An Economic Analysis of Beverage Size Restriction

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Abstract

We model the potential efficiency and distributional consequences of a government beverage-size restriction that is designed to curb or reduce consumption of sugar-sweetened beverages. Unsurprisingly, we find that a credibly implemented restriction can curb consumption, particularly by "high-type" consumers who consume large amounts of sweetened beverages. Surprisingly, we find that for small to moderate restrictions that might be consistent with the magnitude of the NYC soda-ban, consumer welfare will be unaffected by the regulation. Instead, most consumption inefficiency induced welfare losses will be borne by sellers. Thus, policy debates concerning welfare losses from soft-drink sales should focus on business losses rather than consumer welfare losses.

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1 Introduction

We use a nonlinear pricing model to conduct an analytical analysis of public policies that restrict portion sizes for beverage products containing ingredients deemed "harmful" by public health authorities. The most obvious example is the New York City soda ban, which proposed to prohibit sales of soda larger than sixteen ounces. While the NYC soda ban was ultimately overturned in court, policy efforts to reduce sugar and soda consumption are ongoing and contentious. For example, to preempt future regulations on portion sizes, Mississippi passed Senate Bill 2687 (2013) which prevents counties and towns from enacting rules that restrict portion sizes. The governor of Mississippi signed the bill arguing that the bill would protect consumer freedom and choice. The implication is that consumer welfare would be reduced by portion size restrictions.

Our study is the first we are aware of that provides a rigorous theoretical foundation for understanding the economics of portion size restrictions. We use a nonlinear pricing model to study the effectiveness, efficiency, and distributional consequences of size restriction policies. Nonlinear pricing is a subset of a more general class of screening models that rely on the mechanism design concept of the Revelation Principle (Myerson, 1979) to design pricing schemes in the presence of hidden information. Hidden information in this case refers to the fact that retailers cannot precisely identify the willingness-to-pay of each customer and must therefore design menus of price-quantity combinations to induce heterogeneous customers to self-select into the appropriate option. Nonlinear pricing schemes are consistent with stylized observations of actual food marketing practices where, for example, soda-pop is often sold in different price-size cups or containers. These menus of different size-price options allow retailers to engage in second degree price discrimination by incentivizing customers to self-reveal their type through their purchase choices.

Nonlinear pricing is typically profitable for firms because it improves targeting of products to different consumer segments. When firms adopt consumer segmentation strategies, the efficiency and distributional consequences of government policy restrictions are potentially complex as any size-restriction not only has direct effects (i.e. efficiency and distributional consequences from restricting firms' ability to implement a fully optimal nonlinear pricing scheme), but also indirect effects where firms may endogenously switch to a different pricing strategy. As a consequence, a soda-size restriction may have ambiguous effects on consumption and economic welfare of consumers across different segments. Thus, testable hypotheses generated from traditional demand models that do not account for nonlinear pricing may not capture all the nuances of the policy restriction. For example, using a standard textbook demand curve may lead one to mistakenly conclude that any quantity restriction away from the equilibrium level will reduce consumer welfare via a deadweight loss. We show that consumer welfare may not decline under nonlinear pricing.

By isolating both direct and indirect effects, we can more accurately assess the tension between public health objectives and consumer welfare arguments. Our model and analysis can also provide a theoretical foundation for future empirical work as the nonlinear pricing model can yield a richer set of predictions than standard demand models.

We add the caveat that we do not try to mimic any specific statutory proposal in all its detail (e.g. the NYC proposal) but rather conduct a more general analysis of credible portion size restrictions. We do this for two reasons. First, the details of an actual regulatory proposal, like the NYC soda ban, is likely to be very ad hoc and plagued with inconsistencies. In the NYC example, businesses regulated by the NYC Department of Health and Mental Hygiene are subject to the ban. This implies that restaurants would be subject to the regulation but not convenience and grocery stores (which are regulated by the State). This creates a very uneven and confusing regulation, which in turn, complicates economic analysis. Second, proposed regulations often have credibility problems in implementation. For example, in the NYC case, there is no rule to prevent a consumer from purchasing two small beverages that mimics the size of the banned large beverage.

Instead, our focus is to ask the hypothetical question of how a credibly implemented size-restriction would affect consumption, efficiency and distribution of welfare across consumers and sellers. While our scenario is grounded in theory rather than actual statutory rules from specific cases, we also believe that it will serve as a more useful point of reference for future analysis of food/beverage size restrictions. We feel that this approach is more generalizable and can encompass a wide array of proposed size-restrictions that might differ in details but not in substance.

Our analytic results suggest that, unless a restriction is so severe that it eliminates the ability of retailers to engage in market segmentation via nonlinear pricing, consumer welfare will largely be unaffected. Intuitively, while a soda size restriction is likely to limit the range of sizes that can be used to screen consumers, retailers can adjust prices to maintain the same consumer surplus as prior to the policy restriction. This is because high-WTP consumers must be provided with information rents for them to choose the price-size option meant for them. The same level of information rents must be maintained both pre and post-policy restriction. Hence, high-WTP consumer welfare is largely unaffected. Similarly, low-WTP consumers receive only their reservation utility both before and after the policy restriction. However, the restriction will have unambiguous negative effects on seller welfare.

These welfare results suggest that arguments about consumer freedom are misplaced because the restriction will have little to no impact on consumer welfare for most reasonable size restrictions.[1] Rather, a legitimate argument might be that a restriction might harm businesses possibly putting strain on both retailers and soft-drink manufacturers.[2]

Finally, an obviously important question is whether the policy-restriction will achieve its intended objective of reducing consumption of sugar-sweetened beverages. Our key finding are that a regulation implemented in a nonlinear pricing environment will reduce consumption by high-WTP consumers. Interestingly, low-WTP consumers will only have their consumption reduced if the regulation is restrictive enough to eliminate retailers' ability to implement nonlinear pricing schemes. From a policy perspective, this appears to be a desirable outcome since the policy may achieve its intended effect of reducing sugar consumption by those who over-consume while minimizing the impact on those who tend to naturally consume smaller portions.

Our research important because the controversy between consumer welfare losses from restricted consumption and gains in public health is unlikely to abate anytime soon. In 2012, over 70 million Americans were obese. As a proportion of the U.S. population in 2012, 34.9 percent of American adults over twenty years of age and 16.9 percent of children ages two to nineteen were obese (Ogden et al., 2014). Obesity is estimated to cost the United States over $148 billion in deadweight losses (MacEwan et al., 2014), and obese individuals were estimated in 2006 to incur an additional $1,429 in annual medical expenses over normal-weight individuals (Finkelstein et al., 2009).

[1] We offer the caveat that we focus primarily on economic welfare. This does not rule out psychological welfare costs that may arise, for example, because there is a perceived loss in freedom of choice or psychological suffering that may occur if someone feels that their political or economic ideology may have been violated.

[2] We also recognize that policy pandering to consumers might be an effective strategy for those who oppose the policy-restrictions on food/beverage marketing. However, our goal is not to study strategic public campaigns, which might be an interesting topic for future research, but rather to highlight the economic tradeoffs of the beverage restriction policies.

In recent years, soda bans and soda taxes have been proposed due to research linking increased sugar consumption to weight gain, type-two diabetes (Schulze et al., 2004), cardiorenal disease, obesity (Johnson et al., 2007), and metabolic syndrome (Lustig et al., 2012). Proponents of food restriction policies largely base their arguments on such studies linking sugar to negative health outcomes. Among the most prominent articles advocating policy restrictions is a recent *Nature* article by Lustig et al. (2012) who advocate for supply side restrictions on sugar based on public health arguments comparing sugar to alcohol/tobacco. While public health arguments are compelling and critical in policy formation, it is also important to understand potential policy effectiveness (whether it would reduce consumption of the targeted item), efficiency consequences , and who wins and loses (distributional implications).

While some public officials and commentators in the popular press have framed soft drink size restrictions as an issue of consumer freedom and welfare, the impact on food and beverage sellers and their response to such restrictions has largely been ignored. However, viewed through the lens of mechanism design, such regulations are restrictions on nonlinear pricing schemes. Thus, it might be more appropriate to view soft-drink size bans as restrictions on firm strategy rather than restrictions on consumer choice.

2 Model setup without the size-restriction in effect

We start by deriving some benchmark results in the absence of regulation. These results can be used as a basis for comparison after we introduce the regulation so that we can assess how a regulation affects consumption, efficiency, and distribution of welfare.

Our baseline model is a simple nonlinear pricing model in the spirit of Mussa and Rosen (1978) or Maskin and Riley (1984) where a profit-maximizing firm (e.g. beverage retailer) facing two types of consumers: low types (L-type), with willingness-to-pay (WTP) parameter, θ_L, who consume relatively small amounts of of the beverage; and high types (H-type), with WTP parameter, θ_H, who consume large amounts of the beverage. We assume that the retailer cannot observe the types and therefore cannot engage in first-degree price discrimination. Instead, the retailer engages in second degree price discrimination by creating a menu of price-size combinations and letting the consumer "reveal" her type by self-selecting into her preferred price-quantity combination. We will henceforth refer to this as either the *screening* or *segmentation* pricing scheme.

In practice, screening is manifested as the multiple size-price options offered to consumers. These screening pricing schemes are a way of segmenting the market using a second-degree price discrimination scheme. We choose to use a two-type discrete nonlinear pricing model over a continuous-type model because soft-drinks are typically sold with only a few sizes (e.g. small or large). That as, retailers typically don't offer a continuous range of size-price combinations so a simple discrete-type model is more consistent with stylized observations.[3]

We assume that the utility function for each consumer is $U(\theta_i, q) = \theta_i v(q_i) - p_i$

[3] One might also argue that a two-type model might be too simple since soft-drinks often come in three sizes, such as small, medium, and large. However, adding more than two-types significantly complicates the analysis while not adding substantially more insight. For example, a well known result in the nonlinear pricing literature is that, with two-types, high types will receive first-best consumption levels and receive information rents, while low-types will have their consumption levels distorted downwards while receiving no rents above reservation utility. If a third-type is added who falls between the low and high types, the third-type would receive some information rent, though not as large as the high types's while having its consumption distorted downward away from first-best, but not as much as the low-type's distortion. Thus, the results from the two-type case generalize to the intermediate types though the magnitudes are smaller.

where q_i is quantity and p_i is the price paid by the consumer. Moreover, we assume that $v'(q) > 0$ and $v''(q) < 0$ for all q. We assume that i=L,H such that $\theta_H > \theta_L$. This implies that the single crossing condition is satisfied; i.e. $U(\theta_H, q) - U(\theta_L, q)$ is increasing in q. If utility function is differentiable in q, then single crossing implies that $\frac{\partial U(\theta_H,q)}{\partial q} > \frac{\partial U(\theta_L,q)}{\partial q}$. We will also assume that the principal's cost of production is $c(q)$ such that $c'(q) > 0$ and $c''(q) > 0$ so that the cost function is increasing and convex.

With these assumptions in mind, we can now setup the principal's (beverage retailer's) objective function

$$\max_{q_L,q_H,p_L,p_H} \beta\left[p_L - c(q_L)\right] + (1-\beta)\left[p_H - c(q_H)\right] \quad s.t. \tag{1}$$

$$\theta_L v(q_L) - p_L \geq 0 \quad (PC)$$

$$\theta_H v(q_H) - p_H \geq \theta_H v(q_L) - p_L \quad (IC)$$

where β is the probability that the retailer will encounter a L-type consumer. It is well known in the nonlinear pricing literature that only the L-type's participation constraint (PC) and the H-type's incentive compatibility constraint (IC) bind. Therefore, we have omitted the participation constraint for the H-type and the IC constraint for the L-type. Letting these constraints hold with equality and substituting into the objective function yields

$$\max_{q_L,q_H} \pi = \beta\left[\theta_L v(q_L) - c(q_L)\right] + (1-\beta)\left[\theta_H v(q_H) - (\theta_H - \theta_L)v(q_L) - c(q_H)\right] \tag{2}$$

The first order Kuhn-Tucker conditions are

$$(1-\beta)\left[\theta_H v'(q_H) - c'(q_H)\right] \leq 0 \quad where \quad q_H \geq 0 \quad \& \quad \frac{\partial \pi}{\partial q_H} q_H = 0 \tag{3}$$

$$\beta\left[\theta_L v'(q_L) - c'(q_L)\right] + (1-\beta)\left[-(\theta_H - \theta_L) v'(q_L)\right] \leq 0 \quad where \quad q_L \geq 0 \quad \& \quad \frac{\partial \pi}{\partial q_L} q_L = 0 \tag{4}$$

There are two cases of economic interest. In case (i), the retailer serves both types of consumers. In case (ii), the retailer serves only the H-type customer and stops selling to the L-type customer. We will analyze each of these cases separately.

2.1 Unregulated Case i: $q_H > 0$ and $q_L > 0$

This case implies that 3 and 4 both hold with equality which imply that

$$\theta_H v'(q_H) = c'(q_H) \tag{5}$$

$$\theta_L v'(q_L) = c'(q_L) + \frac{(1-\beta)}{\beta}\left[\theta_H - \theta_L\right] v'(q_L) \tag{6}$$

In other words, the retailer should choose container sizes such that the H-type consumer consumes the first-best optimal amount whereas the L-type gets less than first best amount; i.e. a downward distortion. This is because the marginal cost of serving the L-type is driven up by the information rent $\frac{(1-\beta)}{\beta}\left[\theta_H - \theta_L\right] v'(q_L)$. This rent must be paid to the H-type to get the H-type to self-select into the option meant for the H-type. This is a textbook result in nonlinear pricing.

Letting q_H^* and $\tilde{q}_H$ denote the solutions to 5 and 6, we can use the PC and IC constraints to generate the prices for q_H and q_L.

$$p_H = \theta_H v(q_H^*) - (\theta_H - \theta_L) v(\tilde{q}_L) - \overline{u} \tag{7}$$

$$p_L = \theta_L v(\tilde{q}_L) - \overline{u} \tag{8}$$

Note that the prices are set such that the L-type's participation constraint is just satisfied so the L-type makes no rent. The H-type's price, however, is discounted by the information rent $(\theta_H - \theta_L)\tilde{q}_L v'(q_L)$. In other words, the H-type's price is discounted by this amount in order to provide incentives for the H-type to purchase the q_H package rather than the q_L package.

Finally, the prices and quantities can be substituted into the objective functions of the retailer (expected profit), and consumers (utility) to obtain value functions, which allow us to make welfare statements.

We summarize the key benchmark results in the absence of regulation in the following proposition.

Proposition 1. *In the absence of a size-restriction regulation, the retailer's optimal nonlinear pricing strategy yields the following benchmark results:*

1. *A H-type package of (q_H^*,p_H) where q_H^* satisfies condition 5 and p_H is given by 7.*

2. *A L-type package of ($\tilde{q}_L$,p_L) where $\tilde{q}_L$ satisfies condition 6 and p_L is given by 8.*

3. *The retailer's value function (maximized expected profit) is:* $\Pi = (1-\beta)[\theta_H v(q_H^*) - \overline{u} - c(q_H^*) - [\theta_H - \theta_L]v(\tilde{q}_L)] + \beta[\theta_L v(\tilde{q}_L) - \overline{u} - c(\tilde{q}_L)]$

4. *The H-type consumer's value function (welfare under the optimal nonlinear pricing scheme) is* $U_H = \overline{u} + [\theta_H - \theta_L]v(\tilde{q}_L)$.

5. *The L-type consumer's value function is* $U_L = \overline{u}$

We omit the proof because the proposition only summarizes well-known results from the nonlinear pricing literature.

2.2 Unregulated Case ii: $q_H > 0$ and $q_L = 0$

We now examine the case where the retailer chooses to serve only H-types. While this case is not of primary interest since it does not match stylized observations where retailers typically offer menu options for more than one-type, it is nevertheless important to analyze this case for counter-factual reasons. That is, apriori, we cannot rule out the possibility that a retailer might endogenously switch to serving only H-types after a soft-drink restriction is introduced.

There are two textbook reason for why a principal might only serve H-types. First, if β is small, this indicates that there might not be enough L-types in the population to make serving them worthwhile. Second, if the difference types, as measured by $[\theta_H - \theta_L]$, is very large then information rents may outweigh the costs of serving L-types. The Kuhn-Tucker conditions 3 and 4 become

$$\theta_H v'(q_H) = c'(q_H) \tag{9}$$

$$\theta_L v'(q_L) < c'(q_L) + \frac{(1-\beta)}{\beta} [\theta_H - \theta_L] \, v'(q_L) \tag{10}$$

Condition 10 implies that $q_L = 0$. Moreover, this implies a price of $p_H = \theta_H v(q_H) - \overline{u}$.

This is the case where the principal finds it optimal not to serve L-types and focuses its marketing efforts exclusively on H-types. By doing so, it can raise the price of p_H relative to Case i because information rents are no longer required to induce the H-type to choose the option meant for him/her.

In summary, only H-types are served and receive first best consumption level of q_H^* at a price of $p_H = \theta_H v(q_H^*) - \overline{u}$. Then q_H^* and $p_H = \theta_H v(q_H^*) - \overline{u}$ can be substituted into the objective functions to obtain the value functions:

$$\pi_{ii} = (1 - \beta)\left[\theta_H v(q_H^*) - c(q_H^*) - \overline{u}\right] \tag{11}$$

and

$$U_{Hii} = \overline{u} \tag{12}$$

Notice that one advantage of not serving L-types is that the retailer need not pay information rents to the H-types.

3 The impact of a size-restriction regulation

The beverage size-restriction regulation is likely to have two major effects. First, the prices and sizes of the beverages offered to consumers are likely to change. Second, it might cause a discrete shift in the pricing strategies adopted by a retailer. For example, with a stringent size-restriction, the retailer might potentially switch from a segmentation strategy of offering a menu of options to consumers to separate H-types from L-types, to a one-size-fits all strategy that serves both types.

Note that the first effect examines pricing and sizing *holding the discrete pricing strategy fixed*. That is, it refers to how a regulation would affect prices and sizes *within* a major discrete pricing strategy. The second effect examines how the regulation is likely to cause endogenous switching to an altogether different prices strategy. Clearly when retailers are engaged in strategic pricing, examining only small continuous changes in how prices and/or sizes respond to small continuous changes in the stringency of the restriction is potentially misleading. Thus, standard demand models that do not account for both continuous and discrete strategic shifts in strategy may lead to biased conclusions.

Our analyses will proceed as follows. We will first examine the first effect; i.e. we want to know how prices and quantity respond to the introduction of a regulation *within each major discrete pricing strategy*. These price-quantity responses are needed in order to determine how the retailer's value function (profit) will shift in response to a regulation. Once we determine how the retailer's profit is affected under every major discrete pricing strategy, then we can look at the second effect, which is to determine whether the regulation can induce the retailer to shift to a different discrete pricing strategy. The discrete pricing strategy that yields the highest profit will be the strategy adopted by the retailer.

3.1 Policy Effect 1: How do prices and quantities respond to a regulation under each discrete pricing strategy?

In this section, we focus on how price and quantity responds to a regulation *holding the discrete pricing strategy fixed*. The set of possible discrete pricing strategies are:

- Case ib (Screening/segmentation strategy): Sell to both types of consumers with a menu of differentiated H-type and L-type price-size options.
- Case iib: Sell only to H-types.
- Case iiib: Sell to both types using a one-size-fits-all pricing strategy.

In case (ib), the retailer continues to design a menu of drink sizes that segments the H-types and L-types. This is the most important case because selling to both types is the default assumption in the unregulated case. Thus, it is of policy interest to know whether the size regulation will knock the retailer out of this pricing strategy. In case (iib), the retailer only serves the H-type. Cases (ib) and (iib) are analogous to cases (i) and (ii) in the unregulated scenario. The third case, which we call case iiib, is when the retailer serves both the H- and L-types using only one beverage size. We will show that such a pricing scheme will optimally set the size to be $q = q_L^*$. Thus, if $\hat{q}$ creates restrictions on q_H in cases (ib) and (iib), then we want to know whether this one-size-fits all package can be more profitable then the segmentation strategy. Generally, if $\hat{q} \geq q_L^*$, then the profit from case (iiib) is unaffected by the restriction. However, if $\hat{q} < q_L^*$, then the profit from case (iiib) will also depend on the restriction. We will now analyze each of these cases in detail.

3.1.1 Case ib: Sell to both types of consumers with a menu of differentiated H-type and L-type price-size options.

Consider a government mandated restriction on the size of a beverage. For instance, the proposed NYC soda ban put a 16oz restriction on the size of sodas. We can model this using the constraint $q \leq \hat{q}$ where $\hat{q}$ denotes the maximum size of a beverage under the regulation. Then problem 2 is becomes

$$\max_{q_L, q_H} \pi = \beta \left[\theta_L v(q_L) - c(q_L)\right] + (1-\beta) \left[\theta_H v(q_H) - (\theta_H - \theta_L) v(q_L) - c(q_H)\right] \quad s.t. \tag{13}$$

$$q \leq \hat{q}$$

In order for the policy restriction to be economically interesting, we will assume that the policy binds and limits the maximize serving size. The first order Kuhn-Tucker conditions are

$$\theta_H v'(q_H) \geq c'(q_H) \quad where \quad q_H = \hat{q} \tag{14}$$

$$\beta \left[\theta_L v'(q_L) - c'(q_L)\right] + (1-\beta) \left[-(\theta_H - \theta_L) v'(q_L)\right] \leq 0 \quad where \quad q_L \geq 0 \quad \& \quad \frac{\partial \pi}{\partial q_L} q_L = 0 \tag{15}$$

In this case, condition 14 is strictly positive whereas 15 holds with equality so that the optimal $\tilde{q}_L$ satisfies $\theta_L v'(q_L) = c'(\tilde{q}_L) + \frac{(1-\beta)}{\beta}\left[\theta_H - \theta_L\right] v'(q_L)$ which is identical to 6. This suggests that if the retailer continues to use a pricing scheme to segment H- and L-type consumers under the regulation, $\tilde{q}_L$ will be unaffected. Only q_H decreases from q_H^* to $\hat{q}_H$. Because $\tilde{q}_L$ remains unchanged, and q_H^* decreases to $\hat{q}$, it follows from 7 and 8 that p_H drops whereas p_L remains unchanged under the regulation. This is summarized in the following proposition.

Proposition 2. *Suppose that there is a regulatory size-restriction of the form $q_H \leq \hat{q}$ such that the retailer continues to use a screening pricing strategy where $0 < q_L < q_H = \hat{q}$. Then*

1. *$q_H = \hat{q} < q_H^*$ so that the quantity sold to the H-type declines,*
2. *$\tilde{q}_L$ is not affected by the ban,*
3. *p_H drops from $p_H = \theta_H v(q_H^*) - (\theta_H - \theta_L)v(\tilde{q}_L) - \overline{u}$ to $\hat{p}_H = \theta_H v(\hat{q}) - (\theta_H - \theta_L)v(\tilde{q}_L) - \overline{u}$,*
4. *p_L remains unchanged.*
5. *The retailer's profit is: $\Pi_{ib} = \beta\left[\theta_L v(\tilde{q}_L) - c(\tilde{q}_L) - \overline{u}\right] +$*

   $(1-\beta)\left[\theta_H v(\hat{q}) - c(\hat{q}) - (\theta_H - \theta_L)v(\tilde{q}_L) - \overline{u}\right]$
6. *The H-type consumer welfare (utility) is $U_{Hib} = \overline{u} + [\theta_H - \theta_L]v(\tilde{q}_L)$.*
7. *The L-type consumer welfare is $U_{Lib} = \overline{u}$*

Proofs to all propositions are in the Appendix

Essentially, the seller can still use the screening strategy under the lower $\hat{q}$ by correspondingly dropping p_H so that the two types of consumers retain the same incentive compatible levels of welfare as they received in the absence of the regulation.

One condition that must be satisfied, however, in order to make screening possible under $\hat{q}$, is that $\hat{p}_H \geq p_L$. This is easily satisfied, however, since $\hat{p}_H = \theta_H v(\hat{q}) - (\theta_H - \theta_L)v(\tilde{q}_L) - \overline{u} \geq \theta_L v(\tilde{q}_L) - \overline{u} = p_L$ reduces to $v(\hat{q}_H) \geq v(\tilde{q}_L)$, which implies that $\hat{q}_H \geq \tilde{q}_L$ by the assumption that $v'(q) > 0$ for all q.[4]

3.1.2 Case iib: Sell to only high types with $\mathbf{q_L = 0}$

In this case, neither 14 nor 15 hold with strict equality so we have $q_H^* = \hat{q}$ and $\tilde{q}_L = 0$. This is the case where the retailer serves only H-type consumers and decides that it is too costly in terms of information rents to also serve L-type consumers. Because the regulation causes q_H^* to drop to $\hat{q}$, it follows that the price charged to H-types also drops from $p_H^* = \theta_H v(q_H^*) - \overline{u}$ (from case ii) to the now lower $\hat{p}_H = \theta_H v(\hat{q}) - \overline{u}$.

Proposition 3. *Suppose that there is a regulatory restriction of the form $q_H \leq \hat{q}$ and the retailer serves only H-type consumers. Then*

1. $q_H = \hat{q} < q_H^*$ *so that the quantity sold to the H-type declines,*
2. p_H *drops from* $p_H^* = \theta_H v(q_H^*) - \overline{u}$ *to* $\hat{p}_H = \theta_H v(\hat{q}) - \overline{u}$.
3. *The retailer's profit is:* $\Pi_{iib} = (1-\beta)[\theta_H v(\hat{q}) - c(\hat{q}) - \overline{u}]$
4. *The H-type consumer welfare is:* $U_{Hiib} = \overline{u}$

A key point to note is that when the retailer only serves H-type consumers, it no longer needs to pay an information rent because it offers H-types only one price-size option and therefore need not worry about incentivizing H-types to choose the "right" option.

[4] One might also be concerned that, with a regulation, the IC constraint for the low-type may actually be relevant, even though in the unconstrained model, only the high-type IC matters. However, one can easily show that the low-type IC is also implied by $v(\hat{q}_H) \geq v(\tilde{q}_L)$ or $\hat{q}_H \geq \tilde{q}_L$

3.1.3 Case iiib: Sell to both types with a one-sized fits all package

Apriori, one cannot rule out the possibility that a restrictive regulation might make it more profitable for the retailer to use a one-size-fits-all strategy rather than a screening/segmentation strategy. Hence, we must analyze this case to determine whether the retailer might switch to this case under a regulation.

In order to determine the optimal packaging strategy, the retailer solves

$$\max_{p,q} [p - c(q)] \quad s.t. \tag{16}$$

$$\theta_L v(q) - p \geq \overline{u} \tag{17}$$

$$q \leq \hat{q} \tag{18}$$

Note that because $\theta_L < \theta_H$, it follows that H-types will always purchase so long as L-types purchase which is why a participation constraint for H-types was not included in the above optimization problem. Since a profit maximizing seller would not leave money on the table, we can assume that the participation constraint is binding. Solving for p and substituting it into the objective function yields:

$$\max_{q}] [\theta_L v(q) - c(q) - \overline{u}] \tag{19}$$

$$q \leq \hat{q} \tag{20}$$

which yields the first order Kuhn-Tucker conditions:

$$\theta_L v'(q) \geq c'(q) \quad \& \quad q \leq \hat{q} \quad \& \quad \frac{\partial \pi}{\partial q}(\hat{q} - q) = 0 \tag{21}$$

Solving the K-T conditions yields the following proposition.

Proposition 4. *Suppose that there is a restriction of the form $q \leq \hat{q}$ and the retailer uses a one-size-fits-all strategy for both types of consumers. Then*

1. *The quantity offered to both types of consumers is $q = min\{q_L^*, \hat{q}\}$ where q_L^* is the first-best quantity for the L-type consumer.*

2. *The price is $p = \theta_L v(q) - \overline{u}$.*

3. *The retailer's profit is:* $\Pi_{iiib} = \theta_L v(q) - c(q) - \overline{u}$

4. *The H-type consumer welfare is:* $U_{Hiiib} = \overline{u} + [\theta_H - \theta_L]v(\hat{q})$

5. *The L-type consumer welfare is:* $U_{Liiib} = \overline{u}$

Note that, relative to the unregulated segmentation strategy, consumption for H-types drops from q_H^* to q and consumption for L-types increases from $\tilde{q}_L$ to q. Even though consumption by L-types potentially increases, from a public policy perspective, such an outcome might be acceptable because H-type consumption decreases. Presumably, it is the H-type consumers that are likely to overconsume sugar-sweetened beverages.

It is also important to note here that, even if $q_L^* < \hat{q}$ so the restriction is not binding under the one-size-fits-all strategy, it is possible that the restriction induced the retailer to switch to this strategy because the restriction *would have been binding had the retailer stayed with the segmentation strategy.* This is the subject of the next section when we examine policy effect 2, which is how the retailer's discrete pricing strategy endogenously responds to the regulation.

3.2 Policy Effect 2: How does the regulation affect retailers' choice of pricing strategy?

Up to this point, we have investigated the potential impact of the regulatory size-restriction on prices and quantity within various discrete nonlinear pricing strategies, holding the discrete strategy fixed. In this section, we will investigate how the regulation might potentially cause the retailer to make a switch from one type of discrete nonlinear pricing strategy to another. The key to this analysis is to examine how variations in $\hat{q}$ affect the value functions (profits) of the retailer under the various pricing strategies. And then we compare the profits to each other and the retailer will choose the strategy that yields the highest profit.

We assume that, in the absence of the regulation, the retailer adopts the segmentation strategy of offering consumers a menu of options. This is consistent with stylzed observations where beverage retailers offer a variety of sizes and prices for soft-drinks. Essentially, we are assuming that the model parameters are such that the profit identified in proposition 1, part (3),

$$\Pi = (1-\beta)[\theta_H v(q_H^*) - \overline{u} - c(q_H^*) - [\theta_H - \theta_L]v(\tilde{q}_L)] + \beta[\theta_L v(\tilde{q}_L) - \overline{u} - c(\tilde{q}_L)] \quad (22)$$

is highest among all discrete pricing strategies in the unregulated case.

In order to understand the impact of the policy restriction on profits under various endogenous pricing strategies available to the retailer, we will need to examine the stringency of the restriction. This is because increasingly tighter restrictions will limit the pricing strategies available to the retailer. Thus, we will partition the stringency of the restriction into the following regions:

- **Region 1**: $q_L^* \leq \hat{q} < q_H^*$

- **Region 2**: $\tilde{q}_L \leq \hat{q} < q_L^*$
- **Region 3**: $\hat{q} < \tilde{q}_L$.

Note that q_H^* is the first-best level of consumption for the H-type and would be implemented by the retailer in an unregulated market. The quantity q_L^* is the first-best level of consumption for the L-type. The quantity $\tilde{q}_L$ is the optimal L-type size offered to the L-type by the retailer under the unregulated segmentation strategy (see proposition 1). Recall that $\tilde{q}_L < q_L^*$ because the retailer distorts the L-type consumption downward.

3.2.1 Region 1: $q_L^* \leq \hat{q} < q_H^*$

We begin by examining a mild to moderate restriction that is in-between the first-best levels for the H- and L-types. As a practical example, suppose that it is optimal to serve H-types a 32oz beverage and L-types a 12oz beverage under a segmentation strategy. Suppose that 16oz is first best for L-type. In Region 1, a restriction is anywhere between 16oz to 32oz.

We begin by asking whether a regulation would cause the retailer to move from the default position of the segmentation strategy (Case (ib)) to a sell-to-only H-types strategy (Case(iib)). The following proposition provides the answer.

Proposition 5. *Suppose that a retailer chooses the nonlinear pricing strategy outlined in Case (ib) that offers different price-size packages to H-types and L-types. A regulation of the form $q \leq \hat{q}$ cannot cause the retailer to switch to the strategy outlined in Case (iib) where the retailer chooses only to serve H-types.*

Intuitively, the key driver of whether the retailer will switch from a segmentation strategy to a strategy of serving only H-types is the tradeoff between losing L-type profits versus not having to pay information rents to H-types. This is mathematically expressed as:

$$\beta\left[\theta_L v(\tilde{q}_L) - c(\tilde{q}_L) - \overline{u}\right] \geq (1-\beta)[\theta_H - \theta_L]v(\tilde{q}_L) \tag{23}$$

Note that inequality 23 is completely independent of $\hat{q}$ so the regulation in Region 1 would not induce the retailer to switch away from a segmentation strategy toward a H-type only strategy.[5]

The next obvious question is whether the policy restriction would cause a retailer to switch from a nonlinear pricing strategy (Case (ib)) to a single price strategy that serves both types of consumers (Case iiib). To give a practical example of how a switch such as this might work, imagine a retailer who sells soda in 32oz and 12oz sizes at different prices. Suppose that the first best level to L-types is 16oz but the retailer distorts quantity down to 12oz in order to reduce information rents to the H-type to incent them to purchase 32oz sodas. In this case, if a government put a restriction of 20oz as the maximimum soda size, then if the retailer continues to use a screening strategy, it must screen using a 20oz to the H-type. To maintain the information rent to H-types needed to segment the market, the retailer would have to either significantly lower the price of the 20oz soda and/or dramatically reduce the size of the soda to the L-type. Therefore, the retailer may consider switching to a single size soda of 16oz which is priced to serve both types.

To assess whether the retailer will make this switch, we need to determine whether

[5] We must also account for the possibility that serving the high type with the segmentation strategy will still yield positive profits given the information rent. Thus, consider the profits from serving high types under the segmentation strategy, which is $(1-\beta)$ fraction of profits:

$$\theta_H v(\hat{q}) - c(\hat{q}) - (\theta_H - \theta_L)v(\tilde{q}_L) - \overline{u} \tag{24}$$

Consider the most restrictive ban within this region; i.e. $\hat{q} = \tilde{q}_L$. Substituting $\hat{q} = \tilde{q}_L$ into 24, rearranging and canceling terms yields $\theta_L v(\tilde{q}_L) - c(\tilde{q}_L) - \overline{u}$. Note that this is always positive so long as serving the low type yields positive profits. So in general, we don't have to worry about negative profitablity from serving high-types as long as it is profitable to serve low types.

a restriction causes the inequality $\Pi_{ib} \geq \Pi_{iiib}$ (profits from propositions 2 and 4) to be reversed. Writing out this inequality explicitly, we have:

$$\beta\left[\theta_L v(\tilde{q}_L) - c(\tilde{q}_L) - \overline{u}\right] + (1-\beta)\left[\theta_H v(\hat{q}) - (\theta_H - \theta_L) v(\tilde{q}_L) - c(\hat{q}) - \overline{u}\right] \geq \theta_L v(q_L^*) - c(q_L^*) - \overline{u} \tag{25}$$

Note that so long as the above inequality holds, the retailer will stick with the segmentation strategy over the one sized fits all single prize/size strategy. Also note that we replaced q with q_L^* on the right-hand-side of the inequality because in Region 1, the restriction is not tight enough to prevent the retailer from implementing its optimal size of q_L^*.

Proposition 6. *The implementation of a soda ban in the range $q_L^* \leq \hat{q} < q_H^*$ will not cause a retailer to switch from a segmentation strategy to the one-size-fits all single price, single size pricing strategy identified in Case (iiib).*

To summarize, any policy restriction in Region 1 will not cause the retailer to switch from the segmentation strategy of Case ib. The policy restriction would only affect price and quantity for the H-type within the segmentation strategy.

3.2.2 Region 2: $\tilde{q}_L \leq \hat{q} < q_L^*$

If the regulation is restrictive enough to rule out the ability of the retailer to implement a one-size-fits-all strategy that implements the low-type first-best level of q_L^*, then it will be even less likely that the retailer will switch from a segmentation strategy to a one-size-fits all strategy.

Proposition 7. *The implementation of a soda ban in the range $\tilde{q}_L \leq \hat{q} < q_L^*$ will not cause a retailer to switch from a segmentation strategy to the one-size-fits all single price, single size pricing strategy identified in Case (iiib).*

Continuing with our earlier example where 32oz is the first-best size for H-types, 16oz is the first best level for L-types, and 12oz is the optimal size for L-types under the segmentation strategy, a Region 2 restriction would impose a maximum size between 12-16 ounces. A moderate restriction such as this would not cause a retailer to switch from a segmentation to a one-size-fits all strategy.

3.2.3 Region 3: $\hat{q} < \tilde{q}_L$

Region 3 has to do with soda bans that are so restrictive, that it eliminates the ability to screen and hence would cause the retailer to endogenously switch to a single price-size strategy that serves either only H-types or both types. Continuing with our example, this would be a restriction that requires serving size to be less than 12 ounces. However, such restrictions are likely to be political infeasible as they are so onerous and will likely create backlash both from beverage consumers and retailers. The New York City soda ban was a proposal of 16 ounces and even that induced political backlash.

To see why screening would be ruled out by a restriction such that $\hat{q} < \tilde{q}_L$, note that the first-order conditions 14 and 15 would become:

$$\theta_H v'(q_H) > c'(q_H) \quad where \quad q_H = \hat{q} \tag{26}$$

$$\beta\left[\theta_L v'(q_L) - c'(q_L)\right] + (1-\beta)\left[-(\theta_H - \theta_L)v'(q_L)\right] > 0 \quad where \qquad q_L = \hat{q}(< \tilde{q}_L) \tag{27}$$

Thus, the retailer has an incentive to increase both q_H and q_L as much as possible until the ban $\hat{q}$ is reached. Hence, the solutions would be $q_H = q_L = \hat{q}$ so screening is no longer optimal. However, the principal may still decide to serve only H-types rather than both types because the principal would be able to charge a higher price due to the higher willingness-to-pay of H-types. Thus, serving only H-types increases profit margin but serving both types would increase profit through greater volume.

To determine which is the optimal strategy, we must compare retailer profit in proposition 3 to retailer profit in proposition 4.

Proposition 8. *The implementation of a size-restriction in the range $\hat{q} < \tilde{q}_L$ will cause a retailer to switch from a segmentation strategy to the one-size-fits all single price strategy that serves both types if $\frac{[\theta_H - \theta_L]v(\hat{q})}{[\theta_H v(\hat{q}) - c(\hat{q}) - \overline{u}]} \leq \beta$. On the other hand, if $\frac{[\theta_H - \theta_L]v(\hat{q})}{[\theta_H v(\hat{q}) - c(\hat{q}) - \overline{u}]} > \beta$, then the seller will only serve H-types.*

One can see that a large β (higher probability of encountering a L-type) increases the liklihood that the seller will serve both types. However, if there is substantial heterogeneity, as indicated by the spread $[\theta_H - \theta_L]$, then it is more likely that the seller will only serve H-types.

4 Major policy implications: how does the beverage size-restriction affect consumption, consumer welfare and producer welfare?

So far, we have learned that a beverage size-restriction in the small to moderate range (Regions 1 and 2) would not cause a retailer to move away from the segmentation strategy. However, if the restriction is tight enough (e.g. $\hat{q} < \tilde{q}_L$), then the retailer will switch from a segmentation strategy to a strategy of using a single-size-price package to either serve both types of consumers or only the H-type consumer, depending on exogenous parameters.

Now we are ready to address some important policy questions. Specifically, we want to know how the restriction potentially affects consumption of the targeted beverage, consumer welfare, and producer welfare.

4.1 Will the restriction have the intended effect of reducing sweetened beverage consumption?

The primary intent of the regulatory restriction is to the reduce the consumption of excessive amounts of sugared sweetened soft drinks. Thus, the first obvious question is: does our model predict that the restriction will reduce consumption?

Proposition 9. *An enforceable soft-drink size restriction will reduce beverage consumption by H-types and will only reduce consumption by L-types if the regulation is extremely restrictive (i.e. $\hat{q} < \tilde{q}_L$).*

In short, the restriction will likely achieve the intended effect of achieving a reduction in sweetened beverage consumption. However, we do caution that our predictions are based on the assumption that the restriction is credible and enforceable in that rules are in place to prevent consumers from undoing the size-restriction by buying multiple servings. The NYC soda ban did not contain such rules so we are likely to over-estimate the effectiveness of the size-restriction for that particular case. Nonetheless, we also do not believe that the reduction will be zero even in the NYC case since the introduction of smaller packages introduces an inconvenience cost to consumers of consuming larger sodas. Practical barriers such as having to carry two-cups instead of one or having limited cup-holders in vehicles may constrain consumption of multiple smaller sized sodas to some extent. It is also possible that consumer expectations of what constitutes a standard serving sizes may adjust downward over time.

4.2 How will the restriction affect consumer welfare?

Some commentators in the popular press view soda restrictions as an issue of consumer freedom (e.g. Nestle (2012)), which implicitly suggests that consumers would be harmed by such a ban. Thus, one of the hot button issues is how a restriction might affect consumer welfare.

A naive view using standard demand theory might suggest that deadweight losses would be created if consumers are restricted from purchasing the equilibrium level of a soft-drink. However, our model shows that this insight would be misleading when there is nonlinear pricing. The next proposition suggests that, under our strategic pricing model, consumer welfare is largely unaffected by a beverage-size restriction unless the restriction is extreme.

Proposition 10. *For a small to medium size restriction in the range $\hat{q} \in [\tilde{q}_L, q_H^*]$ (Regions 1 and 2), the restriction will have no impact on consumer welfare. However, for highly restrictive regulations where $\hat{q} < \tilde{q}_L$ (Region 3)that causes a seller to endogenously switch from a segmentation strategy to a one-size strategy that serves both types, the H-type consumer's welfare will decline while the L-type consumer's welfare will remain unaffected. If instead, the seller endogenously switches to only a one-size strategy that serves only H-types, then there will be welfare reductions for both types of consumers.*

One of the striking features of our result is that consumer welfare will largely be unaffected by a small to moderate ban. The intuition is that, so long as the ban does not fundamentally change the retailer's selling strategy, the retailer can always compensate the consumer for a smaller quantity by lowering the price. While the H-type earns information rents in the absence of the regulation, the same information rents can be earned post-regulation via a lowering of p_H and full market segmentation can be maintained.

4.3 How will the ban affect seller welfare?

So far, we have seen that the regulation will reduce q away from the optimal levels so that inefficiency arises. However, it appears that consumers don't bear the brunt of the welfare losses particularly for small to moderate restrictions. Instead, our intuition tells us that the seller will bear most of the welfare losses. Our intuition is confirmed by the following result.

Proposition 11. *The seller will unambiguously suffer a loss in expected profits for binding beverage restrictions of any size.*

To summarize, the beverage-size restriction will create an efficiency loss by moving consumption away from optimal levels. However, the welfare loss will largely be borne by beverage sellers rather than consumers. This contrasts both standard demand theory and popular discussions of the implications of soda regulations on consumer welfare.

5 Alternatives to beverage-size-restrictions

Our model also allows us to examine alternatives to the beverage-size-restriction. In particular, comparative statics examining how optimal levels of q_H, q_L, p_L and p_H change with changes in exogenous parameters can give us a rough idea of how changes in the economic environment can influence both consumption of sugar-sweetened beverages and retailers' optimal response to changes in, say, demand conditions.

Note from 5 and 6 that an increase θ_H (H-type WTP) will increase q_H (marginal revenue to increasing q_H increases), while decreasing q_L holding other variables constant. An increase in θ_L will increase q_L. Essentially, when consumer heterogeneity is magnified, this lowers q_L as the retailer must lower q_L to lower information rents used to incent the H-type to accept the q_H size bundle. A practical example of how these comparative statics insights might be useful is to think about how recent scientific studies and public information campaigns that demonize the role of sugar in health affect various model parameters. If these campaigns have affected θ_H and θ_L in a negative and non-proportional way, then this might explain why soda manufacturers have voluntarily reduced soda serving size or sugar content.

Thus, rather than using explicit restrictions or taxes, which might be seen as draconian, it might be more politically feasible to intensify public information campaigns. Our model can provide insight into how information campaigns might affect various parameters, which can in turn, make predictions about firms' endogenous marketing responses.

6 Conclusion

The objective of this paper was provide an economic analysis of beverage size restrictions. Our goal is not to advocate for or against such a regulation but rather to outline the economic effects, including whether the restriction will achieve its objective of reducing consumption, and what the potential welfare effects are both to consumers and sellers. We are also agnostic about whether reducing sweetened soft-drinks will lead to long-term societal health improvements and leave such debates to nutritionists.

Our key findings are that the a regulatory size-restriction on beverage size will likely reduce consumption, particularly to high consumption consumers. Hence, if the intent is to reduce sugar-sweetened beverages, a properly implemented policy that is credibly applied will likely have the intended effect. However, the policy will also create consumption inefficiencies relative to the unregulated case. Firms will be restricted in their implementation of nonlinear pricing strategies and will likely be forced to use price adjustments rather than quantity variation to achieve separation between different types of consumers. Moreover, if the regulation is extremely restrictive, retailers may abandon nonlinear pricing strategies.

Surprisingly, we find that consumers will not suffer welfare losses under restricted nonlinear pricing schemes. Intuitively, in order to create segmentation of different types of consumers, "high-type" consumers must be provided with information rents. If the policy restriction is not so onerous that nonlinear pricing is still possible, retailers will simply adjust prices to high-types downward until they receive the same information rents as they did prior to the regulation. Meanwhile, low-types were held to their reservation utility levels prior to the regulation and will continue to be held at their reservation utility after the regulation. In short, under a nonlinear pricing scheme, all welfare losses from consumption inefficiencies are borne by the seller. One implication of our finding is that debates surrounding soda size restriction that focus on consumer welfare are possibly missplaced. Instead, the focus should be on how much retailers and businesses lose from the regulatory restriction and whether the losses to businesses are worth the potential gain in consumer health benefits.

One caveat to our findings is that we measure consumer welfare in terms of consumer surplus from consuming beverages. We do not focus on possible psychological welfare losses, due to say, mental anguish from the perception that one's freedom has been restricted. Nonetheless, such behavioral studies might be important, which we leave for future research.

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Appendix A - FOR ONLINE PUBLICATION

Proof of Proposition 2

Proof. The proof for part (1) follows trivially from the assumption of a binding restriction, $\hat{q}$, which yields K-T condition 14. Thus, the retailer will increase q_H until $q_H = \hat{q}$.

The proof for part (2) follows from the fact that the first order condition for q_L 15 is unchanged from the unregulated case. Hence, the retailer will still offer the same $\tilde{q}_L$ as the unregulated case.

The proof for part (3) follows from replacing q_H^* with $\hat{q}$ in the the optimal price function. Moreover, since the price function is a function of $v(q_H)$ and $v(q_H)$ is increasing in $q_H \ \forall \ q_H < q_H^*$, it must be true that the new price $\hat{p}_H < p_H$ since $\hat{q} < q_H^*$.

The proof for part (4) from the fact that the L-type price is a function of only q_L (and not q_H) which remains unchanged at $\tilde{q}_L$.

The proofs for parts (5), (6), and (7) follow trivially from plugging in the optimal prices and quantities into the objective functions for the retailer and the two types of consumers. □

Proof of Proposition 3

Proof. The proof for part (1) follows trivially from the assumption of a binding restriction, $\hat{q}$, which yields K-T condition 14. Thus, the retailer will increase q_H until $q_H = \hat{q}$.

The proof for part (2) follows from replacing q_H^* with $\hat{q}$ in the the optimal price function. Moreover, since the price function is a function of $v(q_H)$ and $v(q_H)$ is increasing in q_H $\forall$ $q_H < q_H^*$, it must be true that the new price $\hat{p}_H < p_H$ since $\hat{q} < q_H^*$.

The proofs for parts (3) and (4) follow trivially from plugging in the optimal prices and quantities into the objective functions for the retailer and the two types of consumers. $\square$

Proof of Proposition 4

Proof. To show part (1), referring to the K-T conditions 21, if the size-constraint is not binding so that $q < \hat{q}$, then the first order condition binds with equality so that $\theta_L v'(q) = c'(q)$ so the solution to 21 is clearly equal to the first best level of quantity for L-types, q_L^*. Thus, the size-constraint can only bind if $\hat{q} \leq q_L^*$ in which case $q = \hat{q}$. Hence, $q = min\{q_L^*, \hat{q}\}$

To show part (2), we can use the binding participation constraint to recover the optimal price $p = \theta_L v(q) - \overline{u}$.

Parts (3)-(5) follow trivially from substituting the optimal q and p into the objective functions of the seller, H-type consumer, and L-type consumer. $\square$

Proof of Proposition 5

Proof. The proof involves comparing Π_{ib} from proposition 2 to Π_{iib} from proposition 3. i.e. the retailer will not switch away from the segmentation strategy iff:

$$\Pi_{ib} = \beta \left[\theta_L v(\tilde{q}_L) - c(\tilde{q}_L) - \overline{u}\right] + (1-\beta)\left[\theta_H v(\hat{q}) - c(\hat{q}) - (\theta_H - \theta_L) v(\tilde{q}_L) - \overline{u}\right] \quad (28)$$

$$\geq (1-\beta)\left[\theta_H v(\hat{q}) - c(\hat{q}) - \overline{u}\right] = \Pi_{iib}$$

After some algebra, 28 reduces to

$$\beta\left[\theta_L v(\tilde{q}_L) - c(\tilde{q}_L) - \overline{u}\right] \geq (1-\beta)[\theta_H - \theta_L]v(\tilde{q}_L) \tag{29}$$

Note that 29 is completely independent of $\hat{q}$. Therefore, the regulation cannot reverse the inequality. Consequently, if the retailer chose the segmentation strategy prior to the regulation, it will continue to do so after the regulation. □

Proof of Proposition 6

Proof. Note that, by 14, the left-hand-side of 25 is monotonically increasing in $\hat{q}$ over the range $q_L^* \leq \hat{q} < q_H^*$. Moreover, the right-hand-side of 25 is independent of $\hat{q}$ since $\hat{q}$ does not constrain implementation of q_L^*. Thus, if we can show that 25 is satisfied even when $\hat{q} = q_L^*$, then it must continue to hold for all $\hat{q} \in [q_L^*, q_H^*]$ so that any restriction in this region cannot cause the retailer to switch from a segmentation strategy to a one-size-fits all strategy.

Suppose that $\hat{q} = q_L^*$, the lowest quality level in the range $[q_L^*, q_H^*]$. Then substituting into 25 yields:

$$\beta\left[\theta_L v(\tilde{q}_L) - c(\tilde{q}_L) - \overline{u}\right] + (1-\beta)\left[\theta_H v(q_L^*) - (\theta_H - \theta_L)v(\tilde{q}_L) - c(q_L^*) - \overline{u}\right] \geq \theta_L v(q_L^*) - c(q_L^*) - \overline{u} \tag{30}$$

After rearranging terms and simplifying, 25 reduces to:

$$\beta[c(q_L^*) - c(\tilde{q}_L)] + [v(q_L^*) - v(\tilde{q}_L)][\theta_H(1-\beta) - \theta_L] \geq 0 \tag{31}$$

Note that the left-hand-side of 31 depends on β. We first make the following claim:

Claim 1. *The left-hand-side of 31 is decreasing in β.*

Proof. Taking the derivative with respect to β of the left-hand-side of 31 yields

$$-\{\theta_H[v(q_L^*)-v(\tilde{q}_L)]-[c(q_L^*)-c(\tilde{q}_L)]\}-\beta c'(\tilde{q}_L)\frac{d\tilde{q}_L}{d\beta}-v'(\tilde{q})\frac{d\tilde{q}_L}{d\beta} \tag{32}$$

Note that $-\{\theta_H[v(q_L^*)-v(\tilde{q}_L)]-[c(q_L^*)-c(\tilde{q}_L)]\}$ is non-positive given $\theta_H v'(q) > c'(q)$ for all $q < q_H^*$ and $q_L^* \geq \tilde{q}_L$. Thus, it remains to determine the sign of $\frac{d\tilde{q}_L}{d\beta}$. Assuming an interior solution for $\tilde{q}_L$, implicitly differentiating 15 yields

$$\frac{d\tilde{q}_L}{d\beta}=\frac{-[\theta_H-\theta_L]v'(\tilde{q}_L)}{\beta^2[\theta_L v''(\tilde{q}_L)-c''(\tilde{q}_L)-\frac{1-\beta}{\beta}[\theta_H-\theta_L]v''(\tilde{q}_L)]} \tag{33}$$

Since the denominator of 33 must be negative in order for the principal's objective function to be concave, it follows that 33 is positive so that $\tilde{q}_L$ is increasing in β. Since $\frac{d\tilde{q}_L}{d\beta} > 0$, it follows that 32 must be negative. Hence, the left-hand-side of 31 is decreasing in β. □

Claim 1 suggests that the left-hand-side of 31 is minimized at $\beta = 1$. Hence, if the left-hand-side remains non-negative for $\beta = 1$, then the profit from the segmentation strategy will be at least as high as the profit from the one-size-fits all strategy so that the restriction will not cause the retailer to switch from a segmentation strategy to a one-size-fits all strategy. Note that for $\beta = 1$, the first order condition condition 15 for q_L implies that $\tilde{q}_L = q_L^*$. Hence, at $\beta = 1$, the left-hand-side of 31 reduces to $c(q_L^*) - c(q_L^*) + [v(q_L^*) - v(q_L^*)][-\theta_L] = 0$. Thus, over the range $q_L^* \leq \hat{q} < q_H^*$, a restriction cannot cause the expected profit from the segmentation strategy to fall below the profit from the one-size-fits all strategy. □

Proof of Proposition 7

Proof. The proof can proceed similarly as the proof for Proposition 6. That is, over the range $[\tilde{q}_L, q_L^*)$ we must show that

$$\beta\left[\theta_L v(\tilde{q}_L) - c(\tilde{q}_L) - \overline{u}\right] + (1-\beta)\left[\theta_H v(\hat{q}) - (\theta_H - \theta_L) v(\tilde{q}_L) - c(\hat{q}) - \overline{u}\right] \geq \theta_L v(\hat{q}) - c(\hat{q}) - \overline{u} \tag{34}$$

Comparing 34 to 25, the key difference is that the right-hand-side of 34 is decreasing in $\hat{q}$ over the interval $[\tilde{q}_L, q_L^*)$ whereas 25 is independent of $\hat{q}$ over the interval $[q_L^*, q_H^*)$. Therefore, this inequality becomes more relaxed over the range $[\tilde{q}_L, q_L^*)$. Hence, if the inequality is satisfied over the range $[q_L^*, q_H^*)$ as shown in the proof for 25, then it must be satisfied be satisfied over the range $[\tilde{q}_L, q_L^*)$. □

Proof of Proposition 8

Proof. The proof follows trivially from comparing the profit in proposition 3 to the profit in proposition 4. That is, if $\Pi_{iib} = (1-\beta)[\theta_H v(\hat{q}) - c(\hat{q}) - \overline{u}] \leq \Pi_{iiib} = \theta_L v(\hat{q}) - c(\hat{q}) - \overline{u}$, which implies that $\frac{[\theta_H - \theta_L] v(\hat{q})}{[\theta_H v(\hat{q}) - c(\hat{q}) - \overline{u}]} \leq \beta$, then it is more profitable to serve both types rather than just the H-types. However, if the inequality is reversed, then it is more profitable to serve only H-types. □

Proof of Proposition 9

Proof. By propositions 6 and 7, a ban such that $\hat{q} \in [\tilde{q}_L, q_H^*]$, will not cause the retailer to switch from a segmentation strategy to a single-price strategy. Moreover, by proposition 2, a restriction will decrease q_H from q_H^* to $\hat{q}$ while leaving $\tilde{q}_L$ unchanged. However, by proposition 8, a restriction $q < \tilde{q}_L$ will cause the seller to switch to a single-price/package strategy that either serves both types or only the H-type. In the former case, consumption to both types will drop because $\hat{q} < \tilde{q}_L < q_H^*$. In the latter case, L-types are not served while H-types consume less because $\tilde{q}_L < q_H^*$ $\square$

Proof of Proposition 10

Proof. By proposition 1, consumer welfare in the unregulated case for the H- and L-type consumers are $U_H = \overline{u} + [\theta_H - \theta_L]v(\tilde{q}_L)$ and $U_L = \overline{u}$, respectively.

Suppose that a restriction falls in Region 1 or Region 2; i.e. $\hat{q} \in [\tilde{q}_L, q_H^*]$. Then propositions 6 and 7 suggest that the seller will not switch away from a segmentation strategy. By proposition 2, consumer welfare is $U_{Hib} = \overline{u} + [\theta_H - \theta_L]v(\tilde{q}_L)$ and $U_{Lib} = \overline{u}$. Note that these utility outcomes are identical to U_H and U_L in the absence of the regulation. Therefore, any regulation in Regions 1 and 2 will have no impact on consumer welfare.

Next, a restriction in Region 3; i.e. $\hat{q} < \tilde{q}_L$. Proposition 8 states that such a restriction would cause the retailer to abandon the segmentation strategy and either serve only H-types or both types with a one-size-fits-all strategy. If the retailer serves only H-types, then Proposition 3 states that H-type utility is $U_{Hiib} = \overline{u}$. This represents a welfare loss since the H-type loses information rents $[\theta_H - \theta_L]v(\tilde{q}_L)$ from the regulation. If the retailer serves both types with a one-size-fits-all package, then Proposition 4 states that $U_{Hiiib} = \overline{u} + [\theta_H - \theta_L]v(\hat{q})$ and $U_{Liiib} = \overline{u}$. Note that $U_H - U_{Hiiib} = [v(\hat{q}) - v(\tilde{q}_L)][\theta_H - \theta_L] < 0$ since $\hat{q} < \tilde{q}_L$. Thus, the H-type consumer suffers a welfare loss. Since $U_{Liiib} = \overline{u}$ is identical to U_L, there is no change in welfare to the L-type. $\square$

Proof of Proposition 11

Proof. By proposition 1, the seller's expected profit is $\Pi = (1-\beta)[\theta_H v(q_H^*) - \overline{u} - c(q_H^*) - [\theta_H - \theta_L]v(\tilde{q}_L)] + \beta[\theta_L v(\tilde{q}_L) - \overline{u} - c(\tilde{q}_L)]$, which serves as the unregulated benchmark.

Consider a restriction in Region 1 or 2; i.e. $\hat{q} \in [\tilde{q}_L, q_H^*]$. Then proposition 2 yields the new profit of $\Pi_{ib} = (1-\beta)[\theta_H v(\hat{q}) - \overline{u} - c(\hat{q}) - [\theta_H - \theta_L]v(\tilde{q}_L)] + \beta[\theta_L v(\tilde{q}_L) - \overline{u} - c(\tilde{q}_L)]$. By first-order condition 14, Π_{ib} is increasing in $\hat{q}$ for any $\hat{q} < q_H^*$. Hence, $\Pi_{ib} - \Pi < 0$ so there is seller welfare loss.

If a restriction is in Region 3; i.e. $\hat{q} < \tilde{q}_L$, and the seller serves only H-types, then proposition 3 tells us that $\Pi_{iib} = (1-\beta)[\theta_H v(\hat{q}) - c(\hat{q}) - \overline{u}]$. On the other hand, if the seller serves both types with a one-size-fits-all strategy, then proposition 4 tells us that $\Pi_{iiib} = \theta_L v(\hat{q}) - c(\hat{q}) - \overline{u}$. In either case, the first-order conditions 26 and 27, imply that profit is increasing in both q_H and q_L in Region 3. Hence, $\Pi_{iib} - \Pi < 0$ and $\Pi_{iiib} - \Pi < 0$. $\square$