To Frack or Not to Frack: Option Value Analysis on the U.S. Natural Gas Market

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Abstract

Advances in horizontal drilling, 3-D seismic imaging, and hydraulic fracturing made it highly profitable for firms to produce large quantities of shale gas, making the natural gas boom possible. The number of firms entering the natural gas market from 2000 to present increased tenfold. There are three main testable explanations behind this jump in firm entry: reduced production costs from higher levels of recoverable reserves, lower risk of entry from less natural gas reserve uncertainty, and reduced sunk costs of entry lowering the risk of entry. We develop a real options model exploring the market entry decision of a firm when natural gas prices and reserves are uncertain to examine which of these three explanations impacts the market entry decision the most. The model shows that expected reductions in average costs of production and sunk costs of entry push the firm to optimally enter the natural gas market from 2000 to 2008 when firms were flooding the market. Surprisingly, expectations of lower natural gas reserve uncertainty do not have an influence on the entry decisions made by firms.

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1 Introduction

U.S. natural gas gross withdrawals reached a new high in 2013 at 82 billion cubic feet per day. Shale gas wells became the largest source of total natural gas production, representing 40 percent of total natural gas production (Tran, 2014). Shale gas development exploded on the natural gas scene mostly due to technological advances in horizontal drilling, 3-D seismic imaging, and hydraulic fracturing (fracking). It is now highly profitable for firms to produce large quantities of shale gas, making the natural gas (hydraulic fracturing) boom possible (Wang and Krupnick, 2013). Whether the jump in natural gas gross withdrawals was due to existing firms expanding production or from new firms entering the market is still up for debate.

Wang and Xue (2014) provide evidence of the latter, new firms entering the natural gas market, using data from DrillingInfo.[1] There were 15 active firms drilling natural gas from shale plays in 2000. That number grew to a peak in 2008 with 244 active firms and gradually decreased from 2008 to 2012, leaving the number of active firms in 2012 at 149. Wang and Xue (2014) offer one reason for the boom in firm entry into the natural gas market: the marriage of horizontal drilling, 3-D seismic imaging, and hydraulic fracturing made the cost of production lower, because the level of recoverable reserves steadily increased from these technological advances. We believe there are two other explanations. The number of firms entering the market increased with the boom due to a lower risk of entry into the market. The risk reduction was caused by either lower uncertainty in natural gas reserves or lower sunk costs of entry. Uncertainty surrounding reserves shrank due to 3-D seismic imaging, which provided a better

[1] DrillingInfo is a market research firm that focuses on the oil and gas industry and whose data are often used in trade publications. Wang and Xue (2014) supplement this data with two other sources: Louisiana Department of Natural Resources and Pennsylvania Department of Environmental Protection, which are government agencies that publish shale gas drilling data for their states.

picture of the structure and properties of subsurface rocks than the earilier 2-D method and greatly improved the ability to locate new hydrocarbon deposits (Wang and Krupnick, 2013). Sunk costs were cut, since natural gas producers started outsourcing the drilling and fracking of a well to specialized oil and gas service companies.

Unlike the popular application of real options for optimal production decisions (Kellogg, 2014), a focus on optimal entry into an industry offers two insights to the literature that have been previously missing. The first of which concerns how uncertainty influences market structure for an industry. Do technological advances that reduce the volatility in resource reserves spark firm entry into a market, or does it simply urge existing firms to produce more of the resource? Coupled with exploring the effects of uncertainty in a market is the set of policy implications that go hand-in-hand with determining the forces that foster market entry for a firm. If it is technological advances that sparked the natural gas boom and firm entry into the market, then the government has a possible role to play with policies that subsidize such research. To determine how each of the above explanations accounts for the extraordinary number of firms entering the natural gas market during the boom, we employ option value techniques in a real options entry model. This method is well-equipped to handle both the natural gas reserve uncertainty and the sunk costs of entering the natural gas market. Wellhead price volatility also exists, supplying a second source of uncertainty for firms to consider before deciding to enter the natural gas market.

Our investigation of natural gas market entry and exit forces reveal three key findings. First, we find that the expectation of reducing the average costs of production through increased levels of the long-run mean level of natural gas proved reserves per well is the most likely explanation behind the boom in firms entering the natural gas market. Second, the natural gas boom that caused such large growth in firm entry into the market could have been caused by expectations about lowering the risk of market entry by decreased sunk costs of entry. However, the magnitude in which sunk costs would have had to have been cut leads this explanation to be less feasible. Lastly, reducing the risk of market entry via lower reserve volatility does not impact the decision to enter the natural gas market. Further and contrary to the literature, eliminating the natural gas proved reserves per well uncertainty has the same impact on the entry threshold as an identical removal of natural gas wellhead price uncertainty. The recent natural gas boom in the U.S. displays the type of behavior explained by reductions in average costs of production. Once hydraulic fracturing and horizontal drilling became cost efficient methods of retrieving previously unrecoverable reserves and 3-D seismic technology more accurately identified the amount of reserves, the level of natural gas proved reserves increased greatly - pushing the average costs of production down and explaining the jump in new firm entry into the market.

A review of the literature is presented in the next section. The model is explained in Section 3, while Section 4 describes the data and parameter estimation. Section 5 displays the results, and Section 6 concludes.

2 Literature Review

Real options analysis adopts option value techniques from finance to examine capital budgeting decisions affecting real investments. Real options are similar to options on financial securities in that they are a right, not an obligation, to acquire an asset for an alternative price than the current one (Trigeorgis, 1996). Most investment decisions involve some form of irreversibility, whether it is partial or complete. Typically, the initial cost of investment is sunk; the firm cannot recover it all should they change their mind. Further, there is uncertainty over the future benefits associated with an investment, but the firm does have some flexibility about the timing of its investment. The real options methodology is well suited for analyzing investment decisions that include some level of irreversibility and uncertainty in future payoffs (Dixit and Pindyck, 1994). It allows one to determine the value of flexibility in the timing of investments, which is not accounted for in traditional discounted cash flow calculations.

The real options literature contains many applications in environmental economics and natural resource economics.[2] These papers cover a wide range, including real options applications to climate change policy, renewable resources, and nonrenewable resources. For example, Conrad (1997) develops an option value model that determines the optimal timing and expected value of policies to slow global warming. Similarly, Pindyck (1998, 2000, 2002) use real options analysis to determine how uncertain future evolution of greenhouse gas (GHG) concentrations and uncertain benefits of reduced GHGs affect the level of GHGs that will trigger investment in emission reductions. Pindyck (2007) provides a thorough review of the literature on real options analysis of environmental policy with respect to climate change. Moving to renewable resources, the real options literature consists of papers exploring the time to cut a stand of trees (Clarke and Reed, 1989; Reed and Clarke, 1990; Insley, 2002) all the way to the value of an option to exploit a fishery (Li, 1998; Nostbakken, 2006).

Most of the fascination in using real options deals with the investment in production decisions. One of the more recent examples is that of Kellogg (2014). He estimates the response of investment to changes in uncertainty using data on oil drilling in Texas, rig rental data, and the expected volatility of the future

[2]We adopt the view of Mezey and Conrad (2010), categorizing environmental economics as a separate subfield of natural resource economics. Their paper provides an extensive overview of the applications of real options analysis in resource economics, including environmental economics.

price of oil. There are several problems with this (previous) approach to determining the influence of uncertainty and irreversibility on investment decisions for an industry. First and foremost, focusing on the production decision of a firm does not account for increases in the number of firms entering an industry. Second, papers utilizing real options to determine optimal production decisions have used rig rental rates as an indicator for booms and busts in an industry. However, in March of 2015, the U.S. Energy Information Administration announced that the total U.S. rig count as of March 6 was decreasing while production of both oil and natural gas were increasing (U.S. Energy Information Administration, 2015b). Lastly, hydraulic fracturing technology and horizontal drilling minimize sunk costs tied up in the drilling decision, which means that uncertainty plays a smaller role than before. This is not true for firm entry. For these reasons, we use real options to model entry decisions of firms into an industry instead of applying option value techniques to the popular optimal production decisions.

We are more focused on the application of real options analysis to nonrenewable resources. In one of the seminal papers applying real options to natural resource investments, Brennan and Schwartz (1985) construct a general model of the decision to open, close, or mothball a mine producing a natural resource, copper, whose price is nonstationary stochastic.[3] Dixit (1989) uses the same model of investment as Brennan and Schwartz (1985), but he simplifies the setting, which brings out the generality of the idea and the variety of applications for entry and exit decisions made by a firm. Cortazar and Casassus (1998) extend the Brennan and Schwartz model by assuming a stationary stochastic price process and determining the option to expand copper mine production, instead of the option to delay production.

[3] A project is mothballed when it is put into a state of temporary suspension, allowing it to be reactivated in the future at a sunk cost much less than the original upfront sunk costs.

More recently, Insley (2013) creates a model of a profit maximizing firm that has the option to exploit a non-renewable resource by choosing the timing and pace of development. The resource price is modeled as a regime switching process, which is calibrated to oil futures prices. She applies the model to a problem of optimal investment in a typical oil sands *in situ* operation and determines critical levels of oil prices that would motivate a firm to make a large scale investment, as well to operate, mothball, or abandon the facility.

The previous papers have only considered one stochastic variable, specifically prices of the exhaustible resource. As noted by Slade (2001), fluctuating costs and reserve estimates are largely neglected in the real options literature, most likely because data on those variables are more difficult to obtain. However, she develops a real options model for the decision to temporarily suspend and resume production in Canadian copper mines, where she assumes that not only are spot prices stochastic, but so are production costs and the quantity of remaining reserves.

Our paper deviates from Slade (2001) in that we do not directly model production costs as being stochastic. Instead, we follow Pindyck (1980). He models average production costs as a function of uncertain reserves, thus production costs reflect the endogenous fluctuations of reserves.[4] Further, we explore the effects of uncertain reserves and prices on the entry and exit decision of a firm for the natural gas market. To the best of these authors' knowledge, this is the first real options paper to model reserve uncertainty through average production costs and apply the model directly to the natural gas market. Previous papers have ignored reserve uncertainty, except Slade (2001), and focused on

[4]Pindyck (1980) does not apply real options analysis, but uses stochastic dynamic optimization to determine the optimal rate of extraction of an exhaustible resource when demand and reserves are stochastic. He also assumes that the stochastic processes driving demand and reserves are independent. As explained by Slade (2001), fluctuations in reserves could be entirely due to endogenous responses to price uncertainty. We directly measure the correlation between the price and reserve processes and account for this in our model.

applications to nonrenewable resources like copper (references above) and oil (Cortazar and Schwartz, 1997; Conrad and Kotani, 2005).

Aside from Slade (2001), there is one other paper that closely aligns with our objective. Almansour and Insley (2013) studies the optimal management of a non-renewable resource extraction project when input and output prices follow correlated stochastic processes. Their analysis is applied to an oil sands project when it is currently operating or mothballed. Our model is similar to this one by virtue of modeling reserve uncertainty through its relationship with average production costs. Inherently, this paper has both stochastic output and input prices, but we are accounting for reserve uncertainty and Almansour and Insley (2013) are only focused on adding input uncertainty to output uncertainty, ignoring stochastic reserves altogether. We are more interested in the effect of natural gas price uncertainty and reserve uncertainty on the entry decision of a firm in so much that it explains the reasoning behind the large number of firms entering the natural gas market in the 2000s.

3 The Model

To enter the natural gas market, a firm must acquire the mineral rights for proposed units of land by hiring landmen to negotiate terms with landowners and investigate the mineral interests. The firm must also obtain various permits for drilling for natural gas before entering the market, along with constructing the actual sites where drilling will occur. These are considered sunk costs of entering the natural gas market and act as a barrier. Once the firm overcomes these barriers, the proved natural gas reserves $R(t)$ for the initial well are known, but future levels of reserves for other wells are uncertain. As drilling begins, the firm receives a wellhead price $P(t)$ for the natural gas that is drilled. The wellhead price for the current period is known, but future prices are unknown to the firm. Based on expectations of future prices and reserves, the firm can enter the natural gas market at a sunk cost K at some optimal point in time t_1 and receive a flow of profits. If natural gas prices or reserves per well turn out to be lower than expected, the firm can exit the market at $T_1 > t_1$, which eliminates the flow of benefits and recoups c percent of the sunk costs of entry. If natural gas prices or proved reserves per well are larger at some future time, the firm can enter the market at $t_2 > T_1$, incurring a sunk cost of entry.

The decision problem is presented in terms of a risk-neutral firm whose objective is to determine if and when to enter the natural gas market. This is a problem of optimally switching between entering and exiting the natural gas market. The firm must choose when to move from being out of the market to being in the market by choosing a sequence of times $t_1, t_2, ...$, where the firm enters the market, incurring K and gaining a flow of profits. The firm must also determine when to move from being in the market to out of the market by choosing a sequence of times $T_1, T_2, ...$ at which the flow of profits are relinquished and c percent of K are recouped. Entering and exiting the market in this threshold control setting is made with the knowledge that all future adjustments will be optimal. Using traditional cost-benefit analysis, the firm would enter the market when the expected net present value of profits equals or exceeds the sunk costs associated with hydraulic fracturing. However, if the firm views this decision as partially irreversible, there is an incentive (an option value) to delay entry longer than suggested by cost-benefit analysis in order to further observe how profitable fracking a natural gas well will be.

Produced natural gas is instantaneously sold at price $P(t)$. Wellhead prices evolve randomly around a long-run mean following a geometric mean-reverting (MR) process $\mathrm{d}P = r_1(\bar{P} - P)P\mathrm{d}t + \sigma_1 P\mathrm{d}z_1$. Here, r_1 is the rate of reversion to the mean price level. $\bar{P}$ is the long-run mean price level, and σ_1 is the standard deviation rate. $\mathrm{d}z_1 = \epsilon(t)\sqrt{\mathrm{d}t}$ is the increment of the standard Weiner process, with $\epsilon(t)$ being a standard normal variate. The uncertainty encompassing wellhead prices concerns future prices that are due to fluctuations in the market. The uncertainty is not idiosyncratic to the firm. Pindyck (1999) determines that energy prices, including natural gas, are mean-reverting by testing a century's worth of data. Making natural gas wellhead prices geometric MR prevents any negative value draws.

Similarly, natural gas proved reserve uncertainty is incorporated by modeling reserves per well as geometric MR, $\mathrm{d}R = r_2(\bar{R} - R)R\mathrm{d}t + \sigma_2 R\mathrm{d}z_2$. Allowing the stock of reserves held by the firm to revert back to a long-run mean assumes that exploration, research and development, and technology advances permit discovery and recoverability of natural gas that matches the amount of reserve withdrawals over time. Reserves, here, are not simply the reserves associated with one wellhead. Firms are continuously acquiring new leases, which allows them to replenish what they extract. This means that reserve uncertainty is picking up on the fact that firms do not know what the next play holds reserve-wise. Geometric MR is more in line with a firm's decision to enter the natural gas market than say a declining geometric Brownian motion (GBM) reserve process, which would fit a firm's decision of extraction rate for one well.[5] Real options results critically depend on choosing the correct stochastic process; therefore, a unit root test is used to check the geometric MR assumptions for natural gas wellhead prices. We reject the null hypothesis that the price process follows geometric Brownian motion. This can be found in the Appendix. We cannot complete a unit root test on proved reserves per well due to a lack of data. However, we plot out the set of data points in the Appendix, Figure 7, and determine that they do not appear to follow geometric Brownian motion.

[5]As a robustness check, we model natural gas proved reserves per well as a declining GBM. Switching from geometric MR to declining GBM does not significantly change the results.

Dixit and Pindyck (1994) show that the decision to enter or exit the market is made based on a comparison of the optimal value function that arises when continuing with the status quo, denoted V^C, and the present value of profits when the firm enters the market, denoted V^I, minus the sunk costs associated with hydraulic fracturing K. Comparing V^C and V^I leads to a first-order condition or transversality condition for the objective function. Specifically, V^C is simply the option value, since there is no flow of profits when the firm is not in the market. This option value represents the value of delaying entry into the market to gain more information about the profitability of utilizing hydraulic fracturing technology to drill for natural gas. V^I is the expected net present value of profits plus the option value.[6] The decision rule is simply to enter the market if $V^I - K \geq V^C$.

P and R describe the current state of the market and, thus, determine the relative values of V^I and V^C. This implies the existence of an endogenous threshold level $P^* = P(R) > 0$ for which entering the market is optimal. Since a portion of the sunk costs of entering the natural gas market can be recouped by exiting the market at some future time, the problem is one of optimal switching. The exit threshold is defined by P which is a function of R - analogous to the entry threshold.

Brekke and Øksendal (1994) establish that the optimal switching problem can be rewritten as a set of variational inequalities. Before entering the market, the continuation value function and the enter threshold curve $P^*(R)$ satisfy the following Bellman equation

[6] When the firm enters the market, this option value is terminated, making it an additional opportunity cost of entering. It is this opportunity cost that causes a more cautious response by the firm in the face of uncertainty.

$$\delta V^C \leq r_1(\bar{P} - P)P\frac{\partial V^C}{\partial P} + r_2(\bar{R} - R)R\frac{\partial V^C}{\partial R} + \\ \frac{1}{2}\sigma_1^2 P^2 \frac{\partial^2 V^C}{\partial P^2} + \frac{1}{2}\sigma_2^2 R^2 \frac{\partial^2 V^C}{\partial R^2} + \sigma_1\sigma_2\rho P R \frac{\partial^2 V^C}{\partial P \partial R} \quad (1)$$

and value matching condition

$$V^C[P(t), R(t)] \leq V^I[P(t), R(t)] - K \quad (2)$$

where δ is the discount rate, and $\rho = corr(\mathrm{d}z_1, \mathrm{d}z_2)$ is the correlation coefficient between the two stochastic processes, $P(t)$ and $R(t)$. In financial terms, the firm faces an obligation to the flow of pre-entry benefits. The obligation is treated as an asset whose value V^C must be optimally managed (i.e., maximized). The left-hand side of (1) is the return the firm would require to delay entering the market over the time interval dt. The right-hand side of (1) is the expected return from delaying market entry over the interval dt. The Bellman equation acts as an equilibrium condition ensuring a willingness to delay prior to market entry. The value matching condition in (2) describes where the decision-maker is indifferent between not entering the market and entering it.

One of the conditions is satisfied at each point in the state space of $P(t)$ and $R(t)$. If (1) holds as an equality, it is optimal to delay entering the market. However, if (2) holds as an equality, it is optimal to enter the market immediately. The enter threshold curve is the set of points where both conditions are met. If a firm is already in the market, the exit value function and exit curve $P^*(R)$ satisfy

$$\delta V^I \leq (P - AC(R))\bar{q} + r_1(\bar{P} - P)P\frac{\partial V^I}{\partial P} + r_2(\bar{R} - R)R\frac{\partial V^I}{\partial R} + \frac{1}{2}\sigma_1^2 P^2 \frac{\partial^2 V^I}{\partial P^2} + \frac{1}{2}\sigma_2^2 R^2 \frac{\partial^2 V^I}{\partial R^2} + \sigma_1\sigma_2\rho PR\frac{\partial^2 V^I}{\partial P \partial R} \quad (3)$$

and

$$V^I[P(t), R(t)] \leq V^C[P(t), R(t)] + cK \quad (4)$$

where $\bar{q}$ is the average rate of production of three wellpads for a firm in the natural gas market. If (3) holds as an equality, it is optimal to continue operating in the market (remain in regime 2). If (4) holds as an equality, it is optimal to exit the market (switch to regime 1).

V^C and V^I are the mathematical solutions to the partial differential equations in (1) and (3). V^C contains an option value that delays entering the market. Since there is partial irreversibility here, V^I includes an additional option value related to exiting the market. From (2), the option value in V^I encourages more immediate entry. Having the flexibility to exit the market if the conditions turn out to be unfavorable makes the firm less cautious to enter in the first place.

Numerical methods are required to approximate the unknown value function because of the multi-dimensional nature of the state space and dual entry-exit regimes (Miranda and Fackler, 2002). Using piecewise linear basis functions, we approximate $V^C[P(t), R(t)]$ and $V^I[P(t), R(t)]$ over a subset of the state space (Marten and Moore, 2011). The approximation procedure solves for the 2 x n^2 basis function coefficients which satisfy (1)-(4) and relevant boundary conditions at a set of $n = 100$ nodal points spread evenly over the two-dimensional state space.[7] The necessary boundary conditions are the following: $V^C[0, R(t)] = 0$ and $V^C[P(t), 0] = 0$. These ensure that there is no value of entering the market when natural gas wellhead prices are 0 or when proved reserves per well are 0. The solution is an entry and exit curve $P^*(R)$ containing a set of $n = 100$ points where these conditions are met.

4 Data and Parameter Estimation

Estimates of the parameters included in (1)-(4) are required to approximate $V^C[P(t), R(t)]$ and $V^I[P(t), R(t)]$ over a subset of the state space. U.S. natural gas wellhead price data are drawn from the U.S. Energy Information Administration (EIA) from January 1976 to December 2012 in monthly intervals. Wang and Krupnick (2013) allude to the natural gas boom beginning in the early to mid- 2000s, and Wang and Xue (2014) determine that firms began jumping in the market in 2000. We complete a Zivot-Andrews unit root test that allows for a single break in the intercept of wellhead price data to determine if there is a structural break in the time series due to the boom (Zivot and Andrews, 1992). In January 2000 there is structural break in wellhead prices intercept with a 1 percent significance level. Therefore, we use the pre-boom (pre-structural break) data from January 1976 to December 1999 in our simulations. Wellhead prices are in units of dollars per thousand cubic feet (\$/Mcf). We follow the method explained in Chapter 6 of "The Theory and Practice of Investment Management: Asset Allocation, Valuation, Portfolio Construction, and Strategies" to estimate r_1, $\bar{P}$, and σ_1 found in the Appendix (Fabozzi and Markowitz, 2011).

[7] Upwind finite difference approximations are used to construct a linear spline, which approximates the unknown value function. Miranda and Fackler (2002) presents the entire procedure. We use Matlab along with the CompEcon Toolbox and the smoothing-Newton root finding method to solve the resulting complementarity problem. The approximated state space ranges from 0 to 30 in the P dimension and from 0 to $1,270,000$ in the R dimension. Extending the state space in either the $P(t)$ or $R(t)$ dimension or increasing the number of nodal points beyond 100 does not alter our general results.

The rate of reversion for wellhead prices is 2.35 percent, and the long-run mean is \$2.12. Wellhead price volatility is 6.92 percent.

A Zivot-Andrews unit root test cannot be completed for natural gas proved reserves per well due to a lack of data. Parameter values for r_2, $\bar{R}$, and σ_2 are calculated using EIA data on annual U.S. natural gas (wet after lease separation) proved reserves and U.S. natural gas number of gas and gas condensate wells from 1989 through 1999 as a measure of reserves per well. Again, these parameters are estimated using Fabozzi and Markowitz (2011), and the method for calculation is found in the Appendix. Natural gas proved reserves are in thousand cubic feet per well (Mcf/well). The rate of reversion to the long-run mean for proved reserves per well is 0.0000401 percent. The long-run mean is 572,000 Mcf, and the volatility is 2.77 percent.

It is likely that the stochastic processes for natural gas wellhead prices and proved reserves per well are related, even if that is a lagged relationship. We use the cross-correlation function of the two time series discussed above to calculate the correlation between wellhead prices and proved reserves per well. We observe in Figure 1 that there is a negative relationship between wellhead prices and reserves per well. The correlation coefficient is -0.588 between their levels.

Following Pindyck (1980), average production costs are a function of stochastic proved reserves per well in our model. To numerically approximate the value function, a functional relationship between average production costs and proved reserves per well is estimated using data from EIA. To get a measure of average production cost on a per well basis, we collected annual U.S. nominal cost per natural gas well drilled from 1989 through 1999 and annual U.S. natural gas gross withdrawals in thousand cubic feet over the same time frame divided by the number of producing wells mentioned previously. Taking a ratio of these two variables provides the measure of average production costs per well needed to es-

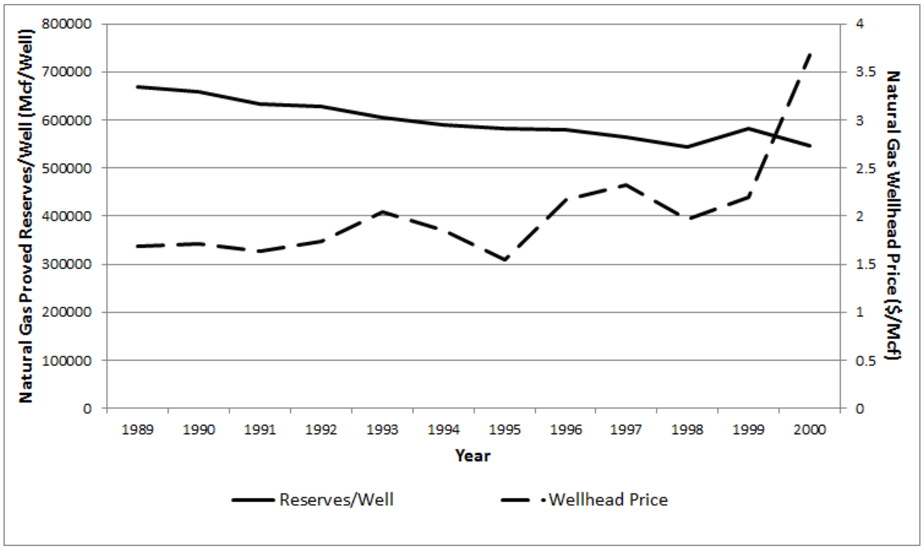

Figure 1: U.S. Natural Gas Wellhead Prices and Wet After Lease Separation Proved Reserves per Well

timate the $AC(R(t))$ relationship. However, the average cost per well data from EIA includes all costs for drilling and equipping wells and for surface-producing facilities. The real options methodology requires a careful accounting of sunk costs and production costs. Sunk costs play a critical role in this analysis, because they account for the irreversibility of the decision. Therefore, we remove the portion of average costs not used in the production of the natural gas well by applying a criteria explained in Ghosal (2010). He determines that capital is more sunk when there is a low ability to lease capital, no or limited second hand markets exist, and if capital depreciates slowly. We use these three criteria to allocate the parts of average production costs to those that are sunk versus not. Utilizing the apportioned average production cost per well and proved reserves per well data, we find $AC(R(t))$ to be an exponential function.

The average rate of production for three wellpads, $\bar{q}$, was calculated using EIA data on U.S. natural gas gross withdrawals in thousand cubic feet and the number of producing wells from 1989 through 1999.[8] Wang and Xue (2014) use proprietary data from DrillingInfo to display the annual number of well drilled versus the annual number of active firms. We utilize their Figure 2 to back-out the average number of wellpads per firm in 2000 equaling approximately 3 (rounded up from 2.6), when the boom began. Our data is in terms of production per one well. By multiplying the average rate of production for one well by the average number of wells on a wellpad (5), we determine that $\bar{q} = 1,030,000$ Mcf (Hefley et al., 2011).

An estimate for the sunk costs of entry into the natural gas market for one well were drawn from a report published by the University of Pittsburgh on "The Economic Impact of the Value Chain of a Marcellus Shale Well" in 2011 (Hefley et al., 2011). This report contains detailed information on all sunk costs required to drill a hydraulic fractured natural gas well. Again, using the Ghosal method, we estimate a value for the sunk cost of entering the natural gas market. This is different than the sunk costs of simply drilling a natural gas well. For example, if a firm's only intention is to drill a natural gas well, it does not need to keep landmen on payroll. Now if that firm's decision is to enter the natural gas market or not, having landmen on staff is a necessity for negotiating the stipulations of leases for land with owners. Given that it takes around a year and a half for a landman to develop a unit for drilling purposes and would not typically be classified as sunk costs in the standard drilling problem, having this type of personnel on staff is more valuable when the firm is making the decision to enter a market, not just for drilling one well. For reasons like this one, the sunk costs associated with market entry and exit are more sunk than those related to the decision to simply drill a well. If that is the case, then stochastic proved reserves and wellhead prices more heavily impact the firm's

[8]We focus on wellpads instead of just wells, because very rarely does a firm construct a wellpad to hold just one well (Hefley et al., 2011). Each wellpad can hold 3-6 wells on it.

Table 1: Natural Gas Entry and Exit Parameter Values

Description	Parameter	Value
Wellhead Price Rate of Reversion	r_1	2.35%
Wellhead Price Long-Run Mean	$\bar{P}$	\$2.12 Mcf
Wellhead Price Volatility	σ_1	6.92%
Proved Reserves Rate of Reversion	r_2	0.0000401%
Proved Reserves Long-Run Mean	$\bar{R}$	$572,000$ Mcf/well
Proved Reserves Volatility	σ_2	2.77%
Average Rate of Production for 3 Wellpads	$\bar{q}$	$1,030,000$ Mcf
Percent Recouped Entry Sunk Costs	c	2.00%
Upfront Sunk Costs of Entry	K	\$5,850,000
Discount Rate	δ	4.00%

decision. Sunk costs associated with entering the market as a firm producing with three natural gas wellpads is $\$5,850,000$.

There is not significant evidence on the percentage of upfront sunk costs that can be recouped if the firm exits the market. Therefore, we assume $c = 2\%$. The final parameter value needed to approximate the value function is the discount rate δ, which we assume to be around 4 percent. All parameter values can be found in Table 1.

5 Results

To illustrate our findings, we apply the model to an average firm choosing to enter the natural gas market in 2000 where the firm produces from three wellpads at any point in time, or thirteen hydraulic fractured wells. This level of production is determined from Figure 1 in Wang and Xue (2014). It is the average level of production for a firm in 2000. There are firms producing from more than three wellpads on a regular basis and those that produce less - we focus on the average firm. Results indicate that the average firm looking to enter the natural gas market, given historical data up to 2000, would not find it optimal to do so, shown in Figure 2. The black line is the market entry threshold, and

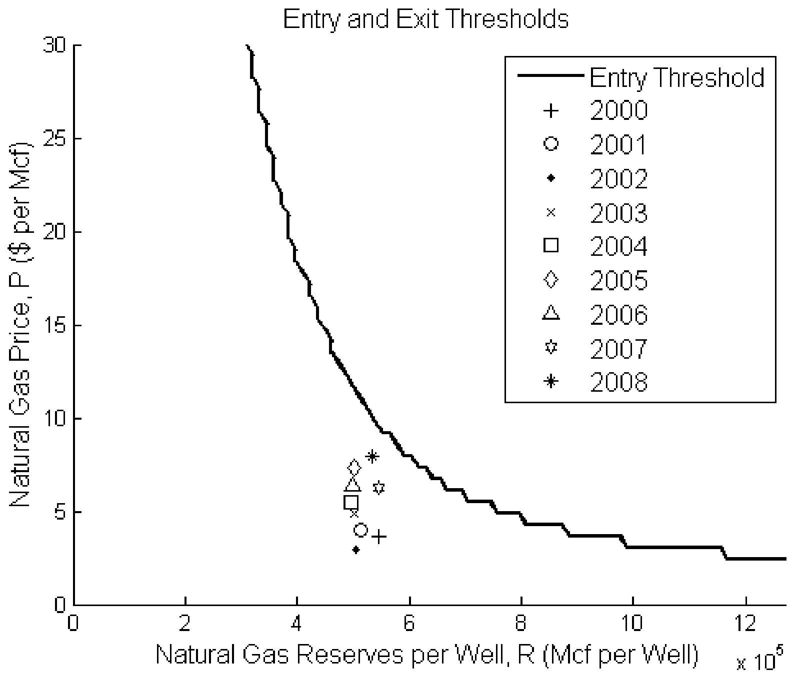

Figure 2: Optimal entry curve for an average firm facing correlated natural gas wellhead price and proved reserves per well of -0.588. The black line represents combinations of P and R that trigger market entry. Each symbol indicates the combination of natural gas wellhead prices (in dollars per thousand cubic feet) and proved reserves per well (in thousand cubic feet per well) from 2000 to 2008.

each symbol represents combinations of P and R for every year starting in 2000 and going through 2008. When natural gas wellhead prices and proved reserves per well reach any combination along the black line and to the right of it signals market entry is optimal. In Figure 2 it is easy to see that all pairs of P and R for 2000 to 2008 fall to the left of the entry threshold, indicating that it is not optimal to enter the natural gas market.

The natural gas boom of the 2000s saw a flooding of the market with new firms entering, specifically starting in 2000; but, Figure 2 suggests that an average firm would not have found it optimal in any year from 2000 to 2008 to enter the natural gas market. Kellogg (2014) finds that firms drilling for natural resources do utilize option value analysis to make optimal decisions. However, our model implies that firms were either not using option valuation for market entry decisions, or firms were expecting a change in the market that is not represented by historical data used to calibrate the model.

Wang and Xue (2014) identify one possible explanation behind this phenomenon, and we hypothesize two others: 1. The technological advances in horizontal drilling, 3-D seismic imaging, and hydraulic fracturing pushed the cost of production down compared to historical averages, because the level of reserves increased. We determine the magnitude change in production costs which a firm would find it optimal to enter the natural gas market for any year between 2000 and 2008 by increasing the long-run mean level of proved reserves $\bar{R}$, ceteris paribus. The greater the $\bar{R}$ value, the lower $AC(R)$ will be. 2. The second explanation behind the jump in firms entering the natural gas market is due to a reduced risk of entry. This risk can be moderated in two ways: 2a. Lower uncertainty in natural gas proved reserves from 3-D seismic imaging technology, horizontal drilling, and hydraulic fracturing and 2b. Lower sunk costs of market entry caused by producers outsourcing actual drilling and fracking to specialized oil and gas companies. We measure the impact of lower natural gas reserve uncertainty by decreasing the volatility or reserves σ_2, ceteris paribus, such that a firm would find it optimal to enter the natural gas market in any year from 2000 to 2008. The effect of reduced sunk costs of entry are examined by lowering K, ceteris paribus, to the point in which one of the symbols in Figure 2 falls to the right of the entry threshold.

Figure 3 displays the market entry threshold for a firm who is drilling three wellpads at any given time. The baseline model is represented by the black curve, where firms are only using historical data from before 2000 to make their decisions. Explanation (1) is depicted by the grey line, showing a 25 percent

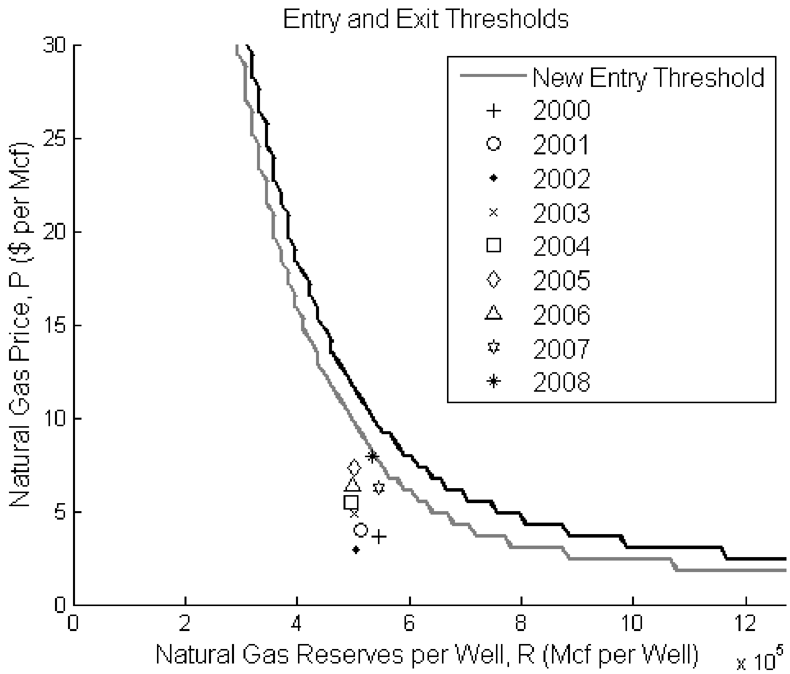

Figure 3: Optimal entry decision for an average firm facing correlated natural gas wellhead price and proved reserves per well uncertainty of -0.588. The black line represents combinations of P and R that trigger market entry in the baseline model, using parameters from Table 1. Combinations of P and R that warrant market entry when the long-run mean level of proved reserves is 25 percent higher ($\bar{R} = 572,000$ Mcf/well $\rightarrow \bar{R} = 715,000$ Mcf/well) is given by the grey line. When the costs of production decrease by 25 percent, due to an increase in $\bar{R}$, an average firm facing natural gas wellhead prices and proved reserves per well in 2008 would find it optimal to enter the natural gas market.

increase in $\bar{R}$, ceteris paribus. This percent reduction in lower production costs pushes the required level of P and R down to the point where a firm facing natural gas wellhead prices (\$7.97 per Mcf) and proved reserves per well ($535,000$ Mcf/well) in 2008 would optimally enter the natural gas market.

Exploring Explanation (2a), completely eliminating natural gas proved reserve volatility does not bring the entry threshold low enough for any firm from 2000 to 2008 to find it optimal to enter the natural gas market. Figure 4 demonstrates this result. Again, the black curve represents the baseline model where

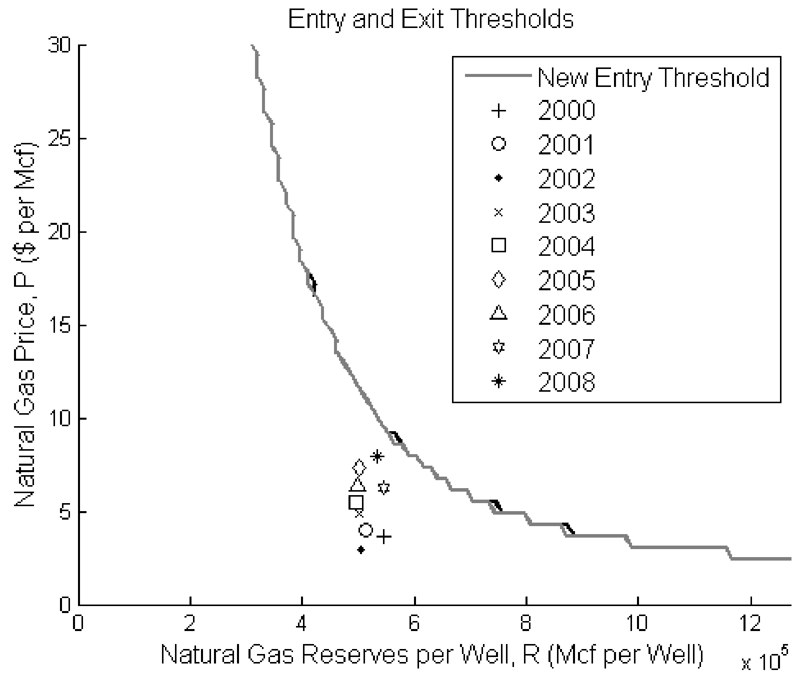

Figure 4: Optimal entry decision for an average firm facing correlated natural gas wellhead price and proved reserves per well uncertainty of -0.588. The black line represents combinations of P and R that trigger market entry in the baseline model, using parameters from Table 1. Combinations of P and R that warrant market entry when proved reserve volatility is 100 percent lower ($\sigma_2 = 2.77\%$ $\rightarrow \sigma_2 = 0\%$) is given by the grey line. When natural gas proved reserve volatility decreases by 100 percent, an average firm facing natural gas wellhead prices and proved reserves per well for any year between 2000 and 2008 would not find it optimal to enter the natural gas market.

firms are only using historical data from before 2000 to make their decisions. The grey line depicts Explanation (2a), revealing a 100 percent removal of σ_2. It appears that the grey line sits almost exactly on top of the baseline model. Eradicating natural gas proved reserve volatility does not push the required level of P and R down to the point where an average firm would optimally enter the natural gas market.

The last explanation we explore focuses on reducing the risk associated with entering the natural gas market by lowering the sunk costs of market entry.

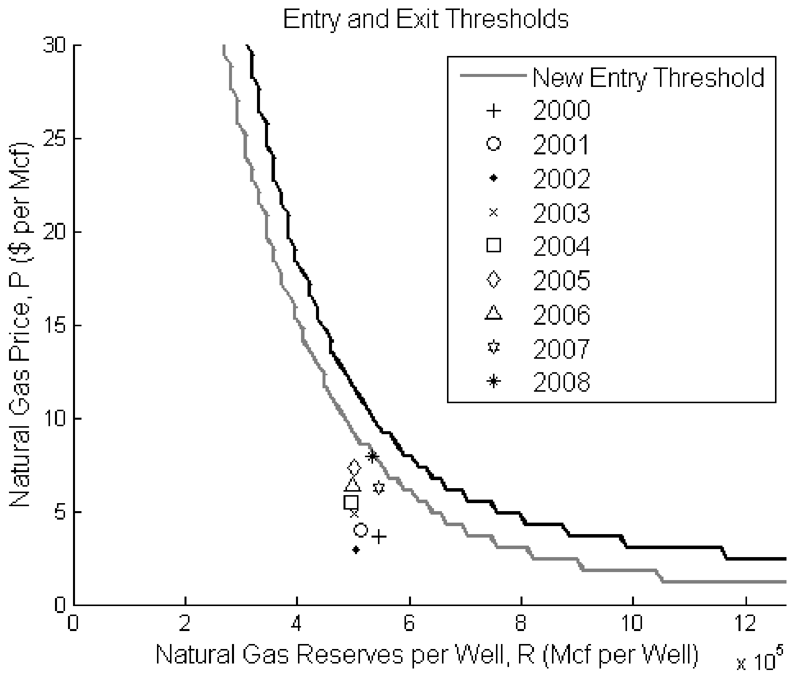

Figure 5: Optimal entry decision for an average firm facing correlated natural gas wellhead price and proved reserves per well uncertainty of -0.588. The black line represents combinations of P and R that trigger market entry in the baseline model, using parameters from Table 1. Combinations of P and R that warrant market entry when sunk costs are 75 percent lower ($K = \$5,850,000 \rightarrow K = \$1,460,000$) is given by the grey line. When the sunk costs of market entry decrease by 75 percent, an average firm facing natural gas wellhead prices and proved reserves per well in 2008 would find it optimal to enter the natural gas market.

Lowering the sunk costs of entry by 75 percent shifts the entry threshold far enough left that an average firm in 2008 would find it optimal to enter the natural gas market. Figure 5 displays this finding. The black line is the baseline model, and the grey line signifies Explanation (2b).

Comparing Figures 3-5, it appears that either lower production costs or lower sunk costs of entry explain the boom in firms entering the natural gas market in the 2000s. While it only takes a 25 percent reduction in productions costs, through increasing the long-run mean level of natural gas proved reserves per well, to push the entry threshold to combinations of natural gas wellhead prices and proved reserves per well where an average firm from 2000 to 2008 would find it optimal to enter the market, it takes a 75 percent reduction in sunk costs to complete the same task. No amount of market entry risk reduction via reduced natural gas proved reserve uncertainty would make an average firm enter the natural gas market.

Explanation (1) seems to be the most likely explanation behind the boom in natural gas firm entry from 2000 to 2008. Firms were expecting the costs of production associated with using hydraulic fracturing technology to drill for natural gas to decrease more than what historical data indicated. A 25 percent increase in long-run mean level of natural gas proved reserves is equivalent to increasing $\bar{R}$ to $715,000$ Mcf/well. In 2013, the proved reserves per well were $726,000$ Mcf/well. Combining this data point with those from 2000 to 2008 found on Figure 2, the long-run mean level of proved reserves per well have been steadily increasing since the mid-2000s, and firms could easily have been expecting this trend to happen even though it had not come about yet. It is feasible to say that the new $\bar{R}$ could reach somewhere near $715,000$ Mcf/well. Further, U.S. Entergy Information Administration (2014) determined that natural gas proved reserves increased by 9.7 percent from 2012 to 2013 and project that this trend will continue for the foreseeable future, meaning that natural gas proved reserves per well is likely to follow the same pattern.

On the other hand, it does not seem feasible that the average firm between 2000 and 2008 was expecting the sunk costs of entering the natural gas market to decrease by 75 percent. This type of reduction would require that the sunk costs of entry to go from $\$5,850,000$ to $\$1,460,000$. Recall that these sunk costs are for an average firm that is producing from 3 wellpads at any given point in time. Hefley et al. (2011) estimates that the sunk costs of entering the natural

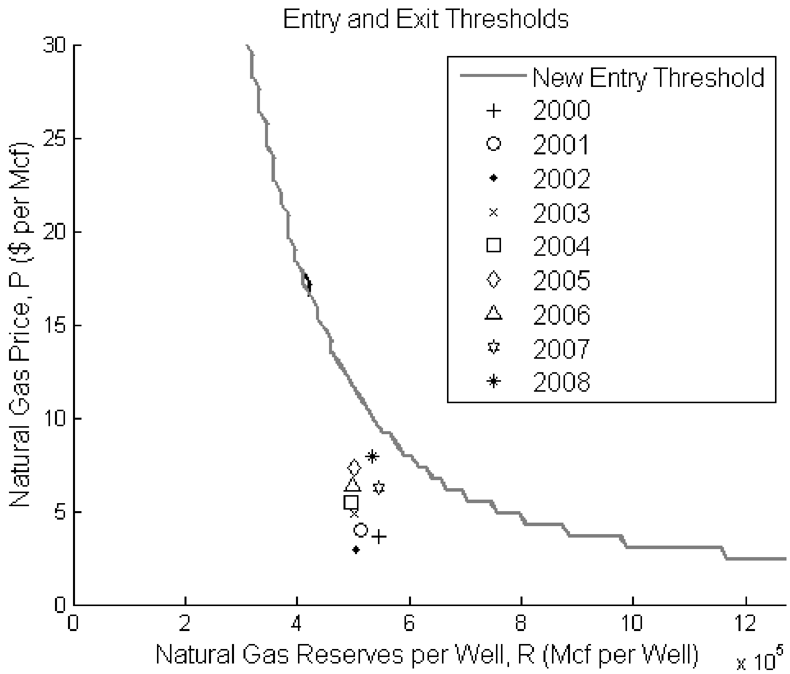

Figure 6: Optimal entry decision for an average firm facing correlated natural gas wellhead price and proved reserves per well versus thresholds when price volatility is set to zero. The black line is the entry threshold in the baseline model when natural gas wellhead price volatility is 6.92 percent. The grey line is the entry curve when wellhead price uncertainty is zero.

gas market for just one wellpad totaled $\$2,250,000$ in 2011.

Our real options model also allows us to determine which stochastic variable impacts the decision made by an average firm to enter the natural gas market. We eliminated wellhead price volatility and compared that to an identical removal in natural gas proved reserve uncertainty. Our results do not confirm previous findings in the literature. It is a common result that reduced price volatility has negligible effects on the threshold curves while an equal reduction in reserve uncertainty causes further delay in the entry and exit decisions (Slade, 2001). This conclusion is typically driven by the fact that the benefit function is linear, and the costs, whether they be production costs like in this paper or otherwise, are convex (Hanemann, 1989). The curvature of the cost function would usually lead to decreased reserve uncertainty having a greater influence than price volatility. This is not true of Figures 4 and 6. We believe the reason behind this result stems from the $AC(R)$ relationship. The data from EIA indicates that the average costs of production and reserves per well fit an exponential relationship slightly better than that of a linear relationship. In fact, the curvature of $AC(R)$ is quite insignificant, leading to a more linear average cost function. We hypothesize that this is driving force behind seeing no change in entry thresholds when we remove natural gas proved reserves per well uncertainty. Figure 6 shows the entry curve when there is no natural gas wellhead price uncertainty (the grey line) lay almost exactly on top of the baseline threshold (the black line). Figure 4 looks nearly identical to Figure 6.

6 Conclusion

The natural gas boom was made possible by technological advances. It became advantageous for firms to extract large quantities of shale gas using hydraulic fracturing and horizontal drilling technology. From the early 2000s, peaking in 2008, to present, the number of active firms in the natural gas market has grown tenfold (Wang and Xue, 2014). There are several reasons why this expansion occurred. We explore three main explanations behind the boom in firm entry. The combination of advances in horizontal drilling, 3-D seismic imaging, and hydraulic fracturing drove the cost of production down. Further, the risk of entering the market decreased because of lower uncertainty surrounding natural gas proved reserves. Or the reduced risk of market entry evolved from moderated sunk costs of entry.

Our results indicate that expectations of reductions in the average costs of production through increasing the long-run mean level of natural gas proved reserves is the most likely explanation behind the boom in firm entry starting in 2000. It takes a 25 percent increase in the long-run mean level of proved reserves to see a firm facing wellhead prices and reserves in 2008 to enter the natural gas market. However, lower sunk costs of entry could explain the drastic number of firms entering the natural gas market but is not feasible, since it would take a 75 percent reduction in sunk costs of entry to land a firm in the entry area in 2008. Surprisingly, reducing the risk of market entry through expectations of lower natural gas proved reserves per well uncertainty would not account for the entry boom. We conclude that the reduction in average costs of production from technological advances in 3-D seismic imaging, horizontal drilling, and hydraulic fracturing influenced market entry the greatest out of the three reasons we explore with the model.

Given that our model incorporates both natural gas wellhead price and proved reserve uncertainty, we determine that reserve uncertainty impacts the market entry decision by firms the same as price volatility. This result is not in line with the literature. Wellhead prices benefit the firm and enter linearly into the model, while proved reserves cost the firm since they enter the model through average costs of production in an exponential fashion. Although this relationship between average costs of production and proved reserves per well does not have a significant curvature based on the data, leading to minimal impacts of reserve uncertainty elimination.

Another explanation behind the boom in firms entering the natural gas market from 2000 to 2008, provided by Wang and Xue (2014), concerns the concept of speculating. It could be that the rush to enter the natural gas market was caused by firms looking to lease land and mineral rights to drill in order to sell them at a higher price later to those suppliers that are completing large scale drilling activities. We are not able to test this explanation with the current model. To test if firms are speculating, the decision by the firm would look at the price of the lease and mineral rights as being a stochastic variable, instead of the natural gas wellhead price and proved reserve uncertainty we incorporate into the model.

Mason and Wilmot (2014) determine that the Henry Hub spot price in the U.S. has a high prevalence of jumps in the data. Incorporating this into our model may lead to natural gas prices having a larger role in the optimal entry and exit decisions of a firm. It should also be noted that the model presented in this paper has several other applications as long as there is a stochastic benefit and cost process involved in the decision-making process. We leave these topics to future research.

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A Appendix

A.1 Unit Root Tests

Given that real options results critically depend on choosing the correct stochastic process, we test whether data are consistent with Brownian motion instead of our assumption of mean reversion. This is typically completed using an augmented Dickey Fuller test (Conrad, 1997; Forsyth, 2000; Insley, 2002; Pindyck and Rubinfeld, 1998). Geometric Brownian motion (GBM) assumes P is lognormally distributed. The logged price level $p = \ln(P)$ is normally distributed and follows an arithmetic Brownian motion (ABM) $\mathrm{d}p = \mu \mathrm{d}t + s\mathrm{d}z$. If p is consistent with ABM, Ito's Lemma ensures P must be consistent with GBM. To test that p are consistent with ABM, we run a restricted regression

$$(p_t - p_{t-1}) = \beta_0 + \beta_1(p_{t-1} - p_{t-2}) + \epsilon_t \tag{5}$$

and unrestricted regression

$$(p_t - p_{t-1}) = \beta_0 + \beta_1(p_{t-1} - p_{t-2}) + \beta_2 t + \beta_3 p_{t-1} + \epsilon_t \tag{6}$$

The null hypothesis that corresponds with p being ABM is $\mathrm{H}_0 : \beta_2 = \beta_3 = 0$.[9] This null hypothesis is rejected at the 1 percent significance level for natural gas wellhead prices, shown in Table 2. This is consistent with previous literature which indicates that natural resource prices exhibit mean reversion tendencies (Dixit and Pindyck, 1994). It is not possible to perform a unit root test for natural gas proved reserves per well, since we are only using data from 1989 to 2000. However, Figure 7 appears to reject the GBM specification.

[9] This set-up is also true for testing if reserves R follow GBM.

Table 2: Unit Root Test for U.S. Natural Gas Wellhead Price Data, Monthly January 1976-December 1999

Unrestricted regression			
Coefficient	Estimate	Std. Error	t-statistic
β_0	0.0186	0.0086	2.17
β_1	0.276	0.0572	4.82
β_2	0.00004	0.00006	0.68
β_3	-0.0392	0.0125	-3.14
Restricted regression			
β_0	0.0035	0.0041	0.84
β_1	0.268	0.0579	4.63
$N = 286$			$F = 5.48$
Prob > F = 0.0046			

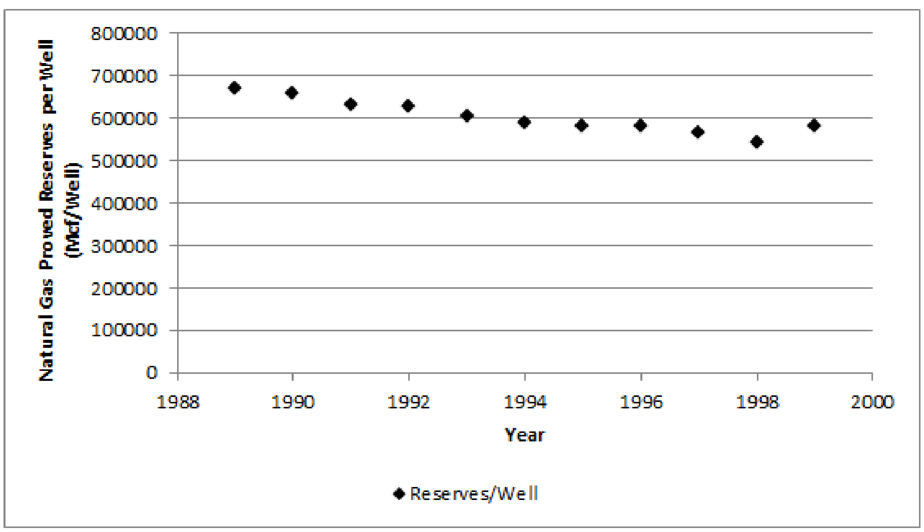

Figure 7: U.S. Natural Gas Proved Reserves per Well, 1989-2000

A.2 Geometric Mean Reversion Parameter Estimation

After determining that Brownian motion is inappropriate for modeling both natural gas prices and reserves, we turn to geometric mean reversion $\mathrm{d}P = r_1(\bar{P} - P)P\mathrm{d}t + \sigma_1 P\mathrm{d}z_1$. The geometric mean reversion model can be written as the following:

$$P_{t+1} = P_t + r_1(\bar{P} - P_t)P_t + \sigma_1 P_t \epsilon_t \tag{7}$$

Here, ϵ_t is a standard normal random variable. To estimate the parameters r_1, $\bar{P}$, and σ_1, we can use a series of 286 historical observations. Completing a Zivot-Andrews unit root test that allows for a single break in the intercept of the time series on data starting in January 1976, we determined that there is a structural break in January 2000 at the 1 percent significance level. Therefore, the 286 observations begin in January 1976 and continue monthly to December 1999. This structural break is consistent with the natural gas (hydraulic fracturing) boom where an influx of firms entered the market. It is with this pre-break data we complete all estimations, including the previous augmented Dickey Fuller test uses the data before the structural break.

Assume that the parameters of the geometric mean reversion remain constant during the time period of estimation. Rewrite the equation for geometric MR as

$$\frac{P_{t+1} - P_t}{P_t} = r_1\bar{P} - r_1 P_t + \sigma_1 \epsilon_t \tag{8}$$

This equation bears characteristics of a linear regression model, with the percentage price change $\frac{P_{t+1}-P_t}{P_t}$ as the dependent variable and P_t as the explanatory variable.

According to Fabozzi and Markowitz (2011), the estimate of r_1 is obtained

Table 3: Geometric Mean Reversion Estimates for U.S. Natural Gas Wellhead Price Data

Coefficient	Estimate	Std. Error	P-Value
$r_1\bar{P}$	0.0499	0.0136	0.000
r_1	-0.0235	0.00720	0.001
$N = 287$			$r_1 = 0.0235$
$\bar{P} = \$2.12$			$\sigma_1 = 0.0692$

as the negative of the coefficient in front of P_t. One way to check if geometric MR is a consistent assumption for natural gas prices is to determine if the coefficient in front of P_t is positive, since the rate of reversion r_1 cannot be a negative number. The estimate for the long-run mean of prices $\bar{P}$ is obtained as the ratio of the intercept term estimated from the regression and the negative of the slope coefficient in front of P_t. The last estimate for volatility σ_1 is obtained as the standard error of the regression.

Two other methods for determining if geometric MR is suitable are the following: 1. The p-value for the coefficient in front of P_t should be small, preferably less than 0.05 and 2. The points in a scatter plot of P_t versus $\frac{P_{t+1}-P_t}{P_t}$ should vary around a straight line with no visible cyclical or other patterns. Table 3 displays the results of the regressions that determine the parameter estimates for prices. Natural gas wellhead prices satisfy all three checks described by Fabozzi and Markowitz (2011). Our results are simulated under the assumption that natural gas wellhead prices follow geometric MR as the literature has determined to be more fitting.

A Zivot-Andrews unit root test allowing for a structural break in the intercept of the natural gas proved reserves per well time series is not possible due to limited observations. Simulations are completed with the time series from 1989 to 2000, following that of natural gas wellhead prices. Using Fabozzi and Markowitz (2011), natural gas proved reserves per well satisfy two of the three

Table 4: Geometric Mean Reversion Estimates for U.S. Natural Gas Recoverable Reserves per Well Data

Coefficient	Estimate	Std. Error	P-Value
$r_2\bar{R}$	0.229	0.146	0.155
r_2	-0.000000401	0.000000241	0.135
$N = 10$			$r_2 = 0.000000401$
$\bar{R} = 572,000$			$\sigma_2 = 0.0277$

checks for geometric MR, shown in Table 4. The p-value associated with the coefficient on R_t is larger than the prescribed 0.05, but that coefficient is negative, as it should be before applying negative 1 to find r_1. We check the sensitivity of our results by modeling natural gas proved reserves per well as GBM and find no change in our results.