Modeling Endogenous Change in Water Allocation Mechanisms: A Non-Cooperative Bargaining Approach

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Abstract

Water allocation in river basins across the world has been historically determined through various institutional arrangements, which are typically hierarchical in nature. But institutions are generally not explicitly recognized in the literature on resources and environmental economics as they do not yield easily to modeling in the conventional framework. This paper proposes a model to assess the potential of efficiency gains possible from institutional change under hierarchical water institutions. We hypothesize two types of institutional change from status-quo; efficient water markets yielding Pareto optimal outcomes, and more interestingly, regional and intra-sectoral water trading under imperfect markets using game theory. Using Banks and Duggan (2006) model of collective decision making to model non-cooperative bargaining, we present here a general framework to compare the outcomes against competitive trading, social planner and an arbitrary status-quo. Application of the hierarchical model to Upper Rio Grande basin calculates the efficiency gains possible from cooperative or non-cooperative bargaining compared to the status-quo. The innovative model also allows for incorporating the impact of climate change, population growth, economic growth and technological change on future supply and demand, which helps study the benefits of plausible institutional changes in otherwise relatively stagnant water allocation institutions. The alternative mechanism of non-cooperative bargaining can be easily applied in large scale water allocation models and the analysis can guide policy making by highlighting the gains feasible institutional reforms can achieve.

Keywords: Water Economics; Institutions; Non-cooperative bargaining theory

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1. Introduction

Markets are commonly used to allocate natural resources that are rival and excludable (e.g., land, fossil fuels, timber), and thus for which exclusive property rights can be defined. Water is not one of these resources. The problem lies in the unique nature of water, which for its traditional uses, has mostly been non-rivalrous and non-excludable, giving it the character of a public good (Libecap, 2008). This creates problems in clearly defining property rights, which is an essential condition for markets to operate successfully. As a result, water allocation in river basins across the world has been historically determined through various institutional arrangements which differ in their degree of complexity (Gopalakrishnan, Tortajada, & Biswas, 2005; Saleth & Dinar, 2004).

The system of prior-appropriation of water in the Western United States is one example of such an institution (Brewer, Glennon, Ker, & Libecap, 2008; Ruml, 2005; Teerink, 1993). As the miners settled and began diverting water for their operations in the 1850s, there appeared a problem of allocation of water. In the Eastern US, where water is relatively abundant, rights have traditionally been linked to the land, but in the West, water is scarce and the riparian water laws of the East were unsuitable. Hence, the system of "first-in-time, first-in-right" was adopted to allocate the limited quantity of water. Over time, the number of stakeholders increased with complicated hierarchies and intricate relationships.

Such an institution makes it difficult to allocate water among different sectors based on their competing uses. Any agent willing to sell or lease his/her water rights need to take into account the impact of water withdrawal on other agents in this system of hierarchy and need to show that "no-harm" will occur to other right-holders (Ruml, 2005). So if a municipality is willing to purchase or lease water rights from a right-holder, it has to make its way through a complicated legal process which not only takes time and increases the transaction costs, but also adds uncertainty.

With these serious pragmatic issues in management and allocation, water does not lends itself to conventional economic analysis. There is even a debate on whether water should be considered as an economic good at all (Briscoe, 1996; Savenije, 2002). Even within a given institution, water poses some complex pricing issues, especially for irrigation use. Given the physical properties of water, volumetric pricing for agricultural use is a costly mechanism to implement, with the alternative mechanism of non-volumetric pricing being cheaper but relatively inefficient (Johansson, Tsur, Roe, Doukkali, & Dinar, 2002; Johansson, 2000). Agricultural water prices are also a political issue, with often noisy lobbying from agriculture to keep the prices low (Cornish, Bosworth, Perry, & Burke, 2004).

The result is a wide disparity between water prices among different economic sectors. In their study of water markets in American West, Brewer et al. (2008) found that farmers in Arizona's Pima County pay around $27 per acre-foot for water while water customers in the nearby city of Tucson pay anywhere between $479 to $3267 per acre-foot. Table 1 below shows the estimated water prices in select OECD nations by broad sector usages. A general trend towards extremely low agricultural water prices, generally by a factor of more than twenty, is clearly apparent.

Table 1.

Estimates of worldwide water prices by broad sector usage

OECD nation	Household water supply	Industrial and commercial	Irrigation and agriculture	Average price of water supply
Netherlands	3.16	1.08	1.44	1.89
France	3.11	0.95	0.08	1.38
Greece	1.14	1.14	0.05	0.78
Spain	1.07	1.08	0.05	0.73
USA	1.25	0.51	0.05	0.6
UK	2.28	1.68	0.02	1.33
Australia	1.64	1.64	0.02	1.1
Portugal	1	1.26	0.02	0.76
Turkey	1.51	1.68	0.01	1.07
Canada	0.7	1.59	0.01	0.77

Source: CWF (2011)

In the American West, the system has led to a situation where urban users place quite a high scarcity value on water, but a significant supply is used to cultivate low-value crops (Brewer, Ker, Glennon, & Libecap, 2006). In California, an acre-foot of water used to grow cotton and alfalfa generates $60 in gross state revenue, while the same quantity of water may generate $980,000 if used in the semi-conductor industry (Brewer et al., 2008; Gleick, 2005). In general, the historically chosen alternative of political allocation of water ignores competing uses of water resulting in an inefficiency such that the marginal valuation of water differs across consumers at the current prices (Olmstead, 2013). This implies that there are potential efficiency gains to be made by reallocating water among different sectors.

Moreover, in a river basin, water institutions are generally hierarchical with water first being allocated to regional authorities, who allocate it further to various economic sectors which are part of it. Agents in the same river basin might value water very differently depending on the rights of not only the sector they belong to, but also the state they belong to and how the institutions in the two hierarchies play out. For example, while the city of San Diego offered $225 per acre-foot to acquire water from southeastern California's Imperial Irrigation District in 2005, a development near the South Rim of Grand Canyon National Park in Arizona was willing to pay $20,000 per acre-foot for the same Colorado River water (Brewer et al., 2008; Glennon, 2002). Unsurprisingly, the water had just cost $15.50 per acre-foot to the irrigation district. Therefore, the hierarchical nature of institutions can be quite important in influencing the marginal valuation differences.

But such institutions are not explicitly recognized in the literature on resources and environmental economics as they do not yield easily to modeling in the conventional framework. Economic models usually ignore water as an input or assume that all demand is met. Even if water is modeled explicitly, it is assumed that the markets are efficient and clear just like any other commodity (Olmstead, 2013). This clearly highlights the need for tractable model of water allocation within an institutional framework which is where this paper seeks to contribute.

The inefficiency of water institutions creates pressure for institutional changes. In the long run, a feasible institutional change might either take the form of changes in allocation parameters to various economic sectors and regions, or it may take the form of a comprehensive change in the allocation mechanism itself. One such change in allocation mechanism will be the creation of perfectly competitive water markets which yields Pareto optimal outcomes. While some water markets in practice, like in Chile (Hearne & Donoso, 2014) and Australia (Wheeler, Bjornlund, & Loch, 2014), have been quite successful, they still cannot be described as perfect. An equitable distribution by a central authority or a social planner will also yield Pareto optimal allocations and can be seen as a form of cooperative bargaining. However, cooperation in water resources allocation is quite difficult to achieve in the real world (Dombrowsky, 2007). Another plausible allocation mechanism would then be the emergence of regional and intra-sectoral water trading under imperfect markets and non-cooperative bargaining.

The objective of this paper is to highlight possible gains from feasible institutional changes by comparing equilibrium outcomes under a baseline institutional allocation with the outcomes under cooperative efficient water markets and imperfect water trading system with hierarchical jurisdictions. In the proposed model, water trading under imperfect market is modeled through a partial equilibrium game-theoretic model of non-cooperative bargaining. In this framework, at the regional level, states can improve on the institutional allocation by negotiating water reallocation in exchange for monetary transfers, and within each state, economic sectors can improve on the institutional allocation by trading water among themselves in exchange for monetary transfers.

The goal is to develop a non-cooperative framework which can be used as an alternative to cooperative water allocation mechanism in numerical models of water resources or economy. Closed-form solutions for the general bargaining model are calculated assuming quadratic welfare functions and applied to the data on Upper Rio Grande basin. We find that since cooperation among different consumers in the water market is practically difficult, non-cooperative bargaining equilibrium removes only a part of the inefficiency, the remaining being the result of non-cooperation. This provides a useful benchmark between the inefficient status-quo allocation and the efficient but difficult to achieve cooperative allocation.

The model can also help study important implications of population increase and climate change on the water sector. With the global population exploding in the last century, the pressure on water resources has increased enormously. Of the total world population of around 7 billion people, 4 billion people were added in the last half century, with the last billion being added in just the last 12 years (UN, 2012). Moreover, it is not only the population increase that is putting pressure on available water resources. Economic development in developing and emerging economies is pulling a large number of households out of poverty, increasing their standard of living which places further demands on the water resources for meeting direct needs, as well as indirect needs through goods and services (UN, 2003).

According to OECD (2012) estimates, water demand is likely to increase by 55% globally between the year 2000 and 2050. A major portion of this demand increase will come from manufacturing sector, which will increase its total demand by 4 times. Electricity generation will increase its demands by 1.4 times and domestic demand is likely to increase by 1.3 times. Without any significant increase in the total supply, there will be tremendous pressure on water allocation, which will witness increased competition among sectors.

Apart from demographics and development, climate change is also expected to impact the availability of water resources across the globe through multiple pathways (Olmstead, 2010). Firstly, long term availability is likely to change through changes in precipitation patterns, temperatures, duration of accumulated snowpack, nature and extent of vegetation, soil, moisture and runoff. Secondly, there will be short term variability in supply with increased frequency and magnitude of droughts and floods. The impact will differ depending on the geography. According to Bell, Zhu, Xie, & Ringler (2014), seasonal runoffs are likely to increase at high latitudes and in some wet tropics, and are likely to decline in dry regions, in mid-latitudes and in dry tropics.

With increasing pressure on demands from non-agricultural sources and uncertainties arising out of climate change, there will be increasing friction between water institutions and competing users for water allocation. Climate change adaptations will not only require institutional responses, but much of the adaptation will itself be undertaken by water institutions (Olmstead, 2013).

Application of the hierarchical model of institutions to data from river basins can highlight the potential efficiencies that feasible institutional changes in the form of efficient water markets or water trading in imperfect markets can gain. In addition, long-run phenomenon like climate change, population growth, economic growth and technological changes will influence the availability and demand of water resources. Water markets are likely to emerge as an adaptive response to these phenomenon, especially climate change (Rosegrant, Ringler, & Zhu, 2014). By taking these exogenous factors in account, the paper seeks to develop a framework for estimating the gains from institutional change, which will now not only depend on the type of change that is undertaken, but also on the timing of the change.

A major innovation in this paper is the use of hierarchical game-theoretic non-cooperative bargaining framework to model water trading in imperfect markets among economic sectors and regional authorities. A significant part of the paper is devoted to developing this hierarchical bargaining model. The plan of the paper is as follows. The next section discusses the relevant literature in the application of game theory to the problem of water allocation. Section 3 presents alternative allocation mechanism in a general model and also under specific functional forms. Section 4 uses the methodology to calculate the allocation level and welfare under status-quo, cooperative Pareto optimal allocation and under the non-cooperative bargaining for the three regions in the Upper Rio Grande Basin. Section 5 concludes the paper summarizing the important findings and arguments.

2. Literature Review

There have been a number of contributions in the broad field of game-theoretic models related to water issues but relatively few papers have focused on the specific question of non-cooperative games for water allocation problem. There are two excellent and relatively recent reviews of the field. Carraro, Marchiori, & Sgobbi (2007) review the literature on the application of non-cooperative bargaining theory to water management problems, with a discussion on groundwater management, surface water management and transboundary allocation problem. They conclude that although considerable gains in water management are possible through negotiated decision making, there is little formal understanding of the forces behind bargaining process, and application of formal negotiation theory to water issues is lacking. Another review by Madani (2010) discusses several cooperative game theory models applied in sharing costs and benefits of development projects in a basin, some non-cooperative models of sustainable extraction of groundwater and only a handful of non-cooperative models of water allocation among users in a basin.

In the literature of cooperative games on water allocation problem, Tisdell & Harrison (1992) looked at the problem of allocating transferable water licenses to farms in Australia and studied the distributional consequences of different initial rights allocations. (Ambec & Ehlers, 2008) considered river water allocation problem for international agents with satiation point for water consumption. Wang, Fang, & Hipel (2003, 2008) develop a Cooperative Water Allocation Model (CWAM) for equitable and efficient water allocation among competing users at the basin level and apply it to South Saskatchewan river basin located in Alberta, Canada. In this model, water rights are initially allocated through a given principle which is not efficient, therefore, in the next stage, reallocation among users takes place taking into account the net benefits of cooperative stakeholders. Mahjouri & Ardestani (2010, 2011) develop a cooperative model of water allocation which aims to fulfill equity, efficiency, and environmental criteria, by deriving allocation shares through optimization on competing demands of different users and on total net benefits.

The dominance of cooperative models in the literature seems to arise from the fact that cooperative games lead to efficient solutions and are inherently attractive. There is also a normative appeal to cooperative games and several international water agreements have shown that cooperation is indeed possible. However, cooperation is likely to be very difficult, if not impossible in an institutional setting like Western US, where water rights are entangled in a complex web of ownerships and legal settlements. In such a setting, noncooperation is likely to be the case and therefore, needs to be studied.

The handful of non-cooperative games of inter-basin water allocation problem are all based on Rausser & Simon (1992) framework of non-cooperative multilateral bargaining. Adams, Rausser, & Simon (1996) applied the model to "three-way negotiations" between agriculture, urban and environmental sectors in California of the 90s. In this framework, there are 3 policy spaces; degree of new infrastructure development, degree of transferability and degree of environmental protection. Preferences of the agents are normalized between 0 and 1 on each of these 3 policy spaces with their ideal points specified exogenously. These preferences are converted into utility through a CES utility function across 3-dimensional policy space. A player is randomly selected to propose a policy vector specifying the quantity of infrastructure development, transferability and environmental protection. Agents accept the proposal if it gives their reservation utility and reject otherwise. The game is solved by simulating through finite iterations of negotiation with a final outcome being also specified exogenously in case the negotiations fail to come to an agreement. Simulations are done for different values of policy preferences in order to understand the behavior of agents. Carraro & Sgobbi (2008) base their model on this framework adding uncertainty over the size of the pie being shared.

There are some major limitations of the existing literature in the non-cooperative water allocation bargaining field. Firstly, the models are very general in the characterization imposed on the agents and therefore, requires simulation across policy space to understand their behavior. Only very little can be understood about the agents analytically. Secondly, in the bargaining game, agents only propose the policy vector (or analogously, water allocation shares) with no possibility of making a transfer payment to other agents in order to reach an agreement. The feature of transfer payments is crucial in water market bargaining because the gulf between marginal valuations of different sectoral users is very wide, making monetary transfers a useful instrument to achieve reallocation through trading.

Thirdly, the applied models lack economic micro-foundations. The benefit function of the agents are derived with little theoretical background on their economic objectives. This implies that the policy recommendations can be very sensitive to the assumptions about the sectors and they cannot be linked with observed data. Finally, there has been no research recognizing the hierarchy of water institutions in a river basin. These shortcomings also imply that the existing non-cooperative bargaining models do not yield easily to application in numerical water models. Therefore, we believe that this paper will make a significant contribution to the existing literature.

In the model proposed below, (Banks & Duggan, 2000, 2006) model of collective decision making is used for non-cooperative bargaining. The starting point for the application of non-cooperative bargaining theory is the Rubinstein (1982) model which is a static game of sharing a pie of size 1 with proposers taking turns, unanimous rule and an allocation of 0 in status-quo. Later models have sought to introduce more features to the model. Binmore (1987) extended the model to random selection of proposers. Baron & Ferejohn (1989) studied a version of the model with majority voting rule. Banks and Duggan (2000) generalize the previous models to include a multidimensional policy space but with the status-quo still remaining 0 untill a solution is reached. Banks and Duggan (2006) present a model where the status-quo is an arbitrary point in the policy space and determines the payoff till the bargaining solution has been reached.

This model is similar to Rausser & Simon (1992) model as both are inspired from Rubinstein (1982) game of sharing a pie of a normalized size. They both share the same feature of multiple players, multidimensional issue space, sequential proposal making and random selection of the agent making a proposal. The main difference is the equilibrium solution method with Banks & Duggan model providing equilibrium outcomes even when the core is empty, which makes it particularly useful for relatively unstructured settings. Banks and Duggan (2000) model has been applied extensively in the field of legislative policy making analysis in political sciences. Saborio-Rodriguez (2013) applied the model in a transboundary environmental externality problem of water pollution in Lerma-Chapala watershed in Mexico.

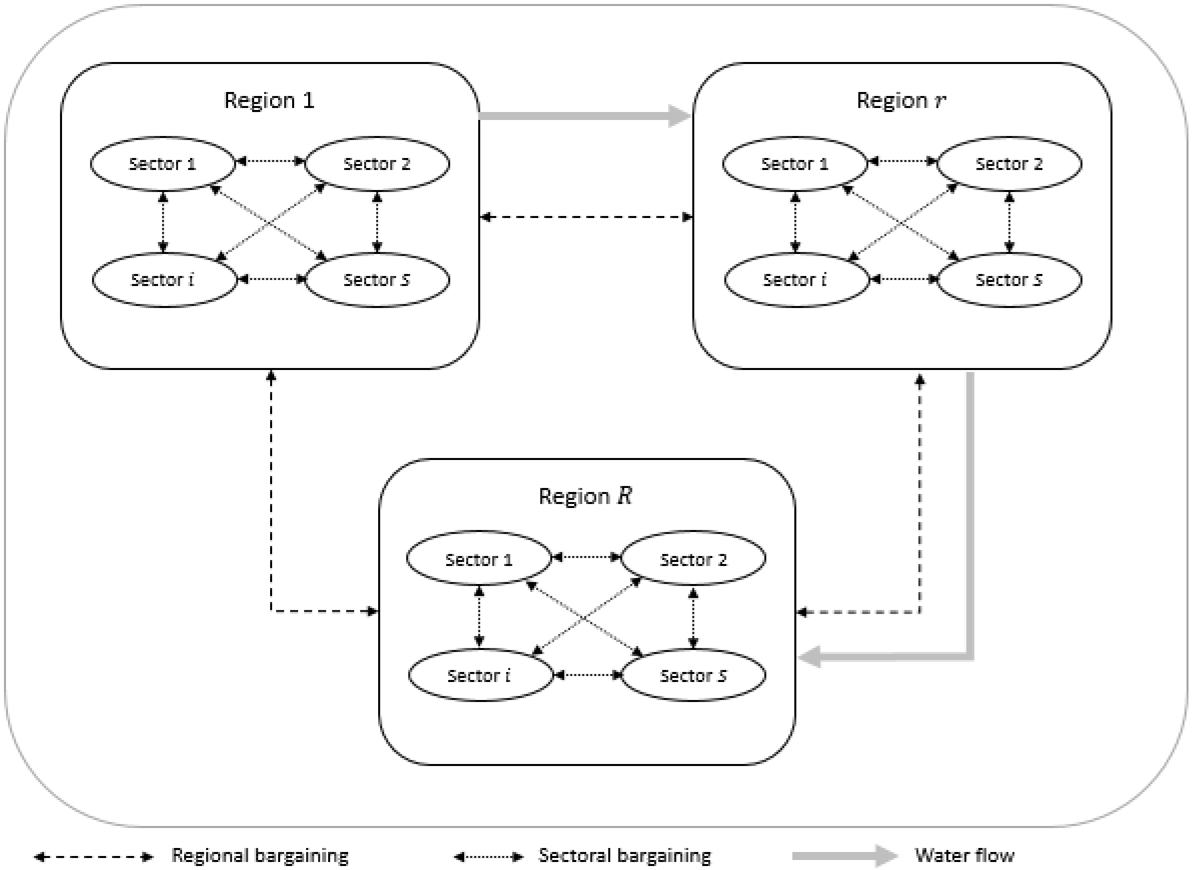

Fig. 1. *Hierarchical Framework for Regions and Sectors in the Basin*

3. The Model

The intra- and inter-sectoral schematics of the river basin and the bargaining model are presented in Figure 1 below. Let r denote a region reliant on surface water supply from the river basin, with $r = 1, 2, \dots, R$. These regions can be states or a sub-jurisdiction of the state in the water basin. For any two regions r and r' in this basin such that $r < r'$, r is defined to be upstream in relation to region r'. The direction of the flow is indicated by the grey line in Figure 1.

Within each region r, there are different sectors undertaking economic activity and demanding water for consumptive purposes. These sectors are denoted by i, with $i = 1, 2, \dots, S$. These sectors can be irrigation districts, municipality, manufacturing, recreation etc. The number of sectors may vary in different regions but is assumed here fixed to S for convenience. These sectors are shown as ovals in Figure 1, totally contained in their respective regions.

To avoid the complexity of water balances, it is assumed that all water withdrawals are net consumption. Water quality is assumed to be uniform across the basin. In addition, a partial equilibrium framework is assumed, in which the regions are small open economies with all the prices, except water prices, being exogenous. Water prices can be either market-driven or fixed administratively, depending on the region under study. The hierarchical structure can easily be expanded to 3-level; countries, states and sectors, depending on the basin under study.

The total volume of water in the river basin available for allocation is W_t in period $t > 0$ and is known at the beginning of the period. This supply is net of any navigational, ecological or treaty obligations required. The total supply is allocated to each region through a politically determined, historical allocation system which allocates a fraction $0 < \omega^r < 1$ to a region r, such that $\sum_r \omega^r = 1$. This forms the basis of 'status-quo' at the regional level and might not necessarily be efficient in the sense that at this allocation, there may be some regions placing a higher marginal valuation on water as compared to some other regions.

A Pareto efficient allocation will be the one at which the marginal valuation of all regions is equal. It can be achieved through a perfectly competitive water trading market in which case the marginal valuation of all regions at the optimal allocation would be 0. However, in some years there may not be adequate water in the river to supply the competitive allocation to all regions. In that case, the perfectly competitive trading requires an exogenous rule to share the water. Alternatively, a social planner allocating the water taking into account the supply constraints can also achieve the Pareto optimal allocation through a cooperative bargaining process.

It is possible and quite likely in the real world that all the regions do not recognize a central authority to undertake the water allocation. In such a case, the allocation may result from negotiations under non-cooperative bargaining. One way to model this negotiation is by letting the regions engage in bargaining among themselves by proposing a water allocation with an associated monetary transfer for compensation. Such a bargaining will help move water from regions which value it less to regions which place a higher value on it. In Figure 1 below, the dashed connections shows the bargaining process relationship among different regions. This is denoted as the regional bargaining problem.

In any region r, the total water allocation, $\omega^r W_t$, is allocated at the beginning of each period to sectors through status-quo allocation fractions $0 < \varepsilon_i^r < 1$, such that $\sum_i \varepsilon_i^r = 1$. These allocation ratios were politically determined in history and are taken as exogenous. The allocation resulting from the status-quo shares need not be Pareto efficient. Just like in the case of regions, the Pareto efficient allocation will result from perfectly competitive water trading or an allocation by the social planner. The sectors can alternatively engage in negotiations over the status-quo water allocation by proposing a change in sectoral water allocation and associated monetary transfers for compensation. For example, under non-cooperative bargaining an agriculture sector using water for producing low-value crops may find it uneconomical to continue if it is promised a monetary compensation higher than the returns from low-value crops in exchange for water rights. In Figure 1, the dotted connections show this negotiation process which is denoted here as the intra-sectoral bargaining problem.

In this section, we present a general framework for comparing the status-quo allocation against the Pareto efficient perfect competition, cooperative bargaining under a social planner, as well as the allocation under non-cooperative bargaining.

3.1 Notation

Denote the set of sectors as $N = \{1, 2, 3, \ldots, S\}$ where $S \geq 2$, belonging to a region r, where each $i \in N$ can be a municipality, industry, irrigation district, recreation sector or any other water consuming sector. In the case where sector i is a municipality or an irrigation district, the number of households or acres under irrigation are denoted as $li = 1, 2, 3, \ldots, L_i$.

Within each such i, the household or farm welfare function is denoted as $W_{i,li}(z_{i,li})$ where $z_{i,li}$ is the annual water consumption per household or acre in farm, measured in acre-feets. This welfare function is assumed to be continuous and concave. The individual agents within each sector are assumed to be identical, therefore we can denote $z_{i,li} = z_i$. The total welfare function in jurisdiction i is the sum-total of all the agents in it, and is denoted as $W_i(z_i)$, where

$$W_i(z_i) = \sum_{li=1}^{Li} W_{i,li}(z_i) = L_i W_{i,li}(z_i)$$

In the case of industrial sector, li can denote the number of firms and in case of recreation, it can denote the number of users. The main idea is to aggregate the individual demand curves in to a sectoral net welfare function.

Let each sector's total demand for water be $Z_i = L_i z_i$. The total water supply available to region r is assumed to be exogenous and fixed at Z_r^0. The water constraint is then

$$\sum_{i=1}^{S} L_i z_i \leq Z_r^0$$

The status-quo allocation is assumed to be $z^0 = \{z_1^0, z_2^0, z_3^0, \ldots z_S^0\}$ with $\sum_i L_i z_i^0 = Z_r^0$. We now derive the allocation under alternative market mechanisms.

3.2 Alternative Allocation Mechanisms

Under a perfectly competitive allocation, each sector maximizes its own welfare function and will demand water at the point where marginal welfare is zero. It is possible that the sum of sectoral demands exceed the available water supply Z_r^0, in which case the river is over appropriated. When this is the case, a cooperative outcome under a social planner allocation can achieve the Pareto optimal allocation. The problem for the social planner is to

$$Max_{z_i} \sum_i W_i(z_i)$$

$$s.t. \sum_i L_i z_i^0 \leq Z_r^0, \quad i = 1, 2, 3, \ldots, S$$

When the supply constraint is not binding, the social planner allocation will be the same as the competitive allocation.

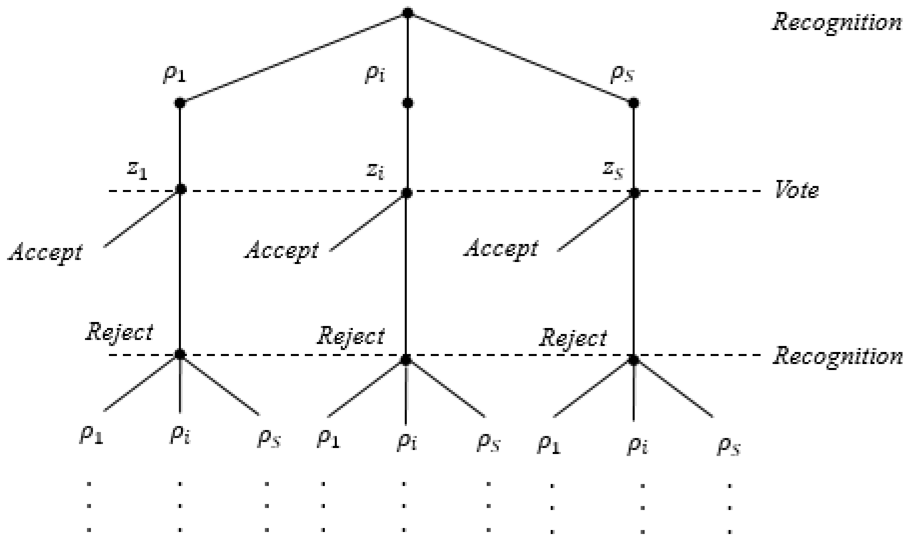

Fig. 2. *Non-cooperative Bargaining Game Tree*

3.2.1 Non-cooperative bargaining

Let $t = \{1, 2, 3, \dots T\}$ denote the time with a finite-horizon. The bargaining allocation results from a non-cooperative negotiation in which at time $t = 1$, a randomly selected sector is asked to put forward a proposal which specifies the division of the water resource among all sectors. Figure 2 shows the tree diagram for the dynamic game. The set of possible allocation is given by a convex, non-empty set of feasible allocation $Z \in \mathcal{R}^d$ and the status-quo allocations are given by $z^0 = \{z_1^0, z_2^0, z_3^0, \dots z_S^0\}$, where $z^0 \in Z$. The sum of the allocation to all sectors must be less than the available supply, i.e., $\sum_{z \in Z} z \leq Z^r$. The recognition probabilities are denoted as $\rho_i \in [0, 1]$ such that $\sum_i \rho_i = 1$ and are assumed to be exogenous and independent of time.

A proposal by sector j is $z_j = \{z_{1j}, z_{2j}, \dots, z_{Sj}\}$ such that $z_j \in Z$ and $\sum_i z_{ij} \leq Z^r$. Each sector $i \in N$ has preferences over Z and for the proposal by j, the welfare function for $i \in N$ is given by $W_i(z_{ij})$, where W is continuous and concave. It is also assumed that each sector has full information on the preferences of other sectors and the total supply available.

Once sector j has put forward the proposal, each sector votes to either *accept* or *reject* it. The voting rule is assumed to be unanimity, $\mathcal{D} = \{N\}$ and if the proposal z_j is accepted, the game ends and it becomes the allocation rule with each sector earning the payoff $W_i(z_{ij})$ for each period till time T. Otherwise the sectors get their status-quo payoff $W_i(z_i^0)$ in that period and game resumes with a new recognition round. If no proposal is accepted in time T, sectors get their status-quo payoff and the game ends.

Although not strictly necessary, we also assume that the status-quo is inefficient in the sense that some sectors have more water than their welfare maximum level while others have less. Therefore, there are possibilities of improvements on the status-quo. In the absence of this assumption, the bargaining will simply yield the trivial solution of status-quo as the allocation.

Let $\delta_i \in (0,1)$ denote the discounting factor for the sector i, then for a proposal z_j accepted at time t, the total payoff for sector i as evaluated at the beginning of the negotiation is the sum of the discounted payoff from status-quo till time $t-1$, and the payoff from proposal allocation from period t to T,

$$\delta_i^0 W_i(z_i^0) + \delta_i^1 W_i(z_i^0) + \cdots + \delta_i^{t-1} W_i(z_i^0) + \delta_i^t W_i(z_{ij}) + \cdots + \delta_i^T W_i(z_{ij})$$

which can also be written as

$$\frac{1-\delta_i^{t-1}}{1-\delta_i} W_i(z_i^0) + \frac{\delta_i^{t-1} - \delta_i^T}{1-\delta_i} W_i(z_{ij})$$

For each sector i, the expected payoff from the next round of bargaining is $U_i(z)$, given as the sum of the payoff resulting from proposal z_j from each $j \in N$ multiplied by the recognition probability of j.

$$U_i(z) = \sum_{j=1}^{S} \rho_j W_i(z_{ij})$$

For a proposal under recognition at any time t, the continuation value C_i^t evaluated at time t for sector i is the expected payoff from *rejecting* the proposal, which is the sum of status-quo payoff at time t and the discounted sum of expected payoff in the next round.

$$C_i^t = W_i(z_i^0) + \frac{\delta_i^t - \delta_i^T}{1-\delta_i} U_i(z)$$

The Acceptance set A_i for each sector i is the set of proposals for which the sector will vote *accept* and is assumed to be stage-undominated such that when the payoff from accepting the proposal is at least as large as the payoff from rejecting, the sector votes to accept. Therefore,

$$A_i = \left\{ z_j \in Z \mid W_i(z_j) \geq \frac{1-\delta_i^{t-1}}{1-\delta_i} W_i(z_i^0) + \frac{\delta_i^{t-1} - \delta_i^T}{1-\delta_i} W_i(z_{ij}) \right\}$$

In identifying the equilibrium strategy for this game, we focus on no-delay, stationery pure strategy sub-game perfect Nash equilibrium (PSNE) the existence and uniqueness of which has been proven in Banks and Duggan (2006). No-delay implies that the decision is reached in the first period. Stationery strategies imply that the equilibrium strategy constitute the equilibrium strategy in each period of time and the sectors have no incentive to deviate from it.

Proposition 1. *The no-delay, pure strategy SPNE in the finite-horizon game is given by a proposal* $z_j^* = \{z_{1j}^*, z_{2j}^*, \ldots, z_{Sj}^*\}$ *such that* $\forall i \neq j$, z_j^* *is such that*

$$W_i(z_{ij}^*) \geq W_i(z_i^0) + \frac{\delta_i - \delta_i^T}{1-\delta_i} U_i(z), \textit{and}$$

$$z_{jj}^* = Z^r - \sum_{i \neq j} z_{ij}^*$$

where j is the randomly selected proposer in time $t = 1$. This outcome will be obtained as a simultaneous solution to the problem of each sector

$$\max_{Z_i \in Z} W_i(z_{ii}),\ such\ that$$

$$\forall i \neq j, \qquad W_j(z_{ij}) \geq W_i(z_i^0) + \frac{\delta_i - \delta_i^T}{1 - \delta_i} U_i(z), and$$

$$z_i \in Z$$

Proof. We prove this proposition by using backward induction. In the last period at time T, the expected payoff from next round of bargaining $U_i(z)$ is simply equal to 0. Which means that for each sector, the continuation value is simply the status-quo payoff. If j is the randomly selected proposer, then the best strategy for j is to allocate everyone else their status-quo payoff and keep the rest. Since the status-quo is inefficient, it ensures that there are enough gains left on the table for j compared to its status-quo payoff. Any proposal giving sectors less than their status-quo will be rejected and j's payoff will be lower than under this strategy.

At time $T - 1$, let j be the randomly selected proposer. The continuation value for each sector is

$$C_i^{T-1} = W_i(z_i^0) + \delta_i U_i(z)$$

The best strategy for j is to allocate other sectors their continuation value and keep the remainder. Continuing the same way, the continuation value for a sector i in the first period $t = 1$ is

$$C_i^1 = W_i(z_i^0) + \frac{\delta_i - \delta_i^T}{1 - \delta_i} U_i(z)$$

If j is the randomly selected proposer then the same strategy of choosing an allocation such that all other sectors get their continuation value, while j keeps the rest is the best strategy in period 1 as well. Therefore, the strategy characterizes the stationery and sub-game perfect Nash equilibrium. It involves no-delay because the proposer selected in the first period will propose such an allocation which everyone will accept and the game will end in the first period.

The strategy is equivalent to each sector maximizing its own payoff given the constraint of assuring each other sector their continuation value and the total supply constraint. It can be easily verified (also shown in the paper later) that the first-order conditions with respect to z_{ij} in each sector i's maximization problem are identical. Therefore, the resulting allocation is symmetric and unique. ■

A similar strategy is also the no-delay PS SPNE in the infinite horizon version of the game where the bargaining continues until a proposal is accepted.

Proposition 2. *The no-delay, pure strategy SPNE in the infinite-horizon game is given by a proposal $z_j^* = \{z_{1j}^*, z_{2j}^*, \dots, z_{Sj}^*\}$ such that $\forall i \neq j$, z_j^* is such that*

$$W_i(z_{ij}^*) \geq W_i(z_i^0) + \frac{\delta_i}{1 - \delta_i} U_i(z), and$$

$$z_{jj}^* = Z^r - \sum_{i \neq j} z_{ij}^*$$

where j is the randomly selected proposer in time $t = 1$. This outcome will be obtained as a simultaneous solution to the problem of each sector

$$\max_{Z_i \in Z} W_i(z_{ii}),\ such\ that$$

$$\forall i \neq j, \qquad W_j(z_{ij}) \geq W_i(z_i^0) + \frac{\delta_i}{1-\delta_i} U_i(z), and$$

$$z_i \in Z$$

Proof. The proof for the proposition is similar to Proposition 1 with some minor changes to incorporate the infinite horizon. At any time t, the continuation value for sector i is

$$C_i^t = W_i(z_i^0) + \frac{\delta_i}{1-\delta_i} U_i(z)$$

In the first period, if j is the randomly selected proposer, then the best strategy is to put forward a proposal which guarantees all the other sectors their continuation value while j keeps the remainder. The rest of the argument is the same as in proof of Proposition 1. ■

The discount factor δ_i denotes the degree of impatience for a sector, with a value close to 0 implying a strong preference towards immediate consumption (future payoff is worth very little) and a value close to 1 implying a strong indifference between consumption in the present or in the future (the payoff in the future is worth close to present payoff). Sectors with lower patience have a continuation value close to the status-quo payoff and therefore require only a small increase over the status-quo to vote accept. The sectors with a high degree of patience however require relatively larger payoff over and above the status-quo to vote accept.

Another thing to note is the condition for the existence of the non-trivial bargaining equilibrium which states that there should be enough inefficiency at the status quo that after giving each sector (other than the proposer) their continuation value, there is enough resource left to yield the proposer more than its own continuation value. This is an indicator of the non-cooperative aspect of the mechanism which is unable to eliminate all the allocative inefficiency. For a status-quo which is not enough inefficient, no proposal might be able to guarantee each sector their continuation value in which case status-quo would result in each period. With specific functional forms, the actual threshold can be calculated and will indicate the level of pure inefficiency attributable to the non-cooperative mechanism relative to a cooperative Pareto efficient allocation of a social planner.

3.3 Water allocation problem assuming functional forms

In this section, we discuss closed-form solutions for a simple model assuming 3 sectors and quadratic welfare functions. For the residential and agricultural sectors $i \in \{1, 2, 3\}$, let the household or farm welfare function be $W_{i,li}(z_{i,li})$ where $z_{i,li}$ is the annual water consumption per household or acre in farm, measured in acre-feets. The welfare function for each agent li in sector i is assumed to take a quadratic form,

$$W_{i,li}(z_{i,li}) = a_{0i} + a_{1i}z_{i,li} - a_{2i}z_{i,li}^2$$

where $a_{0i}, a_{1i}, a_{2i} \geq 0$ are parameter coefficients varying in each sector but same for all the agents within each sector. The first two parameters can be seen as a benefit coefficient from water use, and a_{2i} can be seen as the damage or cost coefficient which excess water use can result in. These damages can be the requirements of new infrastructure, yield loss for crops, and other loss of property which excess water allocation may present. Linear demand curves for water uses in household, agriculture, recreation and industry will yield a quadratic net welfare function where the total welfare peaks at the price of zero, yielding negative marginal welfare for water consumption beyond that point.

For identical agents within each sector, $z_{i,li} = z_i$ and the total welfare function in jurisdiction i is

$$W_i(z_i) = \sum_{li=1}^{Li} W_{i,li}(z_i) = L_i(a_{0i} + a_{1i}z_i - a_{2i}z_i^2)$$

For other sectors like manufacturing or recreation, L_i can be assumed to be number of factories, or number of recreation users respectively. L_i can also be simply assumed to be the weight associated with the welfare. The total water supply for the sector i in region r is as before given by

$$\sum_{i=1}^{3} L_i z_i \leq Z_r^0$$

In a competitive setup, each jurisdiction solves the problem

$$Max_{z_i} W_i(z_i) = L_i(a_{0i} + a_{1i}z_i - a_{2i}z_i^2)$$

which is attained at the allocation level,

$$z_i^* = \frac{a_{1i}}{2a_{2i}}$$

which is the point where marginal benefits associated with the water use are equal to marginal damages. It is however possible that $\sum_{i=1}^{S} z_i^* \leq Z_r^0$ in which case the allocation is not feasible. This might be the case when the river is over appropriated, with the water compact promising each jurisdiction its ideal point, but since the supply is lower, that promise cannot be fulfilled. However, the allocation level is still a useful benchmark and can be seen as an ideal point each sector seeks to achieve.

Looking at the cooperative bargaining, the problem for the social planner is to

$$Max_{z_i} \sum_{i=1} W_i(z_i) \;\; s.t. \;\; L_1z_1 + L_2z_2 + L_3z_3 \leq Z^0, \quad i = 1, 2, 3$$

When the water supply is abundant and the constraint is not binding, then the optimal solution is

$$z_i^* = \frac{a_{1i}}{2a_{2i}}$$

which is the same as competitive solution. But when the constraint is binding, the Pareto efficient allocation for any sector j is given as

$$z_j^* = \frac{Z_r^0 - \sum_{i\neq j} \frac{L_i}{2a_{2i}}\left(a_{1i} - a_{1j}\right)}{L_j + \sum_{i\neq j} L_i \frac{a_{2j}}{a_{2i}}}$$

3.3.1 Bargaining

This 3-sector model of bargaining uses slightly expanded notation system than the section above and a system of monetary payments along with water allocation has been introduced to facilitate bargaining. Let a proposal $x_j = \{\bar{z}_{1j}, y_{1j}, \bar{z}_{2j}, y_{2j}, \bar{z}_{3j}, y_{3j}\}$ be the proposal put forward by agent j proposing allocation levels $\bar{z}_{ij}$ and monetary payments y_{ij}, to be paid by j to i if $y_{ij} > 0$. Let the status quo be $x_j^0 = \{z_1^0, 0, z_2^0, 0, z_3^0, 0\}$ with $\sum_i L_i z_i^0 = Z_r^0$ which is the available water supply for the region.

The sectoral welfare function from proposal x_j for sector i is

$$W_i(x_j) = L_i\left(a_{0i} + a_{1i}\bar{z}_{ij} - a_{2i}\bar{z}_{ij}^2\right) + y_{ij}$$

which the sector compares with the welfare under the allocation level $\dot{z}_i$, which is the ideal point and is assumed to be the competitive maximum allocation. The welfare at this level is given as

$$W_i(\dot{z}_i) = L_i\left(a_{0i} + a_{1i}\dot{z}_i - a_{2i}\dot{z}_i^2\right)$$

The payoff function for the agent i from proposal x_j is denoted as $V_i(x_j)$ and is the difference between $W_i(x_j)$ and $W_i(\dot{z}_i)$.

$$V_i(x_j) = W_i(x_j) - W_i(\dot{z}_i)$$

$$V_i(x_j) = L_i\left(a_{1i}(\bar{z}_{ij} - \dot{z}_i) - a_{2i}(\bar{z}_{ij}^2 - \dot{z}_i^2)\right) + \varphi_i y_{ij}$$

The first part of the payoff function can be seen as the net benefit (loss) from shifting consumption towards (away) the ideal point. The second part is the monetary payment (compensation) for that shift adjusted by a factor $\varphi_i \in \mathcal{R}$ which indicates the willingness of the sector to accept the monetary payment. Since water and money are not perfect substitutes, a sector with a lower value of φ_i will require larger monetary payments to accept any given proposal compared to a sector with higher φ_i. The payoff function is continuous in all its arguments, increasing in the payment made by j to i, and is concave in the proposed allocation $\bar{z}_{ij}$.

To simplify, we assume that the recognition probabilities ρ_i are equal for all three sectors and therefore, $\rho_i = \rho = 1/3$. The discount factors are also assumed to be identical for all the sectors at $\delta_i = \delta \in (0, 1)$.

In this discussion, we assume that the bargaining continued indefinitely with an infinite time horizon. The results are analogous for the finite time horizon version.

Expected payoff from bargaining in the next period, $U_i(x)$ denoted as

$$U_i(x) = \rho\big(V_i(x_1) + V_i(x_2) + V_i(x_3)\big)$$

The continuation value for sector i in the first period, normalized for the discounted value of time for notational convenience, is given as

$$C_i^1 = \left[\sum_{t=1}^{\infty} \delta^{t-1}\right]^{-1} \left(V_i(x^0) + \frac{\delta}{1-\delta} U_i(x)\right) = (1-\delta)V_i(x^0) + \delta U_i(x)$$

And the acceptance set for a sector i becomes

$$A_i = \{z_j \in Z \mid W_i(z_j) \geq (1-\delta)V_i(x^0) + \delta U_i(x)\}$$

Proposition 3. *In this non-cooperative bargaining problem with a unanimity decision rule, for any randomly selected proposer $j \in N$, the no-delay stationery SPNE is an allocation vector x_j^*, where the allocation to a sector $p \in N$ is*

$$\bar{z}_{pj}^* = \frac{Z_r^0 - \sum_{i\neq p} \frac{L_i}{2a_{2i}}\left(a_{1i} - \frac{\varphi_i}{\varphi_p} a_{1p}\right)}{L_p + \sum_{i\neq p} L_i \frac{\varphi_i}{\varphi_p}\frac{a_{2p}}{a_{2i}}}$$

The associated vector of monetary payments is given as

$$y_{jj} = \left(\frac{V_j^0}{\varphi_j} - \frac{\tilde{V}_j}{\varphi_j}\right) - \frac{3-2\delta}{3}\sum_{k=1}^{3}\left(\frac{V_k^0}{\varphi_k} - \frac{\tilde{V}_k}{\varphi_k}\right)$$

$$y_{ij} = \left(\frac{V_i^0}{\varphi_i} - \frac{\tilde{V}_i}{\varphi_i}\right) - \frac{\delta}{3}\sum_{k=1}^{3}\left(\frac{V_k^0}{\varphi_k} - \frac{\tilde{V}_k}{\varphi_k}\right)$$

Proof: See the Appendix. ■

Before applying this bargaining problem to the empirical data in the next section, it would be instructive to highlight some important points from the equilibrium allocation. Firstly, while the framework has been presented for only one sector, it can be easily adopted at the regional level. The hierarchical bargaining will then start with negotiations at the regional level taking into account the total water supply available to the basin. The regional water shares will then act as the exogenous water supply for sectoral bargaining. The allocation can take place annually or with a longer horizon assuming that the required coefficients change exogenously. The outcomes can be easily compared between two periods differing in exogenous coefficients. The bargaining outcome therefore highlights the endogenous institutional change in water allocation in response to a dynamically changing environment.

Secondly, the sectoral population L_p is inversely related to per household or per acre allocation share as $\partial \bar{z}_{pj}^{*}/\partial L_p < 0$. For a sector with larger population, the water allocated to each sub-unit is lower which reflects the division of a finite resource among a larger population. But how does the population affects a sector's bargaining power? The sectoral water allocation is given by $\bar{Z}_{ij}^{*} = L_p \bar{z}_{ij}^{*}$ and $\partial \bar{Z}_{ij}^{*}/\partial L_p \geq 0$ indicating that the population size acts as a weight in the bargaining process and is one determinant of negotiating power. However, if the population increases across

Thirdly, looking at the factor on willingness to accept monetary payment φ_i, its impact on the allocation level is ambiguous. However, it does influences the negotiating power as far as the monetary payments are concerned with $\partial y_{ij}/\partial \varphi_i \leq 0$ if $\left(V_i^0 - \tilde{V}_i\right) \geq 0$ and negative otherwise. $\tilde{V}_i$ is the non-monetary part of the payoff in $V_i\left(x_j\right) = \tilde{V}_i + y_{ij}$. This means what when the allocation is such that the proposal takes the sector closer to the ideal point, a sector with lower willingness to accept the monetary payment will require a larger payment compared to a sector with a higher willingness to accept the payment.

Fourthly, the sectoral coefficient a_{1i} can be interpreted as the technological factor in putting the water to productive uses. A higher value indicates that the same net welfare can be achieved with lower water use. In comparing the allocation between two time period far apart such that the coefficient a_{1i} is larger in the future for each sector, the technological progress implies that each sector will be allocated lower water shares compared to the earlier period but it need not result in a loss of welfare.

Finally, the total supply Z_r^0 is responsive to changes in hydrology resulting from phenomenon such as climate change. In comparing between two time periods such that the future has a lower water supply, the allocation to all the sectors will decrease. If none of the other coefficients have changed, then this results in a lower net welfare for all sectors.

The ability of the bargaining model to deal with many pertinent exogenous changes and allowing the water allocation institutions to change endogenously in response shows the usefulness of the approach in further analysis and policy-making.

4. Application to the Upper Rio Grande Basin

The Upper Rio Grande Basin spans 3 states; Colorado, New Mexico and Texas. Taking into account the flow characteristics, Rio Grande can be treated as two rivers with the dividing line just below Fort Quitman, Texas (Southeast of El Paso). The upper stretch receives water from snowmelt in Colorado and many tributaries while the lower stretch receives water from Rio Conchos and Pecos. The river is therefore allowed to stop flowing at the end of the upper stretch making the Upper Rio Grande basin a closed system amenable to empirical analysis (Rister, Sturdivant, Lacewell, & Michelsen, 2011).

The upper stretch can be further broken down into a hierarchical jurisdiction of 3 states; Colorado (CO), New Mexico (NM) and Texas (TX), and different sectors such as municipalities and industries (M&I) or irrigation districts (AG) under each. Following Ward et al. (2006), the classification in the order of flow is as shown in Table 2.

Table 2.
Economic sectors in the Upper Rio Grande Basin

Location	Label	State	Sector
San Luis Valley	SLV	CO	AG
Albuquqrque	ALB	NM	M&I
Middle Rio Grande Conservancy District	MRGCD	NM	AG
Elephant Butte Irrigation District	EBID	NM	AG
El Paso Municipality	EPMI	TX	M&I
El Paso Irrigation District	EPID	TX	AG

In addition there are 6 reservoirs in New Mexico used for water storage and recreation while there are no reservoirs on the river in Colorado or Texas. For the sake of simplicity, the model does not takes into account reservoir recreation as a negotiating sector because it will require incorporating existing water levels and evaporation rates to calculate their consumptive requirements. In the hierarchical model then, there are 3 states (equivalent to regions in the notation above) and sectors within each of them as shown in Table 2. The bargaining model requires 2 or more players so the intrastate negotiations will take place between sectors in Texas and New Mexico, while the interstate negotiations will take place between the 3 states. We assume the discount rate to be equal for all the 3 states and sectors at $\delta = 0.95$ and the willingness to accept the monetary payment $\varphi = 1$.

4.1 Data

In order to calculate the water resource allocation for regions and sectors, data is needed on the total water availability, status-quo allocations, population in the M&I sectors and irrigated land under agriculture, and the coefficients for the net welfare functions. We use the data from Ward et al. (2006) and Ward & Pulido-Velázquez (2012) which also explain in details how it was obtained. The coefficients on the quadratic welfare functions for the sectors are derived from the linear demand functions which attain a maximum at the price of 0. Table 3 summarizes the supply, households or farm acres and coefficients for the five sectors.

The water consumption column shows the status-quo allocation as average values between 2000 and 2010. The number of households in ALB and EP are for the year 2007 which is the period for which the results are calculated for this model. This data gives the status-quo values for z_i^0 shown in the column of per capita use. Since the data is available only at the sectoral level, it needs aggregation up to the state level for inter-state allocation calculations.

We assume that the states place weights on the sectoral per capita consumption for households or acres, denoted as β_i^r, where $0 < \beta_i^r < 1$ and $\sum_i \beta_i^r = 1$ for each r. This factor reflects the preferences of the state

Table 3.

Sectoral data on water resources utilization and welfare

State	Sector	Water consumption ('000)	Hhd/ acre ('000)	Water consumed per household or acre (ac-ft per yr)	Coefficients				Max Net Welfare consumption per hhd or acre
					a0	a1	a2	β	
CO	SLV	612.00	205	2.99	195	145	14	1	5.18
NM	ALB MI	45.19	107	0.42	0	10843	9627	0.25	0.56
	MRGCD	288.00	63	4.57	-30	67	6	0.375	5.58
	EBID	199.00	91	2.19	137	94	7.12	0.375	6.60
TX	EP MI	43.35	120.56	0.36	0	9507	9392	0.5	0.51
	EP AG	102.00	47	2.17	0	193	21	0.5	4.60

Table 4.

Aggregated state level data on water resources utilization and welfare

State	Water consumption ('000)	Hhd/acre ('000)	Water consumed per household or acre (ac-ft per yr)	Coefficients			Max Net Welfare consumption per hhd or acre
				a0	a1	a2	
CO	612.00	205	2.98	195	145	14	5.18
NM	532.19	261	2.03	41	1130	247	2.28
TX	145.35	167.56	0.87	0	3447	1692	1.02

on per capita consumption level for farm or households and the assumed values are shown in Table 3. In the state welfare function, a higher weight to the agriculture sector is associated with a high water requirement at the ideal point because per acre requirements in agriculture are higher than the residential sector. Analogously, a larger weight to the M&I sector will imply that the ideal water allocation level for the state is on a lower side. Table 4 shows the aggregated data with the total water consumed by each state, the status-quo allocation, the ideal point and the coefficients for the quadratic welfare function. The total available water supply is lower than the ideal point of maximum welfare level, indicating that the river is over-allocated.

Based on this data, four sets of allocations for regions and sectors are calculated using the analytical results from the Section 3. These are:

i. Status-quo allocation
ii. Competitive maximum: since the supply constraint is not taken into account here, this allocation is unattainable under over-allocation but serves as an ideal point.

iii. Social planner's maximum as the Pareto optimal and cooperative bargaining allocation under water supply constraint
iv. Non-cooperative bargaining solution under water supply constraint.

Note that when there are no constraints on water resource, (ii) and (iii) are equivalent. Bargaining outcome is trivial with zero monetary payments and the allocation is also the same as in (ii). The state level allocations are assumed to take place first, with the sectors taking these allocation levels as exogenously determined supply in their bargaining.

4.2 Results

Table 5 presents the allocation and welfare accruing to each state under the status-quo, the unattainable ideal point, the social planner's cooperative bargaining and the inefficient non-cooperative bargaining. The status-quo allocation results in CO being way too far away from its ideal point compared to NM and TX. This is because urban water use in NM and TX works to reduce their ideal point for water consumption, but CO has only an agricultural sector with high water needs. Under the supply constraint, the social planner reduces the amount of water going to CO and increases it for NM and TX. This results in a lower welfare for CO under social planner compared to the status quo. Since CO is a net loser under the cooperative bargaining, it may not accept the outcome.

For the non-cooperative bargaining, first note that the monetary payments and welfare levels differ depending on which state is selected as the proposer. The proposer always seeks to maximize own receipts or minimize own payments. Here we see that irrespective of the proposer, CO receives monetary payments from NM and TX which increases its welfare to more than the status-quo. TX is able to reach very close to its ideal consumption level and makes the major chunk of the payment, and still ends up with a welfare level higher than under the social planner. This is because the urban El Paso dominates the payoff for Texas and is able to bring quite high marginal benefits with a smaller increase in water supply, some of which are used to compensate CO. NM on the other hand receives more water than TX but makes little monetary payments due to the bargaining power from the presence of agricultural sectors lowering its marginal valuation.

The sum of the total welfare across the 3 sectors under non-cooperative bargaining is the same as the total welfare of the social planner but the distribution across states differs. CO gains the most due to the monetary payments and the bargaining power from holding a relatively larger status-quo allocation. There are still efficiency gains as the total welfare is more than the status-quo, but the inter-state inefficiency remains.

Table 6 presents the results from different allocation mechanisms in New Mexico after the state level bargaining has taken place. There are three important things to note in the bargaining results. Firstly, ALB ends up receiving enough water to reach its ideal point. EBID sees an increase in water share as well and MRGCD is the sector with reduced water consumption.

Table 5.

State level allocation under various mechanisms

Sector	Colorado	New Mexico	Texas
Status-quo			
Consumption per acre or hhd	2.98	2.04	0.87
Total water allocated (‘000 af per year)	612	532	145
Welfare ($ ‘000)	103,136	344,041	287,677
Total welfare for 3 sectors ($ ‘000)	734,855		
Total water available (‘000 af per year)	1,289		
Ideal (full supply)			
Consumption per acre or hhd	5.18	2.29	1.02
Total water required (‘000 af per year)	1061	597	170
Welfare ($ ‘000)	116,942	348,020	294,146
Social planner (constrained supply)			
Consumption per acre or hhd	2.74	2.15	1
Total water allocated (‘000 af per year)	561	561	167
Welfare ($ ‘000)	99,850	346,786	294,030
Total welfare for 3 sectors ($ ‘000)	740,667		
Non-cooperative Bargaining (constrained supply)			
Consumption per acre or hhd	2.74	2.15	1
Total water allocated (‘000 af per year)	561	561	167
Payment by column to CO* ($ ‘000)	5,408	5,131	5,131
Payment by column to NM* ($ ‘000)	(900)	(623)	(900)
Payment by column to TX* ($ ‘000)	(4,508)	(4,508)	(4,231)
Welfare for CO* under proposal by column ($ ‘000)	105,258	104,981	104,981
Welfare for NM* under proposal by column ($ ‘000)	345,886	346,163	345,886
Welfare for TX* under proposal by column ($ ‘000)	289,522	289,522	289,799
Total welfare ($ ‘000)	740,667	740,667	740,667

* The state in the column is the proposer

Secondly, ALB has a high valuation of water and ends up making payments to other irrespective of the proposer but in its own proposal, it ends up paying the least. MRGCD ends up with a reduced water allocation and in return receives most of the compensation. EBID receives some compensation too despite an increase in water use. The reason is the higher status-quo allocation to EBID which raises the continuation value and gives bargaining power to EBID for its vote.

Table 6.

Sectoral allocation under various mechanisms in New Mexico

Sector	**ALB MI**	**MRGCD AG**	**EBID AG**
Status-quo			
Consumption per acre or hhd (af per year)	0.42	4.57	2.19
Total water allocated (‘000 af per year)	45	288	199
Welfare ($ ‘000)	306,248	9,507	28,075
Total welfare for 3 sectors ($ ‘000)	343,289		
Total water available (‘000 af per year)	561 after bargaining (532 before it)		
Ideal (full supply)			
Consumption per acre or hhd (af per year)	0.56	5.58	6.60
Total water allocated (‘000 af per year)	60	352	601
Welfare ($ ‘000)	326,687	9,894	40,700
Social planner (constrained supply) – supply increases after state bargaining			
Consumption per acre or hhd (af per year)	0.56	2.35	3.87
Total water allocated (‘000 af per year)	60	148	353
Welfare ($ ‘000)	326,683	5,943	35,891
Total welfare for 3 sectors ($ ‘000)	368,517		
Non-cooperative Bargaining (constrained supply)			
Consumption per acre or hhd (af per year)	0.56	2.35	3.87
Total water allocated (‘000 af per year)	60	148	353
Payment by column to ($ ‘000) ALB MI	(11,422)	(12,597)	(12,597)
Payment by column to ($ ‘000) MRGCD AG	11,401	12,577	11,401
Payment by column to ($ ‘000) EBID AG	21	21	1,196
Welfare for ALB MI under proposal by column ($ ‘000)	315,261	314,085	314,085
Welfare for MRGCD AG under proposal by column ($ ‘000)	17,344	18,520	17,344
Welfare for EBID AG under proposal by column ($ ‘000)	35,912	35,912	37,088
Total welfare ($ ‘000)	368,517	368,517	368,517

Table 7.

Sectoral allocation under various mechanisms in Texas

Sector	El Paso MI	El Paso AG
Status-quo		
Consumption per acre or hhd (af per year)	0.36	2.17
Total water allocated (‘000 af per year)	43	102
Welfare ($ ‘000)	265,728	15,037
Total welfare for 3 sectors ($ ‘000)	280,766	
Total water available (‘000 af per year)	167 after bargaining (145 before)	
Ideal (full supply)		
Consumption per acre or hhd (af per year)	0.51	4.59
Total water allocated (‘000 af per year)	61	216
Welfare ($ ‘000)	290,050	20,842
Social planner (constrained supply)		
Consumption per acre or hhd (af per year)	0.50	2.27
Total water allocated (‘000 af per year)	60	107
Welfare ($ ‘000)	290,019	15,524
Total welfare for 3 sectors ($ ‘000)	305,543	
Non-cooperative Bargaining (constrained supply)		
Consumption per acre or hhd (af per year)	0.50	2.27
Total water allocated (‘000 af per year)	60	107
Payment by column to ($ ‘000) EP MI	(11,312)	(12,492)
Payment by column to ($ ‘000) EP AG	11,312	12,492
Welfare for EP MI under proposal by column ($ ‘000)	278,707	277,527
Welfare for EP AG under proposal by column ($ ‘000)	26,836	28,016
Total	305,543	305,543

Thirdly, the total net welfare for 3 sectors combined under bargaining is the same as the one under social planner but the sectoral distribution is not the same. ALB ends up with a lower net welfare in all the 3 proposal cases, due to the payments it needs to make. MRGCD and EBID end up with higher net welfare than under a social planner because they receive payments.

Table 7 presents the allocations under different mechanisms for the 2 sectors in Texas. We see here again that the status-quo allocation results in a scenario where both the sectors are far away from their ideal point due to water constraint. The social planner allocation makes sure that the El Paso MI achieves its ideal point by taking water from El Paso AG which would result in higher net welfare for both the sectors.

Under bargaining, the allocation results in El Paso MI paying a monetary compensation of $11.3 million to El Paso AG (under El Paso MI's proposal) and a slightly higher payment under El Paso AG's proposal. No matter who gets to propose, El Paso AG stands to receive a higher net welfare than that would result under social planner because it is able to extract rent out of the water resources it holds in the status-quo. The sum of net welfare is the same as under the social planner.

Looking at the overall picture, there are two more observations. Firstly, NM and TX end up making payments to CO. It is assumed here that these payments do not change the sectoral net welfare function in each state. The assumption will not hold true if state level payments are financed from increasing water prices, which may end up changing the sectoral net welfare function and therefore, the state level net welfare function.

Secondly, the weights β_i given to each sector in NM and TX are important determinant of the state's ideal point and will influence the outcome. For example, giving equal weight to 3 sectors in NM will raise the importance of ALB MI and reduce the ideal point to below the status-quo. In that case, NM will try to get rid of the excess water and make payments to reduce its consumption.

6. Conclusion

Given the institutional nature of the water allocation, the goal of this paper was to design an alternative to perfectly competitive and cooperative water allocations mechanisms used for modeling water resources. Using simplifying assumptions, we developed a non-cooperative bargaining model of water allocation where participating sectors can improve on the status-quo allocation by proposing an allocation along with the monetary payments to other sectors. The resulting allocation is able to increase the overall welfare to the same level as under a social planner but due to the non-cooperative nature of the bargaining, the distribution of the gains are dependent on the bargaining power with each sector.

The analytical solutions assuming quadratic welfare functions suggest that the bargaining power result from the status-quo allocation, household population or acres under irrigation, efficiency of water use and the willingness to accept monetary payment in lieu of water. The non-cooperative bargaining is able to endogenize the institutional changes resulting from changes in sectoral characteristics due to population or technological change or changes in water availability due to climate change.

The core idea is to use continuous and concave net welfare functions for water use given the water supply and perfect information. The non-cooperative bargaining solution will be achieved by a constrained optimization problem where each sector maximizes its own welfare taking into account the supply constraint and giving every other sector their continuation value. With these simple requirements, the allocation can be calculated for even complex welfare functions for multitudes of sectors using numerical methods. Further constraints on basin hydrology, surface water, reservoir levels and treaty obligations can be easily incorporated in a larger model. The hierarchical structure is able to adapt to different basin structures from small economic sectors in a single region or transboundary negotiations between different countries.

As a result, the non-cooperative bargaining framework has potential to guide policy-making by suggesting feasible allocations and presenting a suitable benchmark between the inefficient status-quo allocation and the efficient but difficult to achieve cooperative allocation.

References

Adams, G., Rausser, G., & Simon, L. (1996). Modelling multilateral negotiations: An application to California water policy. *Journal of Economic Behavior and Organization*, *30*(1996), 97–111. http://doi.org/10.1016/S0167-2681(96)00844-X

Ambec, S., & Ehlers, L. (2008). Sharing a river among satiable agents. *Games and Economic Behavior*, *64*, 35–50. http://doi.org/10.1016/j.geb.2007.09.005

Banks, J. S., & Duggan, J. (2000). A Bargaining Model of Collective Choice. *The American Political Science Review*, *94*(1), 73–88. http://doi.org/10.2307/2586381

Banks, J. S., & Duggan, J. (2006). A General Bargaining Model of Legislative Policy-making. *Quarterly Journal of Political Science*, *1*(March 2005), 49–85. http://doi.org/10.1561/100.00000001

Bell, A., Zhu, T., Xie, H., & Ringler, C. (2014). Climate-water interactions-Challenges for improved representation in integrated assessment models. *Energy Economics*. http://doi.org/10.1016/j.eneco.2013.12.016

Brewer, J., Glennon, R., Ker, A., & Libecap, G. (2008). Water markets in the west: Prices, trading, and contractual forms. *Economic Inquiry*, *46*, 91–112. http://doi.org/10.1111/j.1465-7295.2007.00072.x

Brewer, J., Ker, A., Glennon, R., & Libecap, G. (2006). Transferring Water in the American West: 1987–2005. *University of Michigan Journal of Law Reform*, *40*, 1021–1043.

Briscoe, J. (1996). Water as an economic good: the idea and what it means in practice. In *World Congress of the International Commission on Irrigation and Drainage* (pp. 177–202). Cairo. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=lhh&AN=19971901344&site=ehost-live

Carraro, C., Marchiori, C., & Sgobbi, A. (2007). Negotiating on water: insights from non-cooperative bargaining theory. *Environment and Development Economics*, *12*(02), 329–349. http://doi.org/10.1017/S1355770X06003536

Carraro, C., & Sgobbi, A. (2008). Modelling negotiated decision making in environmental and natural resource management. A multilateral, multiple issues, non-cooperative bargaining model with uncertainty. *Automatica*, *44*, 1488–1503. http://doi.org/10.1016/j.automatica.2007.12.021

Cornish, G., Bosworth, B., Perry, C. J., & Burke, J. J. (2004). *Water charging in irrigated agriculture: an analysis of international experience*. Food and Agriculture Organization. Retrieved from ftp://ftp.fao.org/agl/aglw/docs/wr28e.pdf\nhttp://books.google.com/books?id=AqSbdR-DsC0C&pgis=1

CWF. (2011). *Water, Water Use and Water Pricing Around the World*. *Canadian water policy Backgrounders*. Calgary, Alberta.

Dombrowsky, I. (2007). *Conflict, cooperation and institutions in international water management: An economic analysis*. Edward Elgar Publishing.

Gleick, P. (2005, February 25). Pending Deal Would Undermine State's Water Solutions. *Sacramento Bee*, p. B7. Sacramento.

Glennon, R. (2002). *Water follies: Groundwater pumping and the fate of America's fresh waters*. Washington, DC: Island Press.

Gopalakrishnan, C., Tortajada, C., & Biswas, A. K. (2005). *Water institutions: policies, performance and prospects*. Springer.

Hearne, R. R., & Donoso, G. (2014). Water Markets in Chile: Are They Meeting Needs? In K. W. Easter & Q. Huang (Eds.), *Water Markets for the 21st Century* (pp. 103–126). Springer.

Johansson, R. (2000). *Pricing irrigation water: a literature survey* (No. 2449). Retrieved from C:\Documents and Settings\djourdain.ECODJOURDAIN\My Documents\endnote\Electronic Papers\PriceWater.pdf

Johansson, R., Tsur, Y., Roe, T. L., Doukkali, R., & Dinar, A. (2002). Pricing irrigation water: A review of theory and practice. *Water Policy*. http://doi.org/10.1016/S1366-7017(02)00026-0

Libecap, G. D. (2008). Transaction Costs, Property Rights, and the Tools of the New Institutional Economics: Water Rights and Water Markets. In E. Brousseau & J.-M. Glachant (Eds.), *New institutional economics: a guidebook* (pp. 272–291). Cambridge University Press.

Madani, K. (2010). Game theory and water resources. *Journal of Hydrology*, *381*(3-4), 225–238. http://doi.org/10.1016/j.jhydrol.2009.11.045

Mahjouri, N., & Ardestani, M. (2010). A game theoretic approach for interbasin water resources allocation considering the water quality issues. *Environmental Monitoring and Assessment, 167*, 527–544. http://doi.org/10.1007/s10661-009-1070-y

Mahjouri, N., & Ardestani, M. (2011). Application of cooperative and non-cooperative games in large-scale water quantity and quality management: A case study. *Environmental Monitoring and Assessment*, *172*, 157–169. http://doi.org/10.1007/s10661-010-1324-8

OECD. (2012). *OECD Environmental Outlook to 2050: The Consequences of Inaction*. *Organisation for Economic Co-operation and Development (OECD)*. Retrieved from http://dx.doi.org/10.1787/9789264122246-en

Olmstead, S. M. (2010). The economics of managing scarce water resources. *Review of Environmental Economics and Policy*, *4*(2), 179–198. http://doi.org/10.1093/reep/req004

Olmstead, S. M. (2013). Climate change adaptation and water resource management: A review of the literature. *Energy Economics*. http://doi.org/10.1016/j.eneco.2013.09.005

Rausser, G., & Simon, L. (1992). A non cooperative model of collective decision making: a multilateral bargaining approach. *Department of Agricultural and Resource Economics (University of California, Berkeley) Working Paper (618)*.

Rister, M. E., Sturdivant, A. W., Lacewell, R. D., & Michelsen, A. M. (2011). Challenges and Opportunities for Water of the Rio Grande. *Journal of Agricultural and Applied Economics*, *3*(2), 367–378.

Rosegrant, M. W., Ringler, C., & Zhu, T. (2014). Water Markets as an Adaptive Response to Climate Change. In *Water Markets for the 21st Century* (pp. 35–55). Springer.

Rubinstein, A. (1982). Perfect Equilibrium in a Bargaining Model. *Econometrica*, *50*(1), 97–109. http://doi.org/10.2307/1912531

Ruml, C. C. (2005). The Coase Theorem and Western U . S . Appropriative Water Rights. *Natural Resources Journal*, *45*, 169.

Saborío-Rodríguez, M. (2013). *Two essays about environment externalities and one essay on trade cost measurement*. Pennsylvania State University.

Saleth, R. M., & Dinar, A. (2004). *The Institutional Economics of Water: A Cross-Country Analysis of Institutions and Performance*. *Edward Elgar*. Retrieved from http://www.amazon.com/dp/1843767775

Savenije, H. H. G. (2002). Why water is not an ordinary economic good, or why the girl is special. *Physics and Chemistry of the Earth*, *27*, 741–744. http://doi.org/10.1016/S1474-7065(02)00060-8

Teerink, J. R. (1993). *Water allocation methods and water rights in the western states, U.S.A.* (No. 198). *Water allocation, rights, and pricing: Examples from Japan and the United States*.

Tisdell, J. G., & Harrison, H. R. (1992). Esimating an Optimal Distribution of Water Entitlements. *Water Resources Research*, *28*(12), 3111–3117.

UN. (2003). *Agriculture, food and water*. *World Water Development Report, 2003*. http://doi.org/10.1016/S0140-6736(53)90577-7

UN. (2012). World Population Prospects: The 2012 Revision. Retrieved March 5, 2015, from http://esa.un.org/unpd/wpp/unpp/panel_population.htm

Wang, L. Z., Fang, L., & Hipel, K. W. (2003). Water Resources Allocation: A Cooperative Game Theoretic Approach. *Journal of Environmental Informatics*. http://doi.org/10.3808/jei.200300019

Wang, L. Z., Fang, L., & Hipel, K. W. (2008). Basin-wide cooperative water resources allocation. *European Journal of Operational Research*, *190*, 798–817. http://doi.org/10.1016/j.ejor.2007.06.045

Ward, F. A., Booker, J. F., & Michelsen, A. M. (2006). Integrated Economic, Hydrologic, and Institutional Analysis of Policy Responses to Mitigate Drought Impacts in Rio Grande Basin. *Journal of Water Resources Planning and Management*, *132*(6), 488–502. http://doi.org/10.1061/(ASCE)0733-9496(2006)132:6(488)

Ward, F. A., & Pulido-Velázquez, M. (2012). Economic Costs of Sustaining Water Supplies: Findings from the Rio Grande. *Water Resources Management*, *26*(10), 2883–2909. http://doi.org/10.1007/s11269-012-0055-8

Wheeler, S., Bjornlund, H., & Loch, A. (2014). Water Trading in Australia: Tracing its' Development and Impact Over the Past Three Decades. In K. W. Easter & Q. Huang (Eds.), *Water Markets for the 21st Century* (pp. 179–202). Springer.

Appendix

Proof of Proposition 3:

Using Proposition 2, the solution for the bargaining problem can be obtained by solving simultaneously the maximization problem for all the 3 sectors. The problem for sector j is to

$$Max_{x_j}\, V_j(x_j)\; s.t.\;\; V_i(x_j) \geq C_i^1 \; for\; i \neq j, \sum_j y_{ij} = 0, \& \sum_i L_i \bar{z}_{ij} \leq Z_r^0$$

For jurisdiction 1, the Lagrangian can be written as

$$\begin{aligned}\mathcal{L}(x_j, \lambda) = L_1 \left(a_{11}(\bar{z}_{11} - \dot{z}_1) - a_{22}(\bar{z}_{11}^2 - \dot{z}_1^2)\right) + \varphi_1 y_{11} \\ + \left[\sum_{i=2}^{3} \lambda_i \left(L_i \left(a_{1i}(\bar{z}_{i1} - \dot{z}_i) - a_{2i}(\bar{z}_{i1}^2 - \dot{z}_i^2)\right) + \varphi_i y_{i1} - C_i^1\right)\right] - \lambda_3(y_{11} + y_{21} + y_{31}) \\ + \lambda_4(Z_r^0 - L_1\bar{z}_{11} - L_2\bar{z}_{21} - L_3\bar{z}_{31})\end{aligned}$$

Assuming interior solution and using FOCs $\frac{\partial\mathcal{L}}{\partial\bar{z}_{i1}} = 0$, $\frac{\partial\mathcal{L}}{\partial y_{i1}} = 0$ and $\lambda_4 \frac{\partial\mathcal{L}}{\partial\lambda_4} = 0$ yields the solution for $\bar{z}_{i1}$ for two cases on $\lambda_4 = 0$ and $\lambda_4 \neq 0$. These conditions are

$$\frac{\partial\mathcal{L}}{\partial\bar{z}_{11}} = a_{11} - 2a_{21}\bar{z}_{11} - \lambda_4 = 0$$

$$\frac{\partial\mathcal{L}}{\partial\bar{z}_{21}} = \lambda_1(a_{12} - 2a_{22}\bar{z}_{21}) - \lambda_4 = 0$$

$$\frac{\partial\mathcal{L}}{\partial\bar{z}_{31}} = \lambda_2(a_{13} - 2a_{23}\bar{z}_{31}) - \lambda_4 = 0$$

$$\lambda_4 \frac{\partial\mathcal{L}}{\partial\lambda_4} = \lambda_4(Z^0 - L_1\bar{z}_{11} - L_2\bar{z}_{21} - L_3\bar{z}_{31}) = 0$$

$$\frac{\partial\mathcal{L}}{\partial y_{11}} = \varphi_1 - \lambda_3 = 0\,; \quad \frac{\partial\mathcal{L}}{\partial y_{21}} = \lambda_1\varphi_2 - \lambda_3 = 0 \quad ; \quad \frac{\partial\mathcal{L}}{\partial y_{31}} = \lambda_2\varphi_2 - \lambda_3 = 0$$

This gives $\lambda_1 = \varphi_1/\varphi_2$, $\lambda_2 = \varphi_1/\varphi_3$, and $\lambda_3 = \varphi_1$. When the water supply constraint is not binding, $\lambda_4 = 0$ and the solution is the same as under competitive equilibrium

$$\bar{z}_{i1}^* = \frac{a_{1i}}{2a_{2i}}$$

But when the constraint is binding, $\lambda_4 \neq 0$, then

$$Z^0 = L_1\bar{z}_{11} - L_2\bar{z}_{21} - L_3\bar{z}_{31}, \text{and}$$

$$\lambda_4 = a_{11} - 2a_{21}\bar{z}_{11} = \frac{\varphi_1}{\varphi_2}(a_{12} - 2a_{22}\bar{z}_{21}) = \frac{\varphi_1}{\varphi_3}(a_{13} - 2a_{23}\bar{z}_{31})$$

There are 4 variables in 4 equations and solving them yields the allocation vector when sector 1 is the proposer. It can be checked that the allocation vector is symmetric and the maximization problem for sector 2 and 3 yield the same outcome which can be generalized for sector $p \in N$ as

$$\bar{z}_{pj}^{*} = \frac{Z_r^0 - \sum_{i \neq p} \frac{L_i}{2a_{2i}} \left(a_{1i} - \frac{\varphi_i}{\varphi_p} a_{1p} \right)}{L_p + \sum_{i \neq p} L_i \frac{\varphi_i}{\varphi_p} \frac{a_{2p}}{a_{2i}}}$$

When $\varphi_1 = \varphi_2 = \varphi_3 = 1$, the allocation vector is the same as under the social planner. However, The overall sectoral welfare will differ because of the monetary payments. The FOCs with respect to $\lambda_1, \lambda_2, \lambda_3$ need to be used along with the continuation value to find the payments. To simplify notation, write the payoff function as

$$V_i(x_j) = L_i \left(a_{1i}(\bar{z}_{ij} - \dot{z}_i) - a_{2i}(\bar{z}_{ij}^2 - \dot{z}_i^2) \right) + \varphi_i y_{ij} = \tilde{V}_i(x_j) + \varphi_i y_{ij}$$

Let us also denote $V_i(x^0) = V_i^0$ for convenience. Since the equilibrium is symmetric, $V_i(x_j) = V_i$ and therefore

$$V_i(x_j) = \tilde{V}_i + y_{ij}$$

The remaining FOCs in the problem of 1 are

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \tilde{V}_2 + \varphi_2 y_{21} - C_2^1 = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = \tilde{V}_3 + \varphi_3 y_{31} - C_3^1 = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = y_{11} + y_{21} + y_{31} = 0$$

Note that the continuation value for sector i is

$$C_i^1 = (1 - \delta) V_i(x^0) + \frac{\delta}{3} \left(V_i(x_1) + V_i(x_2) + V_i(x_3) \right)$$

using which, the equations can be simplified further. There will be two more sets of such conditions from the problem of 2 and 3. In total, there will be 9 equations in 9 variables, which can be solved to get the monetary payments as

$$y_{jj} = \left(\frac{V_j^0}{\varphi_j} - \frac{\tilde{V}_j}{\varphi_j} \right) - \frac{3 - 2\delta}{3} \sum_{k=1}^{3} \left(\frac{V_k^0}{\varphi_k} - \frac{\tilde{V}_k}{\varphi_k} \right); \; y_{ij} = \left(\frac{V_i^0}{\varphi_i} - \frac{\tilde{V}_i}{\varphi_i} \right) - \frac{\delta}{3} \sum_{k=1}^{3} \left(\frac{V_k^0}{\varphi_k} - \frac{\tilde{V}_k}{\varphi_k} \right)$$

■