The Value of Delegated Quality Control and Market Size with an Application to Kyrgyzstan Dairy

Alexander E. Saak

Markets, Trade, and Institutions Division
International Food Policy Research Institute
2033 K Street, NW
Washington, DC 20006-1002 USA
Phone: 202-862-4543
Fax: 202-467-4439
Email address: asaak@cgiar.org

Selected Paper prepared for presentation at the 2016 Agricultural & Applied Economics Association Annual Meeting, Boston, Massachusetts, July 31-August 2

The Value of Delegated Quality Control and Market Size with an Application to Kyrgyzstan Dairy

Abstract

This paper studies the decision of a firm that sells an experience good to delegate quality control to an independent monitor. In an infinitely repeated game consumers' trust provides incentives to (1) acquire information about whether the good is defective and (2) withhold the good from sale if it is defective. If third-party reports are observable to consumers, delegation of monitoring lessens the first and dispenses with the second moral hazard concern but also creates agency costs due to either limited liability or lack of commitment. In equilibrium the firm controls quality without an independent monitor only if trades are sufficiently frequent and consumer information about quality is sufficiently precise. This result holds under different assumptions about feasible contracts, collusion, verifiability of reports, joint inspections, and the number of firms that hire the third-party monitor. If third-party reports are not publicly observed, delegation can be optimal only if two or more firms hire the third-party monitor because then both moral hazard concerns are present under delegation.

Keywords: quality control; trust; repeated game; imperfect monitoring; moral hazard; food safety

1. Introduction

In many food, manufacturing, and service industries, ensuring that a product meets specifications and standards requires expenditures on testing and keeping inferior products out of the market (Ridley et al. 2006; Westmoreland 2014). However, if consumers cannot observe product quality before purchase, there need to be proper incentives to acquire and truthfully report information about quality. The problem of incentivizing quality control can involve the choice of the monitor: a firm can inspect and certify product quality by itself or through a third party such as an independent certifier or an autonomous division within the firm.[1] Food quality control in agricultural value chains in developing countries, where agricultural production tends to be dominated by smallholder farmers, presents particular challenges. Although formal contracting between farmers and agricultural processors is becoming a norm in some developing countries, many smallholders continue to market their products through informal channels. In such cases, a processor who buys products from a smallholder may be the first actor in the value chain to engage in effective quality control. How can agricultural processors better convince their customers that appropriate steps to ensure high quality products are being taken? Should the food processor use its own staff and facilities to conduct testing and assessments of quality and compliance with safety norms? Or should it rely on a third party to monitor and certify product quality? [3]

I develop a repeated game model with long-lived, risk-neutral firm, auditor, and consumers.[4] The setting presented in section 2 assumes that (1) product quality is an exogenous, binary (good/bad), i.i.d. (independently and identically distributed) random variable; (2) there is a costly, perfect inspection technology that generates hard evidence only when quality is bad;[5] (3) the firm either uses the inspection

[1] In addition to financial accounting, third-party certification is common in consumer markets such as childcare, healthcare, organic food, and coffee (Dranove and Jin 2010; Hatanaka, Bain, and Busch 2005; Raynolds, Murray, and Heller 2007; White 2010; Xiao 2010).

[3] For example, Weschler (2014) reports that the share of US food plants that relied on independent food contract laboratories instead of their own laboratories to test food samples for Salmonella gradually increased from 37 percent in 2001 to 63 percent in 2013. According to another survey, 48 percent of food manufacturers conducted internal testing and outsourced to contract laboratories, 30 percent performed tests in-house with their own laboratory technicians, 18 percent exclusively used outside laboratories, and 4 percent did neither (Westmoreland 2014).

[4] Although, to transparently highlight the advantages and disadvantages of delegation, our main result is presented under the assumption that the third-party monitor has a single client, it continues to hold qualitatively if the monitor has multiple clients. Delegation in the setting with one auditor and one firm can also admit an organizational interpretation whereas quality control is delegated to an autonomous unit within the firm as in the case of independent quality control units in the pharmaceutical industry (GMP Guidelines, accessed at http://www.gmp-compliance.org/eca_guidelines.html on November 14, 2015).

[5] As explained in more detail in section 2, the assumption that inspection generates no hard evidence when quality is good/high implies that the so-called unraveling result in Grossman (1981) and Milgrom (1981) that predicts full disclosure in equilibrium does not apply. The assumption that inspection generates hard evidence only when quality

technology itself or irreversibly transfers it to the auditor in the beginning of the game; (4) the firm and the auditor can sign a long-term contract that specifies noncontingent, nonnegative payments to be paid by the firm to the auditor; (5) both the inspection/monitoring effort and the signal are private information of the monitor (firm or auditor); (6) the monitor reports publicly; and (7) consumption (if any) generates an imperfect public signal of quality.

In equilibrium trade can happen only if consumers "trust" that the firm (sufficiently often) keeps bad products out of the market, and the firm's goal is to minimize the total cost of sustaining credibility. In both monitoring regimes, I focus on trigger equilibria where consumption takes place if and only if quality is good (the efficient outcome) until the public signal of quality falls below a certain cut-off point.[6] Delegation of monitoring to the auditor increases credibility because the auditor is less biased than the firm as his or her fee is independent of whether quality is good or bad. On the other hand, delegated quality control also gives rise to agency costs that can offset the benefit of delegation. I characterize conditions such that the firm hires the auditor as a function of the discount factor that parameterizes the frequency of trades or the speed with which information is disseminated among consumers. Section 4 presents my main finding that internal quality control is optimal only if the frequency of trades and precision of the public signal are sufficiently great.

As an illustration, consider a food manufacturer that can assess compliance with a food safety or quality standard on its own or through an independent certifier (Sporleder and Goldsmith 2001; Ollinger et al. 2011; Lytton and McAllister 2014). According to some evidence from markets for bottled water and organic food, smaller firms tended to be certified externally while bigger, multiproduct firms tended to rely on their own (umbrella) brands to convince consumers of their quality (Hakenes and Peitz 2009).[7] This is consistent with the prediction of my model as more popular brands tend to get more media exposure and public scrutiny.

The key trade-off involved in the choice between internal and external quality control can be explained as follows. Because the monitor (firm or auditor) has no reasons to learn quality if it is going to

---

is bad is plausible if, for example, to prove that the quality of a batch is bad one can present a few defective units while to prove that quality is good one has to show that each unit in the batch is not defective. While the assumption that presale information is partially verifiable is made to transparently highlight an economic force that makes delegated monitoring profitable, we also study the case with fully unverifiable presale information in the Fully Manipulable Reports section.

[6] Although in equilibrium buyers sometimes (unfairly) punish the firm (and the auditor), they do not make any statistical inferences about the monitoring effort because in each monitoring regime the monitor is honest. Following some authors (for example, Cabral 2009) we will use the word "trust" to describe an equilibrium of the repeated game where the public history is encapsulated in the customers' belief about the firm's trustworthiness. The assumption that the public signal of quality is noisy is not essential for our results but adds realism and generality to the model.

[7] Motivated by these observations, Hakenes and Peitz (2009) consider the choice between umbrella branding and external certification but assume that certification is honest.

hide bad news, the monitor's ex post truth-telling constraint is not binding in equilibrium. Consider the firm's incentive to inspect quality under internal quality control. The firm's current gain from shirking on monitoring is (1) the cost savings from not putting in the monitoring effort plus (2) the additional sale revenue from not withdrawing bad products from the market. In contrast, the shirking auditor gets only the cost savings from not putting in the monitoring effort but does not get the additional sales revenue. Therefore, compared with the firm, it is easier to ensure that the auditor is immune to the temptation to shirk. This is the positive *monitoring-incentive* effect of delegation. As the auditor sends messages directly to consumers, delegation also has a *commitment* (to allocate the good efficiently) effect since the firm no longer has the option to withhold bad news from consumers. The monitoring-incentive and commitment effects lower the equilibrium cut-off point of the public signal under delegation.

However, the auditor gets an ongoing rent in equilibrium with delegation. Because the firm stops employing the auditor when it can no longer sell to consumers, the auditor shares the firm's concern about the loss of future trading opportunities. As the firm cannot discipline the auditor by other means, the auditor is willing to exert the monitoring effort only if his or her future relationship with the firm is sufficiently valuable. Delegation creates a negative *monitoring-rent* effect because the full auditor's future rent cannot be transferred to the firm at the time of hiring.

As trades become more frequent or consumer information about quality more precise, the necessary consumer punishment softens in both monitoring regimes. As a result, the benefit of delegation decreases because a smaller stopping probability is sufficient to make quality control incentive compatible even without the external monitor. The agency cost, on the other hand, increases when shirking is less likely to be punished. As a result, when public signals of quality are sufficiently precise and frequent, the agency cost exceeds the benefit of delegation. In fact, if consumers do not make mistakes when assessing quality after purchase and trades are sufficiently frequent, the incentive cost (and the benefit of delegation) is zero under internal quality control, but the necessary agency cost remains strictly positive.

Notably, conditions under which delegation takes place in equilibrium do not qualitatively change if I relax (sometimes jointly) the assumptions that (1) the contract between the firm and the auditor is long term with positive, noncontingent payments and the transfer of the monitoring technology to the auditor is irreversible; (2) the firm cannot offer the auditor bribes in secret from consumers; (3) the monitor can overrate but cannot underrate quality and the firm cannot inspect quality itself if it hires the auditor; (4) the auditor is hired by at most one firm; and (5) the auditor reports publicly.

The Unlimited Liability and Short-term Contracts section demonstrates that the negative monitoring-rent effect can persist in the absence of the constraint on the contractual transfers (unlimited liability of the auditor). This can happen if contracting between the firm and the auditor is short term and

the firm can freely regain control of the monitoring technology once it stops employing the auditor (reversible or short-term delegation). Then, if the necessary (dynamically optimal) fee is too high, the firm is not able to credibly promise to keep renewing the contract, and in equilibrium the firm controls quality internally in each period.

If secret transfers between the firm and the auditor are possible, the firm is tempted to bribe the auditor to issue high ratings without monitoring. The Collusion section shows that the possibility of collusion to sell uninspected goods reduces but does not necessarily eliminate the net benefit of delegation because collusion requires that the firm and the auditor share the gains from shirking. If the firm pays a bribe after the auditor approves the good for sale, the firm is tempted to sell an uninspected product but renege on the promised payment to the auditor. If the firm pays a bribe before the auditor issues its report, the auditor is tempted to collect the bribe but renege on the promise to approve an uninspected good for sale.

If the reports are fully manipulable, incentivizing the auditor becomes more costly. Then the auditor must be immune not only to the temptation to overrate quality but, unlike the firm, also to the temptation to underrate quality. As shown in the Punishment of Unfavorable Reports by Consumers section, external quality control can be incentive compatible and profitable if in equilibrium customers punish (by breaking off their relationship with the firm) both low, after-sale signals and unfavorable, presale reports of quality. Contingent contracts or joint monitoring can discipline the auditor more efficiently than the punishment of bad news about quality by consumers. The Report-contingent Fee section demonstrates that auditing fees that are contingent on the report (as in the case where the independent monitor is a supervisor or manager) significantly reduce the agency cost. The Sequential Monitoring section assumes that auditing fees are noncontingent but allows for sequential internal and external inspections and shows that in equilibrium the firm sometimes inspects quality itself before requesting external certification.[8]

While the restrictions on the set of feasible contracts, the possibility of collusion against consumers, and fully unverifiable presale information shift the balance toward internal quality control, allowing the auditor to have multiple clients strengthens the case for delegation. In fact, real-world auditing firms typically have multiple clients. The Multiple Clients section examines a setting with one auditor and several identical, independent firms that exhibits economies of scale in external certification.[9] Although an auditor with multiple clients pools not only the long-run gains from staying honest but also the short-

[8] This is reminiscent of the "monitoring-the-monitor" mechanism in Baliga (1999) or Rahman (2012).

[9] While Strausz (2005) shows that there are economies of scale in external certification in an environment with one certification per period, our monitor inspects and certifies multiple clients simultaneously. However, Cai and Obara (2009) show that there can be some decreasing returns to scale if there is a common shock that affects all items being certified within a period.

run gains from shirking, a larger client base makes it easier for consumers to detect the most profitable deviation, thus lowering the minimum necessary per-client agency cost (the auditing fee).

Another important aspect of external certification is transparency of communication of the results of inspection. The Private Third-party Reporting section assumes that the auditor reports to the firm in private and that the firm in turn communicates to the consumers.[10] Then the commitment (to reveal bad news) effect is absent, and both moral hazard concerns are present under delegation: the auditor is tempted to shirk to save the cost of monitoring, and the firm is tempted to conceal the bad news that it receives from the auditor. As a result, delegation is never optimal if the auditor is hired by one firm. However, if the auditor has multiple clients, delegation with private reporting can be optimal as it makes it is easier to convince consumers that information about quality is acquired in the first place.[11]

Related Literature

The question of why delegation of monitoring can be optimal is examined in different settings in Strausz (1997) and Strausz (2005). Strausz (1997) considers a static principal-agent-supervisor model where both the agent and the monitor need to be induced to exert efforts. In that paper's setting, as in this paper's, there are no direct efficiency gains that are related to delegation, the monitoring effort is the private information of the monitor, and commitment to a disclosure rule is not possible, but unlike here, delegation is costless in equilibrium. Strausz (2005) identifies conditions under which there is demand for external certification by sellers who can also build a reputation themselves. He presents a repeated games model where producers have a more efficient (costless) monitoring technology than the certifier who cannot manipulate the certification outcome unless there is collusion to misrepresent quality. Our paper considers a richer environment where monitoring technology is the same and learning about quality needs to be induced under both internal and external monitoring. We also examine external monitoring in the presence of the possibility of collusion with relational contracting and fully unverifiable presale information.[12]

[10] For example, under current food safety regulation, private third-party food safety auditors may be contractually obligated to keep their findings confidential and not report them to government food authorities and the general public (Costa 2014; Consumer Product Safety Commission 2013). The balance between publicly disclosing commercially sensitive information and achieving credibility of third-party audits is discussed in Food and Drug Administration (2013).

[11] Delegation with private reporting also can be profitable when the auditor has one client if the presale signal of quality is imperfect because then garbling of the presale signal by the auditor can ensure that the firm is not tempted to hide mildly bad news about quality from consumers. The case with an imperfect presale signal is discussed further in section 6 and is left for future research.

[12] Peyrache and Quesada (2011) obtain harmful effects of collusion in a different collusion environment but also assume that only the certifier has reputational concerns.

This paper also contributes to the literature on the reputational incentives of providers of financial information in games with cheap talk. Mathis, McAndrews, and Rochet (2009) show that a monopolist credit rating agency can vary the quality of its ratings over time in a setting with persistent types. Bolton, Freixas, and Shapiro (2012) consider the effects of competition among credit rating agencies for issuers. I also allow for fully unverifiable presale information and consider several disciplining devices such as punishments of unfavorable reports by consumers, contingent contracting, and sequential monitoring.

A related literature abstracts from the issue of credibility of ratings and investigates how intermediaries acquire and certify information under commitment to a disclosure rule. Miklós-Thal and Schumacher (2013) analyze a repeated game model with a long-lived firm and certification intermediary and moral hazard on quality provision but assume that consumers cannot observe the outcomes of past trades.[13] Building on Lizzeri (1999), Faure-Grimaud, Peyrache, and Quesada (2009) show that the seller may prefer to hide an unfavorable rating if the hiring decision is unobservable. Farhi, Lerner, and Tirole (2013) study how the sellers sequentially choose between standards (certifiers) that differ in the level of difficulty and the effects of transparency of ratings.[14] The firm's incentives to acquire and disclose product information on its own in voluntary and mandatory disclosure regimes are studied in Matthews and Postlewaite (1985), Shavell (1994), and Polinsky and Shavell (2012). My information structure with the binary state of the world is less general but allows for a tractable comparison of different monitoring regimes.

This paper also is related to the literature on delegated expertise and credence goods. The optimal organization form when both information gathering and truthful reporting need to be incentivized is studied in Gromb and Martimort (2007). In a setting with multiple experts, they show that the optimal number of experts and structure of incentive contracts depend on the features of the collusion environment. While in my model a single signal about the state of the world is sufficient for efficient allocation, the possibility of collusion also can affect the optimal organization form, that is, the choice between internal and external expert/monitoring. Getting a second opinion (from another expert) can be valuable to consumers in models of markets for credence goods where the expert must exert costly but unobservable effort to diagnose the consumer's need (Pesendorfer and Wolinsky 2003; Dulleck and Kerschbamer 2009). I too allow for the possibility of sequential internal and external inspections but assume that incentives are provided through trust rather than competition among experts.

[13] Stahl and Strausz (2014) study whether a buyer or a seller should pay for certification under the assumption that the seller is informed about quality and uses certification as a signaling device.

[14] In Harbaugh, Maxwell, and Roussillon (2011) sellers observe their own quality and certification is honest, but consumers are unsure about the minimum level of quality required for certification. Gill and Sgroi (2012) also assume that the firm knows its own type and certification provides an incontrovertible signal and investigate the optimal level of stringency of the test in equilibrium where the firm behaves as if it has a good product for signaling reasons.

2. The Basic Model

There is a profit-maximizing auditor (*A*), firm (*F*), and a large number of identical consumers (*C*) at dates $t = 1,2,\ldots$. All players are risk neutral and infinitely lived with a common discount factor $\delta \in (0,1)$ and the outside options of zero. In each period the firm has a good of high or low quality, $\theta_t \in \{l, h\}$. The production cost equals zero. It is common knowledge that quality is drawn from the distribution $\Pr(\theta_t = h) = \alpha \in (0,1)$ and is independent across periods. Initially, θ_t is unknown to all players. The firm, which I assume has the price-setting power, can offer the good to the consumers at price p_t .[15] If the good is consumed, the consumers get $v_h - p_t$ ($v_l - p_t$) if quality is high (low), and the firm gets p_t , where $v_l < 0 < v_h$. I also assume that trade is inefficient if quality is unknown, $\alpha v_h + (1-\alpha)v_l \leq 0$.[16]

Presale Monitoring

There exists a costly presale monitoring technology that can be used by either the firm or the auditor. If the monitor (either the firm or the auditor depending on who controls the technology) decides to monitor, $e_t = 1$, upon incurring cost c the monitor learns a signal $m_t \in \{l, \varnothing\}$, where $\Pr(m_t = l \mid \theta_t = l) = 1$ and $\Pr(m_t = \varnothing \mid \theta_t = h) = 1$, and $0 < c < \alpha v_h$. If $(e_t, \theta_t) = (1, l)$, then monitoring reveals evidence that quality is low, $m_t = l$; if $(e_t, \theta_t) \in \{(1,h), (0,l), (0,h)\}$, then no evidence is revealed to the monitor, $m_t = \varnothing$. Following Tirole (1986), I assume that the monitoring effort, $e_t \in \{0,1\}$, is the monitor's private information and the information that the monitor receives (if any) is hard but concealable. Since the monitor can infer whether quality is high or low, without loss I restrict the space of the monitor's messages to $\{l, h\}$. If the monitor monitored, $e_t = 1$, and observed, $m_t = l$, then he or she can either reveal the evidence, $r_t = l$, or conceal it and report, $r_t = h$; if the monitor monitored, $e_t = 1$, and observed, $m_t = \varnothing$, or did not monitor, $e_t = 0$, the monitor cannot claim that quality is low and must

[15] My main results will continue to hold qualitatively in the presence of competition in the final goods market provided that the firm's price-setting power and the mode of competition among firms are not significantly affected by the monitoring regime. Beginning with Klein and Leffler (1981), and more recently Hörner (2002), there is a large literature that studies the effects of competition on reputational incentives to produce high-quality products.

[16] If $\alpha v_h + (1-\alpha)v_l > 0$ and $c < -(1-\alpha)v_l$, presale monitoring is socially efficient but is not necessary to generate a positive surplus from trade. Then credibility of quality control is more difficult to sustain because the firm can earn a positive profit, $\alpha v_h + (1-\alpha)v_l$, without making any claims about quality. Nonetheless, conditions under which delegation takes place will not be qualitatively different, because in equilibrium with honest monitoring the firm forgoes the profits from selling uninspected products whether or not it hires the auditor.

report, $r_t = h$.[17]

The other possibilities are that presale monitoring generates hard evidence (1) only if quality is high, or (2) always, or (3) never. In cases (1) and (2), there is no scope for external quality control since the standard unraveling result a la Grossman (1981) and Milgrom (1981) applies. The efficient outcome is then attained in the unique equilibrium where consumers buy the good if and only if the firm provides evidence of high quality. Case 3, with fully manipulable reports, is studied in detail in the Fully Manipulable Reports section.

Contracting

In the beginning of the game the auditor proposes a long-term contract to the firm.[18] The contract specifies (1) the monetary transfer from the firm to the auditor (auditing fee) at each date, $\{w_t\}_{t=1}^{\infty}$ and (2) the auditor's obligation to publicly report, $r_t \in \{l, h\}$. Note that the auditor's fee is not contingent on his or her report, which is consistent with real-world contracts of third-party auditor/certifiers who avoid compromising the auditor's independence. I will assume that the auditor cannot make positive transfers to the firm, that is, $w_t \geq 0$. If the firm accepts the contract, the auditor controls the monitoring and reporting technology for the remainder of the game. Otherwise, the technology is controlled by the firm. Either party can terminate the contract at any date *t*. The terms of the contract and the decisions to accept, reject, or terminate the contract are announced publicly.

Postsale Monitoring

Consumers cannot distinguish between high and low quality prior to consumption. However, consumption generates a nonverifiable signal of quality $Y_t \in \{l, h\}$ that is drawn from the distribution

$$\Pr(Y_t = h \mid \theta_t = h) = \Pr(Y_t = l \mid \theta_t = l) = \beta \ ,$$

where $\beta \in (\frac{1}{2}, 1]$ is the precision of the signal and Y_t is conditionally independent across periods. Following Cabral (2009) and Cai and Obara (2009), I assume that there also exists a public randomization device $X_t \sim U[0,1]$, where X_t is independent from Y_t, θ_t and across periods. Without loss I will assume that after consumption a combined signal

$$S_t = \begin{cases} 1, \text{if } Y_t = h \\ X_t, \text{if } Y_t = l \end{cases}$$

[17] While the message $r_t = l$ reveals evidence of low quality, message $r_t = h$ is "cheap talk" and may carry meaning only in equilibrium. MacLeod (2007) terms a good for which the only evidence available comes from low quality a "normal good" (as opposed to an "innovative good" where the evidence comes only from high quality).

[18] The assumption that the auditor proposes a contract is not essential for our results and is made for concreteness.

becomes public information. If $Y_t = h$, the public signal equals 1 with probability one. If $Y_t = l$, the public signal is continuous, which will allow consumers to coordinate their purchasing/punishment strategy after a bad performance.[19] Let s_t denote a typical realization of S_t.[20]

Stage Game

In each period the following sequence of events takes place:

1. Under delegation, the firm or the auditor terminates the contract, $d_t = 0$, or not, $d_t = 1$, and the firm pays the auditor w_t if $d_t = 1$.

2. Nature determines quality, θ_t.

3. The monitor (firm or auditor) privately decides whether to monitor, $e_t = 1$, or not, $e_t = 0$.

4. In the case of monitoring, signal m_t is revealed to the monitor.

5. The monitor (firm or auditor) publicly reports r_t.

6. The firm decides whether to offer the good for sale and sets the price p_t.

7. Consumers decide whether to buy, $b_t = 1$, or not, $b_t = 0$ ($b_t = 0$ if the good is not offered).

8. Signal s_t is publicly observed if the good is consumed, and payoffs are realized.

Equilibrium Concept

I study a perfect public equilibrium (PPE) of the game where players' strategies depend only on the past realizations of the public signals such that following any public history the strategies of the players form a Nash equilibrium.[21] For periods $t = 2,3,\ldots$, the public history consists of the contract termination decisions, reports, prices, purchasing decisions, and public signals in all the preceding periods: $\mathrm{H}_t = \{d_\tau, r_\tau, p_\tau, b_\tau, s_\tau\}_{\tau=1}^{t-1}$. In a regime with delegation, both the firm and the auditor will base their contract termination decisions on H_t. The monitor (the firm or the auditor) will base his or her period t effort decision on H_t and d_t and his or her grading decision on H_t and d_t, e_t, m_t. The firm will base its offering and pricing decision on H_t and e_t, m_t in the regime without delegation and on H_t and d_t, r_t in

[19] A more general specification will slightly complicate the presentation while offering little additional insight.

[20] Board and Meyer-Ter-Vehn (2013) study quality provision in a setting with reputational dynamics when consumers learn only from bad news, only from good news, or both. Fleckinger et al. (2015) also consider costly provision of quality under biased disclosure. Although our information structure is much simpler than in these papers, we allow for learning about quality before and after sale.

[21] I will allow the decisions of the firm and the auditor to depend on their private histories in the games with collusion (Collusion section) and sequential inspections (Sequential Monitoring section).

the regime with delegation. If the good is offered, the customers will base their period t purchasing decision on H_t and d_t, r_t, p_t. Starting from the beginning of period t the expected discounted payoff of the firm is given by

$$\pi_{F,t}^{n} = E[\sum_{\tau=t}^{\infty} \delta^{\tau-t}(b_\tau p_\tau - ce_\tau) \mid \mathrm{H}_t]$$

in the regime without delegation and

$$\pi_{F,t}^{d} = E[\sum_{\tau=t}^{\infty} \delta^{\tau-t}(b_\tau p_\tau - d_\tau w_\tau) \mid \mathrm{H}_t]$$

in a regime with delegation. The auditor's expected discounted payoff is given by

$$\pi_{A,t}^{d} = \sum_{\tau=t}^{\infty} \delta^{\tau-t}(d_\tau w_\tau - ce_\tau) \mid \mathrm{H}_t],$$

and the consumers' expected discounted payoff is given by

$$\pi_{C,t} = \sum_{\tau=t}^{\infty} \delta^{\tau-t} b_\tau (v_{\theta_\tau} - p_\tau) \mid \mathrm{H}_t]$$

in both presale monitoring regimes.

As usual, there are many PPEs in this repeated game, including the repetition of the unique static Nash equilibrium of the stage game where in every period there is no presale monitoring and no trade. I will focus on equilibria where presale monitoring and correct grading/allocation are self-enforcing as part of an equilibrium of the repeated game. Let us define a *trust phase* as all periods during which trade generates a positive expected surplus (presale information about quality is acquired and truthfully reported) and a *punishment phase* as all periods during which the firm and the auditor get their reservation utilities of zero (this is the harshest subgame perfect punishment). Following Cai and Obara (2009) and Cabral (2009), a stationary, trigger-purchasing strategy PPE (I will call it an equilibrium) is defined as follows: (1) the game is in either the trust or the punishment phase, and (2) the game begins in the trust phase and this phase stops forever once the public signal of quality falls lower than a certain cut-off point, s, that is, once $Y_t = l$ and $X_t < s$. As is standard, I will drop the time subscript if a variable is stationary.

It will be convenient to let $F_h(s) = \Pr(Y_t = l \mid \theta_t = h) \Pr(X_t < s) = (1-\beta)\, s$, $F_l(s) = \Pr(Y_t = l \mid \theta_t = l) \Pr(X_t < s) = \beta s$, and $F(s) = \alpha F_h(s) + (1-\alpha) F_l(s)$ denote the conditional and unconditional stopping probabilities. Using this notation, for a given cut-off point of the public signal s, in equilibrium the trust phase continues with the average probability $1 - \alpha F_h(s)$. The continuation probability during the trust phase is obtained by aggregating over the possible outcomes of presale monitoring. With probability α quality is high and there is trade, in which case the public signal falls lower than the cut-off point with probability $F_h(s)$. With the complementary probability $1-\alpha$ trade

does not take place, in which case the game continues with probability one.

I will first characterize an equilibrium that maximizes the firm's discounted payoff under internal monitoring and then consider the firm's decision whether to hire the auditor. Because an individual consumer is small relative to the size of the market, consumers will maximize their current period payoffs based on the public belief about presale monitoring and grading/allocation strategies. Given its full price-setting power, to maximize its profit during the trust phase in any monitoring regime the firm will set the price equal to the customers' expected benefit from the good that successfully passed the inspection: $p = v_h$.

3. No Delegation

Let s^n denote the equilibrium cut-off point of the public signal in the game without delegation. The equilibrium expected discounted payoff of the firm, π_F^n, satisfies the following value-recursive equation

$$\pi_F^n = \alpha v_h - c + \delta[1 - \alpha F_h(s^n)]\pi_F^n. \quad (1)$$

As usual, the firm's payoff is the sum of the expected current profit, $\alpha v_h - c$ (the maximum achievable flow surplus from trade), plus the discounted continuation value, $\delta[1 - \alpha F_h(s^n)]\pi_F^n$.

The firm is willing to monitor if the payoff from monitoring and offering only high quality for sale (truthful reporting) is greater than the payoff from saving the cost of monitoring and selling the good of uncertain quality (reporting *h* without inspection):

$$\pi_F^n \geq v_h + \delta[1 - F(s^n)]\pi_F^n. \quad (2)$$

To understand the right-hand side of equation 2, note that if the firm shirks on its monitoring effort and offers the good for sale anyway, the trust phase continues with probability $1 - F(s^n)$ as the firm does not have any private information about quality.

The firm must also be willing to withhold the good if quality is low (report/reveal *l*):

$$\delta\pi_F^n \geq v_h + \delta[1 - F_l(s^n)]\pi_F^n. \quad (3)$$

The payoff from selling low quality, on the right-hand side of condition 3, consists of the current sales revenue, v_h, plus the reduced continuation value, $\delta[1 - F_l(s^n)]\pi_F^n$. It can be shown that the firm's allocation/reporting incentive compatibility condition 3 is implied by the monitoring-incentive-compatibility condition 2,[22] which can be rewritten as

[22] As in Gromb and Martimort (2007), the monitor's problem combines moral hazard ex ante and adverse selection ex post because both gathering information and reporting it accurately must be incentive compatible. I also find that only the monitoring incentive compatibility constraint binds in equilibrium.

$$\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^n)]} \geq \frac{(1-\alpha)v_h + c}{\delta(1-\alpha)F_l(s^n)}, \tag{4}$$

where I used equation 1 to derive

$$\pi_F^n = \frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^n)]}. \tag{5}$$

Monitoring and efficient allocation are incentive compatible if the firm's discounted future profit, π_F^n, is greater than the current gain from shirking that consists of the incremental expected sale revenue, $(1-\alpha)v_h$, and the monitoring cost, c, divided by the discounted incremental stopping probability due to selling a low-quality good that, had it been inspected, would not have been sold, $\delta(1-\alpha)F_l(s^n)$. Because the profit decreases with s^n, the equilibrium cut-off signal that maximizes the firm's payoff is the smallest s^n that satisfies the firm's monitoring-incentive-compatibility condition 4.

Proposition 1 identifies parameter restrictions such that there exists s^n that solves equation 4. Let $\delta^n \equiv \left(1+(1-\alpha)\beta\frac{\alpha v_h - c}{(1-\alpha)v_h + c} - \alpha(1-\beta)\right)^{-1} < 1$ denote a threshold level of the discount factor, where the inequality holds if $\beta > \beta^n \equiv \left(1+\frac{1-\alpha}{\alpha}\frac{\alpha v_h - c}{(1-\alpha)v_h + c}\right)^{-1} \in [\tfrac{1}{2},1)$.

Proposition 1 *An equilibrium with trade under internal quality control exists if and only if* $\delta \geq \delta^n$, *where* $\delta^n < 1$ *if and only if* $\beta > \beta^n$. *The equilibrium firm's profit,*

$$\pi_F^n = \frac{1}{1-\delta}\left(\alpha v_h - c - \frac{\alpha(1-\beta)}{(1-\alpha)\beta}[(1-\alpha)v_h + c]\right), \tag{6a}$$

and the cut-off point,

$$s^n = \frac{1-\delta}{\delta}\left((1-\alpha)\beta\frac{\alpha v_h - c}{(1-\alpha)v_h + c} - \alpha(1-\beta)\right)^{-1}, \tag{6b}$$

increase (respectively, decrease) if δ, β, v_h *increase, or* c *decreases.*

An equilibrium exists whenever the discount factor is sufficiently great, the public (postsale) signal of quality is sufficiently precise, and the ratio of the monitoring cost and consumption value of high quality is sufficiently small. The threshold level of precision of the public signal such that a sufficiently patient firm can achieve a positive profit, β^n, decreases as the ratio of the honesty payoff,

$\alpha v_h - c$, and the deviation gain, $(1-\alpha)v_h + c$, increases. The comparative statics results all are intuitive and are obtained by differentiating equations 6a and 6b. The firm's per-period equilibrium profit is the surplus from trade, $\alpha v_h - c$, minus an incentive cost, $\frac{\alpha(1-\beta)}{(1-\alpha)\beta}[(1-\alpha)v_h + c]$. The latter increases with the flow deviation gain, $(1-\alpha)v_h + c$, and a measure of how much information a low public signal of quality provides about the monitoring effort, $\frac{\alpha F_h(s^n)}{(1-\alpha)F_l(s^n)} = \frac{\alpha(1-\beta)}{(1-\alpha)\beta}$.

An increase in the probability that the true quality is high, α , has an ambiguous effect on the equilibrium cut-off public signal, s^n , and profits, π_F^n . On the one hand, the expected social surplus from trade and the option value of information generated by presale monitoring increase with α . On the other hand, it is more difficult to detect shirking on the presale monitoring effort because it is less likely that the quality of an uninspected good will turn out to be low.

4. Delegation

I now consider whether there exists an incentive-compatible auditing contract that makes the firm at least as well off as in equilibrium without delegation. In a stationary equilibrium without loss I can restrict my attention to contracts $\{w_t^d\}_{t=1}^{\infty}$ with $w_t^d = w^d$ for all $t = 2,3,\ldots$. As will be shown momentarily, allowing for $w_1^d \neq w^d$ will not have any effect on the incentive compatibility conditions but may increase profits. Let s^d denote the equilibrium cut-off point of the public signal under delegation.

Note that during the punishment phase the firm's best response is to terminate the auditor's contract and stop paying the fee. Therefore, the auditor's equilibrium payoff satisfies the following value-recursive equation:

$$\pi_{A,t}^d = w_t^d - c + \delta[1-\alpha F_h(s^d)]\pi_A^d, \tag{7}$$

where $\pi_{A,t}^d = \pi_A^d \ \forall t \geq 2$. The average continuation probability is again given by $1-\alpha F_h(s^d)$ because the firm offers the good for sale if and only if the auditor reports *h*. The interpretation of equation 7 is analogous to that of equation 1 except the monitor's current profit is now given by $w_t^d - c$ instead of $\alpha v_h - c$.

The auditor's incentive-compatibility conditions can be derived as follows. In the absence of evidence, the auditor has no choice but to report *h*. If the auditor obtains evidence that quality is low, concealing it cannot increase his or her payoff:

$$w_t^d + \delta\pi_A^d > w_t^d + \delta[1-F_l(s^d)]\pi_A^d. \tag{8}$$

It only remains to verify that the auditor is willing to incur the monitoring cost. If the auditor does not monitor, he or she has to report *h* and get $w_t^d + \delta[1-F(s^d)]\pi_A^d$. Thus, shirking on the monitoring effort is not profitable if

$$w_t^d - c + \delta[1-\alpha F_h(s^d)]\pi_A^d \geq w_t^d + \delta[1-F(s^d)]\pi_A^d \tag{9}$$

or

$$\frac{w^d - c}{1-\delta[1-\alpha F_h(s^d)]} \geq \frac{c}{\delta(1-\alpha)F_l(s^d)}, \tag{10}$$

where I used the fact that solving equation 7 yields $\pi_A^d = \frac{w^d - c}{1-\delta[1-\alpha F_h(s^d)]}$.

For a given cut-off point, s^d, the right-hand side of equation 10 increases with the monitoring cost and decreases with the discount factor, probability that the actual quality is low, and precision of the public signal. To further understand the auditor's monitoring-incentive-compatibility condition, note that condition 10 can be rewritten as $\delta([1-\alpha F_h(s)]-[1-F(s)])\,\pi_A^d \geq c$. This says that the monitor's incremental continuation value from monitoring, $\delta([1-\alpha F_h(s)]-[1-F(s)])\,\pi_A^d$, must be greater than the flow cost of monitoring, c. From equation 10 it follows that (1) the ongoing auditor's fee that sustains credibility of third-party monitoring satisfies $w^d \geq \{1-\delta[1-F(s^d)]\}c/[\delta(1-\alpha)F_l(s^d)]$ and (2) the auditor's participation constraints, $\pi_{A,t}^d \geq 0$, are not binding in equilibrium for any $w_1^d \geq 0$.

Under delegation the firm effectively no longer chooses the allocation of the good because consumers update their beliefs about quality based on the auditor's report (which is truthful in equilibrium). Therefore, the firm will hire the auditor if, in addition to the auditor's monitoring-incentive-compatibility condition, the firm's individual-rationality conditions,

$$\pi_{F,1}^d = w^d - w_1^d + \frac{\alpha v_h - w^d}{1-\delta[1-\alpha F_h(s^d)]} \geq \pi_F^n \tag{11}$$

and

$$\pi_{F,t}^d = \frac{\alpha v_h - w^d}{1-\delta[1-\alpha F_h(s^d)]} \geq 0\ ,\ t = 2,3,\ldots\ , \tag{12}$$

are satisfied. Condition 11 ensures that the firm prefers to delegate quality control to the auditor in period $t=1$. Condition 12 ensures that after signing the contract the firm is willing to stay in the market and maintain its relationship with the auditor while the trust phase lasts in periods $t = 2,3,\ldots$.

As in the regime without delegation, all else equal, the equilibrium profits of both the auditor and the firm increase as the cut-off point s^d decreases, that is, if consumers' trust lasts longer. Therefore, the equilibrium ongoing fee, w^d, and the cut-off point of the public signal, s^d, are such that the auditor's monitoring-incentive-compatibility constraint (for $\beta < 1$) and one of the firm's participation constraints are binding. I am now in a position to examine whether and when there exists an incentive-compatible contract that is attractive to the firm. Let $\delta^d \equiv \left(\alpha(1-\alpha)\beta \frac{v_h}{c} + \alpha\beta + (1-\alpha)(1-\beta) \right)^{-1}$, $\beta^d \equiv \left(1 + \frac{1-\alpha}{\alpha} \frac{\alpha v_h - c}{c} \right)^{-1}$, $\beta^{dn} \equiv \left(1 + \frac{4(1-\alpha)(\alpha v_h - c)c}{\alpha[(1-\alpha)v_h + 2c]^2} \right)^{-1}$.

Proposition 2 *If the public signal is sufficiently precise, $\beta > \beta^d$, there exists a threshold value of the discount factor δ^{dn} such that in equilibrium the firm hires the auditor for all $\delta^d \le \delta \le \delta^{dn}$ and does not hire the auditor for all $\delta > \delta^{dn}$, where $\delta^{dn} < 1$ if and only if $\beta > \beta^{dn}$, $\delta^d < \delta^n \le \delta^{dn} \le 1$, $\beta^d < \beta^n < \beta^{dn}$. Under delegation consumers' trust lasts longer, $s^d < s^n$, and the firm gets a rent, $w^d < \alpha v_h$, for $\delta \in [\delta^n, \delta^{dn}]$, but the auditor is a full residual claimant to sale revenues, $w^d = \alpha v_h$, for $\delta \in [\delta^d, \delta^n)$.*

Delegation may or may not be optimal. On the one hand, the auditor is better positioned to maintain customers' trust since his or her fee is independent of the outcome of the inspection. Comparing the right-hand sides of the firm's and auditor's monitoring-incentive-compatibility conditions 4 and 10 reveals that the auditor's current gain from shirking on the monitoring effort, c, is smaller than the firm's, $c + (1-\alpha)v_h$. Furthermore, the efficient allocation of the good is automatically incentive compatible as in equilibrium the firm cannot offer a good that has not received the auditor's stamp of approval. The positive monitoring-incentive and commitment effects of delegation increase the total profit by lowering the equilibrium cut-off point, $s^d < s^n$ (if trust equilibrium without delegation exists).

However, the firm may not be able to extract enough of the benefit of external quality control due to the limited liability constraint.[23] When hiring the auditor the firm gives up at least

$$\pi_{A,1}^d = w_1^d - c + \delta[1 - \alpha F(s^d)]\pi_A^d = \frac{1 - F(s^d)}{(1-\alpha)F_l(s^d)}c, \tag{13}$$

[23] As discussed in the Unlimited Liability and Short-term Contracts section, this is also the case if there is no constraint on transfers but contracting is short term.

to make external quality control incentive compatible, where the second equality follows by setting $w_1^d = 0$ and substituting the binding monitoring-incentive-compatibility constraint 10. The minimum necessary agency cost increases as the cut-off point, s^d , decreases and limits the benefit of delegation.

The firm's continuing participation condition 12 is satisfied for some $\delta < 1$ if and only if $\beta > \beta^d$. For $\beta^d < \beta < \beta^n$, the firm either hires the auditor or stays out of the market because an equilibrium with credible internal quality control does not exist. If the discount factor exceeds a certain threshold, the firm accepts a contract $w_1^d = w^d = \alpha v_h$ that extracts the entire discounted future surplus from trade. For $\beta \geq \beta^n$, equilibria with credible internal and external monitoring exist for sufficiently great values of the discount factor. For $\beta^n \leq \beta \leq \beta^{dn}$, internal quality control is never optimal, but for $\beta > \beta^{dn}$ the minimum necessary agency cost (auditor's rent in period 1) is greater (smaller) than the benefit of delegation for sufficiently great (small) values of the discount factor.

Consider the maximum incremental gain from external quality control at the time when the firm decides whether to hire the auditor or control quality internally:

$$\pi_{F,1}^d - \pi_F^n = \underbrace{\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^d)]} - \frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^n)]}}_{\text{benefit of delegation}} - \underbrace{\frac{[1-F(s^d)]c}{(1-\alpha)F_l(s^d)}}_{\text{cost of delegation}}, \tag{14}$$

where

$$s^d \in \arg\max_s \frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s)]} - \frac{[1-F(s)]c}{(1-\alpha)F_l(s)} \tag{15}$$

maximizes the firm's value under delegation in the first period. It can be verified that (averaged out over time) the benefit of delegation decreases but the agency cost increases with δ. Intuitively, as the discount factor increases, external quality control becomes relatively less valuable because it is easier to achieve credibility in both monitoring regimes. However, the auditor's rent increases because the auditor is more tempted to shirk on monitoring when punishment of bad market outcomes by consumers becomes less severe.

Consider the case with a noiseless public signal, $\beta = 1$. Then, for $\delta^d \leq \delta < \delta^{dn} = \delta^n$, delegation is optimal because only external quality control is credible. For $\delta \geq \delta^n$, the maximum incremental gain from delegation in equation 14 becomes

$$\pi_{F,1}^d - \pi_{F,1}^n = \underbrace{\frac{\alpha v_h - c}{1-\delta} - \frac{\alpha v_h - c}{1-\delta}}_{\text{benefit of delegation}} - \underbrace{\frac{\alpha}{1-\alpha}c}_{\text{cost of delegation}} < 0 \tag{16}$$

because in equilibrium $s^n = s^d = 1$ (the harshest punishment is always optimal if consumers do not make mistakes when assessing quality ex post). The benefit of delegation goes to zero as consumers do not actually punish the firm in equilibrium in both monitoring regimes. However, the discounted auditor's rent in period 1 remains strictly bounded away from zero due to the constraint on admissible transfers. As a result, the firm does not hire the auditor in equilibrium. For sufficiently large δ and $\beta < 1$, the benefit of delegation is also smaller than the cost because on the equilibrium path consumers rarely observe low signals of quality and infrequently punish the firm in both monitoring regimes.

Although the effect of c / v_h on the hiring decision is generally ambiguous as through its effect on the equilibrium cut-off points of the public signal it affects both the benefit and cost of delegation, delegation is always optimal if the ratio of the monitoring cost and consumer valuation of high quality is sufficiently small. As c / v_h goes to zero, the auditor's rent also goes to zero, but the benefit of delegation remains strictly positive and bounded away from zero because in equilibrium without delegation the firm is still tempted to deviate from the efficient allocation.

5. Robustness and Extensions

The setting in section 2 makes particular assumptions about feasible contracts, possibility of collusion against consumers, verifiability and transparency of communication of presale information, and market structure. Next I will investigate whether my main result, that the firm controls quality internally only if the frequency of trades and precision of public signal are sufficiently great, is robust to a relaxation of these assumptions.

Unlimited Liability and Short-term Contracts

Here I study what happens if (1) contractual transfers from the auditor to the firm can be positive and (2) contracting and the transfer of the monitoring technology are short term. If the auditor can make positive transfers to the firm and the transfer of the monitoring technology to the auditor is irreversible, delegation is always optimal as the first-period fee (or a transfer before the game starts) can be used to allocate the incremental discounted surplus from external quality control.

Proposition 3 *Suppose the auditor is not subject to limited liability (that is, can make positive contractual transfers to the firm) and transfer of the monitoring technology is irreversible. Then in equilibrium the firm hires the auditor for any* $\delta \geq \delta^d$, *and the optimal contract is given by* $w_1^{du} = \alpha v_h - \pi_F^n$, $w_t^{du} = w^{du} = \alpha v_h$ *for all* $t = 2,3,\ldots$.

The auditor becomes the residual claimant to future revenues to maximize his or her long-run gain from

maintaining consumers' trust and thus soften the necessary punishment. The equilibrium outcome will not change if the auditor proposes a short-term contract w_t^* in the beginning of each period t (rather than a long-term contract $\{w_t\}_{t=1}^{\infty}$ in the beginning of the game) as long as I continue to assume that the transfer of the monitoring technology is irreversible, where $w_t^* \in \Re$. Then the auditor will again offer $w_1^* = w_1^{du} = \alpha v_h - \pi_F^n$ and $w_t^* = w_t^{du} = \alpha v_h \ \forall t \geq 2$, as deviating from the dynamically optimal transfers cannot increase his or her payoff.

However, the agency cost (monitoring-rent effect) can again limit the scope of delegation if both assumptions (1) and (2) are satisfied, that is, if the firm can (at no cost) regain control of the monitoring technology if it stops employing the auditor. So I now suppose that contractual transfers can be positive or negative, contracting is short-term, and the monitoring technology transfer is reversible (in the sense that the firm is able to monitor quality itself in each period in which it does not hire the auditor). The equilibrium outcome will not change if internal monitoring is not credible because the firm again gets his or her reservation payoff of zero if it does not hire the auditor. However, if internal monitoring is credible, delegation will now take place only if there exists a fee $w_t^* = w^*$ such that simultaneously (1) the auditor's monitoring-incentive-compatibility condition 10 is satisfied and (2) the firm prefers to hire the auditor rather than to monitor quality internally in each period during the trust phase:

$$\frac{\alpha v_h - w^*}{1 - \delta[1 - \alpha F_h(s^d)]} \geq \pi_F^n. \qquad (17)$$

Now the auditor cannot charge the full-revenue-extraction fee, αv_h, when the firm can earn a positive profit on its own. Furthermore, condition 17 is more difficult to satisfy than the participation conditions 11 and 12 in the basic model. Therefore, the set of parameter values such that delegation is optimal is, in fact, smaller than in the model with limited liability of the auditor and irreversible technology transfer.

Proposition 4 *Suppose that contractual transfers from the firm to the auditor can be positive or negative, contracting is short-term, and the transfer of the monitoring technology is reversible. The firm hires the auditor if and only if* $\delta \in [\delta^d, \delta^n)$ *for* $\beta \geq \beta^{dn}$ *or* $\delta \geq \delta^d$ *for* $\beta^d < \beta \leq \beta^{dn}$.

Proposition 4 demonstrates that dispensing with the assumption that the auditor cannot make positive transfers to the firm does not necessarily qualitatively change conditions such that delegation is optimal.

Collusion

I now (for concreteness) return to the setting in section 2 but assume that the firm can offer the auditor noncontractible side payments in secret from consumers and investigate how this affects the conditions under which delegation of quality control is profitable. Secret transfers will not have any effect on the equilibrium outcome if I continue to use PPE as my solution concept. The reason is that in a PPE such transfers cannot affect the future play of the game, which means that the firm cannot use them to manipulate the monitoring and reporting strategies of the auditor. However, when secret transfers between the firm and the auditor are possible, a more appropriate solution concept is perfect Bayesian equilibrium, where the firm and the auditor can use the public history, H_t, and the private history of transfers, $\{z_\tau\}_{\tau=1}^{t-1}$ (that is, a history that is private vis-à-vis consumers but is mutually public) to update their beliefs about how the game will be played in the future.

When secret transfers are possible under delegation the firm and the auditor can collude to shirk on presale monitoring and report *h* with probability one. First, I suppose that the firm promises to secretly pay the auditor $z > 0$ in the beginning of each period as long as the auditor reported *h* in each of the previous periods. Then the auditor is willing to stop monitoring if

$$w_t + z + \delta[1-F(s)]\pi_A^{dc} \geq w_t + z - c + \delta(\alpha[1-F_h(s)]\pi_A^{dc} + (1-\alpha)\pi_A^d),\ t \geq 1, \tag{18}$$

where $w_t = w$ for $t = 2,3,...$ and $\pi_A^{dc} = \dfrac{w+z}{1-\delta[1-F(s)]}$ is the auditor's discounted payoff when the auditor always reports *h* and the good is always offered for sale (the superscript "*dc*" stands for "delegation with collusion"), in which case the continuation probability is given by $1-F(s)$. To understand the no-monitoring condition (condition 18), note that under collusion to report *h* without monitoring, the firm finds out if the auditor has deviated from the secret agreement once the auditor monitors, discovers that the true quality is low, and reports *l*. If this happens and collusion breaks down, the trust phase continues with probability one, and the auditor gets $\pi_A^d = \dfrac{w-c}{1-\delta[1-\alpha F_h(s)]}$, that is, the auditor's payoff without collusion.

Collusion will not take place unless the firm is willing to pay the "upfront" bribe z:

$$v_h - w_t - z + \delta[1-F(s)]\pi_F^{dc} \geq \alpha v_h - w_t + \delta[1-\alpha F_h(s)]\pi_F^d, \tag{19}$$

where $\pi_F^{dc} = \dfrac{v_h - w - z}{1-\delta[1-F(s)]}$ is the firm's discounted payoff when the auditor reports *h* without monitoring and the firm pays the bribe. The right-hand side is the firm's payoff if the firm does not make

the secret upfront payment in the beginning of the period, which is the firm's profit without collusion since collusion immediately breaks down, $\pi_F^d = \frac{\alpha v_h - w}{1-\delta[1-\alpha F_h(s)]}$.

The firm can also offer a report-contingent bribe whereas the firm pays the bribe after (and only if) the auditor reports that quality is high. For collusion to be sustainable in this case, again, the auditor's no-monitoring and the firm's no-reneging conditions must be satisfied. The auditor's no-monitoring condition becomes

$$w_t + z + \delta[1-F(s)]\pi_A^{dc} \geq w_t + \alpha z - c + \delta(\alpha[1-F_h(s)]\pi_A^{dc} + (1-\alpha)\pi_A^d), \quad (20)$$

and the firm's no-reneging condition becomes

$$v_h - w_t - z + \delta[1-F(s)]\pi_F^{dc} \geq v_h - w_t + \delta[1-F(s)]\pi_F^d. \quad (21)$$

The difference between the no-monitoring conditions (conditions 18 and 20) is that now the auditor is less tempted to deviate from collusion because doing so means that the auditor will get the bribe only if he or she discovers that the true quality is high but pretends that he or she has approved an uninspected good for sale. The difference between the no-reneging conditions (conditions 19 and 21) is that the firm now faces a greater temptation to renege on its promise to pay the bribe because the firm can do so after the auditor approved the good.

A contract between the firm and the auditor is collusion proof if there is no $z > 0$ such that (1) the no-monitoring and no-reneging conditions (conditions 18 and 19), and (2) no-monitoring and no-reneging conditions (conditions 20 and 21), are simultaneously satisfied. I look for a contract $(w_1 = \tilde{w}_1^d, w_2 = w_3 = ... = \tilde{w}^d)$ and a cut-off point of the public signal $\tilde{s}^d$ such that

1. the auditor's monitoring-incentive-compatibility condition is satisfied,
2. the contract is collusion proof, and
3. the firm prefers to hire the auditor and stay in the market during the trust phase.

I first consider how the possibility of collusion affects the choice between external and internal quality control in the case of perfect public monitoring. Then in any regime in equilibrium, consumers, who do not make mistakes in assessing quality ex post, maximally harshly punish a bad postsale performance, $\tilde{s}^d = s^d = s^n = 1$.

Proposition 5 *Suppose that* $\beta = 1$. *In the presence of the possibility of collusion against consumers, there exists a threshold level of the discount factor* $\delta^c \in (\delta^d, \delta^n)$ *such that the firm hires the auditor if and only if* $\delta^c \leq \delta < \delta^n$.

For $\beta = 1$, a noncontingent/upfront bribe cannot corrupt the auditor if the auditor's monitoring-incentive-compatibility constraint is satisfied, but collusion can be sustained if the firm pays the bribe after the auditor reports *h.* As a result, the threshold discount factor such that the auditor's promise that it will monitor quality is credible must increase to ensure that the auditor is immune to the temptation to shirk on monitoring for any bribe that the firm can credibly promise to pay. For a certain intermediate range of discount factors, the discount factor is simultaneously (1) sufficiently small for the firm to prefer to shirk on monitoring in equilibrium without delegation and (2) sufficiently great for the auditor to prefer to monitor than collude with the firm given that a bribe must satisfy the firm's nonreneging constraint.

In equilibrium with imperfect public monitoring, the possibility of collusion not only shrinks the range of parameter values such that delegation is optimal (that is, there exists a collusion-proof contract that satisfies the auditor's monitoring-incentive-compatibility and the firm's participation constraints) but also reduces the benefit of delegation in the sense that the consumers' trust is lost sooner.

Proposition 6 *Suppose that* $\beta < 1$. *In the presence of the possibility of collusion, the set of parameter values such that the firm hires the auditor is smaller, and the cut-off point of the public signal is greater (that is, consumers' trust is lost sooner).*

For some (but not all) parameter values the possibility of collusion does not eliminate the benefit of delegation because the firm cannot commit to transfer to the auditor the entire current gain from selling an uninspected good. However, to prevent collusion, the cut-off point of the public signal must increase. If the public punishment is too soft, the firm can corrupt the auditor as the firm's secret promise to share the gain from additional sales with the auditor will be credible.

Fully Manipulable Reports

So far I assumed that the auditor cannot claim that quality is low without evidence. I now consider what happens when the monitor can fully manipulate/falsify his or her signal of quality. This, of course, has no effect on the equilibrium outcome without delegation since the firm cannot gain by misrepresenting high quality as low and not offering the product for sale. However, the problem of incentivizing third-party monitoring and truthful reporting becomes more difficult. As I will show next, hiring the auditor can still be optimal if (1) consumers punish unfavorable third-party reports, (2) the auditor's fee is a function of his or her report, or (3) the firm can inspect quality itself before requesting external certification.

Punishment of Unfavorable Reports by Consumers

I will now assume that presale information is soft and consumers observe $S_t = X_t$ (public randomization/coordination device) when the good is not offered for sale.[24] All other features are the same as in the setting in section 2. Suppose that in equilibrium with delegation consumers stop purchasing forever if (1) the auditor reports *h* (sale takes place) and the public signal falls below s^h or (2) the auditor reports *l* (sale does not take place) and the public signal falls below s^l. Then, on signing a contract with transfers $w_1 = w_1^{ds}, w_2 = w_3 = \ldots = w^{ds}$, the auditor's equilibrium payoff solves the following value-recursive equation:

$$\pi_{A,t}^{ds} = w_t^{ds} - c + \delta[\alpha(1-F_h(s^h)) + (1-\alpha)(1-s^l)]\pi_A^{ds},\ t \geq 1, \tag{22}$$

where $\pi_{A,t}^{ds} = \pi_A^{ds}$ for $t \geq 2$. As before, in equilibrium the trust phase continues with probability $1-F_h(s^h)$ when quality is high. However, if quality is low, although there is no trade, the trust phase continues with probability $1-s^l$. The average continuation probability, $\alpha[1-F_h(s^h)] + (1-\alpha)(1-s^l)$, is obtained by aggregating over these two possibilities.

Delegated monitoring is now incentive compatible if the auditor cannot achieve a greater profit by reporting either *h* or *l* instead of learning and truthfully reporting the true quality:

$$w_t^{ds} - c + \delta[\alpha(1-F_h(s^h)) + (1-\alpha)(1-s^l)]\pi_A^{ds} \geq w_t^{ds} + \delta[1-F(s^h)]\pi_A^{ds}, \tag{23a}$$

$$w_t^{ds} - c + \delta[\alpha(1-F_h(s^h)) + (1-\alpha)(1-s^l)]\pi_A^{ds} \geq w_t^{ds} + \delta(1-s^l)\pi_A^{ds}. \tag{23b}$$

Conditional on having monitored, truthful reporting is incentive compatible if misrepresenting both high and low quality is not optimal:

$$w_t^{ds} - c + \delta[1-F_h(s^h)]\pi_A^{ds} \geq w_t^{ds} - c + \delta(1-s^l)\pi_A^{ds} \tag{24a}$$

$$w_t^{ds} - c + \delta(1-s^l)\pi_A^{ds} \geq w_t^{ds} - c + \delta[1-F_l(s^h)]\pi_A^{ds}. \tag{24b}$$

Given the consumers' stopping strategy, the firm's profit in equilibrium with delegation satisfies the following value-recursive equation:

$$\pi_{F,t}^{ds} = \alpha v_h - w_t^{ds} + \delta[\alpha(1-F_h(s^h)) + (1-\alpha)(1-s^l)]\pi_F^{ds},\ t \geq 1, \tag{25}$$

[24] This has no effect on equilibrium without delegation since consumers can ignore the (uninformative) public signal when they do not trade with the firm.

where $\pi_{F,t}^{ds} = \pi_F^{ds}$ for $t \geq 2$. My next result is an analogue of proposition 2 in the case of soft (fully unverifiable) presale information. Let $\delta^{ds} \equiv \left(1-\alpha(1-\beta)-(1-\alpha)\beta+\alpha^2(1-\alpha)(2\beta-1)\frac{v_h}{c}\right)^{-1}$ and $\beta^{dns} \equiv \left(2-\frac{c}{\alpha^2[(1-\alpha)v_h+c]}\right)^{-1}$, where $\beta^{dns} > \beta^n$.

Proposition 7 *Suppose that presale monitoring never generates evidence (all other assumptions are the same as in the basic model),* $\beta > \beta^{dns}$, *and* $c/v_h < \alpha^2/(1+\alpha)$. *There exists a threshold value of the discount factor* δ^{dns} *such that the firm hires the auditor if and only if* $\delta \in [\delta^{ds}, \delta^{dns})$, *where* $\delta^d < \delta^{ds} < \delta^n \leq \delta^{dns} < \delta^{dn}$. *Furthermore, due to the absence of presale evidence of quality in equilibrium, consumers' trust is lost sooner, and the payoffs are smaller.*

In the proof of proposition 7 it is shown that in equilibrium

$$s^l = F(s^h), \tag{26}$$

and the auditor's ongoing rent is given by

$$\pi_A^{ds} = \frac{c}{\delta\{\alpha(1-F_h(s^h))+(1-\alpha)(1-s^l)-[1-F(s^h)]\}} = \frac{c}{\delta\alpha(1-\alpha)[F_l(s^h)-F_h(s^h)]}. \tag{27}$$

Comparing the denominators on the right-hand sides of equations 10 and 27 reveals that for a given cut-off point of the public signal, $s^h = s^d = s$, the incremental continuation probability due to monitoring is smaller, $\alpha(1-\alpha)[F_l(s)-F_h(s)] < (1-\alpha)F_l(s)$. As a result, delegation of monitoring that generates no hard evidence is less likely to take place and provides fewer benefits in equilibrium. Unlike the firm, the auditor is interested only in maintaining consumers' trust rather than meeting consumers' demand for a high-quality product. To mitigate the auditor's incentive to make unsubstantiated claims that quality is low, in equilibrium the game stops with probability 26 each time the auditor reports *l*.

If the monitoring cost is not too great and the discount factor is in a certain intermediate range, delegation is still beneficial despite the greater stopping probability and agency cost. This happens because under delegation the increase in the stopping probability when there is no sale is offset by the decrease in the stopping probability when the good is offered for sale. However, if the cost of monitoring is sufficiently great, hiring the auditor is not profitable for any value of the discount factor as it becomes too costly to incentivize a third-party monitor whose messages are pure cheap talk.

Report-contingent Fee

So far I assumed that the auditor's compensation cannot be a function of his or her report.[25] Now I suppose that the firm and the auditor can sign a (long-term) contract $\{w_{l,t}, w_{h,t}\}_{t=1}^{\infty}$ with payments that are contingent on the auditor's report as in the case where the auditor is a supervisor or manager, where $(w_{l,t}, w_{h,t}) = (w_l^d, w_h^d),\ t = 2,3,\ldots$.

When presale information is partially verifiable, fees that are contingent on the verifiable messages (whether the auditor reports *h* or *l*) can eliminate the agency costs. Consider the setting in section 2 (where monitoring generates hard evidence when quality is low) with report-contingent contractual transfers. The auditor's equilibrium payoff now satisfies

$$\pi_{A,t}^d = \alpha w_{h,t}^d + (1-\alpha) w_{l,t}^d - c + \delta[1 - \alpha F_h(s^d)]\pi_A^d,\ t = 1,2,\ldots\ , \tag{28}$$

where $\pi_{A,t}^d = \pi_A^d,\ t = 2,3,\ldots$. Conditions 8 and 9, which ensure that the auditor is willing to truthfully report *l* and exert the monitoring effort rather than report *h* without inspection, now become, respectively,

$$w_{l,t}^d + \delta\pi_A^d \geq w_{h,t}^d + \delta[1 - F_l(s^d)]\pi_A^d \tag{29}$$

and

$$\alpha w_{h,t}^d + (1-\alpha) w_{l,t}^d - c + \delta[1 - \alpha F_h(s^d)]\pi_A^d \geq w_{h,t}^d + \delta[1 - F(s^d)]\pi_A^d. \tag{30}$$

It can be verified that under a contract $(w_{l,t}, w_{h,t}) = (w_l^d, w_h^d) = (c/(1-\alpha), 0),\ t = 1,2,\ldots$, the auditor's incentive compatibility conditions 29 and 30 are satisfied, and the auditor gets zero rent, $\pi_{A,t}^d = 0\ \forall t$, as the auditor is paid only if he or she presents evidence that quality is low. In this case, the auditor can always offer an incentive-compatible contract that allows the firm to increase his or her profits compared with those under internal quality control, and in equilibrium the firm always hires the auditor.

If presale monitoring never generates hard evidence (soft presale information), contingent payments also expand the range of parameter values such that hiring the auditor is profitable. This happens because, in the presence of contingent compensation, it is not necessary for consumers to punish unfavorable reports to incentivize third-party monitoring and truthful reporting. A contingent payment scheme ensures that the auditor is willing to (1) truthfully report *h* and (2) exert the monitoring effort rather than report *l* without inspection, if, respectively,

$$w_{h,t}^d + \delta[1 - F_h(s^d)]\pi_A^d \geq w_{l,t}^d + \delta\pi_A^d \tag{31}$$

and

[25] Although I do not consider this possibility here, fees that are contingent on the outcome in the final goods market also can significantly reduce the agency costs (however, under the maintained assumption that the public signal of quality is nonverifiable the firm's nonreneging constraint must be satisfied in equilibrium).

$$\alpha w_{h,t}^d + (1-\alpha) w_{l,t}^d - c + \delta[1-\alpha F_h(s^d)] \geq w_{l,t}^d + \delta \pi_A^d . \quad (32)$$

It can be verified that if the incentive-compatibility conditions (conditions 29–32) are satisfied, then

$$\pi_A^d \geq \frac{c}{\delta \alpha (1-\alpha)[F_l(s^d) - F_h(s^d)]}, \quad (33)$$

and the contractual payments such that the auditor's monitoring constraint 33 is binding are given by

$$w_h^d = (1-\delta(1-F(s^d))\pi_A^d \text{ and } w_l^d = (1-\delta)\pi_A^d . \quad (34)$$

Note that for a given cut-off point of the public signal, $s^d = s^h$, the minimum necessary ongoing rent of the auditor (agency cost) equals that in equilibrium in the model with soft information and noncontingent payments in equation 27 (see the proof of proposition 7 for derivation). However, the trust phase now lasts longer compared with that in equilibrium where consumers punish unfavorable reports in the Punishment of Unfavorable Reports by Consumers section because now the trust phase never stops after the auditor reports *l.* Since condition 33 is more difficult to satisfy than condition 10 it is still the case that delegation is less likely to take place than in the basic model.

Sequential Monitoring

So far I have assumed that in the regime with delegation the firm itself cannot monitor/inspect its products. I now relax this assumption in a setting with soft presale information. Let the long-term contract $\{W_t, w_t\}_{t=1,2,\ldots}$ now specify an upfront fee, W_t, and auditing/certification fee, w_t, in each period. The stage game now consists of the following moves:

1. The firm pays the auditor upfront fee $W_t \geq 0$.

2. The firm privately chooses whether to monitor and (privately) observe quality, $e_t^F \in \{0,1\}$.

3. The firm decides whether to publicly pay $w_t \geq 0$ and request an external inspection.

4. If the firm requests an external inspection, the auditor privately chooses whether to monitor and (privately) observe quality, $e_t^A \in \{0,1\}$, and publicly reports r_t.

5. If the firm does not request an external inspection, the good is automatically withdrawn from the market. Otherwise, the firm can offer the good for sale, and consumers observe the public signal of quality as in section 2.

Next I will show that for some parameter values in equilibrium the firm incentivizes the auditor by occasionally monitoring quality internally before requesting external certification rather than relying on consumers to punish unfavorable reports (under the assumption that report-contingent transfers are not feasible). Let $\gamma = \Pr(e_t^F = 1)$ denote the stationary probability that the firm exerts its own

(precertification) monitoring effort. I will focus on an equilibrium where the auditor, if asked to certify, exerts the monitoring effort with probability one and reports truthfully (this is necessarily the case if $\gamma < 1$ and consumption of low quality generates infinite social loss, $v_l = -\infty$).

Since in equilibrium the firm requests certification unless it detects low quality and, if asked, the auditor inspects with probability one, for a given cut-off point of the public signal, s^{dm}, the game continues with the same probability as in the basic model, $1-\alpha F_h(s^{dm})$. Hence, the firm's profit satisfies the following value-recursive equation:

$$\pi_{F,t}^{dm} = \alpha v_h - W_t - [\gamma\alpha + 1 - \gamma]w - \gamma c + \delta[1-\alpha F_h(s^{dm})]\pi_F^{dm}, \tag{35}$$

where $W_t = W$ and $\pi_{F,t}^{dm} = \pi_F^{dm}$ $\forall t \geq 2$, and $w_t = w$ for $\forall t \geq 1$. The current profit now consists of the expected sale revenue, αv_h, minus the upfront fee, W_t, minus the probability that the firm requests external certification, $\gamma\alpha + 1 - \gamma$ (which happens if the firm learns that quality is high or does not monitor internally) times the auditing/certification fee w, minus the expected cost of internal monitoring, γc, plus the continuation value, $\delta[1-\alpha F_h(s^{dm})]\pi_F^{dm}$.

The key feature of equilibrium with sequential inspections is that the auditor is uncertain about why the firm requests external certification. As long as the firm occasionally, but not always, monitors quality itself, the auditor is much less tempted to claim that quality is low without inspection since the firm may detect a false report and break off its relationship with the auditor. Consider the other possibilities. If the firm never monitors, then the auditor can shirk on the monitoring effort and claim that quality is low without any fear of punishment either by the firm or consumers. If the firm always monitors and requests certification only if quality is high, the auditor has no reason to exert his or her own monitoring effort. I also can rule out the possibility that the firm requests external certification after it discovers that quality is low because then the firm has no incentive to acquire information about quality in the first place.

Therefore, in equilibrium the firm must be indifferent between monitoring and shirking, that is,

$$\alpha[v_h - w] - c - W_t + \delta[1-\alpha F_h(s^{dm})]\pi_F^{dm} = \alpha v_h - w - W_t + \delta[1-\alpha F_h(s^{dm})]\pi_F^{dm} \tag{36}$$

or

$$(1-\alpha)w = c. \tag{37}$$

Because the auditor will not approve a low-quality good for sale, the benefit of precertification monitoring is just the savings on the certification fee, which the firm can avoid if it discovers that quality is low (with probability $1-\alpha$) before incurring the cost of external certification (w).

The auditor's profit now satisfies the following value-recursive equation:

$$\pi_{A,t}^{dm} = W_t + [\gamma\alpha + 1 - \gamma][w - c] + \delta[1 - \alpha F_h(s^{dm})]\pi_A^{dm},\ t = 1,2,\ldots, \tag{38}$$

where $\pi_{A,t}^{dm} = \pi_A^{dm}$ for $t = 2,3,\ldots$. Whether or not the auditor actually inspects the good, he or she gets the upfront fee, W_t. The auditor also earns $w - c$ if his or her services are requested, which happens with probability $\gamma\alpha + 1 - \gamma$.

Next I will derive conditions such that, if asked by the firm, the auditor is willing to exert the monitoring effort and report truthfully. The auditor cannot achieve a greater payoff by shirking on the monitoring effort and reporting that quality is high, if

$$w - c + \delta \frac{\alpha[1 - F_h(s^{dm})] + (1-\gamma)(1-\alpha)}{\gamma\alpha + 1 - \gamma} \pi_A^{dm}$$

$$\geq w + \delta \frac{\alpha[1 - F_h(s^{dm})] + (1-\gamma)(1-\alpha)[1 - F_l(s^{dm})]}{\gamma\alpha + 1 - \gamma} \pi_A^{dm}. \tag{39}$$

Similarly, claiming that quality low without inspection is not optimal, if

$$w - c + \delta \frac{\alpha[1 - F_h(s^{dm})] + (1-\gamma)(1-\alpha)}{\gamma\alpha + 1 - \gamma} \pi_A^{dm} \geq w + \delta \frac{1-\gamma}{\gamma\alpha + 1 - \gamma} \pi_A^{dm}. \tag{40}$$

To understand the deviation payoffs on the right-hand sides of equations 39 and 40, note that if the firm requests external certification, the conditional probability that quality is high is given by $\frac{\alpha}{\gamma\alpha + 1 - \gamma}$, and the conditional probability that the firm shirked on its own monitoring effort is given by $\frac{1-\gamma}{\gamma\alpha + 1 - \gamma}$. The auditor uses these conditional probabilities that quality is high and precertification effort took place to calculate the continuation probabilities from shirking on his or her own monitoring effort. Compared with the basic model, the auditor is more tempted to report that quality is high without monitoring because of the occasional prescreening by the firm. However, the auditor is also less tempted to report low quality without an inspection because the firm will detect a false report if the firm exerted the prescreening effort.

The auditor's truth-telling incentive-compatibility conditions are now given by

$$w - c + \delta[1 - F_h(s^{dm})]\pi_A^{dm} \geq w - c + \delta(1-\gamma)\pi_A^{dm} \tag{41}$$

and

$$w - c + \delta\pi_A^{dm} \geq w - c + \delta[1 - F_l(s^{dm})]\pi_A^{dm}. \tag{42}$$

To understand equation 41, note that, when the auditor learns that quality is high, misreporting will not be detected by the firm with probability $1 - \gamma$ (the conditional probability of being caught by the firm falsely reporting that quality is low increases after the auditor learns that the true quality is high). To understand equation 42, note that, when the auditor learns that the true quality is low, the auditor infers that the firm

shirked on the monitoring effort. Nonetheless, misreporting cannot be profitable because it will reduce the continuation probability from 1 to $1-F_l(s^{dm})$.

The auditor's monitoring-incentive-compatibility conditions 39 and 40 can be equivalently rewritten as

$$\pi_A^{dm} = \frac{W + (\gamma\alpha + 1 - \gamma)[w - c]}{1 - \delta[1 - \alpha F_h(s^{dm})]} \geq \frac{c}{\delta} \frac{\gamma\alpha + 1 - \gamma}{\min\{\alpha[\gamma - F_h(s^{dm})], (1-\alpha)(1-\gamma)F_l(s^{dm})\}}. \quad (43)$$

From equation 43 it follows that in equilibrium the firm randomly conducts its own (precertification) inspection, $\gamma \in (0,1)$, and as before, the auditor's truth-telling-incentive-compatibility conditions are not binding.

From equation 35 and using the fact that in equilibrium the firm is indifferent between monitoring and not monitoring quality before requesting external certification it follows that the firm's profit in period 1 (when it makes the decision to sign a long-term contract) is given by

$$\pi_{F,1}^{dm} = \alpha v_h - w - W_1 + \delta[1 - \alpha F_h(s^{dm})] \frac{\alpha v - w - W}{1 - \delta[1 - \alpha F_h(s^{dm})]}, \quad (44)$$

where $w = c/(1-\alpha)$, and s^{dm}, γ, and W satisfy the auditor's monitoring-incentive-compatibility condition 43 and the firm's ongoing participation condition,

$$\pi_F^{dm} = \frac{\alpha v_h - w - W}{1 - \delta[1 - \alpha F_h(s^{dm})]} \geq 0, \quad (45)$$

which ensures that during the trust phase the firm prefers not to terminate the contract after it has been signed.

By considering the two cases with $\alpha[\gamma - F_h(s^{dm})] > (<)(1-\alpha)(1-\gamma)F_l(s^{dm})$, it can be verified that the auditor's monitoring incentive compatibility condition 43 is easiest to satisfy if $\alpha[\gamma - F_h(s^{dm})] = (1-\alpha)(1-\gamma)F_l(s^{dm})$. This implies that when looking for conditions such that external certification with occasional precertification screening is optimal I can restrict my attention to the following rate of the precertification inspections (per period),

$$\gamma = \frac{F(s^{dm})}{\alpha + (1-\alpha)F_l(s^{dm})}, \quad (46)$$

and ongoing upfront payment,

$$W = \max\left[\frac{1 - \delta[1 - \alpha F_h(s^{dm})]}{\delta(1-\gamma)F_l(s^{dm})} - \alpha, 0\right] \frac{\gamma\alpha + 1 - \gamma}{1 - \alpha} c. \quad (47)$$

Substituting equations 46 and 47 into 44 and 45, the problem of maximizing the firm's profit in equation 44 reduces to finding the smallest value of s^{dm} that satisfies constraints 43 and 45, where $W_1 = 0$. From proposition 1 and proposition 7, it follows that for sufficiently small c the firm can achieve a greater profit in equilibrium with sequential monitoring than under internal quality control or in equilibrium where only the auditor inspects quality and consumers occasionally punish unfavorable reports.

I will identify the precise conditions such that sequential monitoring takes place in equilibrium in the special case with a perfect public signal of quality. Let $\delta^{sm} \equiv \min[\frac{2-\alpha}{1+\alpha(1-\alpha)v_h / c - \alpha(1-\alpha)^2}, \left(1+\alpha^2\right)^{-1}]$.

Proposition 8 *Suppose that the firm can inspect quality before requesting external certification and public signal is perfect,* $\beta = 1$. *Then the firm hires the auditor and randomly inspects quality before requesting third-party certification if and only if* $c / v_h \leq \alpha(1-\alpha)$ *and* $\delta^{sm} \leq \delta < \delta^n$, *where* $\delta^{sm} \in (\delta^d, \delta^{ds})$.

In the proof of proposition 8 it is shown that for sufficiently small values of the monitoring cost, sequential monitoring is a more efficient means of disciplining the auditor than the punishment of unfavorable reports by consumers (considered in the Punishment of Unfavorable Reports by Consumers section). The former entails socially wasteful expenditures on monitoring when the firm monitors quality itself and quality is high, while the latter entails a more frequent loss of (socially valuable) trading opportunities. However, for sufficiently small values of the monitoring cost, occasional unnecessary expenditures on monitoring are less socially wasteful than forgone trades.

Yet the firm never monitors quality itself before requesting external certification if presale monitoring generates hard evidence when quality is low. The goal of an occasional internal monitoring before requesting external certification is to mitigate the auditor's temptation to claim that quality is low without inspection. If such a deviation is not feasible, internal monitoring interferes with external monitoring as the auditor, knowing that the firm sometimes discards low quality itself, is more tempted to report that quality is high without an inspection.

Multiple Clients

In reality auditing firms tend to have many clients. I now return to the setting in section 2 but assume that (1) the auditor can be hired by $N \geq 1$ identical, independent firms with i.i.d. quality draws $\theta_{i,t}$ from the

same distribution as before and (2) the auditor chooses independently the monitoring effort for each client at cost *c* per client.

I first focus on a simpler case with perfect public monitoring, $\beta = 1$. Suppose that the auditor and each firm sign a contract $\{w_t\}_{t=1,2,\ldots}$, where $w_t = w \ \ \forall t \geq 2$. Without loss I can assume that in equilibrium the consumers stop buying from all firms forever (thus destroying all of the auditor's future profits) if one or more firms have sold low quality in the past (the harshest punishment). Then the auditor's equilibrium payoff satisfies the following value-recursive equation:

$$\pi_{A,t}^{d,N} = N(w_t - c) + \delta \pi_A^{d,N}, \tag{48}$$

where $\pi_{A,t}^{d,N} = \pi_A^{d,N} \ \forall t \geq 2$. As in the model with a single firm, the auditor's truth-telling constraints are not binding as the auditor strictly prefers to report truthfully when quality is low (since concealing the evidence of low quality reduces the expected payoff), and the auditor must report *h* if he or she does not have evidence that quality is low.

However, the number of clients does affect the auditor's incentives to exert the monitoring efforts. The auditor is willing to monitor all N firms if

$$\pi_{A,t}^{d,N} \geq N w_t - (N-m)c + \delta \alpha^m \pi_A^{d,N} \text{ for all } m = 1, \ldots, N. \tag{49}$$

The right-hand side of the auditor's monitoring-incentive-compatibility condition 49 is the auditor's payoff from monitoring $N - m$ firms. A deviating auditor saves the cost of monitoring m firms, mc, but reduces the continuation value as the game then continues only if quality of all uninspected goods is high (the auditor has to report *h* for each unmonitored firm), which happens with probability α^m.

Proposition 9 *Suppose that* $\beta = 1$ *and there are* N *independent, identical firms (all other assumptions are the same as in the basic model). Each firm hires the auditor if and only if* $\delta \in [\delta^{d,N}, \delta^n)$, *where* $\delta^{d,N} \equiv \left(1 + (1-\alpha^N)(\alpha v_h - c)/c\right)^{-1} < \delta^n$.

A greater client base (a greater number of firms that hire the auditor) confers an advantage as the smallest discount factor such that the firms hire the auditor, $\delta^{d,N}$, decreases with N (and the auditor's payoff increases with N). The reason external certification exhibits economies of scale can be explained as follows. Although an auditor with multiple clients can shirk on the monitoring efforts for a subset of clients/firms, it can be shown that the *N*-client deviation (that is, simultaneously shirking on all monitoring efforts) is the most profitable deviation for the auditor. Thus, the minimum necessary (per-firm) agency cost is determined by the binding *N*-client deviation constraint:

$$\frac{1}{N}\pi_A^{d,N} = \frac{c}{\delta(1-\alpha^N)}. \tag{50}$$

Because the right-hand side decreases with N, a bigger auditor is more likely to be hired in equilibrium. Intuitively, it is easier for the consumers to detect simultaneous shirking on all monitoring efforts by a larger auditor because the event that none of the firms draw low quality becomes less likely.[26]

External certification also exhibits economies of scale if public signals of quality are imperfect, $\beta<1$, and drawn conditionally independently across firms and periods.[27] Nonetheless, proposition 9 demonstrates that as long as the number of potential clients is finite, internal quality control is optimal for sufficiently large values of the discount factor and precision of public signals.

Private Third-party Reporting

Now I again return to the setting with one firm, $N=1$, but assume that in the regime with delegation of monitoring the auditor reports r_t in secret from consumers to the firm, which in turn communicates with consumers (that is, the firm cannot commit to disclose to consumers the auditor's report that quality is low).

The auditor's profit-maximization problem does not change when reporting is private as long as the firm offers the good for sale if and only if the auditor reports *h.* But now I also need to check the incentive compatibility of keeping low quality out of the market (that is, truthfully reporting bad news to consumers). If the auditor reports *l* to the firm, the firm is willing to withhold the good if

$$-w^d + \delta\pi_F^d \geq v_h - w^d + \delta(1-F_l(s^d))\pi_F^d \tag{51}$$

or

$$\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^d)]} - \pi_A^d \geq \frac{v_h}{\delta F_l(s^d)}. \tag{52}$$

Adding conditions 10 and 52 yields

$$\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^d)]} \geq \frac{(1-\alpha)v_h + c}{\delta(1-\alpha)F_l(s^d)}, \tag{53}$$

[26] It is worth pointing out that in Strausz (2005) the current gains from shirking do not increase with the number of clients.

[27] In equilibrium with multiple clients and imperfect public signals, consumers can punish the auditor based on the average public signal (averaged across certified products). As in Cai and Obara (2009), a specification that is sufficiently tractable for studying the effects of the auditor's size on profits is $S_{i,t} = \theta_{i,t} + \varepsilon_{i,t}$, where $\varepsilon_{i,t} \sim N(0, \sigma_\varepsilon^2)$ and are independent across firms and periods and all other elements of the model.

which is the same as the firm's monitoring-incentive-compatibility condition 4 in the regime without delegation. If the auditor's monitoring and firm's allocation-incentive-compatibility conditions 10 and 52, respectively, are satisfied simultaneously, then $s^d \geq s^n$. Because the firm cannot make trust more long lasting by hiring the auditor when the auditor reports to the firm in private, I obtain the following.

Proposition 10 *Suppose that all assumptions of the basic model are satisfied but the auditor's reports are not observable to consumers. Then delegation is not beneficial.*

Delegation does not increase the joint profit when reports are not observable to consumers because although delegation strengthens the incentive to monitor quality it also weakens the incentive to allocate the good efficiently (no commitment effect). Since the firm has to share its profits (sale revenue) with the auditor to sustain the auditor's incentive to acquire information about quality, consumers need to punish the firm more harshly to make the efficient allocation of the good incentive compatible. In contrast with the case of delegation with public reporting, under private reporting the hardening of the firm's truth-telling-incentive-compatibility constraint offsets the benefits of relaxing the monitoring-incentive-compatibility constraint. Furthermore, note that this result continues to hold in any of the previous extensions where the agency cost is greater than in the basic model (for the same parameter values).

Nonetheless, economies of scale in certification can make delegation with private reporting optimal. Again, for simplicity, I focus on the case with perfect public signals of quality. From substituting the per-firm agency cost for $N \geq 2$ in equation 50 into the firm's truth-telling constraint under delegation (equation 52) it follows that the firm is willing to reveal that the auditor issued a negative report if

$$\frac{\alpha v_h - c}{1-\delta} - \frac{1}{N}\pi_A^{d,N} \geq \frac{v_h}{\delta}, \tag{54}$$

where $\pi_A^{d,N} / N \geq c/[\delta(1-\alpha^N)]$ (recall that when public signals are perfect consumers punish all firms and the auditor when any firm sells a low-quality product). Delegation of monitoring with private reporting when the auditor has many clients can, thus, be optimal because a multiclient auditor imposes a smaller per-firm agency cost than a single-client auditor. This makes it easier to satisfy the firm's truth-telling condition 54 as N increases. Let $\delta^{dp,N} \equiv \left(1+[\alpha v_h - c]/\{v_h + c/[1-\alpha^N]\}\right)^{-1}$.

Proposition 11 *Suppose that $\beta = 1$, auditor's reports are not observed by consumers, and there are $N \geq 2$ identical, independent firms that can hire the auditor. Then in equilibrium there is no trade if $\delta < \delta^{dp,N}$, each firm hires the auditor if $\delta^{dp,N} \leq \delta < \delta^n$, and each firm controls quality internally if $\delta \geq \delta^n$.*

6. Conclusions

I studied an infinitely repeated game where consumers' trust provides incentives to (1) acquire information about whether the good is defective and (2) withhold the good from sale if it is defective. While delegation of monitoring can lessen the first and, under public third-party reporting, dispose of the second moral hazard concern, it also creates agency costs due to either limited liability or lack of commitment to delegation. I showed that in a variety of settings in equilibrium the firm controls quality internally only if the discount factor is sufficiently great and public signal is sufficiently precise.

In addition to the extensions and robustness checks discussed in section 5, several assumptions are worth investigating further. First, I assumed that presale monitoring technology is perfect. If presale signals of quality are imperfect, the cost of maintaining credibility also can be managed by the choice of a grading standard, and in turn, the difficulty of the grading standard may affect the choice between internal and external monitoring. If the grading standard is stringent, it may be easier to incentivize an external certifier to report that the product does not meet the criteria when quality is just below the minimum necessary for successful certification. On the other hand, if the standard is permissive, most of the time the products will meet the criteria necessary for certification, and the firm easily can be incentivized to grade according to the standard without external certification.

Second, I assumed that quality is an exogenous random variable. If the firm can exert effort to improve the average quality (or shift the entire probability distribution), the benefits of delegated monitoring can increase because there will be less room for sophisticated deviations such as joint shirking on efforts to (1) increase the average ex ante quality, (2) monitor ex post quality, and (3) truthfully report the outcome of monitoring. On the other hand, the firm's incentives to exert efforts to improve the average quality can also decrease under external certification because providing adequate incentives to an external monitor is costly.

Appendix A

Proof of Proposition 1 With some minor modifications, my model without delegation is essentially equivalent to the model of provision of quality/effort by a single firm in Cai and Obara (2009), where, in their notation, shirking on effort results in the current gain, $c_h - c_l \equiv (1-\alpha)v_h + c$. Therefore, as in Cai and Obara, without loss I can restrict my attention to stationary equilibria where the trust phase continues as long as the public signal stays above a certain threshold. The monitoring and allocation/grading incentive compatibility conditions 2 and 3 can be equivalently rewritten as, respectively,

$$\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^n)]} \geq \frac{(1-\alpha)v_h + c}{\delta(1-\alpha)F_l(s^n)} \tag{A1}$$

and

$$\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^n)]} \geq \frac{v_h}{\delta F_l(s^n)}, \tag{A2}$$

which shows that equation A2 is implied by equation A1. It can be verified that, when condition A1 is binding, the firm's equilibrium profit can be rewritten as

$$\begin{aligned}\pi_F^n &= \frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^n)]} = \frac{\alpha v_h - c}{1-\delta} - \frac{\delta\alpha F_h(s^n)}{1-\delta}\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^n)]} \\ &= \frac{\alpha v_h - c}{1-\delta} - \frac{\delta\alpha F_h(s^n)}{1-\delta}\frac{(1-\alpha)v_h + c}{\delta(1-\alpha)F_l(s^n)} = \frac{1}{1-\delta}\left(\alpha v_h - c - \frac{\alpha(1-\beta)}{(1-\alpha)\beta}[(1-\alpha)v_h + c]\right) \geq 0,\end{aligned} \tag{A3}$$

where the inequality holds if and only if $\beta \geq \beta^n$. The third equality follows from equation A1, and the fourth equality follows from the definitions of the conditional stopping probabilities. Condition A1 also can be rewritten as

$$\delta\beta s^n \frac{\alpha v_h - c}{1-\delta[1-\alpha(1-\beta)s^n]} - v_h - \frac{c}{1-\alpha} \geq 0. \tag{A4}$$

Thus, the smallest cut-off point of the public signal that satisfies equation A4 is given by

$$s^n = \frac{1-\delta}{\delta}\frac{(1-\alpha)v_h + c}{(1-\alpha)\beta[\alpha v_h - c] - \alpha(1-\beta)[(1-\alpha)v_h + c]} \in (0,1] \tag{A5}$$

if and only if $\delta \geq \delta^n$ and $\beta > \beta^n$. The comparative statics results follow from differentiating the profit in equation A3 and the equilibrium cut-off point of the public signal in equation A5). ■

Proof of Proposition 2 The firm hires the auditor if there exists s^d such that conditions 10 through 12 are simultaneously satisfied. Conditions 10 through 12 can be equivalently rewritten as

$$\pi_A^d \geq \frac{c}{\delta(1-\alpha)F_l(s^d)}, \tag{A6}$$

$$\alpha v_h - w_1^d + \delta[1-\alpha F_h(s^d)]\left(\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^d)]} - \pi_A^d\right) \geq \pi_F^n, \tag{A7}$$

and

$$\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^d)]} - \pi_A^d \geq 0. \tag{A8}$$

Conditions A6 through A8 are the easiest to satisfy if $w_1^d = 0$ and the auditor's monitoring-incentive-compatibility condition A6 binds. Therefore, the firm hires the auditor if and only if

$$\alpha v_h + \delta[1-\alpha F_h(s^d)]\left(\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^d)]} - \frac{c}{\delta(1-\alpha)F_l(s^d)}\right) \geq \pi_F^n \qquad \text{(A9)}$$

and

$$\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^d)]} - \frac{c}{\delta(1-\alpha)F_l(s^d)} \geq 0 \qquad \text{(A10)}$$

for some $s^d \in (0,1]$.

From equations A9 and A10 it follows that there are two cases to consider: $\pi_F^n \leq \alpha v_h$ and $\pi_F^n > \alpha v_h$. By proposition 1, $\pi_F^n = 0$ for all $\delta < \delta^n$, and from the firm's monitoring-incentive-compatibility condition 2 in equilibrium without delegation, it follows that $\pi_F^n > \alpha v_h$ for all $\delta \geq \delta^n$. Therefore, for $\delta < \delta^n$ condition A9 is implied by condition A10, which is satisfied if and only if $\delta \geq \delta^d$ and $\beta \geq \beta^d$, where $\delta^d \equiv \left(\alpha(1-\alpha)\beta v_h / c + \alpha\beta + (1-\alpha)(1-\beta)\right)^{-1} < \delta^n$ and $\beta^d \equiv \alpha\left(\alpha(1-\alpha)v_h / c + 2\alpha - 1\right)^{-1} < \beta^n$. Note that δ^d decreases with the precision of the public signal, β, and the ratio of consumer benefit from high quality and the monitoring cost, v_h / c.

If $\delta \geq \delta^n$, condition A10 is implied by condition A9, which can be rewritten as

$$\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^d)]} - \frac{1-F(s^d)}{(1-\alpha)F_l(s^d)}c \geq \pi_F^n$$

or

$$\frac{\alpha v_h - c}{1-\delta[1-\alpha(1-\beta)s^d]} - \frac{1-\alpha(1-\beta)s^d - (1-\alpha)\beta s^d}{(1-\alpha)\beta s^d}c \geq \pi_F^n$$

or

$$(1-\delta)\{\frac{\alpha v_h - c}{1-\delta[1-\alpha(1-\beta)s^d]} - \frac{c}{(1-\alpha)\beta s^d} + \frac{\alpha(1-\beta)+(1-\alpha)\beta}{(1-\alpha)\beta}c\}$$
$$-\{\alpha v_h - c - \frac{\alpha(1-\beta)}{(1-\alpha)\beta}[(1-\alpha)v_h + c]\} \geq 0 \text{ for some } s^d \in (0,1], \qquad \text{(A11)}$$

where I substituted $\pi_F^n = \frac{1}{1-\delta}\left(\alpha v_h - c - \frac{\alpha(1-\beta)}{(1-\alpha)\beta}[(1-\alpha)v_h + c]\right)$ to obtain equation A11. Since the firm can extract the entire surplus in equilibrium without delegation, condition A11 implies that $s^d < s^n$. Maximizing the left-hand side with respect to s^d yields the following equivalent form of equation A11:

$$G(\delta) \geq 0, \qquad \text{(A12)}$$

where $G(\delta) \equiv \left(\sqrt{\alpha v_h - c} - \sqrt{\delta\frac{\alpha(1-\beta)}{(1-\alpha)\beta}c}\right)^2 + (1-\delta)\frac{\alpha(1-\beta)+(1-\alpha)\beta}{(1-\alpha)\beta}c - \{\alpha v_h - c$
$-\frac{\alpha(1-\beta)}{(1-\alpha)\beta}[(1-\alpha)v_h + c]\}$ and $s^d = \frac{1-\delta}{\delta}\frac{1}{\sqrt{\alpha(1-\alpha)\beta(1-\beta)(\alpha v_h - c)/(\delta c)} - \alpha(1-\beta)}$

$< \frac{1-\delta}{\delta} \frac{1}{(1-\alpha)\beta[\alpha v_h - c]/[(1-\alpha)v_h + c] - \alpha(1-\beta)} = s^n$. Assuming that equation A12 is satisfied, differentiation yields

$$\frac{\partial G}{\partial \delta} = -\sqrt{\frac{\alpha(1-\beta)(\alpha v_h - c)c}{\delta(1-\alpha)\beta}} - c < 0. \quad \text{(A13)}$$

Therefore, there exists a unique $\delta^{dn} \in [\delta^n, 1]$ such that in equilibrium the firm hires the auditor for all $\delta^d \le \delta \le \delta^{dn}$ and does not to hire the auditor for all $\delta > \delta^{dn}$.

Taking the limit yields,

$$\lim_{\delta \to 1} G(\delta) = G(1) = \alpha v_h - c - 2\sqrt{(\alpha v_h - c)\frac{\alpha(1-\beta)}{(1-\alpha)\beta}c} + \frac{\alpha(1-\beta)}{(1-\alpha)\beta}c$$

$$-\{\alpha v_h - c - \frac{\alpha(1-\beta)}{(1-\alpha)\beta}[(1-\alpha)v_h + c]\} < (\ge) 0 \text{ as } \beta > (\le)\beta^{dn}, \quad \text{(A14)}$$

where $\beta^{dn} \equiv \left(1 + \frac{4(1-\alpha)(\alpha v_h - c)c}{\alpha[(1-\alpha)v_h + 2c]^2}\right)^{-1} > \beta^n$. From equations A13 and A14 it follows that $\delta^{dn} < 1$ if and only if $\beta > \beta^{dn}$. Also, from equation A9 it follows that the trust phase lasts longer under delegation, $s^d < s^n$, $\pi_F^d > 0$, and $w^d < \alpha v_h$ for $\delta^n \le \delta \le \delta^{dn}$. It is also easy to see that $G(\delta) \ge 0$ for sufficiently small c / v_h. ■

Proof of Proposition 3 An equilibrium can be recovered by solving the following profit-maximization problem:

$\max_{w_1^d, s^d, \pi_A^d} w_1^d - c + \delta[1 - \alpha F_h(s^d)]\pi_A^d$ subject to equations A6 through A8 and allowing w_1^d to take any positive or negative value. It is straightforward to verify that all constraints bind at optimum, so that the solution to is given by

$$s^{du} = \frac{1-\delta}{\delta} \frac{c}{(1-\alpha)\beta(\alpha v_h - c) - \alpha(1-\beta)c} \le s^d,$$

$$w_1^{du} = \alpha v_h - \pi_F^n,$$

$$\pi_A^{du} = \frac{\alpha v_h - c}{1 - \delta[1 - \alpha F_h(s^{du})]}.$$ ■

Proof of Proposition 4 From the auditor's monitoring-incentive-compatibility and firm's participation conditions 10 and 17, respectively, it follows that the firm hires the auditor if and only if

$$\frac{\alpha v_h - c}{1 - \delta[1 - \alpha F_h(s^d)]} - \frac{c}{\delta(1-\alpha)F_l(s^d)} \ge \pi_F^n \text{ for some } s^d \in [0,1]. \quad \text{(A15)}$$

Therefore, as in the basic model, the firm hires the auditor in each period as long as the trust phase lasts if $\delta \in [\delta^d, \delta^n)$. If $\delta \ge \delta^n$, condition A15 can be rewritten as

$$\max_s (1-\delta)\{\frac{[\alpha v_h - c]}{1 - \delta[1 - \alpha(1-\beta)s]} - \frac{c}{\delta(1-\alpha)\beta s}\} - \{\alpha v_h - c - \frac{\alpha(1-\beta)}{(1-\alpha)\beta}[(1-\alpha)v_h + c]\} \ge 0, \quad \text{(A16)}$$

where $s < s^n$. Solving the optimization problem yields the following equivalent form of equation A16:

$$\left(\sqrt{\alpha v_h - c} - \sqrt{\frac{\alpha(1-\beta)}{(1-\alpha)\beta}c}\right)^2 - \{\alpha v_h - c - \frac{\alpha(1-\beta)}{(1-\alpha)\beta}[(1-\alpha)v_h + c]\} \geq 0, \quad \text{(A17)}$$

where the dynamically optimal cut-off point $s^* = \frac{1-\delta}{\delta[\sqrt{\alpha(1-\alpha)\beta(1-\beta)(\alpha v_h - c)/(c)} - \alpha(1-\beta)]}$

$< \frac{1-\delta}{\delta} \frac{1}{(1-\alpha)\beta[\alpha v_h - c]/[(1-\alpha)v_h + c] - \alpha(1-\beta)} = s^n$, or $\beta < \left(1 + \frac{(1-\alpha)[\alpha v_h - c]c}{\alpha[(1-\alpha)v_h + c]^2}\right)^{-1}$.

Condition A17 is satisfied if and only if $\beta^d < \beta \leq \beta^{dn}$.

■

Proof of Proposition 5 When the public signal is perfect, as shown in proposition 2, the firm does not hire the auditor if $\delta \geq \delta^n$. So suppose that $\delta < \delta^n$, and the firm and the auditor sign the contract that stipulates the auditing fee w_1 in period 1 and w in each period $t = 2,3,...$ until one of the parties terminates the contract (that is, while the trust phase lasts).

As in the absence of the possibility of secret transfers, once the firm and the auditor have signed the contract, the auditor's fee in the first period has no effect on the deviation and collusion payoffs. From equations 18 and 19 in the text it follows that the conditions that sustain collusion against consumers where the firm pays the bribe before the auditor reports *h* are given by

$$w_t + z + \delta\alpha\frac{w+z}{1-\delta\alpha} \geq w_t + z - c + \delta\{\alpha\frac{w+z}{1-\delta\alpha} + (1-\alpha)\frac{w-c}{1-\delta}\} \quad \text{(A18)}$$

and

$$v_h - w_t - z + \delta\alpha\frac{v_h - w - z}{1-\delta\alpha} \geq \alpha v_h - w_t + \delta\frac{\alpha v_h - w}{1-\delta}. \quad \text{(A19)}$$

From conditions A18 and A19 it follows that this form of collusion is not sustainable if and only if

$$\frac{w-c}{1-\delta} - \frac{c}{\delta(1-\alpha)} \geq 0. \quad \text{(A20)}$$

From equations 20 and 21 in the text it follows that the conditions that sustain collusion against consumers where the firm pays the bribe after the auditor reports *h* are given by

$$w_t + z + \delta\alpha\frac{w+z}{1-\delta\alpha} \geq w_t + \alpha z - c + \delta\{\alpha\frac{w+z}{1-\delta\alpha} + (1-\alpha)\frac{w-c}{1-\delta}\}, \quad \text{(A21)}$$

$$v_h - w_t - z + \delta\alpha\frac{v_h - w - z}{1-\delta\alpha} \geq v_h - w_t + \delta\alpha\frac{\alpha v_h - w}{1-\delta}. \quad \text{(A22)}$$

From conditions A21 and A22 it follows that this form of collusion is not sustainable if and only if

$$\frac{w-c}{1-\delta} - \frac{c}{\delta(1-\alpha)} \geq \alpha\{v_h - w - (1-\delta\alpha)\frac{\alpha v_h - w}{1-\delta}\} \quad \text{(A23)}$$

or

$$w \geq \frac{\alpha[1-\delta-\alpha(1-\delta\alpha)]v_h + \frac{1-\delta\alpha}{\delta(1-\alpha)}c}{1-\delta\alpha(1-\alpha)}. \quad \text{(A24)}$$

Note that condition A20 coincides with the auditor's monitoring-incentive-compatibility constraint, and the firm will agree to sign any contract with $w_1 \geq 0, w_2 = w_3 = ... = w \leq \alpha v_h$ because it

will provide the firm with a nonnegative payoff. Substituting the maximum ongoing fee $w = \alpha v_h$, conditions A20 and A24 become, respectively,

$$\alpha v_h \geq \frac{1-\delta\alpha}{\delta(1-\alpha)} c \tag{A25}$$

and

$$\alpha v_h \geq \frac{1}{\alpha+\delta(1-\alpha)} \frac{1-\delta\alpha}{\delta(1-\alpha)} c . \tag{A26}$$

Both conditions are satisfied if and only if $\delta \geq \delta^c$, where

$$\delta^c \equiv \tfrac{1}{2}[-\frac{\alpha}{1-\alpha} - \frac{c}{(1-\alpha)^2 v_h} + \sqrt{\left(\frac{\alpha}{1-\alpha} + \frac{c}{(1-\alpha)^2 v_h}\right)^2 + \frac{4c}{\alpha(1-\alpha)^2 v_h}}] \in (\delta^d, \delta^n) \tag{A27}$$

is the smallest δ that satisfies equation A26. Summarizing, in the presence of the possibility of collusion in equilibrium the firm hires the auditor if and only if $\delta^c \leq \delta < \delta^n$. ■

Proof of Proposition 6 In step 1, I show that in the presence of the possibility of collusion the consumer trust is lost sooner in equilibrium with delegation. In step 2, I establish that there exist parameter values such that delegation takes place.

Step 1. (The effect of the possibility of collusion on trust) First, I obtain an equivalent form of the requirement that conditions 18 and 19 are not satisfied simultaneously. Rewriting the auditor's no-monitoring condition 18 yields

$$\delta(1-\alpha)[1-F_l(\tilde{s}^d)]\pi_A^{dc} \geq -c + \delta(1-\alpha)\pi_A^d \tag{A28}$$

or

$$\frac{\tilde{w}^d + z}{1-\delta[1-F(\tilde{s}^d)]} \geq \frac{1}{1-F_l(\tilde{s}^d)}\left(\frac{\tilde{w}^d - c}{1-\delta[1-\alpha F_h(\tilde{s}^d)]} - \frac{c}{\delta(1-\alpha)}\right), \tag{A29}$$

where I substituted the expressions for π_A^{dc} and π_A^d. Rewriting the firm's no-reneging condition 19 yields

$$\tilde{w}^d - \tilde{w}_1^d + \frac{v_h - \tilde{w}^d - z}{1-\delta[1-F(\tilde{s}^d)]} \geq \tilde{w}^d - \tilde{w}_1^d + \frac{\alpha v_h - \tilde{w}^d}{1-\delta[1-\alpha F_h(\tilde{s}^d)]} . \tag{A30}$$

From adding conditions A29 and A30 it follows that these conditions do not hold simultaneously for any z if and only if

$$H(\tilde{s}^d, \tilde{w}^d) \equiv \frac{\tilde{w}^d - c}{1-\delta[1-\alpha F_h(\tilde{s}^d)]} - \frac{c}{\delta(1-\alpha)} - [1-F_l(\tilde{s}^d)]\left(\frac{v_h}{1-\delta[1-F(\tilde{s}^d)]} - \frac{\alpha v_h - \tilde{w}^d}{1-\delta[1-\alpha F_h(\tilde{s}^d)]}\right) \geq 0 . \tag{A31}$$

The auditor's minimum profit under collusion (the first line on the right-hand side) must be greater than the incremental joint value that the firm is willing to transfer to the auditor (in the second line).

Evaluating function H at $\tilde{w}^d = w^d$ and $\tilde{s}^d = s^d$ yields

$$H(s^d, w^d) = [1-F_l(s^d)]\left(\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^d)]} - \frac{v_h}{1-\delta[1-F(s^d)]}\right) < [1-F_l(s^n)]\left(\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^n)]} - \frac{v_h}{1-\delta[1-F(s^n)]}\right) = 0 , \tag{A32}$$

where to obtain the first equality I used the fact that for $\beta<1$ the auditor's monitoring-incentive-compatibility constraint is binding in equilibrium without the possibility of collusion,

$$\frac{w^d - c}{1-\delta[1-\alpha F_h(s^d)]} - \frac{c}{\delta(1-\alpha)F_l(s^d)} = 0, \tag{A33a}$$

and to obtain the inequality and the second equality I used the facts that

$$s^d < s^n, \tag{A33b}$$

and the firm's monitoring-incentive-compatibility constraint is binding in equilibrium without delegation. This demonstrates that in an equilibrium with delegation $\tilde{s}^d > s^d$, that is, the possibility of collusion decreases the average duration of the trust phase and profits and, hence, the set of parameters values such that delegation is optimal in equilibrium.

Step 2. (The effect of the possibility of collusion on delegation) I now show that there exists a set of parameter values such that delegation takes place in equilibrium with the possibility of collusion. Consider $\delta = \delta^n$. To show that a collusion-proof equilibrium with delegation exists for $\delta < \delta^n$ sufficiently close to δ^n, it suffices to show that (1) $H(\tilde{s}^d, \tilde{w}^d) > 0$; (2) collusion where the firm pays a bribe after a report *h* has been issued provided that the auditor had always approved products for sale in the past is not sustainable, that is, conditions 20 and 21 cannot be satisfied simultaneously; and (3) the auditor is willing to monitor, that is, $\frac{\tilde{w}^d - c}{1-\delta[1-\alpha F_h(\tilde{s}^d)]} > \frac{\tilde{w}^d}{1-\delta[1-F(\tilde{s}^d)]}$, for $\delta = \delta^n$, $\tilde{s}^d = s^n = 1$, and $\tilde{w}^d = \alpha v_h$.

The auditor's no-monitoring-under-collusion condition 20 can be rewritten as

$$(1-\alpha)\left(z + \delta\left(1-F_l(s^n)\right)\frac{\tilde{w}^d + z}{1-\delta[1-F(\tilde{s}^d)]}\right) + c \geq \delta(1-\alpha)\frac{\tilde{w}^d - c}{1-\delta[1-\alpha F_h(\tilde{s}^d)]}, \tag{A34}$$

and the firm's no-reneging-under-collusion condition 21 can be rewritten as

$$\delta[1-F(\tilde{s}^d)]\left(\frac{v_h - \tilde{w}^d - z}{1-\delta[1-F(\tilde{s}^d)]} - \frac{\alpha v_h - \tilde{w}^d}{1-\delta[1-\alpha F_h(\tilde{s}^d)]}\right) \geq z \tag{A35}$$

or

$$z \leq \delta[1-F(\tilde{s}^d)]\left(v_h - \tilde{w}^d - \frac{1-\delta[1-F(\tilde{s}^d)]}{1-\delta[1-\alpha F_h(\tilde{s}^d)]}(\alpha v_h - \tilde{w}^d)\right)$$
$$= \delta[1-F(\tilde{s}^d)]\left(\{1-\alpha\frac{1-\delta[1-F(\tilde{s}^d)]}{1-\delta[1-\alpha F_h(\tilde{s}^d)]}\}v_h + \frac{\delta(1-\alpha)F_l(\tilde{s}^d)}{1-\delta[1-\alpha F_h(\tilde{s}^d)]}\tilde{w}^{dp}\right). \tag{A36}$$

Because from the firm's participation constraint it follows that in equilibrium $\tilde{w}^d \leq \alpha v_h$, condition A36 implies that

$$z \leq \delta[1-F(\tilde{s}^d)](1-\alpha)v_h < (1-\alpha)v_h. \tag{A37}$$

Substituting $z = (1-\alpha)v_h$ and $\tilde{w}^d = \alpha v_h$ into the auditor's no-monitoring-under-collusion condition A34 yields a sufficient condition such that collusion with a postreport bribe is not sustainable:

$$(1-\alpha)\left((1-\alpha)v_h + \delta\left(1-F_l(\tilde{s}^d)\right)\frac{v_h}{1-\delta[1-F(\tilde{s}^d)]}\right) + c \leq \delta(1-\alpha)\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(\tilde{s}^d)]}, \tag{A38}$$

or

$$\frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(\tilde{s}^d)]} - \frac{c}{\delta(1-\alpha)} - \left(1-F_l(\tilde{s}^d)\right)\frac{v_h}{1-\delta[1-F(\tilde{s}^d)]} - \frac{(1-\alpha)v_h}{\delta} \geq 0, \tag{A39}$$

or

$$H(\tilde{s}^d, \alpha v_h) - \frac{(1-\alpha)v_h}{\delta} \geq 0. \tag{A40}$$

But from equation A1 it follows that

$$H(s^n, \alpha v_h) - \frac{(1-\alpha)v_h}{\delta} = F_l(s^n)\left(\frac{v_h}{1-\delta[1-F(s^n)]} - \frac{c}{\delta(1-\alpha)F_l(s^n)}\right) - \frac{(1-\alpha)v_h}{\delta}$$

$$= F_l(s^n)\left(\frac{v_h}{1-\delta[1-F(s^n)]} - \frac{(1-\alpha)v_h + c}{\delta(1-\alpha)F_l(s^n)}\right) = 0. \tag{A41}$$

To verify that the auditor's monitoring-incentive-compatibility condition is satisfied, note that equation A1 also implies that (at $\tilde{w}^d = \alpha v_h$ and $\tilde{s}^d = s^n = 1$) I have

$$\frac{\tilde{w}^d - c}{1-\delta[1-\alpha F_h(\tilde{s}^d)]} = \frac{\alpha v_h - c}{1-\delta[1-\alpha F_h(s^n)]} = \frac{v_h}{1-\delta[1-F_h(s^n)]}$$

$$> \frac{\alpha v_h}{1-\delta[1-F(s^n)]} = \frac{\tilde{w}^d}{1-\delta[1-F(\tilde{s}^d)]}. \tag{A42}$$

Therefore, by continuity, there exists $\tilde{s}^d \in (s^d, 1]$ such that a contract with ongoing payment $\tilde{w}^d = \alpha v_h$ is collusion proof and the auditor's monitoring-incentive-compatibility condition and the firm's individual-rationality conditions (to hire and keep the auditor) are satisfied for all $\delta < \delta^n$ sufficiently close to δ^n. ■

Proof of Proposition 7 Rewriting the auditor's incentive-compatibility conditions 24 and 23 yields, respectively,

$$1-F_h(s_h) \geq 1-s_l, \tag{A43}$$

$$1-s_l \geq 1-F_l(s_h), \tag{A44}$$

$$\pi_A^{ds} \geq \frac{c}{\delta(1-\alpha)[F_l(s_h)-s_l]}, \tag{A45}$$

$$\pi_A^{ds} \geq \frac{c}{\delta\alpha[s_l - F_h(s_h)]}. \tag{A46}$$

As in the basic model, the firm is willing to hire the auditor if it cannot earn a greater profit under internal quality control in period 1 (this is analogous to condition A7),

$$\alpha v_h - w_1^{ds} + \delta[\alpha(1-F_h(s_h)) + (1-\alpha)(1-s_l)]\left(\frac{\alpha v_h - c}{1-\delta[\alpha(1-F_h(s_h)) + (1-\alpha)(1-s_l)]} - \pi_A^{ds}\right)$$

$$\geq \pi_F^n, \tag{A47}$$

and the firm cannot achieve a greater profit by terminating the contract after it has been signed in periods 2,3,… (this is analogous to condition A8),

$$\frac{\alpha v_h - c}{1-\delta[\alpha(1-F_h(s_h)) + (1-\alpha)(1-s_l)]} - \pi_A^{ds} \geq 0. \tag{A48}$$

Again, I seek to identify a set of parameter values under which delegation takes place in equilibrium, that is, the auditor's incentive-compatibility and the firm's individual-rationality conditions A43 through A48 are satisfied. Conditions A43 through A48 are the easiest to satisfy if $w_1^d = 0$ and at least one of the conditions A45 and A46 is binding. If condition A46 does not bind, it must be that $\alpha[s_l - F_h(s_h)] > (1-\alpha)\ [F_l(s_h) - s_l]$. But this is a contradiction because the left-hand sides of conditions A47 and A48 are decreasing in s_l. If condition A45 does not bind, it must be that $\alpha[s_l - F_h(s_h)] < (1-\alpha)[F_l(s_h) - s_l]$. This also yields a contradiction because the left-hand sides of conditions A47 and A48 are decreasing in s_h. Therefore, conditions A43 through A48 are the easiest to satisfy if $\alpha[s_l - F_h(s_h)] = (1-\alpha)[F_l(s_h) - s_l]$ or

$$s_l = F(s_h). \tag{A49}$$

Hence, the truth-telling constraints A43 and A44 are not binding. Substituting the binding conditions A45 and A49 into the firm's hiring and participation constraints A47 and A48, it follows that the firm hires the auditor if and only if there exists $s^h = s \in (0,1]$ such that

$$\alpha v_h + \delta[1 - Bs](\frac{\alpha v_h - c}{1-\delta[1-B]} - \frac{c}{\delta Ds}) \geq \pi_F^n \tag{A50}$$

and

$$\frac{\alpha v_h - c}{1-\delta[1-Bs]} - \frac{c}{\delta Ds} \geq 0, \tag{A51}$$

where $B = \alpha(1-\beta) + (1-\alpha)[\alpha(1-\beta) + (1-\alpha)\beta]$ and $D = \alpha(1-\alpha)(2\beta - 1)$. As in the proof of proposition 2, there are two cases to consider: $\pi_F^n \leq \alpha v_h$ (which happens if and only if $\delta < \delta^n$) and $\pi_F^n > \alpha v_h$ (which happens if and only if $\delta \geq \delta^n$).

If $\delta < \delta^n$, condition A50 is implied by condition A51 since $\pi_F^n = 0$. Condition A51 is satisfied if and only if $\beta > \beta^{ds} \equiv \left(2 - \frac{1}{\alpha[\alpha(1-\alpha)v_h / c + 1]}\right)^{-1} > \beta^n$ and $\delta \geq \delta^{ds} \equiv \frac{1}{1 - B - D + \alpha D v_h / c} > \delta^d$. It can be verified that $\delta^{ds} < \delta^n$ if and only if $\beta > \beta^{dns}$ and $\frac{c}{v_h} < \frac{\alpha^2}{1+\alpha}$.

If $\delta \geq \delta^n$, condition A48 is implied by condition A47 since $\pi_F^n > \alpha v_h$. Condition A47 can be rewritten as

$$\frac{\alpha v_h - c}{1-\delta[1-Bs]} - \frac{1 - [\alpha(1-\beta) + (1-\alpha)\beta]s}{Ds} c \geq \pi_F^n \tag{A52}$$

or

$$(1-\delta)\{\frac{\alpha v_h - c}{1-\delta[1-Bs]} - \frac{c}{Ds} + \frac{\alpha(1-\beta) + (1-\alpha)\beta}{D} c\}$$
$$- \{\alpha v_h - c - \frac{\alpha(1-\beta)}{(1-\alpha)\beta}[(1-\alpha)v_h + c]\} \geq 0\ . \tag{A53}$$

Again, condition A53 can be satisfied only for $s < s^n$ because in equilibrium without delegation the firm extracts the entire surplus. Maximizing the left-hand side of condition A53 with respect to s, and some straightforward manipulating of terms, yields the following equivalent form of condition A53:

$$G^s(\delta) \geq 0, \tag{A54}$$

where $G^s(\delta) \equiv \left(\sqrt{\alpha v_h - c} - \sqrt{\frac{\delta B c}{D}} \right)^2 + (1-\delta)\frac{\alpha(1-\beta)+(1-\alpha)\beta}{D}c - \{\alpha v_h - c - \frac{\alpha(1-\beta)}{(1-\alpha)\beta}$ $[(1-\alpha)v_h + c]\}$. Differentiation yields

$$\frac{\partial G^s}{\partial \delta} = -\sqrt{\frac{B(\alpha v_h - c)c}{\delta D}} - c < 0 \qquad \text{(A55)}$$

whenever condition A54 holds.

From the single-crossing property of functions G^s and G and because $G^s(\delta) < G(\delta)$ it follows that for sufficiently small c/v_h there exists a unique $\delta^{dns} \in [\delta^n, \delta^{dn}]$ such that in equilibrium the firm hires the auditor for all $\delta^n \le \delta \le \delta^{dns}$ and does not to hire the auditor for all $\delta > \delta^{dns}$. Also, note that the smallest cut-off point s that satisfies conditions A50 and A51 is greater than the smallest cut-off point s^d that satisfies conditions A9 and A10; that is, consumers' trust is less long lasting in the presence of soft presale information.

■

Proof of Proposition 8 I first find conditions such that an equilibrium with sequential monitoring exists. Then I verify that if an equilibrium with sequential monitoring and trade is feasible, the firm and the auditor choose an equilibrium with sequential monitoring rather than an equilibrium where only the auditor monitors quality and consumers stochastically punish unfavorable reports.

Step 1. Again, from proposition 1 it follows that if the firm hires the auditor it must be that $\delta < \delta^n$. From equations 46 and 47 in the text it follows that for $\beta = 1$ the values of the firm's monitoring rate and upfront fee that maximize the firm's profit when signing the contract are given by

$$\gamma^{sm} = 1-\alpha, \qquad \text{(A56)}$$

$$W^{sm} = \max\left[\frac{1-\delta}{\delta} - \alpha^2, 0\right]\frac{2-\alpha}{1-\alpha}c. \qquad \text{(A57)}$$

If $\frac{1-\delta}{\delta} \ge \alpha^2$, the auditor's monitoring-incentive-compatibility constraint 43 in the text is binding. On substituting equation A56, equation A57, and $w^{sm} = c/(1-\alpha)$ into equation 44 in the text, the firm's maximum profit in period 1 (when it decides whether to sign the long-term contract) becomes

$$\pi_{F,1}^{sm} = \frac{\alpha v_h - c - \alpha(1-\alpha)c}{1-\delta} - \frac{(2-\alpha)(1+\alpha^2)}{1-\alpha}c, \qquad \text{(A58)}$$

where I set the upfront fee that the firm pays in the beginning of the first period equal to zero ($W_1^{sm} = 0$). Substituting conditions A56 and A57 into condition 45 in the text ensures that, on signing the contract, the firm prefers not to terminate it in $t = 2,3,\ldots$; it becomes

$$\pi_F^{sm} = \frac{\alpha v_h - c - \alpha(1-\alpha)c}{1-\delta} - \frac{2-\alpha}{\delta(1-\alpha)}c \ge 0, \qquad \text{(A59)}$$

where $\pi_F^{sm} < \pi_{F,1}^{sm}$.

If $\frac{1-\delta}{\delta} < \alpha^2$, the auditor's monitoring-incentive-compatibility constraint 43 in the text is not binding (because it is more difficult to incentivize the firm to monitor the auditor than to incentivize the auditor to monitor quality). Then the firm's participation conditions become

$$\pi_{F,1}^{sm} = \pi_F^{sm} = \frac{\alpha v_h - \frac{c}{1-\alpha}}{1-\delta} \geq 0. \quad \text{(A60)}$$

From equations A59 and A60 it follows that if $\alpha(1-\alpha)v_h - c \geq 0$, an equilibrium with sequential monitoring exists if $\frac{1-\delta}{\delta} < \alpha^2$ or $\frac{1-\delta}{\delta} \geq \alpha^2$ and equation A59 is satisfied, that is, if and only if

$$\delta \geq \delta^{sm} \equiv \min\left[\frac{2-\alpha}{1-\alpha}\frac{c}{\alpha v_h - c - \alpha(1-\alpha)c + \frac{2-\alpha}{1-\alpha}c}, \frac{1}{1+\alpha^2}\right]. \quad \text{(A61)}$$

Step 2. Substituting equations A56 and A57 into equations 38 and 43 in the text I find that in equilibrium with sequential monitoring that maximizes the firm's profit, the auditor's payoff is given by

$$\pi_{A,1}^{sm} = (\gamma^{sm}\alpha + 1 - \gamma^{sm})(w^{sm} - c) + \delta\frac{W^{sm} + (\gamma^{sm}\alpha + 1 - \gamma^{sm})(w^{sm} - c)}{1-\delta}$$
$$= \left\{\frac{\alpha^2}{1-\delta} + \max\left[1 - \frac{\delta\alpha^2}{1-\delta}, 0\right]\right\}\frac{2-\alpha}{1-\alpha}c, \quad \text{(A62)}$$

and the joint profit is given by

$$\pi_{F,t}^{sm} + \pi_{A,t}^{sm} = \frac{\alpha v_h - c - \alpha(1-\alpha)c}{1-\delta}, \; t = 1,2,\ldots. \quad \text{(A63)}$$

The firm and the auditor sign the contract $\{W_t, w_t\}$, where $W_1 = 0$, $W_t = W^{sm}$, $t \geq 2$, and $w_t = w^{sm}$, $t \geq 1$, if they cannot achieve a greater joint profit in equilibrium where the firm never inspects the good itself but consumers punish unfavorable reports (see proposition 7). By proposition 7, for $\beta = 1$ the firm hires the auditor in the presence of soft presale signal without sequential monitoring if and only if $\delta^{ds} \leq \delta < \delta^n$, where $\delta^{ds} \equiv \left(\alpha^2(1-\alpha)v_h / c + \alpha\right)^{-1}$, in which case the joint profit is given by

$$\pi_{F,t}^{ds} + \pi_{A,t}^{ds} = \frac{\alpha v_h - c}{1 - \delta[1-(1-\alpha)^2]}, \; t = 1,2,\ldots. \quad \text{(A64)}$$

From equation 22 and binding constraint 27 in the text it follows that the smallest auditor's payoff in period 1 is given by

$$\pi_{A,1}^{ds} = w_1^{ds} - c + \delta(\alpha + (1-\alpha)(1-s_l))\pi_A^{ds} = \frac{c}{1-\alpha}, \quad \text{(A65)}$$

where I used $w_1^{ds} = 0$, $s_l^{ds} = 1-\alpha$, and $\pi_A^{ds} = \frac{c}{\delta\alpha(1-\alpha)}$ to obtain the second quality in equation A65.

From the equilibrium payoffs in equations A62 through A65 it follows that $\pi_{F,t}^{sm} + \pi_{A,t}^{sm} > \pi_{F,t}^{ds} + \pi_{A,t}^{ds}$ if

$$\delta > \hat{\delta} \equiv \frac{\alpha c}{(1-\alpha)\alpha v_h + [\alpha(2-(1-\alpha)^2) - 1]c} \quad \text{(A66)}$$

and $\pi_{A,1}^{sm} > \pi_{A,1}^{ds}$. It is straightforward to verify that $\hat{\delta} < \delta^{ds}$ and $\delta^{sm} < \delta^{ds}$. Because the joint profit is greater in equilibrium with sequential monitoring, the firm cannot compensate the auditor for offering a contract that induces an equilibrium with pure delegation (where consumers punish unfavorable reports and the firm never inspects quality itself) rather than a contract that induces sequential monitoring. ■

Proof of Proposition 9 If $\delta \geq \delta^n$, as in the case with a single firm, none of the firms hires the auditor. This happens because each firm achieves the greatest profit under internal quality control as firms are able to extract the entire surplus from trade. So suppose that $\delta < \delta^n$, and each firm hires the auditor. By equation 49 in the text, third-party monitoring is incentive compatible if for all $m = 1, \ldots, N$

$$Nw_1^d - Nc + \delta\pi_A^{d,N} \geq Nw_1^d - (N-m)c + \delta\alpha^m\pi_A^{d,N} \tag{A67}$$

or

$$\pi_A^{d,N} \geq \frac{mc}{\delta(1-\alpha^m)}. \tag{A68}$$

Treating m as a continuous variable and differentiating the right-hand side of equation A68 with respect to m yields

$$\frac{c}{\delta}\frac{1-\alpha^m + \alpha^m \ln \alpha^m}{(1-\alpha^m)^2}. \tag{A69}$$

I now show that equation A69 is strictly positive. Note that $1-\alpha^m + \alpha^m \ln \alpha^m > 0$ can be equivalently rewritten as

$$H(q) = \frac{1}{q} + \ln q > 1 \text{ for } q = \alpha^m. \tag{A70}$$

Because $H(1) = 1$ and $H'(q) = -\frac{1}{q^2} + \frac{1}{q} < 0$ for all $q < 1$, it follows that the right-hand side of equation A68 achieves its maximum at $m = N$; that is, third-party monitoring is incentive compatible if and only if

$$\pi_A^{d,N} \geq \frac{Nc}{\delta(1-\alpha^N)}. \tag{A71}$$

The auditor is hired in equilibrium if the revenue from sales can cover the auditing fee:

$$\alpha v_h - w_1^d + \delta\frac{\alpha v_h - w^d}{1-\delta} \geq 0 \tag{A72}$$

$$\frac{\alpha v_h - w^d}{1-\delta} \geq 0. \tag{A73}$$

There exists a feasible contract such that the auditor's monitoring-incentive-compatibility A71 and firm's participation constraints A72 and A73 are satisfied if and only if

$$\frac{\alpha v_h - c}{1-\delta} - \frac{c}{\delta(1-\alpha^N)} \geq 0 \tag{A74}$$

or

$$\delta \geq \delta^{d,N} \equiv \left(1 + (1-\alpha^N)\frac{\alpha v_h - c}{c}\right)^{-1}. \tag{A75}$$

■

Proof of Proposition 11 The only difference between the cases wherein auditor's reports are observed and not observed by consumers is the presence of the firm's truth-telling condition (when revealing bad news) in the latter case. Again, the firm does not delegate quality control for $\delta \geq \delta^n$, so assume that $\delta < \delta^n$. When consumers do not observe the auditor's reports, the firm is willing to reveal bad news if

$$\delta \pi_F^{dp,N} \geq v_h \,, \tag{A76}$$

where $\pi_F^{dp,N} = \frac{\alpha v_h - c}{1-\delta} - \frac{1}{N}\pi_A^{d,N}$. As shown in the proof of proposition 9, from equation A71 it follows that the minimum necessary per-firm agency cost is

$$\frac{1}{N}\pi_A^{d,N} = \frac{c}{\delta(1-\alpha^N)}\,. \tag{A77}$$

Substituting equation A77 into equation A76, the firm's truth-telling constraint becomes

$$\delta[\frac{\alpha v_h - c}{1-\delta} - \frac{c}{\delta(1-\alpha^N)}] \geq v_h \tag{A78}$$

or

$$\delta \geq \delta^{dp,N} \equiv \left(1+[\alpha v_h - c]/\{v_h + c/[1-\alpha^N]\}\right)^{-1}. \tag{A79}$$

Note that constraint A78 implies the firm's individual-rationality constraint (willingness not to terminate the contract after it has been signed). The result follows because $\delta^{dp,N} < \delta^n$ for any $N \geq 2$. ■

References

Baliga, S. 1999. "Monitoring and Collusion with Soft Information." *Journal of Law, Economics and Organization* 15:434–440.

Board, S., and M. Meyer-ter-Vehn. 2013. "Reputation for Quality." *Econometrica* 81:2381–2462.

Bolton, P., X. Freixas, and J. Shapiro. 2012. "The Credit Ratings Game." *Journal of Finance* 67:85–111.

Cabral, L. 2009. "Umbrella Branding with Imperfect Observability and Moral Hazard." *International Journal of Industrial Organization* 27:206–213.

Cai, H., and I. Obara. 2009. "Firm Reputation and Horizontal Integration." *RAND Journal of Economics* 40:340–363.

Consumer Product Safety Commission. 2013. *Proposed Rule: Certificate of Compliance.* Federal Register Notices. Accessed [December 1, 2015]. www.cpsc.gov/en/Regulations-Laws--Standards/Federal-Register-Notices/2013/Proposed-Rule-Certificates-of-Compliance.

Costa, R. 2014. "Process and Substance in Third-party Food Safety Audits." *Food Safety News,* July.

Dranove, D., and G. Jin. 2010. "Quality Disclosure and Certification." *Journal of Economic Literature* 48:935–963.

Dulleck, U., and R. Kerschbamer. 2009. "Experts vs. Discounters: Consumer Free-riding and Experts Withholding Advice in Markets for Credence Goods." *International Journal of Industrial Organization* 27:15–23.

Farhi, E., J. Lerner, and J. Tirole. 2013. "Fear of Rejection? Tiered Certification and Transparency." *RAND Journal of Economics* 44:610–631.

Faure-Grimaud, A., E. Peyrache, and L. Quesada. 2009. "The Ownership of Ratings." *RAND Journal of Economics* 40:234–257.

Fleckingery, P., M. Glachantz, and G. Moineville. 2015. *Incentives for Quality in Friendly and Hostile Informational Environments.* Working Paper. Paris: MINES ParisTech & Paris School of Economics.

Food and Drug Administration. 2013. *Accreditation of Third-party Auditors/Certification Bodies to Conduct Food Safety Audits and to Issue Certifications.* Washington, DC.

Gill, D., and D. Sgroi. 2012. "The Optimal Choice of Pre-launch Reviewer." *Journal of Economic Theory* 147:1247–1260.

Gromb, D., and D. Martimort. 2007. "Collusion and the Organization of Delegated Expertise." *Journal of Economic Theory* 137:271–299.

Grossman, S. 1981. "The Informational Role of Warranties and Private Disclosure of Product Quality." *Journal of Law and Economics* 24:461–484.

Hakenes, H., and M. Peitz. 2009. "Umbrella Branding and External Certification." *European Economic Review* 53:186–196.

Harbaugh, R., J. Maxwell, and B. Roussillon. 2011. "Label Confusion: The Groucho Effect of Uncertain Standards." *Management Science* 57:1512–1527.

Hatanaka, M., C. Bain, and L. Busch. 2005. "Third-party Certification in the Global Agrifood System." *Food Policy* 30:354–369.

Hörner, J. 2002. "Reputation and Competition." *American Economic Review* 92:644–663.

Klein, B., and K. Leffler. 1981. "The Role of Market Forces in Assuring Contractual Performance." *Journal of Political Economy* 89:615–641.

Lizzeri, A. 1999. "Information Revelation and Certification Intermediaries." *RAND Journal of Economics* 30:214–231.

Lytton, T., and L. McAllister. 2014. "Oversight in Private Food Safety Auditing: Addressing Auditor Conflict of Interest." *Wisconsin Law Review* 289–335.

MacLeod, W. 2007. "Reputations, Relationships, and Contract Enforcement." *Journal of Economic Literature* 45:595–628.

Mathis, J., J. McAndrews, and J. C. Rochet. 2009. "Rating the Raters: Are Reputation Concerns Powerful Enough to Discipline Rating Agencies?" *Journal of Monetary Economics* 56:657–674.

Matthews, S., and A. Postlewaite. 1985. “Quality Testing and Disclosure.” *RAND Journal of Economics* 16:328–340.
Miklós-Thal, J., and H. Schumacher. 2013. “The Value of Recommendations.” *Games and Economic Behavior* 79:132–147.
Milgrom, P. 1981. “Good News and Bad News: Representation Theorems and Applications.” *Bell Journal of Economics* 12:380–391.
Ollinger, M., M. Muth, S. Karns, and Z. Choice. 2011. *Food Safety Audits, Plant Characteristics, and Food Safety Technology Use in Meat and Poultry Plants*. EIB-82. Washington, DC: U.S. Department of Agriculture, Economic Research Service.
Pesendorfer, W., and A. Wolinsky. 2003. “Second Opinions and Price Competition Inefficiency in the Market for Expert Advice.” *Review of Economic Studies* 70:417–437.
Peyrache, E., and L. Quesada. 2011. “Intermediaries, Credibility and Incentives to Collude.” *Journal of Economics and Management Strategy* 20:1099–1133.
Polinsky, A., and S. Shavell. 2012. “Mandatory versus Voluntary Disclosure of Product Risks.” *Journal of Law, Economics, and Organization* 28:360–379.
Rahman, D. 2012. “But Who Will Monitor the Monitor?” *American Economic Review* 102:2767–2797.
Raynolds, L., D. Murray, and A. Heller. 2007. “Regulating Sustainability in the Coffee Sector: A Comparative Analysis of Third-party Environmental and Social Certification Initiatives.” *Agriculture and Human Values* 24:147–163.
Ridley, D., J. Kramer, H. Tilson, H. Grabowski, and K. Schulman. 2006. “Spending on Postapproval Drug Safety.” *Health Affairs* 25:429–436.
Shapiro, C. 1983. “Premiums for High Quality Products as Returns to Reputation.” *Quarterly Journal of Economics* 98:659–680.
Shavell, S. 1994. “Acquisition and Disclosure of Information Prior to Sale.” *RAND Journal of Economics* 25:20–36.
Sporleder, T., and P. Goldsmith. 2001. “Alternative Firm Strategies for Signalling Quality in the Food System.” *Canadian Journal of Agricultural Economics* 49:591–604.
Stahl, K., and R. Strausz. 2014. *Who Should Pay for Certification?* Working Paper. Department of Economics, University of Mannheim, Germany.
Strausz, R. 1997. “Delegation of Monitoring in a Principal-agent Relationship.” *Review of Economic Studies* 64:337–357.
———. 2005. “Honest Certification and the Threat of Capture.” *International Journal of Industrial Organization* 23:45–62.
Tirole, J. 1986. “Hierarchies and Bureaucracies: On the Role of Collusion in Organizations.” *Journal of Law, Economics, and Organization* 2:181–214.
Weschler, T. 2014. “Food Contract Labs Are Taking a Big Bite out of Food Safety Testing.” Accessed [December 1, 2015]. www.foodsafetymagazine.com/blog/food-contract-labs-are-taking-a-big-bite-out-of-food-safety-testing/.
Westmoreland, K. 2014. “Lab Outsourcing: What You Need to Know.” *FoodSafety Magazine* 20:22–25.
White, L. 2010. “Markets: The Credit Rating Agencies.” *Journal of Economic Perspectives* 24:211–226.
Xiao, M. 2010. “Is Quality Accreditation Effective? Evidence from the Childcare Market.” *International Journal of Industrial Organization* 28:708–721.