T_1,\ \bar{y}_D \leq y_D \Rightarrow \langle \bar{y}_D, y_U, y_R, x_1, x_2, x_3 \rangle \in T_1 \tag{2.7}$$

$$\langle y_D, y_U, y_R, x_1, x_2, x_3 \rangle \in T_1,\ \bar{y}_R \leq y_R \Rightarrow \langle y_D, y_U, \bar{y}_R, x_1, x_2, x_3 \rangle \in T_1 \tag{2.8}$$

$$\langle y_D, y_U, y_R, x_1, x_2, x_3 \rangle \in T_1,\ \bar{x}_1 \geq x_1 \Rightarrow \langle y_D, y_U, y_R, \bar{x}_1, x_2, x_3 \rangle \in T_1 \tag{2.9}$$

$$\langle y_D, y_U, y_R, x_1, x_2, x_3 \rangle \in T_1,\ \bar{x}_2 \geq x_2 \Rightarrow \langle y_D, y_U, y_R, x_1, \bar{x}_2, x_3 \rangle \in T_1 \tag{2.10}$$

$$\langle y_D, y_U, y_R, x_1, x_2, x_3 \rangle \in T_1,\ \bar{x}_3 \geq x_3 \Rightarrow \langle y_D, y_U, y_R, x_1, x_2, \bar{x}_3 \rangle \in T_1 \tag{2.11}$$

For simplicity it is further assumed that the technology T_1 is independent of y_U, which implies that the production of the undesirable output does not have any direct effect on the production of the desirable output y_D.

The set T_2 is also convex and represents the undesirable output generating process. Assuming that q satisfies

$$q_{y_R}(y_R, x_2) < 0 \tag{2.12}$$

$$q_{x_2}(y_R, x_2) > 0 \tag{2.13}$$

and given the definition of T_2 in (2.3), the following properties hold

$$\langle y_D, y_U, y_R, x_1, x_2, x_3 \rangle \in T_2,\ \bar{y}_U \geq y_U \Rightarrow \langle y_D, \bar{y}_U, y_R, x_1, x_2, x_3 \rangle \in T_2 \tag{2.14}$$

$$\langle y_D, y_U, y_R, x_1, x_2, x_3 \rangle \in T_2,\ \bar{y}_R \geq y_R \Rightarrow \langle y_D, y_U, \bar{y}_R, x_1, x_2, x_3 \rangle \in T_2 \tag{2.15}$$

$$\langle y_D, y_U, y_R, x_1, x_2, x_3 \rangle \in T_2,\ \bar{x}_2 \leq x_2 \Rightarrow \langle y_D, y_U, y_R, x_1, \bar{x}_2, x_3 \rangle \in T_2 \tag{2.16}$$

These properties are sometimes referred to costly disposability of undesirable output, socially responsible output, and by-product generating input. The relations expressed in (2.14), (2.15), and (2.16) simply describe the fact that the undesirable output y_U is a by-product of the production process whose disposability is not free. As shown in (2.12) and (2.15), the trade-off between the undesirable output and the socially responsible output y_R is negative, capturing the mitigating effect that socially responsible activities have on the production of the undesirable output. On the other hand, as shown in (2.13) and (2.16), the trade-off between the undesirable output y_U and the input x_2, which is responsible for the generation of y_U, is non-negative, capturing the fact that the optimal level of undesirable output is increasing in the use of the by-product generating input x_2.

Given the properties of T_1 and T_2 derived above, it is now possible to rationalize the properties of the joint technology T. Specifically, T is convex and satisfies free disposability of the desirable output and the first and third input because T_1 satisfies the same condition with respect to y_D, x_1 and x_3 and T_2 does not impose any restrictions on them. However, T violates free disposability of the socially responsible output and of the second input because, while T_1 satisfies the free disposability condition with respect to y_R and x_2, T_2 violates free disposability with respect to y_R and imposes a restriction on x_2 that is, in fact, the exact opposite of free disposability. Finally, T displays costly disposability with respect to the undesirable output because T_1 does not impose any restriction on y_U while T_2 implies that y_U can be mitigated only through a decrease in the use of the undesirable output generating input x_2, or the costly implementation of the socially responsible activities y_R.

The weakly efficient points of T are defined as the quantity vectors $\langle y_D, y_U, y_R, x_1, x_2, x_3\rangle \in T$ satisfying $f(y_D, y_R, x_1, x_2, x_3) = 0$ and $y_U = q(y_R, x_2)$. This is because, if a quantity vector $\langle y_D, y_U, y_R, x_1, x_2, x_3\rangle \in T$ satisfies $f(y_D, y_R, x_1, x_2, x_3) < 0$, it is technologically feasible to decrease the usage of the inputs x_1 and x_3 without affecting the production levels of the desirable output y_D and the usage of x_2, therefore such a vector cannot be efficient. Similarly, a quantity vector $\langle y_D, y_U, y_R, x_1, x_2, x_3\rangle \in T$ satisfying $y_U > q(y_R, x_2)$ cannot be efficient because it is technologically feasible to decrease the production level of the undesirable output y_U without modifying the usage of the inputs and the production level of the desirable output.

Consider the quantity vector $\langle \hat{y}_D, \hat{y}_U, \hat{y}_R, \hat{x}_1, \hat{x}_2, \hat{x}_3\rangle$ which is a weakly efficient point of T since it satisfies $f(\hat{y}_D, \hat{y}_R, \hat{x}_1, \hat{x}_2, \hat{x}_3) = 0$ and $y_U - q(\hat{y}_R, \hat{x}_2) = 0$. Let $f_{y_D}(\hat{y}_D, \hat{y}_R, \hat{x}_1, \hat{x}_2, \hat{x}_3) \neq 0$ and $q_{y_R}(\hat{y}_R, \hat{x}_2) \neq 0$, then the matrix

$$\begin{bmatrix} f_{y_D}(\hat{y}_D, \hat{y}_R, \hat{x}_1, \hat{x}_2, \hat{x}_3) & f_{y_R}(\hat{y}_D, \hat{y}_R, \hat{x}_1, \hat{x}_2, \hat{x}_3) \\ 0 & -q_{y_R}(\hat{y}_R, \hat{x}_2) \end{bmatrix} \tag{2.17}$$

has full rank and, by the implicit function theorem, there exists a neighborhood V around $\langle \hat{y}_U, \hat{x}_1, \hat{x}_2, \hat{x}_3\rangle \in \mathbb{R}_+^{N+M_U}$, a neighborhood W around $\langle \hat{y}_D, \hat{y}_R\rangle \in \mathbb{R}_+^{M_D+M_R}$ and continuously differentiable mappings $\psi : V \to \psi(V)$ and $h : W \to h(W)$ with images $y_D = \psi(y_U, x_1, x_2, x_3)$ and $y_R = h(y_U, x_2) = q^{-1}(y_U, x_2)$ such that $\langle \psi(y_U, x_1, x_2, x_3), h(y_U, x_2)\rangle \in W$ and

$$\begin{aligned} & f(\psi(\cdot), h(\cdot), x_1, x_2, x_3) = 0 \\ & y_U - q(h(\cdot), x_2) = 0 \end{aligned} \tag{2.18}$$

Then the trade-off between the desirable and undesirable output at the weakly efficient point $\langle \hat{y}_D, \hat{y}_U, \hat{y}_R, \hat{x}_1, \hat{x}_2, \hat{x}_3\rangle$ is given by:

$$\frac{\partial \psi(\hat{y}_U, \hat{x}_1, \hat{x}_2, \hat{x}_3)}{\partial y_U} = -\frac{f_{y_R}(\hat{y}_D, \hat{y}_R, \hat{x}_1, \hat{x}_2, \hat{x}_3) h_{y_U}(\hat{y}_U, \hat{x}_2)}{f_{y_D}(\hat{y}_D, \hat{y}_R, \hat{x}_1, \hat{x}_2, \hat{x}_3)} \geq 0 \tag{2.19}$$

This is because (2.4) and (2.5) establish that $f_{y_D} \geq 0$ and $f_{y_R} \geq 0$, and (2.12) imposes that $q_{y_R} < 0$, thus $h_{y_U} < 0$ given that $q^{-1}(y_U, x_2) = h(y_U, x_2)$. Intuitively, the trade-off between the desirable and undesirable output is non-negative because in a local neighborhood of the weakly efficient point $\langle \hat{y}_D, \hat{y}_U, \hat{y}_R, \hat{x}_1, \hat{x}_2, \hat{x}_3\rangle$ of the technology T, holding the levels of all the inputs fixed, an increase in y_U can be generated only by a reduction in socially responsible activities and hence, as the input usage is constant, the resources must be diverted from the production of the socially responsible output to the production of the desirable output.

Defining $\check{f}(y_D, y_U, x_1, x_2, x_3) = f(y_D, h(y_U, x_2), x_1, x_2, x_3)$, where $h(y_U, x_2) = y_R$, the technology T can be reformulated as

$$T = \{\langle y_D, y_U, y_R, x_1, x_2, x_3\rangle \in \mathbb{R}_+^{M+N} | \check{f}(y_D, y_U, x_1, x_2, x_3) \leq 0 \wedge y_R \geq h(y_U, x_2)\} \tag{2.20}$$

and the function $\check{f}(\cdot)$ can be used to analyze the trade-off between the desirable output and the second input, i.e. the input that is responsible for the generation of the undesirable output. This trade-off can be expressed as

$$-\frac{\check{f}_{x_2}(y_D, y_U, x_1, x_2, x_3)}{\check{f}_{y_D}(y_D, y_U, x_1, x_2, x_3)} = -\frac{f_{x_2}(y_D, y_R, x_1, x_2, x_3) + f_{y_R}(y_D, y_R, x_1, x_2, x_3)h_{x_2}(y_U, x_2)}{f_{y_D}(y_D, y_R, x_1, x_2, x_3)} \tag{2.21}$$

Because the technology T violates free disposability of x_2 by exhibiting the opposite costly disposability property, the sign of the trade-off in (2.21) is ambiguous. This is because an increase in x_2 has a composite effect on the desirable output for fixed levels of inputs x_1 and x_3 and undesirable output y_U. On the one hand, an increase in x_2 generates the standard non-negative effect on y_D given by $-\frac{f_{x_2}(y_D, y_R, x_1, x_2, x_3)}{f_{y_D}(y_D, y_R, x_1, x_2, x_3)} \geq 0$, which directly depends on the conventional free disposability properties of the technology T_D expressed in (2.4) and (2.6). On the other hand, an increase in x_2 generates a non-positive effect given by $-\frac{f_{y_R}(y_D, y_R, x_1, x_2, x_3)h_{x_2}(y_U, x_2)}{f_{y_D}(y_D, y_R, x_1, x_2, x_3)} \leq 0$, which depends on the fact that the technology T_U displays costly disposability in x_2, as verified by the condition in (2.13). Intuitively, this second effect is non-positive because an increase in x_2 needs to be compensated by an increase in socially responsible activities in order to keep the level of undesirable output constant. However, since the levels of the other inputs x_1 and x_3 is also constant, implementing more socially responsible activities requires resources to be diverted from the production of the desirable output to the production of the socially responsible output. Thus the term $-\frac{f_{y_R}(y_D, y_R, x_1, x_2, x_3)h_{x_2}(y_U, x_2)}{f_{y_D}(y_D, y_R, x_1, x_2, x_3)}$ reflects the fact that the costly disposability of the input x_2 has negative repercussions on the production of desirable output. Nonetheless, not knowing the relative magnitudes of these two opposite effects, the sign of the trade-off between the costly disposable input x_2 and the desirable output cannot be determined.

In addition, the function $\check{f}(\cdot)$ allows for analyzing the trade-off between the third input and the undesirable output. Recall that input x_3 can be considered as a "socially responsible" input because, while contributing to the production of the desirable output as a conventional input, it simultaneously contributes to the production of socially responsible activities as well. Differentiating $\check{f}(\cdot)$ with respect to x_3 and y_U yields

$$\begin{aligned} -\frac{\check{f}_{x_3}(y_D, y_U, x_1, x_2, x_3)}{\check{f}_{y_U}(y_D, y_U, x_1, x_2, x_3)} &= -\frac{f_{x_3}(y_D, y_R, x_1, x_2, x_3)}{f_{y_U}(y_D, y_R, x_1, x_2, x_3) + f_{y_R}(y_D, y_R, x_1, x_2, x_3)h_{y_U}(y_U, x_2)} \\ &= -\frac{f_{x_3}(y_D, y_R, x_1, x_2, x_3)}{f_{y_R}(y_D, y_R, x_1, x_2, x_3)h_{y_U}(y_U, x_2)} \leq 0 \end{aligned} \tag{2.22}$$

The non-positive trade-off between the socially responsible input and the undesirable output captures the fact that an increase in x_3 decreases the undesirable output through the mitigating effect of socially responsible activities, given fixed levels of inputs x_1 and x_2 and desirable output.

3 Empirical Methodology

3.1 Primal problem: measuring efficiency

The set representation of the technology illustrated so far is conceptually useful in characterizing the properties of the transformation process and the relationships between inputs and outputs, but it is not very helpful from an empirical perspective. To this end it is useful to turn to a function representation of the technology that is computationally accessible while maintaining the same assumptions of convexity, feasibility, and disposability discussed in the basic model set-up. The function representation chosen here is the directional output distance function, a more general and flexible variation of Luenberger's shortage function[4], and it is defined as

$$\begin{aligned}&\overrightarrow{D_O}(y_D, y_U, y_R, x_1, x_2, x_3; g_{y_D}, -g_{y_U}, g_{y_R}) = \\ &\max\ \{\beta \mid \langle y_D + \beta g_{y_D}, y_R + \beta g_{y_R}, x_1, x_2, x_3\rangle \in T_1, \langle y_U - \beta g_{y_U}, y_R + \beta g_{y_R}, x_2\rangle \in T_2\} \qquad (3.1)\end{aligned}$$

where $g_y = (g_{y_D}, -g_{y_U}, g_{y_R})$ is a vector that determines the direction in which $\overrightarrow{D_O}$ is defined.

This function seeks to simultaneously expand the good outputs (desirable and socially responsible output) while contracting the bad output (undesirable output). The fact that the directional vector is preassigned allows for expanding or contracting any output in different directions making the directional output distance function particularly suitable in the presence of bads. The same suitability does not apply to standard output distance functions[5] because they only allow for expanding every output proportionally and at the same rate as much as it is feasible, which is certainly not desirable when undesirable outputs are produced along with the desirable ones.

The directional output distance function measures the distance, in the preassigned direction g_y, to the boundary of the technology T, therefore it can be interpreted as a measure of inefficiency, i.e. by how much desirable and socially responsible outputs can be expanded and undesirable output contracted and still be feasible. In other words, a firm i with output bundle (y_D, y_U, y_R) producing inside T operates efficiently if, given the direction vector g_y, it is able to expand the desirable and socially responsible outputs and contract the undesirable output to the boundary of T at the point $\langle y_D + \beta^* g_{y_D}, y_U - \beta^* g_{y_U}, y_R + \beta^* g_{y_R}\rangle$, where $\beta^* = \overrightarrow{D_O}(y_D, y_U, y_R, x_1, x_2, x_3; g_{y_D}, -g_{y_U}, g_{y_R})$.

For each firm i the problem of maximizing efficiency (or minimize inefficiency) in the primal (quantity)

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[4] Luenenberger (1992 and 1995)

[5] Shepard (1970).

space consists of finding β^{i*}, which is the directional output distance function, and can be formalized as

$$\max_{\beta^i,\lambda^j} \quad \beta^i \tag{3.2}$$

$$\text{s.t.} \quad y_D^i + \beta^i g_{y_D} \leq \sum_{j=1}^{I} \lambda^j y_D^j \tag{3.3}$$

$$y_U^i - \beta^i g_{y_U} \geq \sum_{j=1}^{I} \lambda^j y_U^j \tag{3.4}$$

$$y_R^i + \beta^i g_{y_R} = \sum_{j=1}^{I} \lambda^j y_R^j \tag{3.5}$$

$$x_n^i \geq \sum_{j=1}^{I} \lambda^j x_n^j \quad n = 1, 3 \tag{3.6}$$

$$x_2^i = \sum_{j=1}^{I} \lambda^j x_2^j \tag{3.7}$$

$$\lambda^j \geq 0 \quad \forall j = 1, ..., I \tag{3.8}$$

Note that this problem is perfectly consistent with the axiomatic approach developed in section 3.1 as the constraints reflect the properties of the joint technology. In particular, (3.3) and (3.6) represent free disposability of desirable output and inputs x_1 and x_3; (3.4) represents costly disposability of undesirable output; (3.5) and (3.7) represent the fact that free disposability of socially responsible output and input x_2 is violated in T because the technology T_1 satisfies free disposability with respect to y_R and x_2 while the technology T_2 satisfies the opposite condition, i.e. costly disposability.

The maximization problem above is equivalent to the following

$$
\begin{aligned}
\max_{\beta^i, \lambda^j} \quad & \beta^i && (3.9) \\
\text{s.t.} \quad & -\sum_{j=1}^{I} \lambda^j y_D^j \leq -y_D^i - \beta^i g_{y_D} && (3.10) \\
& \sum_{j=1}^{I} \lambda^j y_U^j \leq y_U^i - \beta^i g_{y_U} && (3.11) \\
& \sum_{j=1}^{I} \lambda^j y_R^j \leq y_R^i + \beta^i g_{y_R} && (3.12) \\
& -\sum_{j=1}^{I} \lambda^j y_R^j \leq -y_R^i - \beta^i g_{y_R} && (3.13) \\
& \sum_{j=1}^{I} \lambda^j x_n^j \leq x_n^i \quad n = 1, 3 && (3.14) \\
& \sum_{j=1}^{I} \lambda^i x_2^j \leq x_2^i && (3.15) \\
& -\sum_{j=1}^{I} \lambda^i x_2^j \leq -x_2^i && (3.16) \\
& -\lambda^j \leq 0 \quad \forall j = 1, ..., I && (3.17)
\end{aligned}
$$

This is because any equality constraint can be also expressed as a pair of opposite inequality constraints. Then, rewriting the same problem in matrix form yields

$$
\begin{aligned}
\max_{z} \quad & c'z && (3.18) \\
\text{s.t.} \quad & Az \leq b && (3.19) \\
& z \geq 0 && (3.20)
\end{aligned}
$$

which is

$$\max_{z} \overbrace{\underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}}_{1\times(1+I)}}^{c'} \overbrace{\underbrace{\begin{bmatrix} \beta^i \\ \lambda^1 \\ \vdots \\ \lambda^I \end{bmatrix}}_{(1+I)\times 1}}^{z} \tag{3.21}$$

$$\text{s.t.} \quad \overbrace{\underbrace{\begin{bmatrix} g_{y_D} & -y_D^1 & \cdots & -y_D^I \\ g_{y_U} & y_U^1 & \cdots & y_U^I \\ -g_{y_R} & y_R^1 & \cdots & y_R^I \\ g_{y_R} & -y_R^1 & \cdots & -y_R^I \\ 0 & x_1^1 & \cdots & x_1^I \\ 0 & x_2^1 & \cdots & x_2^I \\ 0 & -x_2^1 & \cdots & -x_2^I \\ 0 & x_3^1 & \cdots & x_3^I \end{bmatrix}}_{(M_D+M_U+2M_R+N_1+2N_2+N_3)\times(1+I)}}^{A} \overbrace{\underbrace{\begin{bmatrix} \beta^i \\ \lambda^1 \\ \vdots \\ \lambda^I \end{bmatrix}}_{(1+I)\times 1}}^{z} \leq \overbrace{\underbrace{\begin{bmatrix} -y_D^i \\ y_U^i \\ y_R^i \\ -y_R^i \\ x_1^i \\ x_2^i \\ -x_2^i \\ x_3^i \end{bmatrix}}_{(M_D+M_U+2M_R+N_1+2N_2+N_3)\times 1}}^{b} \tag{3.22}$$

3.2 Dual problem: internal values

Duality theorems allow for deriving the dual to the previous problem that, in matrix form, is given by

$$\min_{\gamma} \quad \gamma' b \tag{3.23}$$

$$\text{s.t.} \quad \gamma' A \geq c' \tag{3.24}$$

$$\gamma \geq 0 \tag{3.25}$$

which is

$$\min_{\gamma} \underset{1\times(M_D+M_U+2M_R+N_1+2N_2+N_3)}{\overbrace{\begin{bmatrix} p_D^i & p_U^i & p_R^i & \hat{p}_R^i & w_1^i & w_2^i & \hat{w}_2^i & w_3^i \end{bmatrix}}^{\gamma'}} \underset{(M_D+M_U+2M_R+N_1+2N_2+N_3)\times 1}{\overbrace{\begin{bmatrix} -y_D^i \\ y_U^i \\ y_R^i \\ -y_R^i \\ x_1^i \\ x_2^i \\ -x_2^i \\ x_3^i \end{bmatrix}}^{b}} \tag{3.26}$$

$$\text{s.t.} \quad \underset{1\times(M_D+M_U+2M_R+N_1+2N_2+N_3)}{\overbrace{\begin{bmatrix} p_D^i & p_U^i & p_R^i & \hat{p}_R^i & w_1^i & w_2^i & \hat{w}_2^i & w_3^i \end{bmatrix}}^{\gamma'}} \underset{(M_D+M_U+2M_R+N_1+2N_2+N_3)\times(1+I)}{\overbrace{\begin{bmatrix} g_{y_D} & -y_D^1 & \cdots & -y_D^I \\ g_{y_U} & y_U^1 & \cdots & y_U^I \\ -g_{y_R} & y_R^1 & \cdots & y_R^I \\ g_{y_R} & -y_R^1 & \cdots & -y_R^I \\ 0 & x_1^1 & \cdots & x_1^I \\ 0 & x_2^1 & \cdots & x_2^I \\ 0 & -x_2^1 & \cdots & -x_2^I \\ 0 & x_3^1 & \cdots & x_3^I \end{bmatrix}}^{A}} \geq \underset{1\times(1+I)}{\overbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}}^{c'}} \tag{3.27}$$

With some manipulation, the problem of each firm i in the dual (price) space can be formalized as

$$\max_{p^i,w^i} \; p_D^i y_D^i - p_U^i y_U^i + (\hat{p}_R^i - p_R^i) y_R^i - w_1^i x_1^i - (w_2^i - \hat{w}_2^i) x_2^i - w_3^i x_3^i \tag{3.28}$$

$$\text{s.t.} \; p_D^i g_{y_D} + p_U^i g_{y_U} + (\hat{p}_R^i - p_R^i) g_{y_R} \geq 1 \tag{3.29}$$

$$p_D^i y_D^1 - p_U^i y_U^1 + (\hat{p}_R^i - p_R^i) y_R^1 - w_1^i x_1^1 - (w_2^i - \hat{w}_2^i) x_2^1 - w_3^i x_3^1 \leq 0 \tag{3.30}$$

$$\vdots$$

$$p_D^i y_D^I - p_U^i y_U^I + (\hat{p}_R^i - p_R^i) y_R^I - w_1^i x_1^I - (w_2^i - \hat{w}_2^i) x_2^I - w_3^i x_3^I \leq 0 \tag{3.31}$$

$$p_D^i, p_U^i, p_R^i, \hat{p}_R^i, w_1^i, w_2^i, \hat{w}_2^i, w_3^i \geq 0 \tag{3.32}$$

The interpretation of this problem is insightful and quite straightforward. For each firm minimizing inefficiency is equivalent to finding a system of optimal, relative (to the numeraire bundle), internal/shadow values that rationalize profit maximization. The shadow prices p^i and w^i that solve the dual problem are different for each firm as they are not market prices but internal valuations that each firm assigns to its outputs and inputs, consistently with profit maximization, representing the contribution of each output and input in creating value for the firm.

Because the technology is characterized by the presence of an undesirable output, y_U, the internal value of this output is negative, as expected, since disposing of y_U represents actually a cost for the firm. It is also interesting to analyze the internal values associated with the socially responsible output y_R and the by-product generating input x_2, i.e. $(\hat{p}_R - p_R)$ and $-(w_2 - \hat{w}_2)$, respectively. Putting in place socially responsible activities is costly for the firm and this is represented by the negative value p_R. At the same time the production of CSR positively contributes to the mitigation of the undesirable output, as reflected in the positive value of $\hat{p}_R$. As a result, the overall value of CSR for the firm depends on the relative magnitude of these two opposite effects. A similar argument applies for the input x_2. In fact, this input represents a cost to the firm when it is used in the production of the desirable output, as reflected by the negative sign of w_2. Nonetheless, the same input has also a beneficial effect, as shown by the positive value of $\hat{w}_2$, because reducing the usage of this input also reduced the amount of undesirable output produced. Once again, the total contribution of the by-product generating input in terms of profits depends on the relative magnitude of these two contrasting effects.

Finally, note that the constraints of the dual problem provide an alternative characterization of the technology. Specifically, the constraint in (3.29) is a normalization implying that all the internal values derived in the dual problem are expressed in terms of the numeraire bundle g_y. This is because the directional distance function in the primal problem can be also interpreted as a collection of outputs and inputs, thus it can be thought as a numeraire bundle whose price in terms of itself is always one. In addition, the constraints (3.30)-(3.31) reflect the fact that, since the system of internal/shadow prices that solves the dual problem for firm i is optimal only for firm i, this set of constraints holds at equality for firm i only if evaluated at firm's i optimal prices p^i and w^i. For every other firm, p^i and w^i cause this set of constraints to hold with inequality because at firm's i optimal internal prices every other firm is inefficient in the sense that it is not able to match the internal cost of the input bundle with the internal value of the output bundle. That is, p^i and w^i are necessarily inconsistent with profit maximization for any firm other than firm i.

3.3 Shadow value and marginal impact of CSR

In economics the concept of marginal value refers to the change in a value associated with a specific change in some controlled variable, or the measure of the worthiness of a good in terms of other goods. In many instances marginal values are more insightful than overall values as they allow to isolate the effects of single variables variations and to quantify trade-offs. Usually marginal values are derived by differentiating smooth functions used to characterize the environment of interest (e.g. production, profits, costs, utility, expenditure).

DEA technologies are conservative approximations derived as convex hulls of observed data points and present, by construction, kinks. This lack of smoothness renders DEA models not amenable to conventional differential arguments, at least for the extreme efficient firms. More specifically, the kinks associated with efficient units in the primal (quantity) space map into flat portions in the dual (price/internal value) space. Thus, the non-differentiability at the kinks in the primal problem translates into non-unique internal/shadow values in the dual. Simply put, the dual problem described in section 3.3 has multiple optimal solutions (p, w) for extreme efficient firms. Chambers and Färe (2008) show how to apply generalized differential arguments, namely directional derivatives and superdifferentials, to DEA representations of technologies to infer marginal/shadow values based on the concept of willingness to pay and willingness to accept. In the context of the proposed study, their approach provides a very useful methodology for deriving the shadow prices that each firm attaches to producing or forgiving one more unit of CSR.

In what follows we illustrate the *calculus* for DEA proposed by Chambers a Färe using the directional output distance function in (3.1). To facilitate the exposition this function is now simply redefined as $\overrightarrow{D_O}(y,x,g_y)$. Recall that $\overrightarrow{D_O}$ is a function representation of the technology thus the assumptions on the joint technology T determine the properties of $\overrightarrow{D_O}$. Specifically, since T is convex, $\overrightarrow{D_O}(y,x,g_y)$ is concave in (y) and satisfies the translation property

$$\overrightarrow{D_O}(y+\delta g_y,x,g_y)=\overrightarrow{D_O}(y,x,g_y)-\delta,\quad \delta\in\mathbb{R} \tag{3.33}$$

and the representation property

$$\overrightarrow{D_O}(y,x,g_y)\geq 0\Leftrightarrow y\in T \tag{3.34}$$

Because $\overrightarrow{D_O}(y,x,g_y)$ is concave in y, its directional derivative[6]

$$\overrightarrow{D_O}'(y,x,g_y;y^0)=\lim_{\delta\to 0^-}\left\{\frac{\overrightarrow{D_O}(y+\delta y^0,x,g_y)-\overrightarrow{D_O}(y,x,g_y)}{\delta}\right\} \tag{3.35}$$

is a superlinear function of y^0 satisfying $\overrightarrow{D_O}'(y,x,g_y;0)=0$ and $-\overrightarrow{D_O}'(y,x,g_y;-y^0)\geq\overrightarrow{D_O}'(y,x,g_y;y^0)$. By the translation property in (3.33) and the definition of directional derivative in (3.35)

$$\begin{aligned}\overrightarrow{D_O}'(y,x,g_y;g_y)&=\lim_{\delta\to 0^+}\left\{\frac{\overrightarrow{D_O}(y+\delta g_y,x,g_y)-\overrightarrow{D_O}(y,x,g_y)}{\delta}\right\}\\&=\lim_{\delta\to 0^+}\left\{\frac{\overrightarrow{D_O}(y,x,g_y)-\delta-\overrightarrow{D_O}(y,x,g_y)}{\delta}\right\}\\&=-1\end{aligned} \tag{3.36}$$

Similarly,

$$\begin{aligned}\overrightarrow{D_O}'(y+\beta g_y,x,g_y;y^0)&=\lim_{\delta\to 0^+}\left\{\frac{\overrightarrow{D_O}(y+\beta g_y+\delta y^0,x,y^0)-\overrightarrow{D_O}(y+\beta g_y,x,g_y)}{\delta}\right\}\\&=\lim_{\delta\to 0^+}\left\{\frac{\overrightarrow{D_O}(y+\delta y^0,x,g_y)-\beta-\overrightarrow{D_O}(y,x,g_y)+\beta}{\delta}\right\}\\&=\overrightarrow{D_O}'(y,x,g_y;y^0)\quad\forall\beta\in\mathbb{R}\end{aligned} \tag{3.37}$$

The derivation in (3.37) simply implies that directional derivatives for directional distance functions are translation invariant with respect to the direction defining the directional distance function.

The superdifferential of $\overrightarrow{D_O}$ in y, denoted as $\partial\overrightarrow{D_O}(y,x,g_y)$, is given by

$$\partial\overrightarrow{D_O}(y,x,g_y)=\left\{v\in\mathbb{R}^{\mathbb{N}}\mid\overrightarrow{D_O}(y,x,g_y)+v'(y^0-y)\geq\overrightarrow{D_O}(y^0,x,g_y)\quad\forall y^0\in\mathbb{R}^{\mathbb{N}}\right\} \tag{3.38}$$

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[6] See Rockafellar (1970), Theorem 23.1.

which can be also expressed as[7]

$$\partial\overrightarrow{D_O}(y,x,g_y)=\left\{v\mid v'y^0\geq\overrightarrow{D_O}'(y,x,g_y;y^0)\quad\forall y^0\right\}\tag{3.39}$$

or equivalently

$$\overrightarrow{D_O}'(y,x,g_y;y^0)=\inf\left\{v'y^0\mid v\in\partial\overrightarrow{D_O}(y,x,g_y)\right\}\tag{3.40}$$

Denoting $\nabla\overrightarrow{D_O}(y,x,g_y)$ as the gradient of $\overrightarrow{D_O}$ in y and considering that when $\overrightarrow{D_O}(y,x,g_y)$ is differentiable in y $\overrightarrow{D_O}'(y,x,g_y;y^0)$ is the inner product of the gradient and y^0, i.e. $\overrightarrow{D_O}'(y,x,g_y;y^0)=\nabla\overrightarrow{D_O}(y,x,g_y)'y^0$, it can be proven[8] that if $v\in\overrightarrow{D_O}(y,x,g_y)$

$$v'g=1\tag{3.41}$$

$$v\in\partial\overrightarrow{D_O}(y+\beta g_y,x,g_y)\quad\forall\beta\in\mathbb{R}\tag{3.42}$$

These two mathematical results have important economic implications. First, the fact that the inner product of any element of the superdifferential $\partial\overrightarrow{D_O}(y,x,g_y)$ and g_y must be equal to one reflects the fact that $\partial\overrightarrow{D_O}(y,x,g_y)$ contains the shadow prices of the output bundle normalized by the shadow value of the numeraire bundle g. Second, not only directional derivatives but also superdifferentials of directional distance functions are translation invariant in the direction of g.

To see how the concepts of directional derivatives and superdifferentials allow for deriving shadow prices for extreme efficient units consider the revenue function associated with T for given output prices $p\in\mathbb{R}^{\mathbb{N}}_+$

$$R(x,p)=\max\{p'y\mid y\in T\}\tag{3.43}$$

As long as there exists a y such that $y+\beta g_y\in T$ for some β, by the representation property

$$\begin{aligned}R(x,p)&=\max\left\{p'(y+\overrightarrow{D_O}(y,x,g_y)g_y)\right\}\\&=\max\left\{p'y+\overrightarrow{D_O}(y,x,g_y)p'g_y\right\}\end{aligned}\tag{3.44}$$

Now take any solution to (3.44) and denote it as y^* which is the efficient level of output maximizing revenue. The directional derivative of (3.44) in an arbitrary direction y^0 away from y^* is given by

$$\begin{aligned}&\lim_{\delta\to0^+}\left\{\frac{p'(y^*+\delta y^0)+\overrightarrow{D_O}(y^*+\delta y^0,x,g_y)p'g_y-p'(y^*+\overrightarrow{D_O}(y^*,x,g_y)g_y)}{\delta}\right\}\\&=p'y^0+\overrightarrow{D_O}'(y^*,x,g_y;y^0)p'g_y\end{aligned}\tag{3.45}$$

If y^* is optimal, the directional derivative $p'y^0+\overrightarrow{D_O}'(y^*,x,g_y;y^0)p'g_y$ is non-positive in every possible

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[7] See Rockafellar (1970), Theorems 23.3 and 23.4.

[8] See Lemma 1 and its proof in Chambers and Färe (2008).

direction so that

$$\frac{p'y^0}{p'g_y} \leq -\overrightarrow{D_O}'(y^*, x, g_y; y^0) \tag{3.46}$$

which implies that $\frac{p}{p'g_y} \in \partial\overrightarrow{D_O}(y^*, x, g_y)$ for every y^0. As mentioned before, this means that for efficient firms (those efficiently selecting y^*) $\partial\overrightarrow{D_O}(y^*, x, g_y)$ contains all the possible normalized shadow prices for $\overrightarrow{D_O}$ at y^*.

Recall that $\overrightarrow{D_O}'(y^*, x, g_y; g_y) = -1$ by (3.36), then for $y^0 = g_y$ (3.45) becomes

$$p'y^0 + \overrightarrow{D_O}'(y^*, x, g_y; g_y)p'g_y = p'g_y - p'g_y = 0 \tag{3.47}$$

Hence, translations of y^* in the direction of g_y do not have any impact on the objective $R(x, p)$, thus if y^* solves the revenue maximization problem so does any translation of y^* in the direction of g_y. That is, for extreme efficient firms on the primal kinks of the technology there are multiple optimal solutions (i.e. any y^* and any translation of it in the direction of g_y) to the dual revenue maximization problem. This solution indeterminacy is simply solved by setting $D_O(y^*, x, g_y)$ to ensure that y^* is on the frontier of T.

Denote e_m as the mth element of the standard orthonormal basis and consider an increase in the production of y_m^*[9] by one unit, which implies a movement from the efficient point y_m^* in the direction of e_m, then (3.46) becomes

$$\begin{aligned} \frac{p_m}{p'g_y} &\leq -\overrightarrow{D_O}'(y^*, x, g_y; e_m) \\ &= -\inf\left\{v'(e_m) \mid v \in \partial\overrightarrow{D_O}(y^*, x, g_y)\right\} \\ &= -\inf\left\{v_m \mid v \in \partial\overrightarrow{D_O}(y^*, x, g_y)\right\} \end{aligned} \tag{3.48}$$

Therefore, any normalized price $\frac{p_m}{p'g_y}$ at which y^* is efficient is a lower bound for $-\overrightarrow{D_O}'(y^*, x, g_y; -e_m)$ implying that $-\overrightarrow{D_O}'(y^*, x, g_y; e_m)$ represents *willingness to gain*, i.e. a measure of what an extreme efficient firm would be willing to receive for engaging in the production of one extra unit of y_m. In the same fashion, considering a movement in the direction of $-e_m$, which is associated with holding off the production of one unit of y_m and forfeit the revenue from that unit, yields

$$\begin{aligned} \frac{-p_m}{p'g_y} &\leq -\overrightarrow{D_O}'(y^*, x, g_y; -e_m) \\ \frac{p_m}{p'g_y} &\geq \overrightarrow{D_O}'(y^*, x, g_y; -e_m) \\ &= \inf\left\{v'(-e_m) \mid v \in \partial\overrightarrow{D_O}(y^*, x, g_y)\right\} \\ &= -\sup\left\{v_m \mid v \in \partial\overrightarrow{D_O}(y^*, x, g_y)\right\} \end{aligned} \tag{3.49}$$

which establishes that $\frac{p_m}{p'g_y}$ is an upper bound for $\overrightarrow{D_O}'(y^*, x, g_y; -e_m)$. Thus, $\overrightarrow{D_O}'(y^*, x, g_y; -e_m)$ can be interpreted as *willingness to lose*, i.e. a measure of what an extreme efficient firm would be willing to give

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[9] In the empirical analysis we will focus specifically on CSR so $y_m = y_R$.

up to forgive the production of one unit of y_m.

Since directional derivatives are positively linearly homogeneous and concave functions of y

$$\overrightarrow{D_O}'(y^*, x, g_y; -e_m) \leq -\overrightarrow{D_O}'(y^*, x, g_y; e_m) \tag{3.50}$$

which formally represents the gap between willingness to gain and willingness to lose generated by the non-smoothness of the technology. Intuitively, for a firm operating efficiently the marginal gain of producing one additional unit of y_m should be higher than the marginal loss of relinquishing one unit of it. Note that, even if there are potentially infinitely many (normalized) shadow prices for y_m, this approach allows for identifying the only two prices that are economically relevant: the shadow gaining price and the shadow losing price. At the kinks these two prices diverges but are still uniquely identified by $-\overrightarrow{D_O}'(y^*, x, g_y; e_m)$ and $\overrightarrow{D_O}'(y^*, x, g_y; -e_m)$, respectively.

It is important to keep in mind that the interpretation of the directional derivatives $-\overrightarrow{D_O}'(y^*, x, g_y; e_m)$ and $\overrightarrow{D_O}'(y^*, x, g_y; -e_m)$ as willingness to gain and willingness to lose, respectively, applies only to extreme efficient units that are at the kinks of the technological frontier. For efficient firms that are on the technological frontier, but not at the kinks, willingness to gain and willingness to lose coincide as $-\overrightarrow{D_O}'(y^*, x, g_y; e_m) = \overrightarrow{D_O}'(y^*, x, g_y; -e_m)$. For inefficient firms the interpretation of $-\overrightarrow{D_O}'(y^*, x, g_y; e_m)$ is still insightful but different. Specifically, if y^* is not efficient, $-\overrightarrow{D_O}'(y^*, x, g_y; e_m)$ simply measures the change in the directional distance function resulting from a small move in the direction of e_m.

3.4 Implications of the empirical analysis

The proposed empirical framework has several practical implications for evaluating the impact of CSR on firms' production structure and value. First, the directional output distance function provides a parsimonious and computationally accessible way of describing a joint technology accommodating for desirable, undesirable, and mitigating outputs. Second, the primal problem generates a measure of inefficiency for each firm that provides an implicit ranking and permits the identification of the leaders in the industry, i.e. the firms that are able to produce the product mix (of desirable, undesirable, and CSR outputs) in the most efficient way. Third, the dual problem allows for deriving an actual measure of the (internal/shadow) value of CSR for each firm even if CSR is a non-marketed output. Comparing this value of CSR to the internal value of the desirable output and the total imputed profits is useful to understand the relative worthiness of CSR activities in terms of the other outputs produced and the total value created by the firm. Lastly, deriving the shadow value of CSR as a measure of willingness to gain and willingness to lose allows for quantifying the effect of CSR activities at the margin. More importantly, the fact that for the extreme efficient firms willingness to gain and willingness to lose differ is crucial to recognize that these firms face both a value of doing more and a cost of doing less CSR. The value and the cost are asymmetrical since the benefit of doing more CSR should exceed the damage of doing less.

4 Data

Collecting data to conduct an empirical analysis based on a multi-input, multi-output model is not necessarily straightforward as detailed disaggregated measures of inputs and outputs at the micro level are usually not available. With respect to inputs the task is less demanding since the popularity of the KLEM (capital, labor, energy, materials) model in production economics has established the practice of collecting input data, or more often data on input expenditures, at least for the general input categories of capital, labor, energy, and materials. With respect to outputs the task is more difficult as aggregate sales at the firm level are the most commonly available measure of output. This last consideration emphasizes the challenge of finding good measures of non-marketed outputs, namely outputs that are produced but are not sold in a market, like the undesirable output and the CSR output in the joint production model proposed here.

Data on CSR production are particularly difficult to acquire for the following reasons. First, even if firms seem to agree on the fact that CSR activities are essential for their business and increasingly engage in their production, they have yet to develop a consistent and precise way of recording the resources they actually devote to CSR. Second, the need for bringing CSR to the core of the business is clearly accompanied by the need for transparent communication however, in the absence of mandatory criteria and strict guidelines, firms' reporting on CSR activities is not homogeneous and easily comparable across firms. Lastly, the strategic importance that CSR has achieved has triggered the proliferation of consulting firms and institutions working on providing scores and rankings that summarize in one final number the CSR performance of each firm. Unfortunately, since this final number is usually obtained as some weighted combination of inputs and outputs involved in the generation of CSR, data on scores and rankings are normally not appropriate in a multiple input/output framework. Even so, this kind of data are practically the only available information on CSR performance at the firm level.

Sustainalytics[10] is a global responsible investment research firm dedicated to support investors with the development and implementation of responsible investment strategies. Sustainalytics' research focuses on developing a reliable and structured scoring system for firms with respect to their ESG/CSR performance and it is based on a methodology that identifies specific issues for each industry, scores every issue for each firm belonging to the same industry, and provides a CSR ranking that evaluates the relative performance of each firm with respect to their peers in the industry. Even if consisting of scores, the dataset provided by Sustainalytics is particularly suitable for the analysis developed in this paper because it consists of detailed scores for different CSR indicators along with a final ranking. These detailed CSR scores are available for each firm included in the sample, so that each firm presents with several disaggregated data points. Moreover, firms belonging to the same industry are scored on the same issues, so that firms can be consistently evaluated and compared and the occurrence of missing values is minimized. In addition, their methodology focuses on identifying strengths and weaknesses for every CSR category (environment, social, governance) in which the single indicators are organized. The fact that scores for detailed indicators are available together with the distinction of these indicators between favorable/positive and controversial/negative aspects of CSR is extremely helpful in identifying measures of mitigating CSR outputs and socially responsible inputs (scores for the positive indicators) and measures of undesirable outputs and socially irresponsible inputs (scores for the negative indicators).

Data on desirable, marketed output and conventional production inputs are more easily available through datasets, such as Orbis from Bureau van Dijk, ThomsonOne from Thomson Reuters, or Compustat from

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[10] http://www.sustainalytics.com/ "Sustainalytics is an award-winning global responsible investment research firm specialized in environmental, social and governance (ESG) research and analysis. The firm offers global perspectives and solutions that are underpinned by local expertise, serving both values-based and mainstream investors that integrate ESG information and assessments into their investment decisions."

Standard&Poor's that are based on the information included in companies' annual financial reports.

In the empirical analysis we concentrate the investigation on the food and beverages manufacturing sector because it presents peculiar and interesting production and CSR characteristics. This sector is populated by firms producing very differentiated products, thus competing on different grounds in terms of desirable outputs. This high differentiation in marketed products is not equally prominent with respect to CSR activities as all food manufacturers face similar challenges concerning food safety controversies, demand for healthier food products, responsible sourcing of raw materials. Along with these specific issues, firms in the food manufacturing sector also face CSR issues common to every manufacturing sector such as responsible usage of water and energy, as well as supply chain and labor standards management.

To carry out the analysis we construct a customized dataset matching information on CSR performance from Sustainalytics with information on desirable, marketed output and conventional production inputs from Orbis (Bureau van Dijk) and ThomsonOne (Thomson Reuters). Specifically, sales, fixed assets, and cost of goods sold are obtained from Orbis and number of employees from Orbis, ThompsonOne or firms' online accessible reports depending on where this information was available. Sales are used as a measure of the desirable, marketed output y_D, while fixed assets, number of employees and cost of goods sold are used as measures of capital, labor and variable inputs, respectively, and constitute the elements of the conventional inputs of production vector x_1. The construction of the remaining part of the dataset, i.e. measures of the undesirable output y_U, the socially irresponsible input x_2 and the socially responsible input x_3 is one of the innovations and contributions of this paper and deserves a thorough illustration.

Recall that the information provided by Sustainalytics is in the form of scores. For each industry a certain (usually quite large) number of indicators across the three Environment, Social and Governance dimensions of CSR performance are chosen and assigned a raw score from 0 to 100 where 0 denotes a very poor performance and 100 denotes an excellent performance. Along with the raw scores Sustainalytics provides also a system of industry-specific weights for each indicator that reflect the importance of each indicator in characterizing the overall ESG performance in each sector. The weights are sector-specific to capture the idea that different indicators might matter more or less for CSR depending on the industry. For example, while managing emissions and toxic waste could be very important in the chemical sector, it certainly is not as important in the banking sector. To construct the measure of socially responsible output y_R we select 9 indicators among those available and aggregate them into one weighted score using a re-scaled system of weights that reflects the relative importance given to these indicators in the original Sustainalytics dataset. Similarly, we construct measures of undesirable output y_U, socially irresponsible input x_2 and socially responsible input x_3 aggregating 10, 3 and 2 indicators, respectively. Note that, as the scores assigned by Sustainalytics are increasing in the performance, i.e. the better the performance the higher the score, for the undesirable output and socially irresponsible input I use the inverse of the original score (100-original score) to be consistent with the theoretical framework. Table 1 provides further details on the variables constituting the dataset used in the empirical analysis.

Because of the difficulty of matching the data from several different sources, the presence of missing values, and the necessity of being particularly careful in selecting indicators from the Sustainalytics dataset that can be meaningfully characterized as inputs and outputs measures, the final dataset utilized to implement the empirical analysis consists of a cross-section of 175 publicly traded firms. The sample includes the major players in the food and beverages manufacturing industry worldwide. For each firm we have data on a total of 8 variables - one desirable output, one undesirable output, one socially responsible output, three conventional inputs, one socially irresponsible input, and one socially responsible input. All the data refer to 2014, the latest available year in our data sources with the most complete information. Table 2 reports descriptive statistics of the variables used in the analysis specifying in what units they are expressed[11].

The information provided in table 2 and displayed in figures 1 and 2 suggest the following about the data. Desirable output (sales) and conventional production inputs (capital, labor, variable inputs) have very high variability and present with a considerable number of outliers while undesirable output, socially irresponsible input and socially responsible input have a less disperse distribution with almost no outliers[12]. All the variables do not seem to be normally distributed with the exception of the socially responsible output y_R. This is clearly showed in figure 1 where a normal curve is imposed over the histogram of each variable and confirmed statistically by the result of a test for normality[13] which rejects the null hypothesis of normality with a 5 percent confidence level for y_R and with a 1 percent confidence level for all the other variables.

Figures 3 - 5 present scatter plot matrices of desirable output with the other outputs (undesirable and socially responsible), desirable output with conventional production inputs (capital, labor and variable inputs), and desirable output with the other production inputs (socially irresponsible and socially responsible), respectively. The matrix diagonal contains the kernel density of each variable. The scatter plot matrix in 4 shows a positive correlation between conventional inputs and desirable output, as expected. However, the nature of the correlation between the other inputs of production and the desirable output in figure 5 is not as clear. Figure 3 suggests the existence of a positive correlation between the desirable and undesirable output as predicted in the theoretical model where y_D and y_U are positively correlated because y_U is a by-product of y_D. The correlation between the desirable output and the socially responsible output, on the other hand, is not clearly positive reflecting the mechanism that in the theoretical model makes socially responsible efforts beneficial in terms of mitigation but costly in terms of resources.

5 Results

5.1 Efficiency measure

The solution to the primal problem[14] in (3.2) - (3.8) yields a measure of inefficiency that quantifies how much desirable and socially responsible outputs can be expanded and undesirable output contracted within the feasibility constraint imposed by the technology T. The expansion of y_D and y_R and the contraction of y_U are in the pre-assigned direction of $g_y = (g_{y_D}, -g_{y_U}, g_{y_R})$. In this case g_y has been arbitrarily chosen to be $g_y = (1, -1, 1)$ which simply means that all the outputs are considered equally important when moving toward the frontier. Note that when the directional vector is chosen such that it enters the constraints of the primal problem additively, as in this case, the inefficiency score β has a lower bound at 0 and an upper bound that depends on the scale and magnitude of the data. Therefore, $\beta = 0$ indicates efficiency while $\beta > 0$ indicates margin for technical improvement where the higher the value of β the higher the inefficiency.

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[11] Understanding the units in which the variables are expressed is important to understand the results of the empirical analysis

[12] This is partially due to the fact that even if the scale of the raw scored assigned by Sustainalytics is from 0 to 100 the scores are usually assigned in quintiles, i.e. 0, 25, 50, 75, 100.

[13] Test proposed by D'Agostino, Belanger, and D'Agostino (1990) with the empirical correction developed by Royston (1991).

[14] We assume variable returns to scale so the primal problem is solved for each firm imposing the additional constraint that $\sum_{j=1}^{I} \lambda^j = 1$.

In the sample of food and beverages manufacturing firms analyzed here efficiency levels are very high. Approximately 75 percent of the firms are technically efficient, among them only 30 are just efficient while the remaining 102 are extreme efficient. Extreme efficient firms are not simply located on the technical frontier but define its shape by characterizing the vertexes of the technology convex hull. Less than 25 percent of the firms (43 firms) are found to be inefficient. In figure 6 the distribution of the efficiency scores is illustrated through a histogram and a nonparametric kernel density. The distribution is clearly concentrated around zero since 3/4 of the firms are technically efficient.

This result is not surprising for several reasons. First, the empirical analysis is carried out with three outputs and five inputs but each of these outputs and inputs have a specific and peculiar role in the production process. This means that, while firms have more freedom in articulating the scope of their production along different dimensions, their decisions are also necessarily more complex as these different dimensions can be conflicting. For example, more desirable outputs generates more undesirable output, which then needs to be mitigated. Similarly, socially responsible activities can add value to the firm but are costly in terms of resources that need to be allocated to their production. These trade-offs translate into the constraints defining the technology 'pulling' the boundaries of the feasibility set in different, sometimes opposite, directions. Therefore, the more freedom and more choices available to firms generate a very peculiar technology set that can accommodate different production 'recipes' and make it easier for firms to be efficient. Second, while the variability in conventional inputs and output is quite large in the data this is not the case for undesirable output, socially responsible output and socially irresponsible and responsible inputs. This is in part due to data limitations but also to the fact that the CSR performance of firms seems to be much more homogeneous - that is, there seems to be minimum standards that every firm strives to achieve. Hence, even firms that are not extremely competitive in terms of sales or conventional productive resources (capital, labor) are on the contrary very competitive in terms of socially responsible efforts. Because of the linkages between the different elements of the production structure, this also generates higher levels of efficiency.

5.2 Shadow values

For inefficient and just efficient[15] firms the solution to the dual problem in (3.28)-(3.32) is unique and provides a measure of the internal value that each firm assigns to its input-output bundle. These values are firm-specific, consistent with profit maximization, and represents the contribution of each input and output in creating value for the firm.

Table 3 reports summary statistics for the outputs shadow prices[16]. The main conclusions that can be drawn from these results are the following. The average shadow price of desirable output p_D is positive implying that, as expected, producing the desirable output contributes positively in creating value for the firm. As expected, the average shadow price of undesirable output p_U is negative and considerably higher that the other outputs shadow prices. suggesting that the production of the undesirable output represents a considerable cost for the firm. The average shadow price of the socially responsible output is positive, meaning that the cost of producing CSR is compensated by the benefit of its mitigating effect. This result is reasonable because, given the hight negative value attached to the undesirable output, engaging in socially

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[15] Just efficient firms are those located on the technology frontier but not on a vertex of the technology convex hull.

[16] Note that these outputs are expressed in different units, i.e. million of USD for the desirable output and scores for the undesirable and socially responsible output, and their shadow prices are relative to the numeraire bundle g_y which makes the interpretation of the results not immediate.

responsible activities is necessary and overall firms are able to extract value from these activities even if their implementation is costly. It is interesting to point out that the shadow price of CSR is exactly zero for all the 30 just efficient firms, which indicates that these firms are capable of balancing the costs associated with socially responsible efforts with their mitigating nature in a perfectly efficient way.

5.3 Marginal value of socially responsible activities

In this section we focus on extreme efficient firms (those on the vertexes of the technology hull) because the solution of the dual problem for these firms delivers the most interesting insight for understanding the value of CSR at the margin. Recall that the dual problem in (3.28)-(3.31) does not have a unique solution for extreme efficient firms. However, focusing on the highest and lowest shadow price for the socially responsible output allows for obtaining the only two prices that are economically relevant, i.e. the shadow gaining price (the price an extreme efficient firm is willing to receive to produce one more unit of CSR) and the shadow losing price (the price an extreme efficient firm is willing to 'pay'/give up to forgive the production of one unit of CSR). Recall that these prices are normalized and expressed in the units of the numeraire bundle g_y.

Because the measure of socially responsible output used here is a score from 0 to 100, identifying the units and understanding the meaning of producing one more or one less unit of CRS is not straightforward. Technically, one unit of CSR corresponds to one score point but, since the raw scores are mostly given in quintiles (0, 25, 50, 75, 100) and the raw scores are weighted and aggregated into one single measure of socially responsible output, establishing the precise magnitude of one score point is potentially complicated. Nonetheless, to understand the results it is sufficient to loosely interpret the shadow prices of CSR discussed here as a measure of how much an extreme efficient firm is willing to gain for improving its CSR performance (thus getting a higher score) or lose for worsening its CSR performance (thus getting a lower score). Table 4 provides some descriptive statistics for the upper and lower bound of p_R.

The average shadow gaining price[17] of CSR is positive and equal to 0.666. Even if this number might be complicated to interpret in terms of units and magnitude, its sign is indicative of the fact that, on average, extreme efficient firms attach a positive value to CSR activities and are willing to increase their socially responsible efforts for a positive price. Note that, since CSR is a mitigating yet costly activity, its price does not need to be necessarily positive. The fact that the average shadow gaining price is positive indicates that extreme efficient firm consider a higher socially responsible commitment to be beneficial for adding value to their business.

On the other hand, the average losing price[18] of CSR is negative and equal to -0.071. This result is both somewhat unanticipated but also very insightful. First, a negative losing price of CSR implies that, on average, extreme efficient firms are not willing to 'pay' any price for reducing their socially responsible effort, instead they want to realize a gain. This suggests that engaging in less CSR is considered so costly

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[17] This upper bound for p_R is calculated averaging over the maximum shadow price of CSR obtained by solving a modified version of the dual problem for the 102 extreme efficient firms present in the sample. The modified dual problem calls for maximizing p_R under the same constraints of the standard dual problem presented in (3.28)-(3.32) and the additional constraint that $p_D y_D^i - p_U y_U^i + (\hat{p}_R - p_R) y_R^i - w_1 x_1^i - (w_2 - \hat{w}_2) x_2^i - w_3 x_3^i = 0$.

[18] This lower bound for p_R is calculated averaging over the minimum shadow price of CSR obtained by solving a modified version of the dual problem for 55 extreme efficient firms. In this case, the modified version of the dual problem consists of minimizing p_R under the same constraints of the standard dual problem presented in (3.28)-(3.32) and the additional constraint that $p_D y_D^i - p_U y_U^i + (\hat{p}_R - p_R) y_R^i - w_1 x_1^i - (w_2 - \hat{w}_2) x_2^i - w_3 x_3^i = 0$. Note that the modified dual problem that generates the lower bound of p_R can be optimally solved only for 55 out of the 102 extreme efficient firms present in the sample. For the remaining 47 firms this problem is unbounded.

and damaging that firms want to be compensated for doing so. This result is particularly interesting and it is in line with the increasingly pervasive evidence that CSR has become an activity that firms feel compelled to do and, more importantly, that certain minimum standards/levels of CSR are perceived as necessary.

6 Concluding Remarks

The conviction that CSR should be a prominent business practice has undoubtedly reached a wide consensus among firms, consumers, investors, and policy makers. While the academic literature in recent years has provided formal and rigorous support to this conviction, the existing research has been almost exclusively focused on *why* CSR is done. This study takes a very different perspective and attempts to shed light on *how* CSR is done incorporating it into a formal production framework.

To this extent, we develop a joint production model for characterizing the technology and representing the transformation process of multiple inputs into multiple outputs. Specifically, each firm is assumed to produce a desirable output but, because the production of this desirable output may require the use of some undesirable input, an unwanted output can be generated along with the desirable one. Thus, the firm needs to engage in socially responsible activities to mitigate the unwanted output. The overall technology supporting this joint production is obtained as a composition of two distinct technologies: one describing the desirable-output production and the other describing the generation of the undesirable output. CSR is the link between these two technologies as it simultaneously represents the opportunity cost of producing socially responsible activities in terms of desirable output and its mitigating effect with respect to undesirable output.

Empirically, the implementation of the analysis is based on a parsimonious non-parametric DEA approach. DEA techniques allow for constructing the joint technology as the intersection of the desirable production and the undesirable production technology. Once the technology is fully characterized, a set of internal/shadow values for inputs and outputs can be derived which reveals how much the production of CSR is worth to the firm in terms of the other outputs produced. For extreme efficient firms, the Chambers-Färe *calculus* method for DEA technologies is used to identify unique shadow values for CSR as measures of willingness to gain for producing one more unit of CSR and willingness to lose for giving up the production of one more unit of CSR.

Our results indicate that in the sample of 175 firms included in the analysis efficiency levels are very high as approximately 75 percent of the firms are found to be technically efficient. For inefficient and just efficient firms the average shadow value of socially responsible activities is positive, implying that the cost of implementing these activities is compensated by their mitigating effect. For extreme efficient firms the average marginal value of increasing their socially responsible commitment is positive indicating that more CSR is considered beneficial for adding value to the firm. Conversely, the average marginal value of decreasing the CSR effort is negative indicating that lower levels of CSR are perceived as costly and damaging so that firms want to be compensated for reducing their socially responsible performance.

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Tables and Figures

Table 1: List of variables included in the empirical analysis

Variable	Description	Indicator	Source
y_D	desirable output	Sales	Orbis
y_U	undesirable output	Operations Incidents	Sustainalytics - E
		Environmental Supply Chain Incidents	Sustainalytics - E
		Product and Services Incidents	Sustainalytics - E
		Employee Incidents	Sustainalytics - S
		Social Supply Chain Incidents	Sustainalytics - S
		Product and Services Incidents	Sustainalytics - S
		Society and Community Incidents	Sustainalytics - S
		Business Ethics Incidents	Sustainalytics - G
		Governance Incidents	Sustainalytics - G
		Public Policy Incidents	Sustainalytics - G
y_R	socially responsible output	Environmental Policy	Sustainalytics - E
		Environmental Management System	Sustainalytics - E
		Sustainable Agriculture Programs	Sustainalytics - E
		Freedom of Association Policy	Sustainalytics - S
		Discriminatory Policy	Sustainalytics - S
		Supply Chain Monitoring	Sustainalytics - S
		Bribery and Corruption Policy	Sustainalytics - G
		Global Compact Signatory	Sustainalytics - G
		Board Independence	Sustainalytics - G
x_{1_k}	conventional input	Capital - Fixed Assets	Orbis
x_{1_l}	conventional input	Labor - Number of Employees	Orbis/ThomsonOne
x_{1_v}	conventional input	Variable Inputs - Cost of Goods Sold	Orbis
x_2	socially irresponsible input	Water Management Programs	Sustainalytics - E
		GHG Reduction Programs	Sustainalytics - E
		Scope of Social Supply Chain Standards	Sustainalytics - S
x_3	socially responsible input	Green Procurement Policy	Sustainalytics - E
		Diversity Programs	Sustainalytics - S

Notes: The scores for the indicators used to construct measures of undesirable output and socially irresponsible input have been transformed as100-original score to be consistent with the theoretical framework.
Sustainalytics - E, S or G signifies that the indicator comes from either the E (Environment), S (Social) or G (Governance) category, in which the indicators in the Sustainalytics database are organized.

Table 2: Descriptive statistics of the variables used in the empirical analysis

Variable	Description	Mean	Median	Trimmed Mean 10%	Standard deviation	MAD
y_D	desirable output - sales (USD)	8233.86	3486.88	3557.63	13921.03	2318.59
y_U	undesirable output (score)	2.30	0.07	0.22	4.66	0.07
y_R	socially responsible output (score)	37.71	34.51	35.41	23.63	17.12
x_{1_k}	capital - fixed assets (USD)	7278.93	2474.88	2497.23	15091.47	1605.00
x_{1_l}	labor - number of employees	28394.50	12700.00	12433.58	47236.23	8490.00
x_{1_v}	variable inputs - cost of goods sold (USD)	5010.73	2046.09	2224.33	9406.09	1366.61
x_2	socially irresponsible input (score)	54.78	62.50	56.43	34.12	27.50
x_3	socially responsible input (score)	24.54	26.40	20.05	22.38	19.60

Notes: The variables expressed in USD are in millions of USD; whenever the values were expressed in other currencies they have been converted into USD using the Dec 31 2014 exchange rate provided by the IMF.
The variables expressed in scores are expressed on a potential scale from 0 to 100.

Table 3: Descriptive statistics of the outputs shadow values

Variable	Mean	Median	St. dev	Min	Max
p_D	0.00017	0.00000	0.00036	0.00000	0.00220
p_U	-0.95530	-1.00000	0.17996	-1.05970	0.00000
p_R	0.04456	0.00000	0.17971	-0.06070	0.99930

Note: The statistics are calculated for the subsample of 43 inefficient and 30 just efficient firms.

Table 4: Descriptive statistics of the marginal value of CSR

Variable	Mean	Median	St. dev	Min	Max
p_R *upper bound*	0.66563	0.99550	0.43619	-0.21440	1.00000
p_R *lower bound*	-0.07141	-0.00380	0.55956	-3.52720	0.78010

Note:The statistics for the upper bound of p_R are calculated for all the 102 extreme efficient firms present in the sample while the statistics for the lower bound p_R are calculated for a subsample of 55 extreme efficient firms.

Figure 1: Histogram of the variables used in the empirical analysis

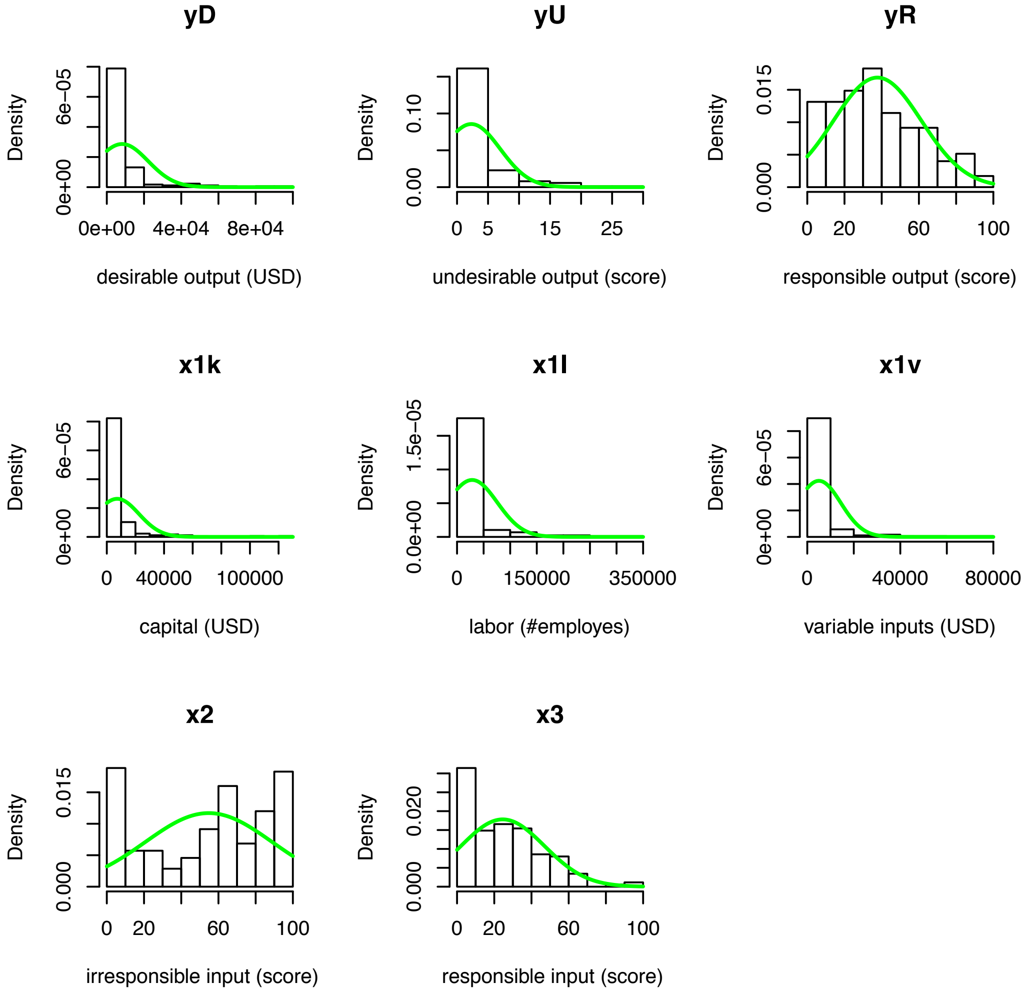

Figure 2: Box plots of the variables used in the empirical analysis

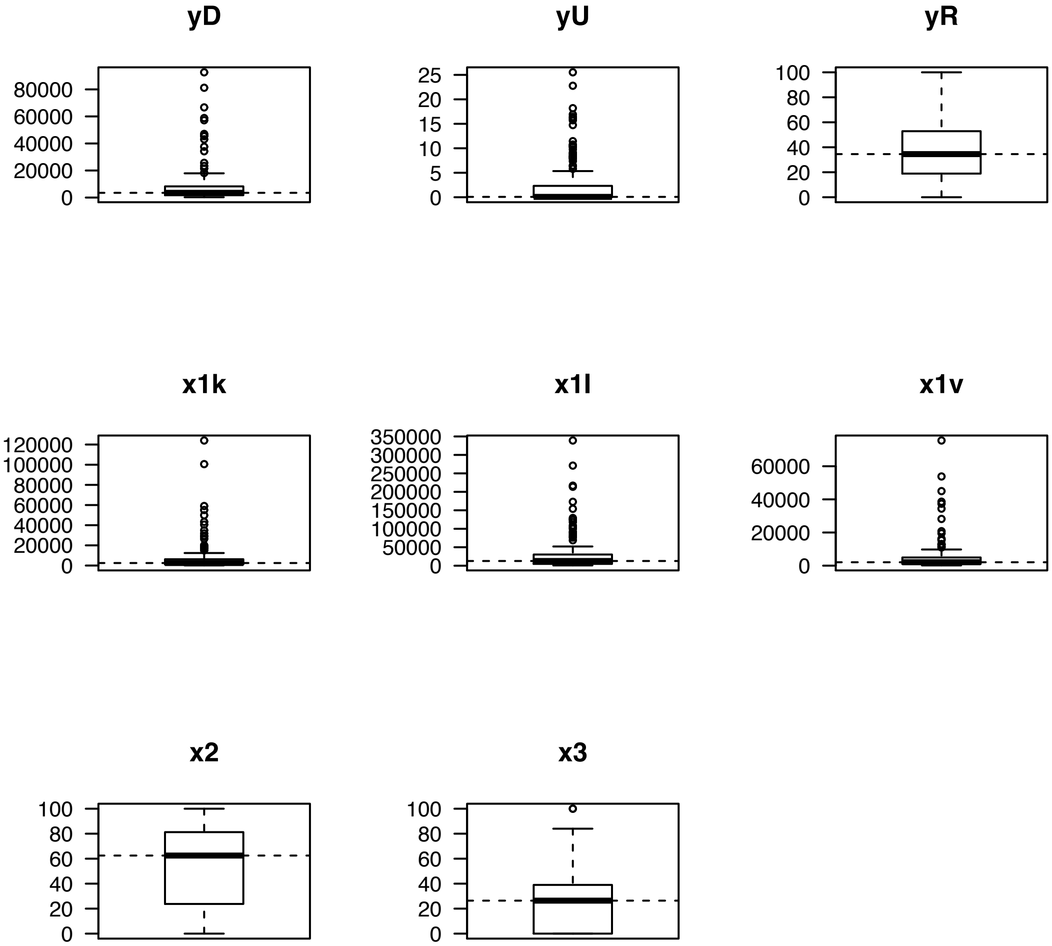

Figure 3: Scatter plot matrices of the desirable output with the other outputs

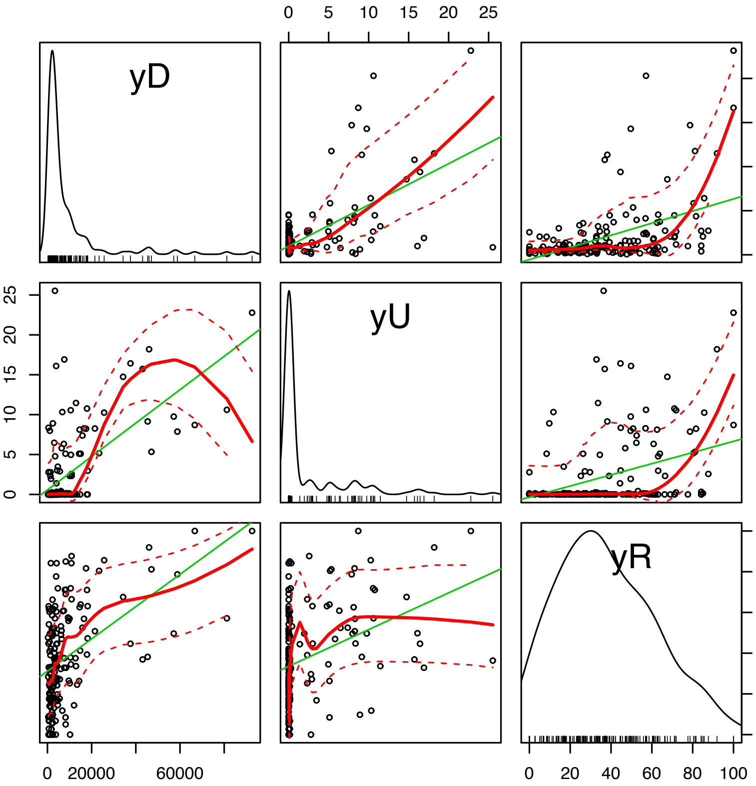

Figure 4: Scatter plot matrices of the desirable output with the conventional production inputs

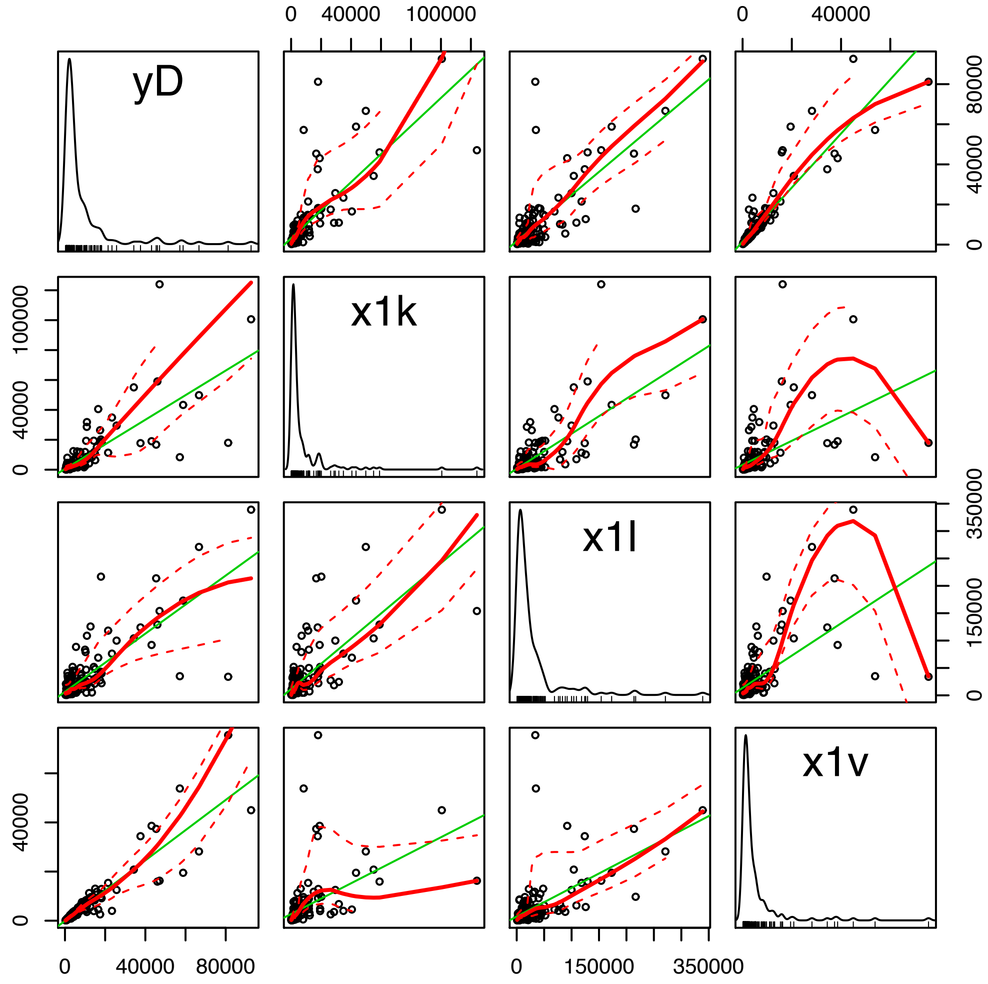

Figure 5: Scatter plot matrices of the desirable output with the other production inputs

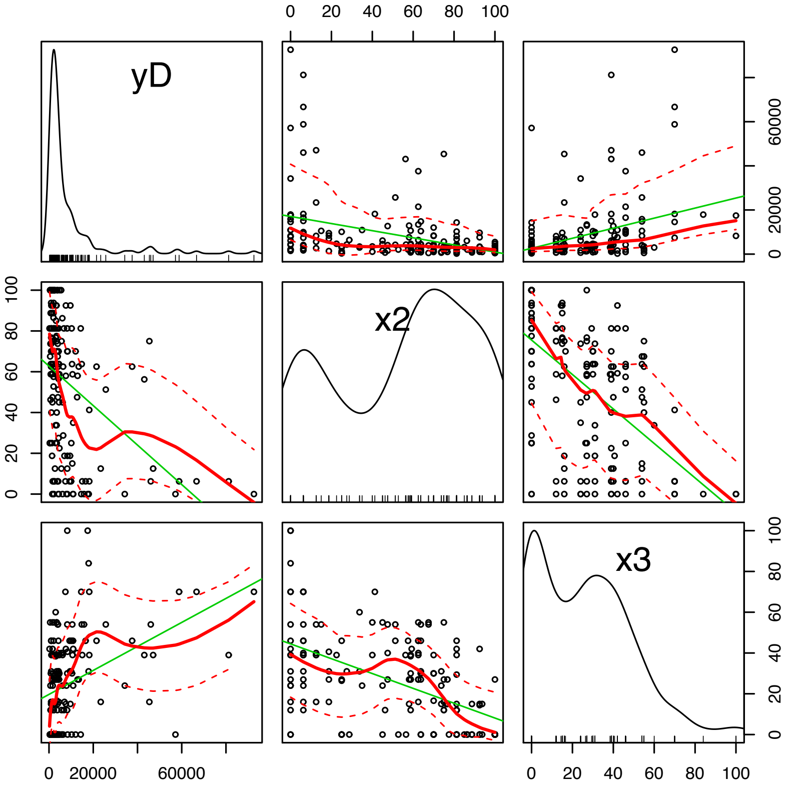

Figure 6: Distribution of efficiency scores