Valuing Natural Resources Allocated By Dynamic Lottery*

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Abstract

"Preference point" lotteries—under which the probability an individual is drawn increases with their stock of preference points earned over time by being unsuccessful in past drawings—are widely used to allocate access to many economically important natural resources (e.g., big game hunting opportunities). Lotteries form a natural choice experiment: by observing the opportunities for which an individual applies, the alternatives not chosen, the associated costs, the probability of winning a permit, etc., statistical inferences can be made about how individuals trade off site characteristics for cost. Knowledge of these trade-offs can then be used to estimate applicants' willingness to pay for site quality characteristics and site access. Two key features of recreationalists' choices under preference point lottery are (i) forward-looking behavior (since the odds of winning a permit depend on the accumulated stock of preference points) and (ii) equilibrium sorting (whereby individuals decide where to apply based on their expectations of others' choices and vice versa). We develop a novel revealed preference method for estimating individuals' willingness to pay for access to recreational opportunities allocated by preference point lottery that accounts for these two features. We apply our model to the case study of black bear hunting in Michigan. We estimate total willingness to pay for access to a small site to be nearly $150,000.

Keywords: dynamic discrete choice model; equilibrium sorting; lottery; preference points; revealed preference; travel cost

JEL Codes: C2, C5, D9, Q26, Q51

1 Introduction

Resource management agencies often use permits to regulate access to recreational opportunities (e.g., big game hunting or river rafting). These permits are typically priced at rates below those that would arise in the marketplace. This guarantees greater equity of access for low-income resource users, but also excess demand for permits. Hence, wildlife management agencies in many states and provinces throughout North America allocate permits via lotteries.

Many agencies rely on "dynamic lotteries" (e.g., "preference point" and "weighted" lotteries) to allocate permits.[1] These lotteries differ from a simple drawing where the probability an applicant wins a permit is simply the permit quota divided by the number of applicants. The probability an applicant is drawn in a dynamic lottery changes over time depending on whether the applicant was successful in past drawings. For example, preference point lotteries award points to unsuccessful applicants. Each year, applicants for permits to access a particular site are sorted by the number of preference points they possess. Permits are then allocated to those with the highest number of preference points until the permit quota is filled. The successful applicant's preference point balance is reset to zero upon being issued a permit. Each unsuccessful applicant earns a single point towards next year's application, thereby increasing their odds of future success. Weighted lotteries work similarly, except that each unsuccessful applicant is given an additional draw for the following year's lottery.

Lotteries provide an opportunity to estimate individuals' willingness to pay (WTP) for access to recreational opportunities and their characteristics. Indeed, lotteries form a natural choice experiment: by observing the opportunities for which an individual applies, the alternatives not chosen, the associated costs, the probability of winning a permit, etc., statistical inferences can be made about how individuals trade off site characteristics for cost.

[1] For example, dynamic lotteries are commonly used to allocate big game hunting permits, including those for black bear (Michigan), elk (Colorado, Michigan, Montana), deer (California, Montana), and even alligators (Louisiana). Dynamic lotteries are also used to allocate river noncommercial rafting permits in Grand Canyon National Park.

Knowledge of these trade-offs can then be used to estimate applicants' WTP (Holms and Adamowicz, 2003).

Several prior studies have estimated WTP for access to natural resources when permits are rationed by lottery. Most focus on the case of simple lotteries (e.g., Boxall 1995; Loomis 1982; Nickerson 1990; Scrogin and Berrens 2003; Scrogin et al. 2000), although Akabua et al. (1999) and Buschena et al. (2001) estimate WTP for site access under preference point lotteries. Yet, this prior work is of limited use for estimating WTP for dynamic lotteries because it does not account for (i) the intertemporal trade-off inherent in these lotteries and/or (ii) equilibrium sorting by applicants.

A defining feature of dynamic lotteries is the intertemporal trade-off between site access and site quality. Because an applicant's odds of success vary from year to year with her stock of preference points or extra draws, and because this stock is reset to zero upon being drawn for a permit, she has an incentive to manage her stock across time by optimally choosing where *and when* to apply for a permit (Buschena et al., 2001). Put differently, a trade-off exists between the expected utility from winning access to a lower quality site in the current period versus the discounted expected utility from winning access to a higher quality site in the future. Ignoring this trade-off can lead to biased WTP estimates whenever applicants place a positive value on future recreation opportunities.

Buschena et al. (2001) recognize the dynamic nature of the application decision and use a hedonic approach to estimate hunt values based on the cost of building one's preference point stock to the level required to win a permit. Specifically, the authors use a Poisson model to estimate the number of preference points required to win a permit for a specific site based on a vector of site quality characteristics. The marginal value of the permit is the amortized total cost of accumulating enough preference points to guarantee a license. Though novel, their approach is of limited use in counterfactual analysis because it does not capture the equilibrium sorting behavior of applicants under changes to site quality or access. These changes affect the present value of expected indirect utility from applying for a given permit, which also depends endogenously on the share of applicants that apply for the permit (and thus the probability of success) in Nash equilibrium. Changes to expected utility will affect WTP, so a fully generalizable method for estimating WTP must account for equilibrium sorting.

Intertemporal trade-offs and equilibrium sorting have each been treated separately in prior site choice studies. Provencher and Bishop (1997) and Hicks and Schnier (2006) use Rust's "nested fixed point" (NFXP) algorithm to estimate a site choice model for fishing trips. This approach nests a contraction mapping within a maximum likelihood routine; the contraction mapping calculates the maximized present value net benefits from fishing over time (thereby accounting for the intertemporal trade-off), and the maximum likelihood routine estimates the objective function parameters, given the results from the contraction mapping nest. Congestion plays no role in these analyses, in contrast to our model of application choice under a dynamic lottery. Timmins and Murdock (2007) use Berry's (1994) equilibrium sorting model to estimate a site choice model of recreational fishing where congestion (caused by large numbers of other fishermen visiting the same site) reduces fisherman utility directly via a linear disutility term. Berry's approach also uses a nested algorithm in which the contraction mapping nest solves for an objective function parameter that ensures the predicted share of individuals making a given choice matches the observed share (thereby accounting for equilibrium sorting). The maximum likelihood routine then estimates the remaining objective function parameters. In contrast with our problem, the choice of where to go fishing is static in Timmins and Murdock's model, and so their approach does not account for intertemporal trade-offs.

We develop a model of application choice under a dynamic lottery by marrying Rust's (1987) NFXP and Berry's (1994) equilibrium sorting model into a single, three-nest algorithm that simultaneously accounts for both the intertemporal trade-off and equilibrium sorting by applicants. The inner nest utilizes a contraction mapping to calculate the maximized present value net benefits from managing one's stock of preference points over time, following Rust (1987). The second nest uses the results from the first to solve for a utility function parameter that ensures the predicted share of applicants for a given hunt matches the observed share, following Berry (1994). The final nest uses maximum likelihood to estimate the remaining utility function parameters. For concreteness, we derive our model in the context of black bear hunting in Michigan, permits for which are allocated via preference point lottery, although the method is generalizable to other contexts.[2]

We begin our analysis by providing background on black bear hunting in Michigan; this background provides context for our modeling approach and estimation procedure, discussed in the following section. We then present estimation results, followed by a discussion and conclusion.

2 Black Bear Hunting in Michigan

The Michigan Department of Natural Resources (DNR) has allocated permits for black bear hunting via preference point lottery since 2000. Hunts take place in late summer through mid-autumn. The black bear range in Michigan is divided into twelve bear management units (BMUs) (Figure 1). The Lower Peninsula BMUs that are open to hunting each host a single hunt lasting one week in mid-September. Drummond Island, on the eastern tip of the Upper Peninsula, hosts a single hunt that lasts six weeks over September and October. The remaining Upper Peninsula BMUs each host three hunts over the course of the autumn. There are 22 total hunts each year (Table 1). Hunt quality characteristics vary across each hunt and BMU; Table 2 summarizes several key characteristics of each BMU, including population, total forest land open to hunting (the sum of private commercial forest and state forest and wildlife area acreage), the number of hunts each BMU hosts, the season duration, and mean success rate (i.e., the proportion of hunters who took a bear). The mean success rate reported in Table 2 is similar across BMUs, although these figures mask

[2] For example, similar algorithms have been used to estimate demand for durable and semi-durable goods like automobiles (Schiraldi, 2011), digital cameras (Carranza, 2010), and video game consoles (Nair, 2007).

considerable variation within a given BMU over the course of the season (Figure 2).

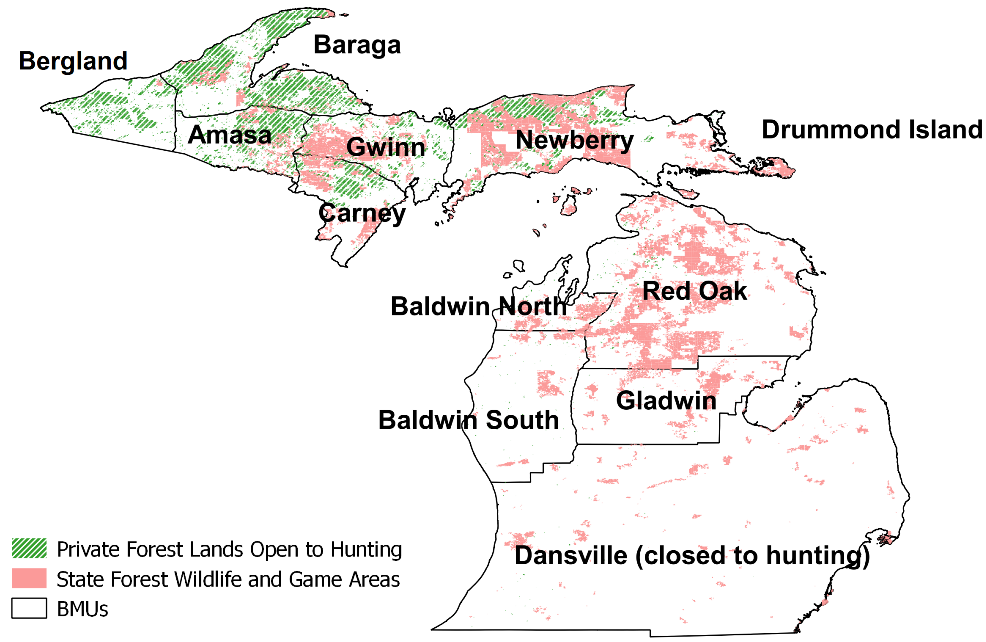

Figure 1: Michigan bear management units (source: DNR 2015a; 2015b)

Applicants pay an application fee and a license fee up front. The application fee is $4 for all applicants except Comprehensive Lifetime License holders, for whom the application fee is waived. License fees are $15 for Michigan residents and $150 for nonresidents. Unsuccessful applicants are refunded the license fee, but not the application fee. Applicants also have the option of applying for the "preference point-only" option. Those who take this option are automatically awarded a preference point for use in future drawings and pay only the $4 application fee, but cannot hunt in the current season. With the preference-point only option, an applicant can choose from 23 total hunt choices.[3]

The number of licenses available for each hunt, or the "permit quota," ranges from only

[3]The bear permit drawing in Michigan is actually divided into two rounds; applicants apply for a first and second choice of hunt before the drawing takes place. If an applicant is not drawn for his first choice, then he is entered in the drawing for his second choice if any permits for that hunt remain after the first round. We model only a single round here. Including a second round makes estimation infeasible due to large number of choice alternatives it implies. (The choice set balloons from 23 alternatives if considering the first round only to 485 if considering both rounds). However, fewer than half of applicants even entered a second choice on the 2009 application, and approximately 2 percent of those that did were awarded a permit. Hence, ignoring the second round is unlikely to have a significant effect on our results.

Table 1: 2009 Michigan Bear Hunting Seasons and Quotas (Source: DNR 2009)

Hunt	Bear management unit	Season	Quota	Applicants
1	Bergland	9/10–10/21	350	1221
2		9/15–10/26	605	693
3		9/25–10/26	625	474
4	Baraga	9/10–10/21	380	2070
5		9/15–10/26	690	1190
6		9/25–10/26	1270	1141
7	Amasa	9/10–10/21	135	1157
8		9/15–10/26	190	555
9		9/25–10/26	355	694
10	Carney	9/10–10/21	205	1256
11		9/15–10/26	435	603
12		9/25–10/26	540	408
13	Gwinn	9/10–10/21	250	1660
14		9/15–10/26	360	831
15		9/25–10/26	860	785
16	Newberry	9/10–10/21	400	4316
17		9/15–10/26	490	1753
18		9/25–10/26	1420	2055
19	Drummond Is.	9/10–10/21	3	227
20	Red Oak	9/18–9/26	1700	12427
		(10/2–10/8 archery)		
21	Baldwin	9/18–9/26 (all)	60	2684
		9/11–9/26 (north area)		
22	Gladwin	9/18–9/26	150	909

Table 2: Bear Management Unit Characteristics and Summary Statistics (Source: DNR 2009; 2015b)

BMU	Population	Total forest land open to hunting (ac)	Hunts yr^{-1}	Mean success rate	Total season duration (days)
Bergland	16,452	340,020	3	0.28	47
Amasa	23,636	500,823	3	0.43	47
Baraga	78,327	974,399	3	0.29	47
Gwinn	48,954	574,025	3	0.27	47
Carney	57,362	436,796	3	0.25	47
Newberry	60,591	1,333,215	3	0.31	47
Drummond Island	457	22,550	1	0.36	42
Red Oak	294,981	1,394,083	1	0.26	7
Baldwin	467,081	288,297	1	0.48	7
Gladwin	303,693	231,582	1	0.10	7
Mean	135,153.40	609,579	—	0.30	34.5
Standard deviation	160,231.18	468,865	—	0.10	19.04

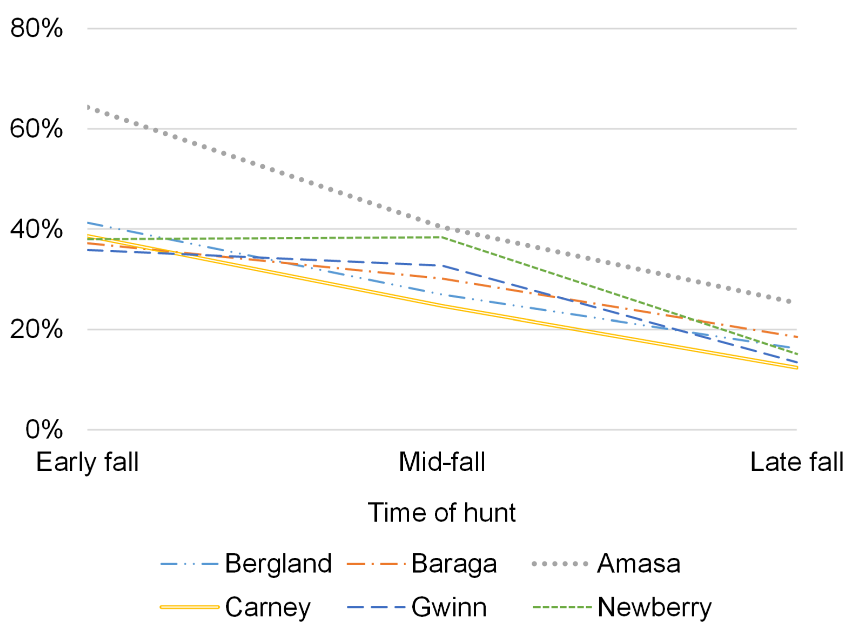

Figure 2: Hunt success rates by bear management unit and time of season

3 per year in the Drummond Island BMU to 1,700 for the Red Oak BMU (Table 1). For BMUs with multiple hunts, the quota increases for seasons that open later in the year, and application numbers tend to decrease for these later hunts. This is likely because bears are less active later in the autumn, and because bears become more easily "spooked," or more difficult to hunt, as the season wears on. The license quota for each hunt is made available on the DNR's website before the drawing. The DNR also publishes the previous year's drawing success rate for each hunt conditional one's stock of preference points each year before the drawing.

A total of 56,762 individuals participated in Michigan's preference point lottery for bear permits in 2009. Data describing each applicant—including hunt choices, addresses, and their preference point stock—were provided by the DNR. Figure 3a shows a histogram of applicants' preference point stocks; most applicants have fewer than three preference points, and only two applicants have ten—the maximum possible.

We estimate round-trip travel costs for each applicant using US Census data on median annual income for each applicant's ZIP code. We assume the opportunity cost of travel time to be one third of each applicant's imputed hourly wage. This opportunity cost was then multiplied by the travel time to the BMU, which was calculated using PC*Miler (ALK Technologies, Inc., 2015) assuming an average travel speed of 55 mi hr^{-1}.[4] To this cost were added the application fees and mileage costs calculated for a four-wheel drive truck (American Automobile Association, 2009). Figure 3b shows a histogram of applicants' round-trip travel costs to each BMU. The vast majority are less than \$750 trip^{-1}, although the distribution is skewed by out-of-state hunters whose costs total more than \$1,500 trip^{-1} in some cases.

[4] Our data are limited in that we do not have information on exactly where each applicant would hunt within a given BMU. We therefore calculate travel time based on the distance between the applicant's address and the centroid of the BMU. Many of the BMUs are very large (e.g., the Red Oak BMU in the Lower Peninsula; Figure 1), and hence there is likely to be considerable error in our estimates (Ji et al., 2016). That said, we discretize the travel cost data as part of our estimation procedure (described below). This should negate some (although likely not all) of this error.

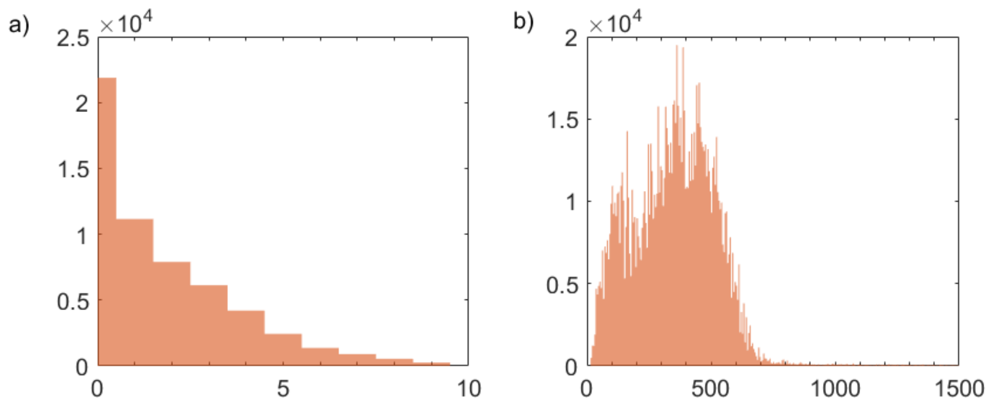

Figure 3: Histograms of applicants' a) preference point stocks and b) travel costs in dollars

3 Hunt Application Choice Under a Preference Point Lottery

We now develop a model of application choice for bear hunting permits in Michigan. Suppose the set of all bear hunters is $\mathcal{N}$. Suppose also that the maximum number of preference points is $\bar{p}$. We can then partition $\mathcal{N}$ as $\mathcal{N} = \{\mathcal{N}_1, ..., \mathcal{N}_{\bar{p}}\}$, where $\mathcal{N}_p$ is the set of applicants with p preference points. Let N_p and N be the cardinality of $\mathcal{N}_p$ and $\mathcal{N}$, respectively.

Each applicant makes an application choice $j \in \{1, \ldots, J\} = \mathcal{J}$, where $\mathcal{J}$ is the set of all application choices. Let $\sigma_j = [\sigma_{j0} \cdots \sigma_{j\bar{p}}]$ be a vector whose elements describe the observed share of hunters with $p = 0, ..., \bar{p}$ preference points that make application choice j.

The probability an applicant for hunt j wins a permit in year t with p preference points is

$$\begin{aligned}\phi_{jpt} =& 1\left(\sum_{p'=p}^{\bar{p}} N_{p'}\sigma_{jp'} \leq q_j\right) \\ &+ 1\left(\sum_{p'=p}^{\bar{p}} N_{p'}\sigma_{jp'} > q_j, \sum_{p'=p+1}^{\bar{p}} N_{p'}\sigma_{jp'} \leq q_j\right)\frac{q_j - \sum_{p'=p+1}^{\bar{p}} N_{p'}\sigma_{jp'}}{N_p\sigma_{jp}},\end{aligned} \tag{1}$$

where q_j is the permit quota for hunt j and $1(\cdot)$ is an indicator function that takes a value of one if the argument is true and zero otherwise. Consider an applicant with p preference points. If the number of applicants with p or more preference points, given by $\sum_{p'=p}^{\bar{p}} N_{p'}\sigma_{jp'}$, is less than the quota for hunt j, then the first right-hand side term in (1) evaluates to one and the second indicator function evaluates to zero, so that the applicant receives a hunting permit with certainty. If the number of applicants with p or more preference points is greater than the quota, but the number of applicants with more than p preference points is less than the quota (so that the second right-hand side indicator function evaluates to one and the first indicator function evaluates to zero), then there is a simple lottery for all permits that remain after the applicants with more than p points are issued permits. The probability of winning a permit is then $\left(q_j - \sum_{p'=p+1}^{\bar{p}} N_{p'}\sigma_{jp'}\right)/N_p\sigma_{jp}$. If neither of these conditions hold, then the probability the applicant wins a permit is zero.

Let the indirect expected utility applicant i receives from hunt j be

$$v_{ijpt}(\delta_{jpt}, \mu) = \delta_{jpt} + \phi_{jpt}\mu TC_{ij}, \tag{2}$$

where: $\delta_{jpt} = \phi_{jpt}X_j^{\mathrm{T}}\beta$ represents the baseline expected indirect utility from applying to site combination j; X_j is a vector of observable characteristics for site j (the superscript "T" denotes "transpose"), with elements x_{jk} representing individual site characteristic $k = 1, \ldots, K$; β is a vector of marginal utility parameters; μ is the marginal utility of income; and ϵ_{ijt} is an idiosyncratic, conditionally-independent error term (Rust, 1987). We make the dependence of v_{ijpt} on δ_{jpt} and μ explicit since these terms play an important role in our estimation procedure, described below.

Each applicant makes application choices to maximize the present value of his expected indirect utility from hunting over an infinite time horizon:

$$V_{ijpt} = v_{ijpt}(\delta_{jpt}, \mu) + \rho E\{R_{ip't+1}\}, \tag{3}$$

where ρ is an exogenous discount factor and $R_{ip't+1}$ is the "conditional value term," which measures the maximized value of i's future expected indirect utility from hunting, conditional on his choice in period t and his updated stock of preference points, p'. The expectation in (3) is taken with respect to p and ϵ_{ijt}. If ϵ_{ijt} is independent and identically distributed over time and p follows a Markovian transition process,[5] then we can assume "conditional independence" (i.e., the Markov transition density is independent of ϵ_{ijt}; Rust 1987), which facilitates taking expectations over future periods and is commonly employed in dynamic discrete choice modeling (Aguirregabiria and Mira, 2010). Specifically, we can rewrite (3) as

$$V_{ijpt} = v_{ijpt}(\delta_{jpt}, \mu) + \rho \sum_{p'} \Pi_{jpp't} R_{ip't+1}, \tag{4}$$

where $\Pi_{jpp't}$ is the probability the applicant's stock of preference points transitions from p to $p' = 0, \ldots, \bar{p}$ conditional on choice j. Note that

$$\Pi_{jpp't} = \begin{cases} \phi_{jpt} & \text{for } p' = 0 \\ 1 - \phi_{jpt} & \text{for } p' = p + 1 \\ 0 & \text{otherwise} \end{cases}$$

since an applicant can only (i) lose all her preference points (if she wins a permit) or (ii) earn one additional preference point (if she is not drawn).

For ease of exposition, we will continue with the notation employed in (3). Assuming stationarity allows us to drop the time subscript. We further assume ϵ_{ij} follows a type-1 extreme value distribution so that we can write the probability hunter i chooses application

[5]Indeed, an applicant's preference point stock next period depends only on his current stock and the decision he or she makes this period, suggesting p follows a Markovian transition process.

combination j as[6]

$$P_{ijp} = \frac{\exp\left(v_{ijp}(\delta_{jp}, \mu) + \rho E\{R_{ip'}\}\right)}{\sum_{j' \in \mathcal{J}} \exp\left(v_{ij'p}(\delta_{j'p}, \mu) + \rho E\{R_{ip'}\}\right)} = \frac{\exp\left(V_{ijp}\right)}{\sum_{j' \in \mathcal{J}} \exp\left(V_{ij'p}\right)}. \tag{5}$$

Likewise, our distributional assumption allows us to write

$$\begin{aligned} E\{R_{ip}\} &= E\left\{ \ln\left(\sum_{j \in \mathcal{J}} \exp\left(v_{ijp}(\delta_{jp}, \mu) + \rho E\{R_{ip'}\}\right)\right)\right\} + \gamma, \\ &= E\left\{ \ln\left(\sum_{j \in \mathcal{J}} \exp\left(V_{ijp}\right)\right)\right\} + \gamma \end{aligned} \tag{6}$$

where $\gamma \approx 0.5772$ is Euler's constant. We can then write the estimated share of applicants with p preference points that apply to hunt j as

$$\tilde{\sigma}_{jp} = \frac{1}{N_p} \sum_{i \in \mathcal{N}} P_{ijp}. \tag{7}$$

At Nash equilibrium, the estimated shares from (7) will equal the observed shares from the data: $\tilde{\sigma}_{jp} = \sigma_{jp} \; \forall j, p$.

Equations (5)–(7) comprise our model of application choice under a preference point lottery. We now derive an algorithm for estimating μ and β.

4 Estimation

Estimation of (5)–(7) proceeds by marrying Rust's (1987) NFXP algorithm—which can be used to calculate V_{ijp}—with Berry's (1994) model of equilibrium selection—which can be used to estimate the δ_{jp} that equate $\tilde{\sigma}_{jp}$ and σ_{jp} at Nash equilibrium. Specifically, we use

[6] Our model is essentially a dynamic extension of the canonical conditional logit model. Static versions of these models are limited by the "independence of irrelevant alternatives" (IIA) assumption, which imposes unrealistic cross elasticities of substitution between choices (Holms and Adamowicz, 2003); specifically, the relative probability of choosing any two choice alternatives is unaffected by the characteristics of other alternatives under IIA. Provencher and Bishop (1997) show that IIA does not hold for dynamic site choice models since the term $E\{R_{ip'}\}$ in (5) depends on the attributes of all other sites.

a two-stage estimation procedure. The first stage comprises an algorithm with three nests: an "outer" nest that uses maximum likelihood to estimate μ; a "middle" nest that uses a contraction mapping to calculate the δ_{jp} that imply the equilibrium σ_j; and an "inner" nest that uses a contraction mapping to calculate V_{ijp}. The second stage estimates the marginal utility parameters β using Timmins and Murdock's (2007) instrumental variables approach.

Our model requires some slight modifications to (5)–(7) in order to ensure convergence of the first-stage algorithm. We begin by discussing these modifications, and then describe each stage in detail.

4.1 Normalizing the "Preference Point-Only" Option

The middle nest in our estimation procedure utilizes a contraction mapping (see equation (11) below) to calculate the δ_{jp} that imply the equilibrium σ_j. For technical reasons (see Berry 1994), δ_{jp} must be normalized to some value for a particular j to achieve convergence. We therefore normalize the preference point-only option to zero. Assuming without loss that this corresponds to alternative $j = 1$, then $\tilde{\delta}_{1p} = \delta_{1p} - \delta_{1p} = 0 \; \forall p$. This normalizes the other δ_{jp} terms to $\tilde{\delta}_{jp} = \delta_{jp} - \delta_{1p}$, which can be interpreted as the baseline expected indirect utility from applying to site combination $j \neq 1$, *relative to the preference-point only option.*

The applicant's expected net present value from applying to hunt j under this normalization is

$$\tilde{V}_{ijp} = v_{ijp}(\tilde{\delta}_{jp}, \mu) + \rho \sum_{p'} \Pi_{jpp'} R_{ip'}. \tag{8}$$

The choice probability (5) under this normalization is then

$$\tilde{P}_{ijp} = \frac{\exp\left(\tilde{V}_{ijp}\right)}{\sum_{j'} \exp\left(\tilde{V}_{ij'p}\right)}, \tag{9}$$

where the We use the choice probability in (9) in lieu of (5) in our nested estimation procedure. We now describe this procedure, starting with the inner nest.

4.2 Stage 1, Inner Nest

The inner nest is used to calculate $\tilde{V}_{ijp}$. We initialize the model with a "guess" of μ^O, $\tilde{\delta}_{jp}^{OM}$, and $\tilde{V}_{ijp}^{OMI}$, where the superscripts O, M, and I denote the iteration of the outer, middle, and inner nests, respectively. We also calculate ϕ_{jp} from (1) using data describing hunt quotas and the observed shares of applicants that apply for each hunt with each preference point level.[7]

Given this initial guess and the probabilities, we can rewrite (8) as

$$\tilde{V}_{ijp}^{OMI} = v_{ijp}(\tilde{\delta}_{jp}^{OM}, \mu^O) + \rho \sum_{p'} \Pi_{jpp'} R_{ip'}^{OMI}, \tag{10}$$

where $R_{ip}^{OMI} = \ln\left(\sum_j \exp\left(\tilde{V}_{ijp}^{OMI}\right)\right) + \gamma$. Rust (1987) shows that (10) is a contraction mapping that can be solved using a fixed point algorithm for the unique vector $\left[\tilde{V}_{i1p}^{OM*} \ \cdots \ \tilde{V}_{iJp}^{OM*}\right] \forall i, p$.

4.3 Stage 1, Middle Nest

The middle nest is used to calculate the $\tilde{\delta}_{jp}$ that equates the observed shares with the estimated shares. We substitute $\tilde{V}_{ijp}^{OM*}$ and our initial guesses μ^O and δ_{jp}^{OM} into (9) to calculate the associated choice probabilities. From this, we can then predict the share of applicants with p preference points who choose hunt j from (7).

[7] Technically, ϕ_{jp} depends on σ_j, which we calculate in the middle nest *after* calculating the fixed point V_{ijp}^{OM*}. Substituting the observed σ_j here allows us to calculate V_{ijp}^{OM*} (which would be impossible otherwise), and in any case the estimated $\hat{\sigma}_j$ implies the observed ϕ_{jp} upon convergence of our algorithm.

Berry (1994) shows that the following contraction mapping can be used to calculate the $\tilde{\delta}_{jp}^{O*}$ that equate the predicted shares, $\tilde{\sigma}_{jp}^{O*}$, to the observed equilibrium shares, σ_{jp}:[8]

$$\tilde{\delta}_{jp}^{OM+1} = \tilde{\delta}_{jp}^{OM} + [\ln(\sigma_{jp}) - \ln(\tilde{\sigma}_{jp}^{OM})]. \tag{11}$$

Note that the inner nest will be rerun for every iteration M of the middle nest since $\tilde{\delta}_{jp}^{OM}$ affects the value of $\tilde{V}_{ijp}^{OMI}$. Denote the value of $\tilde{V}_{ijp}^{OMI}$ associated with $\tilde{\delta}_{jp}^{O*}$ as $\tilde{V}_{ijp}^{O**}$.

4.4 Stage 1, Outer Nest

The outer nest uses the calculated values of $\tilde{V}_{ijp}^{O**}$ and $\tilde{\delta}^{O*}$ along with the initial guess μ^O to estimate the marginal utility of income, μ, via maximum likelihood. The log-likelihood function for (9) is

$$L = \sum_i \sum_j \ln\big(1(d_i = j)\tilde{P}_{ijp}^{O*}\big), \tag{12}$$

where d_i represents the application chosen by applicant i. The values $\tilde{V}_{ijp}^{O**}$, $\tilde{\delta}_{jp}^{O*}$, and μ^O are substituted into (9), and an optimization routine updates μ^O until convergence, where L is maximized at μ^*, $\tilde{\delta}_{jp}^{**}$, and $\tilde{V}_{ijp}^{***}$. Note again that the middle nest must be rerun for each iteration of the outer nest.

The gradient for (12) is

$$\sum_i \sum_j 1(d_i = j)\left(\frac{\partial \tilde{V}_{ijp}^{O**}}{\partial \mu} - \sum_{j'} \frac{\partial \tilde{V}_{ij'p}^{O**}}{\partial \mu}\tilde{P}_{ij'p}^{O*}\right) \tag{13}$$

where, following Rust (1994), $\partial \tilde{V}_{ijp}^{O**}/\partial \mu = \left(1 - \frac{\partial \Psi_{ijp}}{\partial \tilde{V}_{ijp}^{O**}}\right)^{-1} \frac{\partial \Psi_{ijp}}{\partial \mu}$ and Ψ_{ijp} is the contraction mapping from (3) that satisfies $\tilde{V}_{ijp}^{O**} = \Psi_{ijp}\big(\tilde{V}_{ijp}^{O**}\big)$.

[8]The shares $\sigma_{jp} = 0$ for some j,p in our data. The contraction mapping (11) breaks down in this case since ln(0) is undefined. Timmins and Murdock (2007) suggest a "numerical patch," i.e., increasing the number of applications to each site by a small increment ψ (e.g., $\psi = 10^{-12}$). This allows (11) to be calculated, and the authors show that small values of ψ have a negligible effect on choice probabilities for sites that are actually chosen in the data.

4.5 Stage 2

The final stage of our estimation procedure recovers the remaining marginal utility parameters, β. Recall from (2) that $\delta_{jp} = \phi_{jp} X_j^T \beta$. After normalizing δ_{0p}, we can estimate the beta by regressing $\tilde{\delta}_{jp}^{**}$ on the site quality characteristics interacted with the success probabilities plus a constant, κ, to account for the arbitrary normalization:

$$\tilde{\delta}_{jp}^{**} = \phi_{jp} X_j^T \beta + \kappa + \epsilon_{jp}. \tag{14}$$

The error term in (14) is correlated with the $\phi_{jp} X_j$; to see this, note that any omitted factor that positively contributes to baseline expected utility would decrease ϕ_{jp}, all else equal.

Timmins and Murdock (2007) suggest an instrumental variables approach for avoiding this endogeneity problem. The approach is based on the logic underlying the standard assumptions underlying any revealed preferences exercise. Specifically, we assume the characteristics of sites other than j contribute to the expected utility from site j only by altering the success probability ϕ_{jp}; indeed, the hunter success rate, say, at BMU j will not affect the utility from hunting at site j' except by influencing the number of hunters who apply to site j and, hence, the probability of winning a permit at a given site. Hence, we can use the site quality characteristics of sites other than j to form an instrument for the $\phi_{jp} X_j$ terms. We follow Timmins and Murdock (2007) by first estimating β via (14) using OLS, ignoring the endogeneity. Denote the estimator as $\hat{\beta}$. Next, we use $\hat{\beta}$ to calculate σ, using only the exogenous explanatory variables:

$$\hat{\sigma}_{jp} = \frac{1}{N_p} \sum_{i \in \mathcal{N}_p} \frac{\exp\left(X_j^T \hat{\beta} + \mu^* TC_{ij}\right)}{\sum_{j'} \exp\left(X_{j'}^T \hat{\beta} + \mu^* TC_{ij'}\right)}. \tag{15}$$

We use the estimated $\hat{\sigma}_{jp}$ to calculate the success probability $\hat{\phi}_{jp}$ using (1). We then calculate instruments as the predicted values that result from regressing $\phi_{jp} x_{jk}$ on the vector $\hat{\phi}_{jp} X_j$;

denote these predicted values as $\widehat{\phi_{jp}X_j}$.[9] Finally, we regress δ_{jp}^{**} on $\widehat{\phi_{jp}X_j}$ using OLS to arrive at a consistent estimator of β.[10]

4.6 Issues in Estimation

The sheer size of our lottery application problem makes the estimation of (9) impractical without additional modifications. To see this, note that 56,762 hunters with eleven possible preference point stocks applied for one of 23 combinations of bear hunting permits in Michigan in 2009. Taken to the extreme, this means that, for every iteration of the middle nest, we would need to solve for $56,762 \times 23 \times 11 = 14,360,786$ fixed points V_{ijp}^{OM*} using our inner nest. Of course, our algorithm must rerun the middle nest for every iteration O of the maximum likelihood routine in the outer nest (which in turn reruns the inner nest at each iteration), so the computational resources needed to estimate μ can quickly become unreasonable.

We address this issue by discretizing applicants' travel costs. Specifically, applicants' travel costs are converted to discrete units of \$100 by dividing each TC_{ij} by 100 and rounding the result to the nearest whole number. Hence, each applicant is defined by (i) his or her stock of preference points, ranging from 0 to 10 and (ii) the costs of traveling to hunt in each site, ranging from 0 to $\overline{TC}_{ij}$.[11] This discretization reduces the number of unique applicants from 56,764 to 194, dramatically reducing the computational resources needed to solve the inner nest. In addition, each unique applicant's Bellman equation (3) is independent of other applicants' choices in the inner nest (since the success probabilities ϕ are held fixed at this

[9]We use $\widehat{\phi_{jp}X_j}$ as instruments instead of simply multiplying $\hat{\phi}_{jp}$ by the X_j terms since the latter is nonlinear in the endogenous variable ϕ_{jp}. Indeed, this latter approach is an example of Hausman's "forbidden regression" (Hausman, 2001).

[10]Timmins and Murdock (2007) use median regression in the second stage. This is because the numerical patch used to account for zero shares in the data (see footnote 8) results in arbitrarily small values of δ_{jp}^{**} for those sites. We drop these arbitrarily small δ_{jp}^{**}s and use OLS instead. Doing so generates some selection bias, but avoids other potential biases arising from the nonlinearity of the GMM estimator used in quantile regression.

[11]We let $\overline{TC}_{ij} = 5$ in our algorithm since the majority of applicants' round-trip travel costs to each site are less than \$500 in our dataset (see Figure 3b).

stage of the estimation procedure). Hence, the inner nest can be run in parallel, further reducing the wall time for estimation.

5 Results

We programmed our first-stage algorithm in MATLAB (Mathworks, Inc., 2015) and ran it in parallel using twenty processors at Michigan State University's High Performance Computing Center.[12]

Estimating μ^* requires knowledge of applicants' discount factor, ρ. This information is not available *a priori*, although Brookshire et al. (1983) finds evidence of discount factors for big game hunting opportunities ranging from $\rho \in [0.95, 0.99]$. We therefore run the first-stage algorithm over three values of ρ—0.95, 0.975, and 0.99—and compare the resulting models using the Akaike Information Criterion (AIC), following Hicks and Schnier (2006). The first-stage estimates are presented in Table 3. The algorithm failed to converge for $\rho = 0.95$. The estimated marginal utility of income is highly significant for the remaining models, and is of the expected sign. The estimate is an order of magnitude smaller than prior estimates for big-game hunting (although these studies examine different species; Boxall 1995; Knoche and Lupi 2007), although it is similar in magnitude to estimates found in studies of general recreation site choice (e.g., Boxall et al. 2001). The log-likelihood for the model with $\rho = 0.975$ is slightly greater—and the AIC is slightly smaller—relative to the estimate with $\rho = 0.99$. We therefore proceed by assuming $\rho = 0.975$ so that $\mu^* = -0.0027$.

The remaining marginal utility parameters, β, are recovered using the instrumental variables procedure outlined in Section 4.5. We hypothesize that baseline expected utility from hunt j depends on the season duration, the time of hunt (i.e., whether the hunt takes place in early-, mid-, or late-fall), the forest area open to hunting in each BMU, average past hunter success rates (i.e., the mean proportion of hunters who successfully take a bear), and the

[12]Estimation required approximately 90 minutes for convergence using MATLAB's trust-region-reflective constrained optimization algorithm. The interior point algorithm would not successfully converge due to the flat shape of the log-likelihood function near the optimum.

Table 3: Stage 1 Estimation Results

Discount factor, ρ	Marginal utility of income, μ^* (Std. error)[b]	Log-likelihood	AIC
0.95[a]	——	——	——
0.975	-0.002747^{***} (0.0000)	–128,641	257,284
0.99	-0.002502^{***} (0.0000)	–128,833	257,664

[a] The algorithm failed to converge for $\rho = 0.95$.
[b] *** denotes estimates are significant at the 1 percent level or smaller.

population of residents living in the BMU. A constant term is included to account for the normalization used in Stage 1. The estimated marginal utility parameters associated with each of these variables are presented in Table 4.

Table 4: Stage 2 Estimation Results

Explanatory variable	Coefficient[a]	t[b]
Season duration (days)	-0.490^{***}	–10.88
Early-fall hunt = 1	5.583^{**}	2.27
Mid-fall hunt = 1	2.962^{*}	1.74
Forest land open to hunting (acres)	$3.63\text{E}{-6}^{**}$	2.50
Hunter success rate	11.23^{*}	1.76
BMU population (individuals)	$-3.5\text{E}{-5}^{***}$	–4.86
Constant	-3.60^{***}	–3.10
Observations	186	
$F_{6,21}$	36.78	
Prob. > F	0.000	
R^2	0.25	

[a] *** and ** denote the estimate is significant at the 1 and 5 percent level, respectively.
[b] Standard errors are clustered by BMU.

Our results indicate that baseline hunt utility is higher for earlier hunts. This is in line with our hypothesis that bears become less active and more skittish later in the year (see Figure 2). Baseline hunt utility also increases with the number of forest acres open to hunting. This is unsurprising since larger BMUs offer a greater diversity of locations in which to hunt. Hunter success rate also has a positive effect on baseline utility; this relationship is significant both statistically and—especially—economically. In contrast, utility decreases with the population residing in the BMU. This suggests hunters value more remote hunting opportunities. Baseline expected utility also decreases with the season duration. The explanatory variables are jointly highly significant and explain approximately 25 percent of the variation in the calculated δ_{jp}^{**}.

6 WTP for Changes to Hunt Access and Quality

An advantage to estimating a structural model of lottery choice is the ability to perform counterfactual analysis (Arcidiacono and Ellickson, 2011), wherein the estimated marginal utility parameters β^*, ξ_j^*, and μ^* can be used to estimate applicants' WTP for hunting opportunities and site characteristics. Consider first the welfare changes that occur if the DNR closes a BMU to hunting permanently and without warning.

Prior to the closure, each applicant's expected present value of indirect utility from applying is V_{ijp}^{***}, calculated in the first stage or our estimation procedure. Our distributional assumptions allow us to calculate a money metric value of the expected present value from bear hunting using the familiar log-sum expression,

$$WTP_i^0 = \frac{\ln\left(\sum_{j\in\mathcal{J}} \exp(V_{ijp})\right)}{|\mu^*|}. \tag{16}$$

After the closure, all choice alternatives that contain the closed BMU will be removed from the applicants' choice set. Closing a site has two effects on applicants' indirect expected present value of utility. First, site closure reduces utility directly by reducing the number of choice alternatives available to the applicant (i.e., by reducing the summation term in (6)). Second, site closure may change utility indirectly by causing applicants to change their application choices, reflected by σ_j, leading to new success probabilities that arise endogenously via equilibrium sorting.

Timmins and Murdock (2007) suggest using yet another contraction mapping to calculate the equilibrium shares under the restricted choice set. Specifically, they show that the function describing the share of individuals that choose site j (which is analogous to equation (7) in our model) is a contraction mapping under certain regularity conditions. This contraction mapping can be solved in a manner similar to (11) in the middle nest of our estimation procedure to yield the updated equilibrium shares.

Unfortunately, (7) does not satisfy the necessary conditions for a contraction mapping, and so we require an alternative means for calculating the updated σ_j.[13] In the Appendix, we propose an alternative approach for estimating σ_j under the restricted choice set using a global optimization technique known as a genetic algorithm (GA). GAs are computationally-intensive and hence are extremely time consuming; we will conduct more rigorous counterfactual policy analyses in future research.

It is still possible to roughly estimate the value of a BMU assuming the σ_j and, hence, the ϕ_{jp} do not change *ex post*. This may occur, for instance, if a hunt with relatively few applicants, each of whom had a relatively few preference points, were to close. As an example, consider the change in value from closing final hunt in the Carney BMU (hunt 12, Table 1), for which only 408 applicants applied in 2009. The vast majority of these applicants had one or fewer preference points (Figure 4), and hence closing the site (thereby forcing these applicants to apply elsewhere) is likely to have a negligible effect on the probability of being drawn for other sites. Removing the V_{ijp} associated with the final Carney hunt from (16) results in the restricted choice set $\mathcal{J}'$; the money metric value of the expected present value from bear hunting under the restricted choice set is then

$$WTP_i^1 = \frac{\ln\left(\sum_{j\in\mathcal{J}'}\exp(V_{ijp})\right)}{|\mu^*|}. \tag{17}$$

Subtracting WTP_i^1 from WTP_i^0 yields individual i's WTP access to the third Carney hunt.

[13] Specifically, (15) is not Lipschitz continuous (Nadler, Jr., 1969). To see this, note that σ_{jp} in (7) depends on ϕ_{jp} $\forall p$, which in turn depends on σ_j in (1). Small changes to σ_j can cause discrete "jumps" in the probability that an applicant wins a permit under a preference point lottery, and so (7) is not continuous.

The mean WTP is only $2.64 per applicant; aggregated over the entire population of applicants, the total WTP for access to Drummond Island is $149,185.

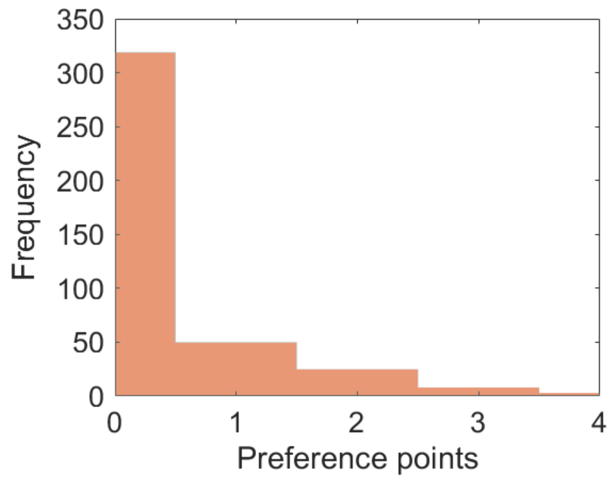

Figure 4: Preference point stocks of applicants for the third hunt in the Carney BMU, 2009

An analogous procedure can be used to elicit values from small changes in site quality characteristics; a full explanation is omitted for conciseness.

7 Conclusion

Preference point lotteries are an important mechanism for allocating access rights to natural resources. These lotteries also serve as a natural choice experiment that can be used to value access to hunting sites and their characteristics. Being able to estimate these values is of critical importance for effective resource management.

We develop a novel method for valuing access to natural resources that are governed by preference point lotteries. Our estimation procedure accounts for the intertemporal trade-offs and equilibrium sorting behavior inherent in these lottery types. As a result, our approach improves on those used in prior studies which either (i) assume myopic decision-making or (ii) ignore the endogenous equilibrium sorting behavior of applicants that arises from changes to site access and site quality characteristics.

Our approach does suffer some key limitations. Our ability to perform counterfactual policy analysis is hampered by the discrete nature of the success probabilities (1). Accurately valuing changes to site quality characteristics or site access requires computationally-intensive approaches which are both expensive and time-consuming. On-going work is focused on simplifying the process of simulating the process of equilibrium sorting to enhance our potential for performing counterfactual policy analysis. Furthermore, the need to discretize travel costs means that the WTP measures estimated here are approximations; future studies that employ our method must balance computational tractability with numerical precision in order to use the method most effectively.

Appendix: Counterfactual Policy Analysis with a Genetic Algorithm

Genetic algorithms (GAs) can be used to calculate the updated equilibrium application shares, σ_j, after a site closure or changes to site quality characteristics. GAs are a global optimization technique that are increasingly used to optimize highly nonlinear or discontinuous problems in applied economics (e.g., Dietz and Asheim 2012; Rabotyagov et al. 2009, 2014; Reeling and Gramig 2012; Richards et al. 2014). GAs mimic the biological process of evolution: an initial "population" of candidate solutions is ranked according to their "fitness," or how well they satisfy an objective function. The fittest solutions are then combined, and new "child" solutions are formed. "Mutations," or random changes to the child solutions, introduce variety to the candidate solutions. Over enough "generations," or iterations, the GA converges to the global optimum.

We can solve for the updated equilibrium shares by using the GA to minimize

$$\Delta = \max(|\sigma_{jp}^0 - \hat{\sigma}_{jp}|) \ \forall j, p, \tag{18}$$

where σ_{jp}^0 is a "guess" about the updated equilibrium share and $\hat{\sigma}_{jp}$ is the calculated equilibrium share implied by σ_{jp}^0, defined as in (7). A sufficiently small value of Δ implies convergence to the new equilibrium under the restricted choice set. Given the updated σ_j, we can use (6) to calculate the expected present value of indirect utility under the new equilibrium. WTP can then be estimated using (17), as above.

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