Comovements and Volatility Spillover in Commodity Markets

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Sihong Chen*  Ximing Wu†

Abstract

This paper analyzes comovements and connectedness of commodity futures in the past two decades. We apply dynamic conditional correlation model (DCC) to capture time-varying dependence structure of a variety of commodities across different sectors. We propose to estimate network connectedness of commodity markets by the modeling framework of Diebold and Yilmaz (2014) that studies direction and magnitude of volatility spillover using reduced-form vector autoregression (VAR) models and generalized forecast error variance decomposition. We find that both DCC and VAR models present consistent results: while comovements and connectedness of commodity markets have dramatically increased during 2007-2009 financial distress, they have returned to the pre-crisis levels after. We also find that recent downward movement of commodity prices does not necessarily indicate stronger connection between commodity markets, which poses challenges on recent studies in commodity financialization.

JEL classification: G01 ; G10 ; Q02

Keywords: Dynamic conditional correlation, VAR, generalized forecast error variance decomposition, volatility spillover, market connectedness

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1 Introduction

Commodity markets have attracted much attention in both academia and industry since early 2000s, but some interesting problems still remain unknown and are little studied so far. First of all, large inflows into commodity markets, termed “financialization of commodity markets”, has dramatically increased the correlation between a large number of commodity futures (Tang and Xiong, 2012). Though recent research provides some evidence of structural change in correlation, most of them do not fully account for the information of higher moments or model the joint distribution of futures returns, partly due to the paucity of flexible multivariate distributions in the literature. Since many assets like stocks, bonds and commodities that show low correlation in the history tend to crash together during the recent financial crisis, understanding time-varying comovements of commodities in a large portfolio is of great importance from the perspective of risk management.

Secondly, transmission mechanism of volatility between commodities is still not clear. Adams and Glück (2015) show significant risk spillover from stocks to commodities, but they only measure spillover using value-at-risk and do not consider how shocks in one market may affect volatility in another. By modeling volatility spillover we may quantify the magnitude and direction of volatility shocks of various commodities with a causal interpretation, which is not readily available by modeling comovements alone. As a result, measures of markets connectedness based on volatility spillover may shed light on portfolio construction and diversification. This paper attempts to study the dynamics of comovements and volatility spillovers in commodity futures by employing high dimensional dynamic conditional correlation models and develop a variety of new connectedness measures in recent literature for commodity markets based on intraday ranged-based volatility.

We make two primary contributions to the current literature in this paper. First, we explore the joint dynamics of dependence structure of 20 commodities. Tang and Xiong (2012) find increasing correlation since 2004, but they model dynamics of correlations by rolling-window for all pairwise combinations of commodities one after another, which is inefficient as they do not explicitly take all information into account and not necessarily robust to the structural change in correlations. Adams and Glück (2015) consider structural breaks in correlations but their sample only include 8 commodities and also do not provide a joint estimation of dependence structure in futures returns. Second, we investigate volatility spillover in these commodities as measures of markets connect-
edness. Adams and Glück (2015) study spillover in the left tails of return distributions (Value-at-Risk or VaR) and regress individual VaR on many other VaR, say, those of stock market, commodity market and emerging market. Their method directly model shocks from various markets to individual commodity, but do not consider spillover between commodity futures. We show that our analysis based on Diebold and Yilmaz (2014, 2015) is more comprehensive and provide additional information in quantifying dynamics of volatility spillover in commodity futures. We also discuss how to estimate a total connectedness in the commodity markets, which can be readily viewed as a “fear gauge” of investors, a measure similar to the VIX for stock markets.

The paper proceeds as follows. Section 2 provides a brief outline of DCC model, VAR model, generalized forecast error variance decomposition and the related network connectedness in range-based volatility. Section 3 discusses how we construct rolling future contracts to meet data requirement for DCC and VAR models, and presents the major findings in our empirical analysis. Section 4 concludes.

2 Modeling framework

In this section we present the basics of our modeling framework for estimation of co-movements and connectedness of commodity futures markets. We first present the DCC model that describes time-varying dependence structure of futures returns. We briefly discuss how DCC model can incorporate recent development of maximum composite likelihood estimation for high dimensional data, which imposes a great deal of computational burden on the traditional maximum likelihood estimation. Second, we show how reduced-form VAR models and generalized variance decomposition can be used to construct static and dynamic connectedness tables of range-based volatility, which is not identical to the GARCH-type volatility in the DCC model.

2.1 Dynamic conditional correlation model

We assume $R_{i,t}$ is the log returns of commodity future $i$ at period $t$, $\sigma_{i,t}$ follows generalized autoregressive conditional heteroscedasticity (GARCH) process, and $z_{i,t}$ is innovation term. The order of ARIMA models is selected by AIC or BIC, and the GARCH(1,1) model is estimated by QMLE. The univariate models is described as:

$$R_{i,t} = \mu_{i,t} + \epsilon_{i,t} = \mu_{i,t} + \sigma_{i,t}z_{i,t}$$

(1)
\[ \sigma^2_{i,t} = \omega_i + \alpha \varepsilon^2_{i,t-1} + \beta \sigma^2_{i,t-1} \]  

Motivated by the seminal paper of Engle (2002) and a recent application by Christoffersen et al. (2014a), we propose to estimate the dynamic conditional correlation which drives dynamics in the multivariate distribution. Since covariance is the product of correlations and standard deviations, we can write:

\[ \Sigma_t = D_t \Gamma_t D_t \]  

where \( D_t \) has the standard deviations \( \sigma_{i,t} \) on the diagonal and 0 elsewhere, and \( \Gamma_t \) has 1 on the diagonal and conditional correlation off the diagonal. The dynamics of \( \Gamma_t \) is driven by a GARCH-type process:

\[ \tilde{\Gamma}_t = (1 - \alpha \Gamma - \beta \Gamma) \tilde{\Gamma} + \alpha \Gamma (z_{t-1} z'_{t-1}) + \beta \tilde{\Gamma}_{t-1} \]  

and we use it to define the conditional correlation by the following normalization to ensure correlations are all in the \([-1, 1]\) interval:

\[ \Gamma_{ij,t} = \frac{\tilde{\Gamma}_{ij,t}}{\sqrt{\tilde{\Gamma}_{ii,t} \tilde{\Gamma}_{jj,t}}} \]  

We obtain the estimated \( \tilde{\varepsilon}_{i,t} \) from the GARCH models and use them to measure the “targeting correlation” \( \tilde{\Gamma} \) by \( \frac{1}{T} \sum_{t=1}^{T} \tilde{\varepsilon}_{i,t} \tilde{\varepsilon}'_{i,t} \) in formula (4). This is a direct modeling of correlation dynamics and has the potential to capture precisely the time-varying nature of correlations. Engle et al. (2008) propose to estimate this dynamic process with composite likelihood to avoid computational burden with traditional likelihood. To be specific, the composite likelihood is defined as:

\[ CL(\alpha, \beta) = \sum_{t=1}^{T} \sum_{i=1}^{N} \ln f(\alpha, \beta; R_{i,t}, R_{j,t}) \]  

where \( \ln f(\alpha, \beta; R_{i,t}, R_{j,t}) \) is the bivariate distribution of pair \( i \) and \( j \) with variance defined by formula (3). We can denote \( f \) as normal, student or other variants of elliptical distribution family that explicitly take variance into the likelihood function. We now maximize composite likelihood by summing over all possible pairs in each period \( t \), which is numerically fast and efficient, and we will consider both normal and student distributions in our empirical applications.

### 2.2 VAR models for network connectedness of volatility

Following the seminal work of Diebold and Yilmaz (2012, 2014, 2015), we estimate reduced-form VAR approximating models to construct connectedness measures from
the $H$-step ahead generalized forecast error variance decomposition. To be specific, we consider the following $N$-dimensional VAR($p$) model:

$$y_t = \sum_{i=1}^{p} \Phi_i y_{t-i} + \epsilon_t$$

(7)

where $\epsilon_t \sim (0, \Omega)$, and $\Omega$ is the stationary covariance of $\epsilon$. The related moving average representation is

$$y_t = \sum_{i=0}^{\infty} A_i \epsilon_{t-i}$$

(8)

where the $N \times N$ coefficient matrices $A_i$ is supposed to be $A_i = \sum_{j=1}^{p} \Phi_j A_{i-j}$. $A_0$ is an $N \times N$ identity matrix and $A_i=0$ for $i < 0$. We transform these moving-average coefficient matrices to obtain variance decomposition such that we can split $H$-step-ahead forecast error variances of each commodity returns and account for the system shocks in the VAR($p$) model.

We avoid to use the popular variance decomposition such as structural VAR or Cholesky factor since they both requires orthogonalization of VAR shocks, but they assume some other conditions (structural VAR) or depend on the ordering of variables (Cholesky factor). We propose to use the generalized forecast error variance decomposition of Koop et al. (1996) and Pesaran and Shin (1998), which is robust to the ordering of variables and does not assume additional conditions, to take correlated shocks into account. In particular, we define variable $j$’s contribution to variable $i$’s $H$-step-ahead generalized forecast error variance as:

$$\theta_{ij}^g(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (e_i^A_h \Omega e_j)^2}{\sum_{h=0}^{H-1} (e_i^A_h \Omega A_h^t e_i)}$$

(9)

where $e_j$ is the selection vector with $j$-th element as 1 and 0 elsewhere, and $\sigma_{jj}$ is the standard deviation of $\epsilon$ for variable $j$. We proceed to normalize this variance decomposition matrix such that the row sums are one:

$$\hat{\theta}_{ij}^g(H) = \frac{\theta_{ij}^g(H)}{\sum_{j=1}^{N} \theta_{ij}^g(H)}$$

(10)

This normalization facilitates interpretation of variance decomposition and provides a directional measure of pairwise connectedness from $j$ to $i$ with predictive horizon $H$. To simplify notation we can write $C_{i \leftarrow j}(H) = \hat{\theta}_{ij}^g(H)$. It is natural to define net pairwise directional connectedness $C_{ij}(H) = C_{i \leftarrow j}(H) - C_{j \leftarrow i}(H)$. We can also derive aggregate
"from" and "to" connectedness measures such that we can investigate the total "influence" an arbitrary variable \( i \) exerts or receives in the system of VAR\((p)\) model:

\[
C_{i \leftarrow \bullet}(H) = \frac{\sum_{j=1, j \neq i}^{N} \theta^g_{ij}(H)}{N} \times 100 \tag{11}
\]

and

\[
C_{\bullet \leftarrow i}(H) = \frac{\sum_{j=1, j \neq i}^{N} \theta^g_{ji}(H)}{N} \times 100 \tag{12}
\]

and we may calculate the net total directional connectedness for variable \( i \) simply as (6)-(5):

\[
C_i(H) = C_{\bullet \leftarrow i}(H) - C_{i \leftarrow \bullet}(H) \tag{13}
\]

We can eventually aggregate all "to" or "from" measures and take their means as a system-wide measure of total connectedness:

\[
C(H) = \frac{\sum_{i=1}^{N} C_{\bullet \leftarrow i}(H)}{N} = \frac{\sum_{i=1}^{N} C_{i \leftarrow \bullet}(H)}{N} \tag{14}
\]

We are especially interested in these measures and will discuss them with more details in next section. Following Diebold and Yilmaz (2015) we use Garman and Klass (1980)'s approach for intraday range-based volatility which we are going to estimate in the VAR\((p)\) model:

\[
\hat{\sigma}^2 = 0.511 (h - l)^2 - 0.019 [(c - o) (h + l - 2o) - 2(h - o) (l - o)] - 0.383 (c - o)^2 \tag{15}
\]

where \( h, l, o \) and \( c \) stand for the log of daily high price, low price, opening price and close price respectively. Volatility is always treated as "fear gauge" or sentiment of investors, and we focus on the interdependence of volatility in various commodity futures to explore the transmission mechanism of commodity markets sentiments since it is little studied in the current literature.

3 Empirical results

In this section we present major empirical results using two datasets from different sources. We first introduce how we construct the rolling commodity prices to handle futures with various expiry dates. We then discuss the estimation results from dynamic conditional correlation models with normal and student distributions. Lastly we analyze the results of network connectedness from both static and dynamic VAR\((p)\) models.
and show that it is critical to consider dynamics in the connectedness measures, which are extremely sensitive to market sentiment and should be examined carefully across financial cycles.

3.1 Data

We consider 20 commodities in the Goldman Sachs Commodity Index (GSCI) that serves as a benchmark for investment in the commodity markets. In particular we have 3 commodities in energy sector (WTI crude oil, Brent crude oil and natural gas), 5 commodities in grains sector (corn, soybean, wheat, soybean oil and rough rice), 6 commodities in softs sector (coffee, cotton, sugar, cocoa, lumber and orange juice), 3 commodities in livestocks sector (feeder cattle, lean hogs and live cattle) and 3 commodities in metals sector (gold, silver and copper).

We obtain the first dataset from Datastream using ticker “CS04” from 2nd Jan 1995 to 23rd May 2015 for all commodity futures. These are continuous returns which are rolled when the first-nearest to expire future contracts have reached expiry date. On this date the second-nearest to expire future contracts returns are used to ensure these returns are all based on the same contract. As mentioned in subsection 2.2, we need to have daily high price, low price, opening price as well close price to measure intraday range-based volatility, which are not provided in the Datastream. We get the second dataset from Bloomberg from 2nd Jan 1996 to 26th Feb 2016. Following Christoffersen et al. (2014b) we construct rolling futures by comparing the trading volumes of the first-nearest to expire contract and the second-nearest to expire contract, and roll to the second contract if its trading volume is greater.

3.2 Dependence structure of commodities

From section 2.1 we know that the DCC model is actually a two-stage estimation. In the first stage we use quasi maximum likelihood estimation (QMLE) to model the dynamics of GARCH volatility for univariate log returns, and in the second stage we use estimated GARCH volatility and the proposed maximum composite likelihood estimation to obtain \(\alpha\), \(\beta\) or other parameters that drive the dynamics of high dimensional correlation matrix. To save space we omit the GARCH results for all twenty commodities below and focus on the DCC models. We estimate both normal and student distributions and the results is in Table 1:
Table 1: Results of dynamic conditional correlation models

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>Degree of freedom</th>
<th>Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.0184</td>
<td>0.9841</td>
<td></td>
<td>-3239941</td>
</tr>
<tr>
<td>Student</td>
<td>0.0475</td>
<td>0.9574</td>
<td>15.65</td>
<td>-3216431</td>
</tr>
</tbody>
</table>

Table 1 suggests student model is preferred to the normal model in terms of likelihood, and this should not be too surprising since student distribution has one additional parameter (degree of freedom) to control for tail behavior of returns, which is quite common in the bear markets. We next present the estimated dynamic conditional correlation using student model for all commodities. Since there $N(N - 1)/2 = 190$ correlations in our sample, we cluster commodities by their groups and present dynamic conditional correlations at group level, reducing the within-group and cross-group correlation to 15 only. We also average all dynamic correlations at the same period $t$ to have an overall correlation for these commodities.

[INSERT Figure 1 & Figure 2 ABOUT HERE]

Figure 1 and 2 show the dynamics of correlations in two decades. It is obvious to see most of them have increased and peaked during the 2007-2009 financial crisis, but they declined sharply after. Since mid-2014 there have seen dramatic decreases in energy prices and we can see that these group-correlations have gone up in a not very significant way.

3.3 Network connectedness of commodities

In this subsection we will present empirical results of static and dynamic estimation of volatility connectedness using the modeling framework in section 2.2. In section 3.3.1 we first show the static measures of connectedness based on all observations in the sample, then we discuss in section 3.3.2 how to derive dynamics of these measures and use them to monitor and describe evolution of volatility connectedness in commodity markets during the 2007-2009 financial distress and the recent downward movement of commodity prices since mid-2014.
3.3.1 Static (unconditional, full-sample) analysis of connectedness

We estimate a VAR(3) model with all observations in the sample from January 1996 to February 2016. Since there are 20 commodities in our sample, it is not concise to present a $20 \times 20$ connectedness table below. We again cluster all these futures based on the commodity sectors they belong to and aggregate their measures of connectedness. We eventually arrive at a $5 \times 5$ connectedness table below:

Table 2: Full-sample connectedness table

<table>
<thead>
<tr>
<th>To\From</th>
<th>Energy</th>
<th>Grains</th>
<th>Softs</th>
<th>Livestocks</th>
<th>Metals</th>
<th>Sum of From</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>30.11</td>
<td>0.53</td>
<td>0.23</td>
<td>0.49</td>
<td>1.37</td>
<td>32.74</td>
</tr>
<tr>
<td>Grains</td>
<td>1.05</td>
<td>16.47</td>
<td>1.13</td>
<td>0.46</td>
<td>2.12</td>
<td>21.23</td>
</tr>
<tr>
<td>Softs</td>
<td>0.66</td>
<td>1.3</td>
<td>14.69</td>
<td>0.24</td>
<td>0.91</td>
<td>17.79</td>
</tr>
<tr>
<td>Livestocks</td>
<td>2.31</td>
<td>1.16</td>
<td>0.45</td>
<td>26.90</td>
<td>1.30</td>
<td>32.11</td>
</tr>
<tr>
<td>Metals</td>
<td>1.90</td>
<td>1.89</td>
<td>0.67</td>
<td>0.51</td>
<td>26.43</td>
<td>31.40</td>
</tr>
<tr>
<td><strong>Sum of To</strong></td>
<td><strong>36.04</strong></td>
<td><strong>21.33</strong></td>
<td><strong>17.17</strong></td>
<td><strong>28.59</strong></td>
<td><strong>32.13</strong></td>
<td><strong>27.05</strong></td>
</tr>
<tr>
<td>Net</td>
<td>3.30</td>
<td>0.11</td>
<td>-0.62</td>
<td>-3.52</td>
<td>0.73</td>
<td></td>
</tr>
</tbody>
</table>

Note: The sample is from Jan 2, 1996 to Feb 26, 2016, and the predictive horizon is 12 days. The $ij$-th entry of this $5 \times 5$ matrix represents pairwise directional connectedness from $j$ to $i$. The rightmost column is the sum of volatility spillover received by different sector. The bottom “Sum of To” row is the sum of volatility spillover from any sectors to the others. The bottommost “Net” row is simply the difference between “Sum of To” and “Sum of From”. The intersection of “Sum of From” and “Sum of To” is the total connectedness measure in the VAR model system.

From Table 2 we can find some notable features of connectedness in commodity markets. For example, the diagonal element is always much higher than the other elements in the same row, suggesting that most of the volatility shocks come from the commodities in the same sector. It seems that commodity markets are quite segmented since at least 80% of the sum gives or received by a sector is from the same sector. Energy sector has the highest “Net”, implying that it has the greatest influence on the other four sectors, while livestocks sector has the lowest “Net”, indicating that its influence on the other commodities is minimal. The total connectedness of the system is 27.05, which is relatively small compared to the same measure (which is 78.3) in Diebold and Yilmaz (2014) who consider the stock returns of U.S. financial companies. As a next step we seek to check these measures in a dynamic framework such that we can examine the

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1Robustness check shows that varying orders in the VAR models only has little impact on our results.
impacts of financial distress or other significant events on connectedness of commodity markets.

3.3.2 Dynamic (conditional, rolling-window) analysis of connectedness

We still use the same VAR(3) model and 12-day-ahead forecast error for generalized decomposition, but now estimate with a 252-day\(^2\) (all trading days in a year) rolling-window to model dynamics in the volatility connectedness. The total connectedness plot is showed in Figure 3 below. We can see dramatic increase in total connectedness during the 2007-2009 financial crisis but it has decreased since late 2009. The peak of total connectedness is in the week of Oct 26th, 2008 which is one month later than the Lehman Brothers’ bankruptcy on Sep 15th, 2008. It appears that during financial crisis stock markets have led commodity markets since Diebold and Yilmaz (2014, 2015) who study the stock returns of financial institutions in the U.S. and Europe find this dynamic connectedness has reached its peak right after the bankruptcy of Lehman Brothers. The most striking feature in the past two years is the significant decline in crude oil prices, but our plot shows that this decline does not imply increasing connectedness in all 20 commodity markets in the sample.

![Figure 3: Dynamic total connectedness with a 252-day rolling window and 12-day-ahead predictive horizon for variance decomposition.](image)

Recall that the static measure of connectedness in our full-sample unconditional analysis is only 27.05, which is apparently much smaller than the ranges of dynamic mea-

\(^2\)We have used 150 days, 200 days and 300 days as alternative rollwing-window and the results are still similar.
sures most of the time. This suggests that static analysis may underestimate connectedness and it is critical to take dynamics into consideration. Next we explore the dynamic measures of volatility connectedness at sector level, again due to the fact that we have 20 commodities and cannot present all 380 pairwise directional connectedness in a very concise way. We omit the diagonal-elements in the dynamic connectedness table to focus on cross-sector connectedness.

[INSERT Figure 4, Figure 5 & Figure 6 ABOUT HERE]

We can see from figures 4-6 that cross-sector volatility spillover have increased dramatically during the 2007-2009 financial crisis, but they returned to the pre-crisis level after. Recent downward price movements of energy markets have increased livestocks, softs and metals markets, but these effects are not as significant as those in the financial distress. These plots show that the directional pairwise volatility spillover are quite volatile even when the economy is expanding, posing challenge on the recent research that assert commodity markets are only highly integrated during the crisis period.

4 Concluding remarks

We characterize dependence structure and volatility spillover using a large sample of daily commodity futures in the past two decades. We show that correlations between various commodities have increased sharply during financial crisis, but they returned to the pre-crisis levels after 2011. We also find that volatility spillover has peaked during financial distress, but these spillovers seem to be much more volatile than the dynamic correlations. We also find that market connectedness based on volatility spillover has declined in the past three years, despite the fact that there is a dramatic downward movements in commodity prices trigger by energy markets. Our results have important implications for risk management and portfolio construction with commodity futures.

It may prove interesting to investigate and extend the models we use for other topics in the future. Kilian (2009) shows that depending on the driven demand or supply shocks oil price may have different impacts on macroeconomy and commodity markets using a structural VAR model. To deepen our understanding in volatility spillover driven by different mechanisms across financial cycles, we wonder if we may follow Kilian (2009) and employ the structural VAR model that helps explain the time-varying
nature of spillover within a macroeconomic framework. Secondly, since we propose to estimate comovements and volatility spillovers sequentially, one may ask if and how the empirical results of time-varying high dimensional DCC models and connectedness measures could be combined to produce useful insights for risk management. In what way can these models guide financial market participants in hedging and risk management? Can volatility spillover shed light on interpreting dynamics in comovements in the near future? Can we identify the sources of systemic risk and price them using multifactor asset pricing models? Last but not least, the practical value of connectedness measure mainly relies on rolling window estimation of VAR model, which is not necessarily robust to the choice of window length. Although we have verified robustness of dynamic VAR model results using various rolling-window in our analysis, it is appealing to use a data-driven approach to select the rolling-window such. Therefore, how to choose an optimal window length is another problem we could consider. A robust time-varying approach that allows for dynamics in the VAR model is the key to precise estimates of connectedness across financial cycles. We conclude by raising these questions and hope they can be extended to a wider range of future studies.

References


Figure 1: Dynamic conditional correlations of overall, within and cross-groups
Figure 2: Dynamic conditional correlations of cross-groups
Figure 4: Volatility spillover from energy and grains markets to other markets
Figure 5: Volatility spillover from softs and livestocks markets to other markets
Figure 6: Volatility spillover from metals to other markets