DEVELOPING CONSISTENT ESTIMATES OF MARGINAL EFFECTS IN A SIMULTANEOUS EQUATION MODEL WITH LIMITED DEPENDENT VARIABLES

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Selected Paper prepared for presentation at the 2016 Agricultural & Applied Economics Association Annual Meeting, Boston, Massachusetts, July 31-August 2

Developing Consistent Estimates of Marginal Effects in a Simultaneous Equation Model with Limited Dependent Variables

Summary: We demonstrate that Theil-type variance corrections are required to obtain consistent marginal effect estimates in Nelson-Olsen's two-stage limited dependent variable (2SLDV) model. Theil's residuals-based corrections are infeasible with 2SLDV. We present a new variance correction procedure that is virtually equivalent in the 2SLDV context to Theil's 2SLS corrections for continuous models. Monte Carlo simulations are used to demonstrate that the proposed variance correction procedures generate consistent marginal effect estimates. The relevance and potential empirical importance of the correction procedures are illustrated in an analysis of technology adoption by Ethiopian farmers.

Nelson and Olson's (1978) two-stage procedures for estimating simultaneous equation structural models with one or more limited dependent variables (2SLDV) continue to be used extensively by economists in a wide range of applications.[1] However, those procedures have two limitations that affect their usefulness. The first widely recognized problem is that closed form expressions for the standard errors of estimated parameters are difficult to derive in higher dimensions (Amemiya 1979; Maddala 1983). However, advances in computer capacity and bootstrapping techniques have largely obviated the need for closed form representations in the estimation of parameter variances (Cameron and Trivedi 2005).

The second limitation that, to our knowledge, has not been addressed is that, while reduced form parameter estimates obtained using the Nelson-Olsen procedures are consistent, the estimated marginal effects of right-hand side variables obtained for the second stage structural models are inconsistent and biased. We show that to obtain consistent estimates of those marginal effects second stage "Theil-like" structural variance correction procedures are required. This issue, which has not previously been addressed, is the focus of this article.

We develop a new set of computationally efficient variance correction procedures that can be readily be applied to a wide range of simultaneous equations models involving different forms of limited dependent variables. [2] Our procedures are applicable in situations where monotonicity between Pearson correlations in latent variable model residuals and Spearman rank correlations in the observed dependent variables exist. Monte Carlo simulations are used to demonstrate that the proposed variance correction procedures generate consistent marginal effect estimates in simultaneous equations model with continuous and limited dependent variables. The relevance and potential empirical importance of the correction procedures are illustrated in an analysis of technology adoption by Ethiopian farmers in a simultaneous equations probit-probit setting.

As discussed above, estimating parameter variances through the use of closed form expressions is generally infeasible in 2SLDV models. In linear simultaneous equations models estimated using two-stage least squares (2SLS) procedures, it is well known that using fitted instrumental variables in the second stage results in biased variance estimates unless Theil-like correction procedures are applied (Theil 1971, p. 451; Greene 1995, p.735-739). However, Theil's residuals based corrections cannot be directly applied in a 2SLDV model because, for some observations, the values of the latent residuals cannot be obtained. We present an alternative correction procedure we show to be virtually equivalent to Theil's corrections for a 2SLS model but can also be implemented efficiently in 2SLDV models.

The new procedure requires an estimate of the reduced form latent residual covariance matrix which is obtained by exploiting the monotonicity between the rank correlations of the observed limited variables and pairwise Pearson correlations between the latent residuals. Using Nelson and Olsen's original example, the results from our Monte Carlo simulations illustrate that uncorrected structural 2SLDV marginal effect estimates are inconsistent and biased, while the proposed variance correction procedures generate consistent conditional marginal effect estimates.

The severity of bias and inconsistency that results when variance correction procedures are not used depends on the specific underlying model. In simulations based on a modified version of the original Nelson-Olson model using one thousand draws of 10,000 observations, we found if the variance correction procedure is not implemented, 8 percent of the time the model *overestimates* the probability that the dependent variable (y_1) equals its true value by 5 or more percentage points. Further, 19 percent of the time the model *underestimates* the probability that y_1 equals its true value by 5 or more percentage points; and 14.3 percent of the time, the model would *underestimate* the probability that y_1 equals its true value $y_1 = 1$ by 10 or more percentage points. At its most extreme, the model would *underestimate* the probability that y_1 equals its true value by 19 percentage points.

The potential implications of ignoring the issues of bias and consistency are further illustrated in the context of an empirical model of technology adoption by rural Ethiopian farmers – estimated using cross-sectional household-level survey data. The decisions to apply chemical fertilizer and use improved seed are widely reported to be simultaneously determined (Debertin 2002; Foster and Rosenzweig 2010; Smale, Heisey, and Leathers 1995). In this study, chemical fertilizer and improved seed use are both modeled as binary choice processes, yielding a two probit equation simultaneous system with two limited dependent variables.

Results estimated using Nelson-Olson's 2SLDV procedures indicate that there is a noticeable difference in the estimated marginal effects of exogenous variables in the structural equations when a Theil-like correction procedure is applied. For example, when the impact of using improved seed upon on the use of chemical fertilizer is calculated using the corrected variance procedure, on average farmers who use improved seed are estimated to be 4.9 percentage points more likely to use chemical fertilizer than when the same result is calculated using the uncorrected variance covariance matrix. Thus, failure to use the proposed variance correction procedure results in underestimates of the extent of chemical fertilizer adoption in Ethiopia.

The importance of using the variance correction procedure is highlighted at extreme values in the data. The maximum difference between the uncorrected and corrected marginal effects for improved seed use on the probability of chemical fertilizer use is approximately 11.4 percentage points. This difference exceeds 9.2 percentage points for nearly 25 percent of producers.

Nelson-Olson 2SLDV Procedures

The first stage of the Nelson-Olson 2SLDV process involves estimating reduced-form estimates for each equation in the system using appropriate continuous (C), Tobit (T), or Binary (B) probit structures and recovering continuous linear predictors $\hat{y}_j^*$ from each equation in the model[3]. In the second stage, structural equations are estimated by replacing each endogenous right-hand-side variable, y_j, with its continuous fitted instrument $\hat{y}_j^*$. As with 2SLS where all dependent variables are continuous, the structural parameter standard error estimates reported by most econometric packages are inaccurate due to the use of fitted instruments in the second stage. Direct procedures (Amemiya 1979; Maddala 1983; Greene 1995) or bootstrapping procedures (Goodwin and Smith 2003; Chakir and Hardelin 2010) for estimating the asymptotic covariance matrices of the parameter estimates have been developed. Obtaining accurate 2SLDV marginal effect estimates is more difficult.

Consistent Marginal Effect Weights in 2SLDV Models

Using Greene's notation, the estimated marginal effect of variable k in a probit equation j can be written as:

(1-a) $$\frac{\partial E[y_j|\boldsymbol{x_j}]}{\partial x_{j,k}} = \phi(\boldsymbol{\beta}'\boldsymbol{x_j})\beta_k = \phi\left(\frac{\hat{y}_j^*}{\hat{\sigma}_j=1}\right)\beta_k,$$

where $\phi(\cdot)$ denotes the density function of the standardized normal distribution evaluated at value $(\cdot)$. In expression (1-a), the probit model is rescaled under the assumption of a unit variance. We show that the unit variance assumption is invalid when the marginal effects of exogenous variables are estimated in the second stage structural models. Accurately identifying the second stage marginal effects in a probit model requires the use of a corrected standard error estimator $\tilde{\sigma}_j$, which is defined below. Using the corrected standard error the marginal effects are consistently computed as:

(1-b) $$\frac{\partial E[y_j|\boldsymbol{x_j}]}{\partial x_{j,k}} = \phi\left(\frac{\hat{y}_j^*}{\tilde{\sigma}_j}\right)\beta_k.$$

For a censored-Tobit model, the following computation is required to obtain consistent marginal effects:

(1-c)
$$\frac{\partial E[y_j|x_j]}{\partial x_{j,k}} = \Phi\left(\frac{\hat{y}_j^*}{\tilde{\sigma}_j}\right)\beta_k,$$

where $\Phi(\cdot)$ denotes the cumulative normal density function computed at value $(\cdot)$.

In expressions (1-b) and (1-c), the values $\phi(\cdot)$ and $\Phi(\cdot)$ can be viewed as "weights" used to convert parameter estimates into marginal effects; where the weights vary by observation $i = 1,2, \ldots, n$. Since weights for observation i do not differ across parameters k, in the following section we examine the effects of variance corrections upon the marginal effect "weights" $\phi\left(\frac{\hat{y}_j^*}{\sigma_j}\right)$ and $\Phi\left(\frac{\hat{y}^*}{\sigma_j}\right)$ rather than focusing on the marginal effects of any given right hand side variable in the structural equations of the model.

A modified Nelson-Olson Numerical 2SLDV Example

To demonstrate the inconsistency of uncorrected marginal effect estimates in 2SLDV models we utilize a variant of Nelson and Olson's original latent variable simulation example. Specifically we modify Nelson-Olson's original Continuous-Tobit (C-T) model to a Binary-Tobit (B-T) model and use a probit model to estimate the binary equation. Nelson-Olson's original latent variable example is:

(2)
$$\begin{aligned} y_1^* &= 1.0y_2^* + 1.0x_1 + 2.0x_2 + 0.5x_3 + u_1 \\ y_2^* &= 0.5y_1^* + 2.0x_1 + 0.5x_4 + 1.0x_5 + u_2 \end{aligned}.$$

The exogenous x_j variables were generated as $N(0,1)$ and held fixed in repeated samples.

Nelson-Olson generated the latent errors u_1 and u_2 as multivariate normal with

$Cov(u) = \Sigma_u = \begin{bmatrix} 2.25 & 0.75 \\ 0.75 & 2.25 \end{bmatrix}$.

The system of structural equations (2) can be mathematically represented in two forms, both of which will be used in the following discussions.

Observation i representation:

(3-a) $$\boldsymbol{\Gamma y_i^*} = \boldsymbol{B x_i} + u_i.$$

Equation j representation:

(3-b) $$y_j^* = \boldsymbol{Y_j^* \gamma_j} + \boldsymbol{X_j \beta_j} + u_j = \boldsymbol{Z_j \delta_j} + u_j.$$

Properties of the reduced-form estimates and residuals are utilized in the following bias-correction procedures. The reduced-form of expression (2) is:

(4) $$\begin{aligned} y_1^* &= 6.0x_1 + 4.0x_2 + 1.0x_3 + 1.0x_4 + 2.0x_5 + v_1 \\ y_2^* &= 5.0x_1 + 2.0x_2 + 0.5x_3 + 1.0x_4 + 2.0x_5 + v_2 \end{aligned},$$

with $Cov(v) = \Sigma_v = \begin{bmatrix} 24 & 18 \\ 18 & 14.25 \end{bmatrix}$.

The reduced-forms also have two mathematical representations:

Observation i representation:

(5-a) $$y_i^* = \Gamma^{-1} B x_i + \Gamma^{-1} u_i = A x_i + v_i,$$

with $Cov(v) = \Sigma_v = (\Gamma^{-1})\Sigma_u(\Gamma^{-1})'$.

Equation j representation:

(5-b) $$y_j^* = X\alpha_j + v_j.$$

The covariance terms in (5-a) can be rearranged as:

(6) $$Cov(u) = \Sigma_u = \Gamma\Sigma_v\Gamma',$$

a form that motivates the variance correction estimator discussed below.

The observed variables in the (B-T) example are generated as:

(7) $$\begin{aligned} y_1 &= h_1(y_1^*) = I(y_1^* > 0) \\ y_2 &= h_2(y_2^*) = \max(y_2^*, 0) \end{aligned},$$

where h_i is a transformation function and $I(\cdot)$ is a zero-one indicator function.

In the following variance correction procedures, we use probit models to estimate equations with binary dependent variables. When estimating the probit equations in the 2SLDV system, the system is implicitly rescaled so that the reduced form probit equations have unit variance. Following Maddala (1983, p. 244-247), the rescaled structural and reduced form equations in expressions (2) and (4) can be written as the structural model:

(8) $$\begin{aligned} y_1^* &= 0.204y_2^* + 0.204x_1 + 0.408x_2 + 0.102x_3 + u_1 \\ y_2^* &= 2.449y_1^* + 2.000x_1 + 0.500x_4 + 1.000x_5 + u_2 \end{aligned},$$

with covariance matrix $\Sigma_u = \Gamma\Sigma_v\Gamma' = \begin{bmatrix} 0.094 & 0.153 \\ 0.153 & 2.250 \end{bmatrix}$ and the reduced form:

(9) $$\begin{aligned} y_1^* &= 1.225x_1 + 0.817x_2 + 0.204x_3 + 0.204x_4 + 0.408x_5 + v_1 \\ y_2^* &= 5.000x_1 + 2.000x_2 + 0.500x_3 + 1.000x_4 + 2.000x_5 + v_2 \end{aligned},$$

with covariance matrix $\Sigma_v = \begin{bmatrix} 1.00 & 3.67 \\ 3.67 & 14.25 \end{bmatrix}$.[4] In equation (9), the reduced-form covariance matrix has been rescaled to have unit variance in the first equation while maintaining the original correlation structure (where $\rho = 0.973$) of equation (4). The structural covariance matrix is recovered from the rescaled reduced form covariance matrix as $\Sigma_u = \Gamma\Sigma_v\Gamma'$.

To numerically examine the sampling properties of the Nelson-Olson two-stage process we used R to generate joint observations[5] of (y_1^*, y_2^*) and (y_1, y_2) using reduced form system (4) and transformation functions (7) with four sample sizes ranging from 250, 500, 1,000, and 10,000 observations. Using Nelson-Olson's 2SLDV process, we estimated each equation's parameters, including the reduced form model's estimated residual variance or squared Tobit scale parameters ($VVAR1$ and $VVAR2$), the uncorrected estimated structural variance parameters ($UVAR1$ and $UVAR2$). We then use the corrected structural variance estimates ($C\text{-}UVAR1$ and $C\text{-}UVAR2$) calculated as described below. The process was repeated 1,000 times at each sample size to numerically approximate confidence intervals on the parameter estimates. Tables 1 and 2 present the simulation summary statistics for the B-T model.

Table 1 presents and contrasts the true reduced form parameters of the model and the covariance terms from expression (9) with the mean values of the parameter estimates obtained from 1,000 simulations at each of the four sample sizes. The results show that reduced form parameter estimates are consistently estimated in the first stage of Nelson and Olson's 2SLDV procedure. While reduced form equation parameter estimates are not the focus here, note that the elements of the reduced form covariance matrix $\Sigma_v = \begin{bmatrix} 1.00 & 3.67 \\ 3.67 & 14.25 \end{bmatrix}$ are consistently estimated using the proposed variance correction procedure.[6]

Procedures for estimating the latent covariance terms from Nelson-Olson's equation by equation estimations, described below, yield accurate estimation of the reduced form latent covariance matrix Σ_v. Obtaining this estimate permits the use of equation (6) to obtain Theil-like corrected structural variance estimates in the 2SLDV system.

Table 2 contrasts the structural equation and covariance parameters from expression (8) to the means of the structural equation parameter estimates from the simulations. With the exception of the uncorrected variance estimators $UVAR1$ and $UVAR2$ associated with the structural models, the results reported in tables 1 and 2 support Nelson and Olson's conclusion about bias and consistency with respect to their two-stage procedure parameter estimates. As Theil noted, however, uncorrected variance estimates from the second stage estimation are strongly biased by the use of fitted instruments in the second stage. The results presented in table 2 indicate that the Theil-like corrected structural variance estimators $C\text{-}UVAR1$ and $C\text{-}UVAR2$ are consistent estimates of the diagonal elements of Σ_u.

The differences between the marginal effect weights obtained using the uncorrected and corrected standard error estimates for the Binary-Tobit (B-T) variant of Nelson-Olson's numerical example are shown in the graphs presented in figure 1 where histograms are presented for the B-T model's linear predictors $\hat{y}_1^*$ and $\hat{y}_2^*$. Plots of the uncorrected (red) and corrected (blue) marginal effect weights for the probit equation, $\left(\frac{\hat{y}_1^*}{\hat{\sigma}_1=1}\right)$ and $\phi\left(\frac{\hat{y}_1^*}{\tilde{\sigma}_1=0.306}\right)$, are shown in the first panel; similar plots of the uncorrected (red) and corrected (blue) marginal effect weights for the Tobit equation, $\Phi\left(\frac{\hat{y}_2^*}{\hat{\sigma}_2=3.774}\right)$ and $\Phi\left(\frac{\hat{y}_2^*}{\tilde{\sigma}_2=1.5}\right)$, are shown in the second panel.

In the probit model, the use of uncorrected standard errors overstates the range for the latent variable, $\hat{y}_1^*$, over which the probability of observing $y_1 = 1$ changes as $\hat{y}_1^*$ varies. In this model, changes in the probability of observing $y_1 = 1$ actually occur when $\hat{y}_1^*$ lies between -1 and +1. Observations where $\hat{y}_1^* < -1$ have virtually zero probability of observing $y_1 = 1$ (with zero marginal effect) while observations where $\hat{y}_1^* > 1$ have almost unit probability of observing $y_2 = 1$ (again with zero marginal effects). For the probit model, using the uncorrected variance estimated in the second stage structural equations results in biased marginal effect estimates over most of the range of the $\hat{y}_1^*$ domain. Similar results hold with respect to the second stage Tobit model with the Tobit model's marginal weights changing primarily in the $\hat{y}_2^*$ interval $(-3.86, 3.86)$ when corrected standard errors are used rather than the broader $(-9.72, 9.72)$ interval obtained when the uncorrected standard errors are used.

A Visual Contrast of the Marginal Effect Weight Estimates

In the simulation analysis, we compute the true structural marginal effect weights, the uncorrected estimated marginal effect weights, and the corrected estimated marginal effect weights for each observation. Figure 2 contrasts the actual and estimated marginal effect weights for the probit structural equation. Figure 3 contrasts the actual and estimated marginal effect weights for the censored structural Tobit equation. The plots show individual marginal effect weights for each of the 10,000 observations generated in the first repetition of the simulations.

The first frame in figure 2 plots both the uncorrected estimated marginal effect weights (in red) and the variance corrected estimated marginal effect weights (in blue) against the true marginal effect weights for each observation. If estimated weights were to match the true marginal effect weights, all values would lie on the black line in that frame. The plots demonstrate that, while the estimated corrected marginal weights have noise, they align much more closely with the true values of the marginal weights than do the uncorrected marginal effect weight estimates. The first frame in figure 3 demonstrates that similar results hold for the structural Tobit equation. In both equations, the uncorrected marginal effect weights are clearly biased and inconsistent while the variance corrected marginal weight estimates are consistent.

The second frames in figures 2 and 3 contain plots that examine the incidence of the marginal effects weights using frequency histograms. The black histogram plots the actual marginal effect weights, the red histogram plots the uncorrected weights, and the blue histogram plots the corrected weights. In both cases, the corrected weight estimates match the actual weights much more closely. The third plot in figures 2 and 3 present frequency histograms of marginal effect estimation errors calculated as the estimated marginal effect weight minus the true marginal effect weight. The variance corrected marginal estimates (blue) are substantially more efficient than the uncorrected marginal effect weights (red).

Estimating Latent Reduced Form Correlations used in Second Stage Variance-Corrections

The above discussion has demonstrated that incorporating Theil-like variance corrections are needed to obtain consistent structural marginal effect estimates. The second stage

Theil-like variance correction procedure for the 2SLDV model utilizes the diagonal elements of the covariance estimator

$$\tilde{\Sigma}_u = \hat{\Gamma}\hat{\Sigma}_v\hat{\Gamma}' \tag{10}$$

presented in expression (6) when constructing the marginal weights in equations (1-b) and (1-c). Appendix A demonstrates that the diagonal elements of expression (10) are virtually identical to Theil's residuals-based corrected variances $\tilde{\sigma}_j^2$ for a 2SLS system if $\hat{\Sigma}_v$ is computed using reduced form fitted residuals.

Estimating the alternative variance corrections presented in Appendix A in a system with one or more limited dependent variables requires an estimate of the cross-equation latent error correlations in the reduced form system. In a system of continuous dependent variables, cross-equation residual correlations can be consistently estimated using pairwise Pearson correlations between the residual vectors $(\hat{v}_j, \hat{v}_k)$. This approach cannot be used when one or more of the endogenous variables are truncated or binary as the resulting latent residual vector cannot be directly estimated. However, we can often recover information with respect to the latent correlations by examining the joint dependency structure of the observed dependent variables induced by the joint dependency structure of the unobserved latent residuals.

Consider a reduced form system with latent dependent variables where the vector of equation j's latent variable realizations can be written as:

$$y_j^* = X\alpha_j + v_j. \tag{11}$$

and the vectors of residuals $V = [v_1, v_2, v_3, \dots . v_J]$ are multivariate normal; i.e., $V \sim MVN(0, \Sigma_v)$. The observed dependent variables are derived or transformed from the latent variables via a function or operator $h_j(\cdot)$ such that:

(12) $$y_j = h_j(y_j^*).$$

Examples of $h_j(\cdot)$ functions include possible combinations of a complete or continuous (C) operator (i.e., $h(y_j^*) = y_j^*$), a lower truncating (T) operator (i.e., $h(y_j^*) = \max(y_j^*, \tau_j)$), and a binary (B) operator (i.e., $h(y_j^*) = I(y_j^* \geq \tau_j)$where $I(\cdot)$ is an indicator function. Note that with either the (T) or (B) operator, latent realizations are not fully observed and the residual vectors $\hat{v}_j$ cannot be estimated. In the case of the (B) operator, the truncation point τ_j is also unobserved and non-estimable.

Simulations available from the authors demonstrate that if the quantile positions of the latent τ_k and τ_l values relative to y_j^* do not differ widely between any pairs of equations k and then an estimation algorithm exists can generate consistent estimates of the across equation covariance terms, l[7].

Our procedure first estimates pairwise cross-equation latent Pearson correlations by searching for the Pearson correlation that generates a Spearman rank correlation that best matches the Spearman rank correlation between the dependent variables the original observations in the data set. The following steps are used to estimate correlations between pairs of reduced form latent residuals:

An Algorithm for Estimating Reduced Form Latent Residual Correlations

(Step 1) Estimate the Spearman rank correlation $\hat{\rho}^{S}_{k,l} = Srank(y_k, y_l)$ between the observed data vectors of sample size n.

(Step 2) Separately estimate the parameters $\hat{\alpha}_j, \hat{\tau}_j, \hat{\sigma}^2_j$ and linear predictors $\hat{y}^*_j$ for each reduced form equation using the appropriate continuous, Tobit, and/or probit models. The probit model will implicitly rescale and translate the model such that $\hat{\tau}_j = 0$ and $\hat{\sigma}^2_j = 1$. The joint pairs of linear predictors $[\hat{y}^*_k \quad \hat{y}^*_l]$ are replicated a large number of times obtaining a set of joint reduced form predicted values $\hat{Y}^*$ with a large sample size N (N should be much larger than the original sample size n for modest sample sizes).

(Step 3) Estimate the Pearson correlation $\hat{\rho}^{P}_{k,l}$ by selecting a trial starting value $\tilde{\rho}^{P}_{k,l}$:

(a) Generate the trial covariance matrix $\tilde{\Sigma}_v = \begin{bmatrix} \hat{\sigma}^2_k & \tilde{\rho}^{P}_{k,l}\hat{\sigma}_k\hat{\sigma}_l \\ \tilde{\rho}^{P}_{k,l}\hat{\sigma}_l\hat{\sigma}_k & \hat{\sigma}^2_l \end{bmatrix}$.

(b) Generate an $(N\ x\ 2)$ multivariate normal sample $\tilde{V} \sim MVN\left(0, \tilde{\Sigma}_v\right)$.

(c) Construct an $(N\ x\ 2)$ matrix of simulated latent variables $[\tilde{y}^*_k \quad \tilde{y}^*_l] = \tilde{Y}^* = \hat{Y} + \tilde{V}$.

(d) Construct a joint sample of simulated "observed" data as $\tilde{y}_k = h_k(\tilde{y}^*_k, \hat{\tau}_k)$ and $\tilde{y}_l = h_l(\tilde{y}^*_l, \hat{\tau}_l)$.

(e) Compute the Spearman rank correlation between the vectors of simulated observed data i.e., $\tilde{\rho}^{S}_{k,l} = Srank(\tilde{y}_k, \tilde{y}_l)$.

(f) Compute the estimated "Spearman rank error" as the absolute difference between the Spearman rank correlation estimated from the original data (Step

1) and the Spearman rank correlation estimated from the simulated data i.e., $SRankErr(\tilde{\rho}^P_{k,l}) = |\tilde{\rho}^P_{k,l} - \hat{\rho}^S_{k,l}|$.

(g) Repeat steps (3-a) through (3-f) to obtain the Pearson correlation $\hat{\rho}^P_{k,l}$ that minimizes $SRankErr$.

In figure 4, we map the Pearson-LDV Spearman rank correlation (in black) and the corresponding $SRankErr$ (in red) for the simulations whose results are plotted in figures 2 and 3. The LDV Spearman rank correlation is not a direct estimate of the Pearson correlation, but the mapping between the Pearson and the LDV Spearman correlations (plotted in blue) is monotonic, allowing us to recover a Pearson correlation estimate of 0.9702 from the data's original LDV Spearman rank correlation of 0.842 for this sample. The procedures described in the above algorithm allow us to consistently estimate the reduced form covariance matrix $\hat{\Sigma}_v$ which, combined with the structural endogenous parameter estimates $\hat{\Gamma}$ allow us to obtain a consistent Theil-like corrected structural covariance matrix estimate as $\tilde{\Sigma}_u = \hat{\Gamma}\hat{\Sigma}_v\hat{\Gamma}'$. Theil-like corrected variances $\tilde{\sigma}_j^2$ can be recovered from the diagonals of $\tilde{\Sigma}_u$ and used in expression (1) to obtain consistent marginal effects estimates in the 2SLDV system.[8]

An Empirical Application: Chemical Fertilizer and Improved Seed Use in Ethiopia

To explore the effects of using the above Theil-like correction procedure to obtain marginal effects in 2SLDV structural models, we examine technology adoption by farmers in Ethiopia where agriculture currently accounts for approximately 40 percent of the country's GDP and engages 85 percent of the country's households, most of whom live in extreme poverty (Ferenji 2004; Byerlee *et al*. 2007). In an effort to improve crop yields, agricultural policy initiatives have focused on the determinants of adoption rates of technologically advanced agricultural inputs – specifically, chemical fertilizer and improved seed.

Agricultural technologies are often introduced as a complementary package of inputs such that decisions to use chemical fertilizer entail joint decisions to use other agricultural innovations such as pesticides or improved seed (Debertin 2002; Foster and Rosenzweig 2010; Smale, Heisey, and Leathers 1995). In the survey data set used for this analysis, the decision to adopt agricultural inputs is simply recorded as either "yes" or "no," yielding a zero-one indicator variable (equal to one if the input is used anywhere on the farm).

Description of Data

The primary data for this analysis were obtained from the 2001/02 Annual Agricultural Sample Survey (AgSS)[9] conducted by the Central Statistical Agency (CSA) of Ethiopia and made available to us by HarvestChoice[10]. To estimate the effects of the decision to adopt chemical fertilizer and improved seed we model the two binary variables representing chemical fertilizer and seed adoption using the following sets of explanatory variables: farmer and farm household demographic characteristics; farm cultivation practices and land characteristics; and characteristics that define the biophysical nature and infrastructure of the wereda (or districts)[11] in which the farm is located. Variable definitions and sample characteristics are presented in table 3. The majority of farmers included in the sample are illiterate, middle-age men who head an average household of five people. In terms of agricultural inputs, 35.4 percent of the sampled farmers used chemical fertilizer and 11.4 percent used improved seed, and 9 percent used both inputs. The sample size is 28,554 households.

Empirical Model and Methodology

Following Train (2009), we use a random utility model (RUM) that assumes Ethiopian producers exhibit utility-maximizing behavior. Given that the dependent variables, CFERT and SEED, are both indicator variables, we estimate a probit-based Binary-Binary (B-B) model. Identification of the model is obtained by paralleling the instrumental variable choices made by Nkonya, Schroeder, and Norman (1997)[12]. The econometric model is:

$$\begin{aligned} CFERT_i &= \gamma_1 SEED_i + \beta_{10} + \beta_{11}\boldsymbol{W}_i + \beta_{12}\boldsymbol{X}_i + \beta_{13}\boldsymbol{Z}_{i,j} + \beta_{14} CATTLE_{i,j} + u_{1i} \\ SEED_i &= \gamma_2 CFERT_i + \beta_{20} + \beta_{21}\boldsymbol{W}_i + \beta_{22}\boldsymbol{X}_i + \beta_{23}\boldsymbol{Z}_{i,j} + \beta_{25} MAIZEYLD_{i,j} + u_{2i} \end{aligned} \quad (13)$$

where

$CFERT_i$	is a measure of chemical fertilizer adoption by farmer i,
$SEED_i$	is a measure of improved seed adoption by farmer i,
$CATTLE_i$	is a measure of the cattle owned by farmer i,
$MAIZEYLD_i$	is a measure of the average maize yield in farmer i's wereda,
$\boldsymbol{W}_i$	is a vector of the characteristics that describe farmer i's farm,
$\boldsymbol{X}_i$	is a vector of demographic characteristics that describe farmer i, and
$\boldsymbol{Z}_{i,j}$	is a vector of characteristics that define wereda j where farmer i resides[13].

The system is identified by omitting the variable CATTLE from the seed equation and the variable MAIZEYLD from the fertilizer equation. The parameter estimates from this system are presented in table 4 where the reported standard errors were calculated using a jackknife procedure with 1,000 repetitions.

Since the dependent variables in the simultaneous equation model are binary, it is not possible to measure the underlying, latent utility associated with the choice to use chemical fertilizer or improved seed (Train 2009). Thus, we cannot accurately measure the latent residuals associated with the resulting reduced form model. As discussed above, first-stage parameter estimates can be scaled using conventional practices, but the use of linear predictors in the second-stage requires that the variance be adjusted to calculate marginal effects from the probit model.

The variance in the second stage of the 2SLDV model is associated with the latent utility from the endogenous variables. If this fact is ignored then the estimates of marginal effects on the probability of technology adoption obtained from the second stage of the 2SLDV model will be inconsistent and biased. The marginal effects obtained for the structural chemical fertilizer and improved seed regressions with and without using Theil-like corrections are presented in tables 5 and 6[14].

Empirical Results

In themselves, the marginal effect estimates presented in tables 5 and 6 are of economic interest and relevant to policy debates[15]. However, here the focus is on the impact of using Theil-like variance corrections in a 2SLDV model on the estimated marginal effects associated with exogenous variables in the structural equations. Uncorrected estimates of marginal effects are substantially different than corrected marginal effects for both the structural chemical fertilizer use and improved seed use models. The differences between the corrected and uncorrected marginal effects from the simultaneous regression model are presented at the quartile ranges for the entire sample in table 7. According to this table, failure to account for the variance correction in the second-stage of the 2SLDV model would result in an average misestimation of the probability to use chemical fertilizer, given the use of improved seed, by 4.9 percentage points. The resulting bias is most severe near the extremes. Failure to account for the variance correction could result in a maximum error in the estimate of the probability to use chemical fertilizer, given the use of improved seed, by 11.4 percentage points. The severity of misestimation depends on the individual farmer.

Figure 5 provides a visual comparison of the uncorrected and corrected marginal effect weights from the second-stage for the two 2SLDV. The first plots for chemical fertilizer and improved seed use are scatterplots of the uncorrected marginal effect weights against the corrected marginal effect for each individual. These two plots show that uncorrected marginal effects are biased upward (downward) for lower (higher) levels of variance-corrected marginal effects. From a policy perspective, failure to account for the variance-correction could result in a serious misallocation of resources. For example, if the Ethiopian government is interested in the effects of increased investment in infrastructure on chemical fertilizer use, it could examine the marginal effects of the average distance to market variable. If the marginal effects of increased market isolation are biased upward in a particular region the government would allocate more resources than necessary to reach a technology adoption target.

Most farmers in Ethiopia cultivate less than one hectare of land. A one hectare increase in land (essentially doubling the farmer's operation) is estimated to increase the probability of chemical fertilizer adoption by approximately 3.4 percentage points when the variance correction procedure is used to obtain marginal effects. If the variance-correction is ignored, the estimate falls to 3.1 percentage points. While there is only a 9.4 percent difference between the average uncorrected and corrected marginal effect weights for chemical fertilizer use, there is a much larger bias at the extreme. A one hectare increase in the size of a farm that faces the maximum marginal effect will result in a 5.7 point increase in the probability of chemical fertilizer use. This estimated consequence of changes in farm size on chemical fertilizer use is 20.7 percent larger than the estimate calculated with an uncorrected variance.

The histograms in figure 5 present distributions of the uncorrected and corrected individual marginal effect weights for both the inputs examined. Similar conclusions can be drawn from these plots as those drawn from the simulation plots in figures 2 and 3. The distribution of the weights for both inputs is wider when the variance correction is applied. Additionally, for improved seed use, once the variance-correction is taken into consideration, there is a higher frequency of low marginal effects weights than when the correction is absent.

Summary and Conclusions

In this study, procedures have been developed that enable Theil-like corrections to be used to obtain unbiased and consistent estimates of conditional marginal effects in a system with one or more endogenous and limited dependent variables. Monte Carlo simulations showed that failing to apply such variance-correction procedures biases the estimates of the marginal effects while variance-correction procedures enable the development of consistent and unbiased estimates of the marginal effects for structural conditional exogenous variables. An empirical application to a two equation probit-probit model of technology choice by Ethiopian smallholder farmers indicates that failing to apply the variance-correction procedures can substantially affect the estimated marginal effects and estimated probabilities of technology adoption for substantial proportions of the sample population. The approach we describe in this article is relatively easy to implement and enables more accurate identification of the marginal effects in structural endogenous choice models containing one or more limited dependent variables.

Footnotes

[1] See the papers by Auten and Joulfaian, 2001; Brooks, Cameron, and Carter 1998; Chakir and Hardelin, 2010; Christoffersen, 2001; Dennis, Nandy, and Sharpe 2000; Gagné, 2003; Goodwin and Smith, 2003; Ida and Goto, 2009; and Nandy, 2010.

[2] An alternative approach to solving this problem could be to use Markov Chain Monte Carlo FIML procedures, but that approach has two shortcomings. First, these procedures remain computationally expensive; second, every combination of limited and continuous dependent variable models requires its own fairly extensive set of recoding

[3] Some authors have used logit models for the binary (B) equations. However, the covariance matrix based correction procedures described below are more easily implemented when using probit models.

[4] The use of probit to estimate the first equation of the 2SLDV model implicitly rescales the variables so that the variance in the first reduced form equation is one.

[5] The R code used in the article's simulations and estimations are available from the authors upon request.

[6] The diagonal terms of Σ_v can be easily obtained from most standard regression packages.

[7] The key to the following procedure is whether there is a monotonic mapping between the latent Pearson correlation and the LDV Spearman correlation calculated between the limited dependent variables. While the LDV Spearman correlation cannot be used as a direct estimate of the latent Pearson correlation, the latent Pearson correlation can be recovered by searching over the Pearson-LDV Spearman mapping. Simulations

demonstrate that if the quantiles of the τ_k and τ_l, relative to the underlying y_k^* and y_l^* values, differ greatly, the mapping becomes non-monotonic or "flat" over some range of the underlying Pearson correlation. We note however, that FIML bivariate probit models also do poorly at identifying the underlying Pearson correlation in these circumstances.

[8] Simulations contrasting these results to those obtained from a Full Information Maximum Likelihood (FIML) bivariate probit model indicated that the efficiency of the procedures based on the estimation algorithm were within one percent of the efficiency of the FIML estimates for sample sizes as small as one hundred and improved with larger sample sizes.

With a multivariate probit, our pairwise correlation estimates are consistent but less efficient than FIML multivariate probit estimates but can be obtained in a fraction of the solution time for the multivariate probit model.

[9] We choose to use the 2000/2001 (1993 Ethiopian Calendar) AgSS survey because climatic patterns were relatively stable in this particular time period. The AgSS data were supplemented with a measure of isolation extracted from a global map of travel time to major cities developed by the European Commission and the World Bank (2008), as well as wereda-level data (such as geographical identifiers) that accompanied the Ethiopian Development Research Institute's 2006 *Atlas of the Ethiopian Rural Economy* (Tadesse *et al*).

[10] HarvestChoice is a Gates Foundation funded project, details of which, and data from, can be found at www.harvestchoice.org/.

[11] Weredas are the third-level administrative division of Ethiopia.

[12] Nkonya, Schroeder, and Norman (1997) use a Tobit-Tobit (T-T) model to examine the determinants of the number of hectares planted to improved maize seed and quantity of nitrogen fertilizer used on farms in northern Tanzania. They argued that chemical fertilizer is expensive relative to improved seed, and thus, farmers with greater financial resources at their disposal could afford to apply chemical fertilizer. In agrarian societies such as Tanzania and Ethiopia, wealth is often signified by livestock ownership, specifically cattle. In our analysis, we do not have access to information on individual livestock ownership, so we use CATTLE, the proportion of cattle ownership in a wereda, as a proxy. Improved maize seed is the most frequently used improved seed in Ethiopia. Therefore, MAIZEYLD, the average maize yield in a wereda (quintals per hectare), proxies the individual yield difference between local and improved seeds.

[13] In the system of structural equations (13), vector W includes the total area and number of parcels and fields under cultivation by the farmer. The farmer-specific variables included in vector *X* are age, sex, education level, and household size. The variables included in vector *Z* represent the bioclimatic and economic nature of the region where the farmer resides. Biophysical variables are average elevation, average slope, tree coverage, all-weather road density (meters per km^2), and a measure of rainfall in the wereda (average number of months with a rainfall over 100mm). Variables that describe a farmer's ability to access services from his respective wereda are the distance to market (measured as the average time, in hours, it takes to travel to a market center of 50,000 people or greater from the wereda), population density of the wereda (hundreds of people

per km^2) and number of banks, primary schools, and secondary schools present in the wereda.

[14] The marginal effects presented are average marginal effects (taken over all observations), as opposed to marginal effects at the mean. Additionally, the associated standard errors were calculated using a jackknife procedure with 1,000 repetitions.

[15] See Byerlee *et al*. (2007); Ferenji (2005); and Nkonya, Schroeder, and Norman (2007)

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APPENDIX A: An Alternative Procedure for Deriving Theil-Like Corrected Variance Values

Theil (1971, p. 451-458) demonstrated the need for a variance correction procedure when computing the parameter covariance matrix from the two stage estimation of a continuous simultaneous equation system. Modifying Greene's (1995, p.735-739) notation slightly, Theil's estimated asymptotic parameter covariance matrix can be written as:

$$\hat{\Sigma}_{\delta_j} = \tilde{\sigma}_j^2\left[\hat{Z}_j{}'\hat{Z}_j\right]^{-1} \tag{A1}$$

where $\tilde{\sigma}_j^2 = \frac{1}{T}\tilde{u}_j{}'\tilde{u}_j$ and $\tilde{u}_j$ are corrected residuals. This appendix contrasts the diagonal elements of an alternative consistent estimator:

$$\tilde{\tilde{\Sigma}}_u = \hat{\Gamma}\hat{\Sigma}_v\hat{\Gamma}' \tag{A2}$$

to Theil's residual-based corrected variance estimators $\tilde{\sigma}_j^2$. With a continuous system, we note that the diagonal elements of Estimator (A2) can also be constructed using a set of transformed residuals constructed as[16]:

$$\tilde{\tilde{u}}_j = \left(\hat{V}\hat{\Gamma}'\right)_j \tag{A3}$$

where $\left(\hat{V}\hat{\Gamma}'\right)_j$ denotes the j'th column of the matrix $\hat{V}\hat{\Gamma}'$, $\hat{V}$ is the matrix of stage I reduced form fitted residuals, and $\hat{\Gamma}$is the matrix of second stage estimated endogenous parameters.

Theil's corrected and the uncorrected residuals can respectively be written as:

$$\tilde{u}_j = y_j - [Y_j \quad X_j]\begin{bmatrix}\hat{\gamma}_j \\ \hat{\beta}_j\end{bmatrix} = y_j - Z_j\hat{\delta}_j \tag{A4}$$

and

$$\hat{u}_j = y_j - \left[\hat{Y}_j \quad X_j\right]\begin{bmatrix}\hat{\gamma}_j \\ \hat{\beta}_j\end{bmatrix} = y_j - \hat{Z}_j\hat{\delta}_j \tag{A5}$$

where $Y_j = [y_k]$ and $\hat{Y}_j = [\hat{y}_k]$ for $k \neq j$.

Subtracting (A4) from (A5) and rearranging gives:

$$\tilde{u}_j = \hat{u}_j - \left[Y_j - \hat{Y}_j\right]\hat{\gamma}_j = \hat{u}_j - \hat{V}_j\hat{\gamma}_j \tag{A6}$$

where $\hat{V}_j$ is a submatrix of the first stage residuals matrix $\hat{V}$. Adding and subtracting the vector of first stage residuals $\hat{v}_j$ gives:

$$\tilde{u}_j = \left(\hat{u}_j - \hat{v}_j\right) + \left(\hat{v}_j - \hat{V}_j\hat{\gamma}_j\right) = \left(\hat{u}_j - \hat{v}_j\right) + \left(\hat{V}\hat{\Gamma}'\right)_j = \left(\hat{u}_j - \hat{v}_j\right) + \tilde{\tilde{u}}_j. \tag{A7}$$

For identified and over-identified SE-CDV systems, the differences between $\tilde{u}_j$and$\tilde{\tilde{u}}_j$ (and thus $\tilde{\sigma}_j$ and $\tilde{\tilde{\sigma}}_j$) will tend to be small as both the non-adjusted second stage residuals $\hat{u}_j$ and the first stage residuals $\hat{v}_j$ are estimated using information from the entire column space of X. Monte Carlo simulation code (in R) is available from the authors and demonstrates that the differences between the two estimators are trivial in the SE-CDV models examined.

[16] As indicated in the main body of the article, the advantage of using estimator (A2) rather than Theil's residual based estimator arises when residual based procedures are non-applicable.

Table 1: Comparison of Actual Parameters and Covariance Terms to Simulation Values for B-T Model – Reduced Form Equations

Parameter	Actual Value	NOBS=250 Mean	NOBS=250 Std. Dev	NOBS=500 Mean	NOBS=500 Std. Dev	NOBS=1,000 Mean	NOBS=1,000 Std. Dev	NOBS=10,000 Mean	NOBS=10,000 Std. Dev
				EQUATION 1					
a10	0	-0.002	0.114	0	0.077	-0.001	0.053	0	0.017
a11	1.225	1.281	0.168	1.25	0.114	1.239	0.076	1.225	0.024
a12	0.817	0.853	0.148	0.833	0.091	0.828	0.069	0.816	0.02
a13	0.204	0.213	0.101	0.206	0.07	0.207	0.057	0.204	0.016
a14	0.204	0.211	0.108	0.213	0.08	0.209	0.054	0.204	0.017
a15	0.408	0.421	0.115	0.414	0.085	0.413	0.058	0.407	0.018
VVAR1	1	1	0	1	0	1	0	1	0
				EQUATION 2					
a20	0	-0.012	0.374	-0.007	0.255	-0.009	0.184	-0.002	0.06
a21	5	5.009	0.38	4.999	0.253	4.999	0.179	4.999	0.057
a22	2	1.995	0.32	1.999	0.221	2.004	0.16	1.998	0.048
a23	0.5	0.505	0.271	0.493	0.186	0.504	0.149	0.5	0.045
a24	1	0.998	0.302	1.012	0.22	1.007	0.145	1.001	0.049
a25	2	1.993	0.288	2.003	0.23	2	0.145	1.998	0.049
VVAR2	14.247	13.81	1.93	14.07	1.298	14.14	0.918	14.23	0.294
rhoV12	0.973	0.976	0.02	0.976	0.016	0.974	0.013	0.974	0.005
covV12	3.674	3.621	0.267	3.656	0.184	3.663	0.134	3.673	0.044

Table 2: Comparison of Actual Parameters and Covariance Terms to Simulation Values for B-T Model – Structural Equations

Parameter	Actual Value	NOBS=250 Mean	NOBS=250 Std. Dev	NOBS=500 Mean	NOBS=500 Std. Dev	NOBS=1,000 Mean	NOBS=1,000 Std. Dev	NOBS=10,000 Mean	NOBS=10,000 Std. Dev
				EQUATION 1					
g12	0.204	0.211	0.047	0.207	0.032	0.207	0.023	0.204	0.006
b10	0	-0.004	0.078	0	0.051	0	0.036	0	0.011
b11	0.204	0.223	0.219	0.212	0.153	0.205	0.107	0.206	0.031
b12	0.408	0.43	0.125	0.417	0.083	0.412	0.063	0.409	0.018
b13	0.102	0.106	0.078	0.103	0.056	0.103	0.045	0.102	0.013
UVAR1	0.094	1	0	1	0	1	0	1	0
C-UVAR1	0.094	0.114	0.083	0.101	0.054	0.097	0.037	0.095	0.012
				EQUATION 2					
g21	2.449	2.37	0.39	2.408	0.255	2.426	0.189	2.449	0.057
b20	0	0.002	0.253	-0.005	0.169	-0.005	0.123	-0.001	0.039
b21	2	2.006	0.477	2.003	0.302	1.998	0.226	2.002	0.067
b24	0.5	0.507	0.252	0.502	0.18	0.502	0.122	0.502	0.038
b25	1	1.01	0.267	1.01	0.2	0.999	0.147	1.001	0.043
UVAR2	2.25	13.88	1.94	14.1	1.3	14.16	0.918	14.23	0.294
C-UVAR2	2.25	2.362	0.991	2.296	0.629	2.28	0.479	2.243	0.157
rhoU12	0.333	0.278	0.483	0.328	0.379	0.328	0.312	0.344	0.112
covU12	0.153	0.15	0.274	0.16	0.198	0.151	0.153	0.157	0.048

Table 3: Variable Descriptions and Sample Characteristics

Variable	Description	Mean	Std. Dev.	Min	Max
CFERT	1 if Farmer Adopted Chemical Fertilizer	0.35	0.48	0.00	1.00
SEED	1 if Farmer Adopted Improved Seeds	0.11	0.32	0.00	1.00
TTIME	Avg. Travel Time to a Population of 50,000	7.75	4.56	1.09	36.05
SEX	1 if Male	0.81	0.39	0.00	1.00
AGE	Age of Farmer	42.93	15.20	11.00	89.00
EDUCATION	Education of Farmer	1.48	0.95	1.00	7.00
ILLIT	Illiterate	0.73	0.45	0.00	1.00
ED_1-3	1st - 3rd Grade	0.14	0.34	0.00	1.00
ED_4-6	4th - 6th Grade	0.09	0.28	0.00	1.00
ED_7-8	7th - 8th Grade	0.03	0.17	0.00	1.00
ED_9-11	9th - 11th Grade	0.01	0.11	0.00	1.00
ED_12	12th Grade	0.00	0.07	0.00	1.00
ED>12	Beyond 12th Grade	0.00	0.04	0.00	1.00
HHSIZE	Household Size	5.24	2.27	1.00	15.00
FARM_AREA	Area Cultivated (in hectares)	0.96	0.83	0.00	7.27
FIELDS	Number of Fields Cultivated	8.20	5.38	1.00	62.00
PARCELS	Number of Parcels Cultivated	3.04	2.07	1.00	24.00
CROP_CAT	Major Crop Category Grown	2.23	1.18	1.00	8.00
CASHCRP	Cash Crops	0.10	0.30	0.00	1.00
CEREALS	Cereals	0.81	0.39	0.00	1.00
FRUIT	Fruit Crops	0.00	0.06	0.00	1.00
HRBSPC	Herbs and Spices	0.02	0.05	0.00	1.00
OILSEED	Oilseeds	0.02	0.13	0.00	1.00
PULSES	Pulses	0.05	0.22	0.00	1.00
ROOT	Root Crops	0.01	0.09	0.00	1.00
VEG	Vegetables	0.04	0.06	0.00	1.00
IRR	1 if Farmer Adopted Irrigation	0.07	0.25	0.00	1.00
ELEV	Avg. Elevation of Wereda (100 meters above sea level)	17.98	4.61	4.04	29.40
SLOPE	Avg. Slope of Wereda (percentage rise)	6.55	2.82	0.30	16.50
TREES	Percentage of Tree Cover in Wereda (of total ground cover)	18.80	14.39	1.07	70.28
RAIN	Number of Months with Rainfall > 100mm	4.34	1.85	0.00	8.70
ROADDEN	All-Weather Road Density in Wereda (meters per km2)	30.83	28.55	0.00	145.07
PRIM_SCH	Number of Primary Schools in Wereda	29.35	13.05	2.00	75.00
SEC_SCH	Number of Secondary Schools in Wereda	1.52	2.82	0.00	17.00
POPDEN	Population Density of Wereda (hundreds of people per km2)	2.39	4.57	0.03	29.42
BANKS	Number of Banks in Wereda	0.84	1.88	0.00	11.00

INST	Number of Micro-finance Institutions in Wereda	2.58	1.41	1.00	7.00
CATTLE	Proportion of Cattle Ownership in Wereda	72.56	16.10	6.20	96.80
MAIZEYLD	Average Maize Yield in Wereda (quintals per hectare)	15.85	8.59	0.26	50.57

Table 4: Empirical Application Results -- Parameter Estimates

	Dependent Variable: Chemical Fertilizer				Dependent Variable: Improved Seed			
	Coeff.	Std. Error	P-Value		Coeff.	Std. Error	P-Value	
(Intercept)	-0.475	0.291	0.052	*	-1.775	0.628	0.002	***
SEED	1.094	0.095	0.000	***	---	---	---	
CFERT	---	---	---		0.302	0.175	0.040	**
TTIME	0.002	0.006	0.382		-0.027	0.008	0.000	***
SEX	-0.078	0.047	0.044	**	0.064	0.039	0.048	**
AGE	-0.003	0.001	0.008	***	0.001	0.001	0.130	
ED_1-3	0.020	0.045	0.325		0.104	0.049	0.024	**
ED_4-6	0.086	0.055	0.056	*	0.131	0.073	0.038	**
ED_7-8	0.111	0.090	0.098	*	0.197	0.106	0.031	**
ED_9-11	0.151	0.119	0.098	*	0.221	0.138	0.054	*
ED_12	-0.037	0.206	0.432		0.477	0.204	0.013	**
ED>12	-0.313	0.460	0.207		0.630	0.372	0.030	**
HHSIZE	0.019	0.007	0.002	***	0.007	0.009	0.228	
FARM_AREA	0.114	0.026	0.000	***	0.040	0.046	0.182	
FIELDS	0.002	0.004	0.343		0.022	0.007	0.002	***
PARCELS	-0.010	0.010	0.170		0.037	0.010	0.000	***
CEREALS	0.301	0.069	0.000	***	0.133	0.132	0.146	
FRUIT	-1.136	1.551	0.158		-0.414	1.315	0.424	
HRBSPC	1.697	1.889	0.276		-1.772	1.713	0.161	
OILSEED	0.095	0.139	0.244		-0.041	0.118	0.377	
PULSES	0.408	0.123	0.000	***	-0.206	0.118	0.050	**
ROOT	0.475	0.348	0.034	**	-0.373	0.309	0.042	**
VEG	-0.059	0.766	0.396		0.026	0.605	0.355	
BIRR	-0.524	0.073	0.000	***	0.426	0.051	0.000	***
ELEV	0.104	0.005	0.000	***	-0.047	0.016	0.005	***
SLOPE	-0.017	0.008	0.026	**	-0.023	0.011	0.024	**
TREES	-0.035	0.002	0.000	***	0.016	0.006	0.005	***
RAIN	0.063	0.022	0.005	***	0.081	0.044	0.033	**
ROADDEN	0.000	0.001	0.434		-0.001	0.001	0.023	**
PRIM_SCH	-0.019	0.002	0.000	***	0.008	0.002	0.002	***
SEC_SCH	0.057	0.008	0.000	***	0.007	0.019	0.356	
POPDEN	-0.005	0.007	0.232		0.001	0.006	0.391	
BANKS	-0.014	0.016	0.171		-0.018	0.018	0.143	
INST	0.007	0.012	0.260		0.018	0.012	0.079	*
CATTLE	0.004	0.001	0.001	***	---	---	---	
MAIZEYLD	---	---	---		0.013	0.004	0.001	***
Uncorrected Variance:	1.000				1.000			
Corrected Variance:	0.631				0.654			
Significance levels:	* (10%); ** (5%); *** (1%)							

Table 5: Empirical Application Results – Marginal Effects (Dependent Variable: Chemical Fertilizer)

	Average Marginal Effects Uncorrected				Average Marginal Effects Corrected			
	Marginal Effect	Std. Error	P-Value		Marginal Effect	Std. Error	P-Value	
(Intercept)	-0.129	0.079	0.052	*	-0.142	0.087	0.052	*
SEED	0.297	0.026	0.000	***	0.328	0.029	0.000	***
TTIME	0.001	0.002	0.382		0.001	0.002	0.382	
SEX	-0.023	0.014	0.044	**	-0.024	0.015	0.044	**
AGE	-0.001	0.000	0.008	***	-0.001	0.000	0.008	***
ED_1-3	0.007	0.015	0.325		0.007	0.015	0.325	
ED_4-6	0.030	0.019	0.056	*	0.030	0.019	0.056	*
ED_7-8	0.040	0.033	0.098	*	0.040	0.033	0.098	*
ED_9-11	0.055	0.044	0.098	*	0.055	0.044	0.098	*
ED_12	-0.009	0.069	0.432		-0.009	0.069	0.432	
ED>12	-0.081	0.118	0.207		-0.081	0.118	0.207	
HHSIZE	0.005	0.002	0.002	***	0.006	0.002	0.002	***
FARM_AREA	0.031	0.007	0.000	***	0.034	0.008	0.000	***
FIELDS	0.000	0.001	0.343		0.000	0.001	0.343	
PARCELS	-0.003	0.003	0.170		-0.003	0.003	0.170	
CEREALS	0.083	0.019	0.000	***	0.091	0.021	0.000	***
FRUIT	-0.123	0.143	0.158		-0.123	0.143	0.158	
HRBSPC	0.333	0.362	0.276		0.333	0.362	0.276	
OILSEED	0.034	0.049	0.244		0.034	0.049	0.244	
PULSES	0.152	0.048	0.000	***	0.152	0.048	0.000	***
ROOT	0.177	0.108	0.034	**	0.177	0.108	0.034	**
VEG	-0.009	0.171	0.396		-0.009	0.171	0.396	
BIRR	-0.154	0.018	0.000	***	-0.154	0.018	0.000	***
ELEV	0.028	0.001	0.000	***	0.031	0.002	0.000	***
SLOPE	-0.005	0.002	0.026	**	-0.005	0.002	0.026	**
TREES	-0.010	0.001	0.000	***	-0.011	0.001	0.000	***
RAIN	0.017	0.006	0.005	***	0.019	0.007	0.005	***
ROADDEN	0.000	0.000	0.434		0.000	0.000	0.434	
PRIM_SCH	-0.005	0.000	0.000	***	-0.006	0.000	0.000	***
SEC_SCH	0.015	0.002	0.000	***	0.017	0.002	0.000	***
POPDEN	-0.001	0.002	0.232		-0.001	0.002	0.232	
BANKS	-0.004	0.004	0.171		-0.004	0.005	0.171	
INST	0.002	0.003	0.260		0.002	0.004	0.260	
CATTLE	0.001	0.000	0.001	***	0.001	0.000	0.001	***

Significance levels: * (10%); ** (5%); *** (1%)

Table 6: Empirical Application Results – Marginal Effects (Dependent Variable: Improved Seed)

	Average Marginal Effects Uncorrected				Average Marginal Effects Corrected			
	Marginal Effect	Std. Error	P-Value		Marginal Effect	Std. Error	P-Value	
(Intercept)	-0.290	0.103	0.002	***	-0.263	0.093	0.002	***
CFERT	0.049	0.029	0.040	**	0.045	0.026	0.040	**
TTIME	-0.004	0.001	0.000	***	-0.004	0.001	0.000	***
SEX	0.010	0.006	0.048	**	0.009	0.006	0.048	**
AGE	0.000	0.000	0.130		0.000	0.000	0.130	
ED_1-3	0.016	0.008	0.024	**	0.016	0.008	0.024	**
ED_4-6	0.021	0.012	0.038	**	0.021	0.012	0.038	**
ED_7-8	0.034	0.020	0.031	**	0.034	0.020	0.031	**
ED_9-11	0.039	0.027	0.054	*	0.039	0.027	0.054	*
ED_12	0.099	0.052	0.013	**	0.099	0.052	0.013	**
ED>12	0.149	0.087	0.030	**	0.149	0.087	0.030	**
HHSIZE	0.001	0.002	0.228		0.001	0.001	0.228	
FARM_AREA	0.007	0.008	0.182		0.006	0.007	0.182	
FIELDS	0.004	0.001	0.002	***	0.003	0.001	0.002	***
PARCELS	0.006	0.002	0.000	***	0.005	0.002	0.000	***
CEREALS	0.021	0.021	0.146		0.019	0.019	0.146	
FRUIT	0.038	0.158	0.576		0.038	0.158	0.576	
HRBSPC	-0.042	0.042	0.161		-0.042	0.042	0.161	
OILSEED	-0.005	0.016	0.377		-0.005	0.016	0.377	
PULSES	-0.025	0.013	0.050	**	-0.025	0.013	0.050	**
ROOT	-0.037	0.019	0.042	**	-0.037	0.019	0.042	**
VEG	0.024	0.057	0.355		0.024	0.057	0.355	
BIRR	0.079	0.012	0.000	***	0.079	0.012	0.000	***
ELEV	-0.008	0.003	0.005	***	-0.007	0.002	0.005	***
SLOPE	-0.004	0.002	0.024	**	-0.003	0.002	0.024	**
TREES	0.003	0.001	0.005	***	0.002	0.001	0.005	***
RAIN	0.013	0.007	0.033	**	0.012	0.007	0.033	**
ROADDEN	0.000	0.000	0.023	**	0.000	0.000	0.023	**
PRIM_SCH	0.001	0.000	0.002	***	0.001	0.000	0.002	***
SEC_SCH	0.001	0.003	0.356		0.001	0.003	0.356	
POPDEN	0.000	0.001	0.391		0.000	0.001	0.391	
BANKS	-0.003	0.003	0.143		-0.003	0.003	0.143	
INST	0.003	0.002	0.079	*	0.003	0.002	0.079	*
MAIZEYLD	0.002	0.001	0.001	***	0.002	0.001	0.001	***

Significance levels: * (10%); ** (5%); *** (1%)

Table 7: Resulting Independent Variable Marginal Effect Differences (Absolute) between Corrected and Uncorrected Marginal Effects from Simultaneous B-B Regression of Chemical Fertilizer Use and Improved Seed Use

	Improved Seed Use	Chemical Fertilizer Use
Minimum Marginal Effect Difference	0.000	0.000
1st Quartile Marginal Effect Difference	0.030	0.007
Mean Marginal Effect Difference	0.057	0.010
3rd Quartile Marginal Effect Difference	0.092	0.012
Maximum Marginal Effect Difference	0.114	0.029

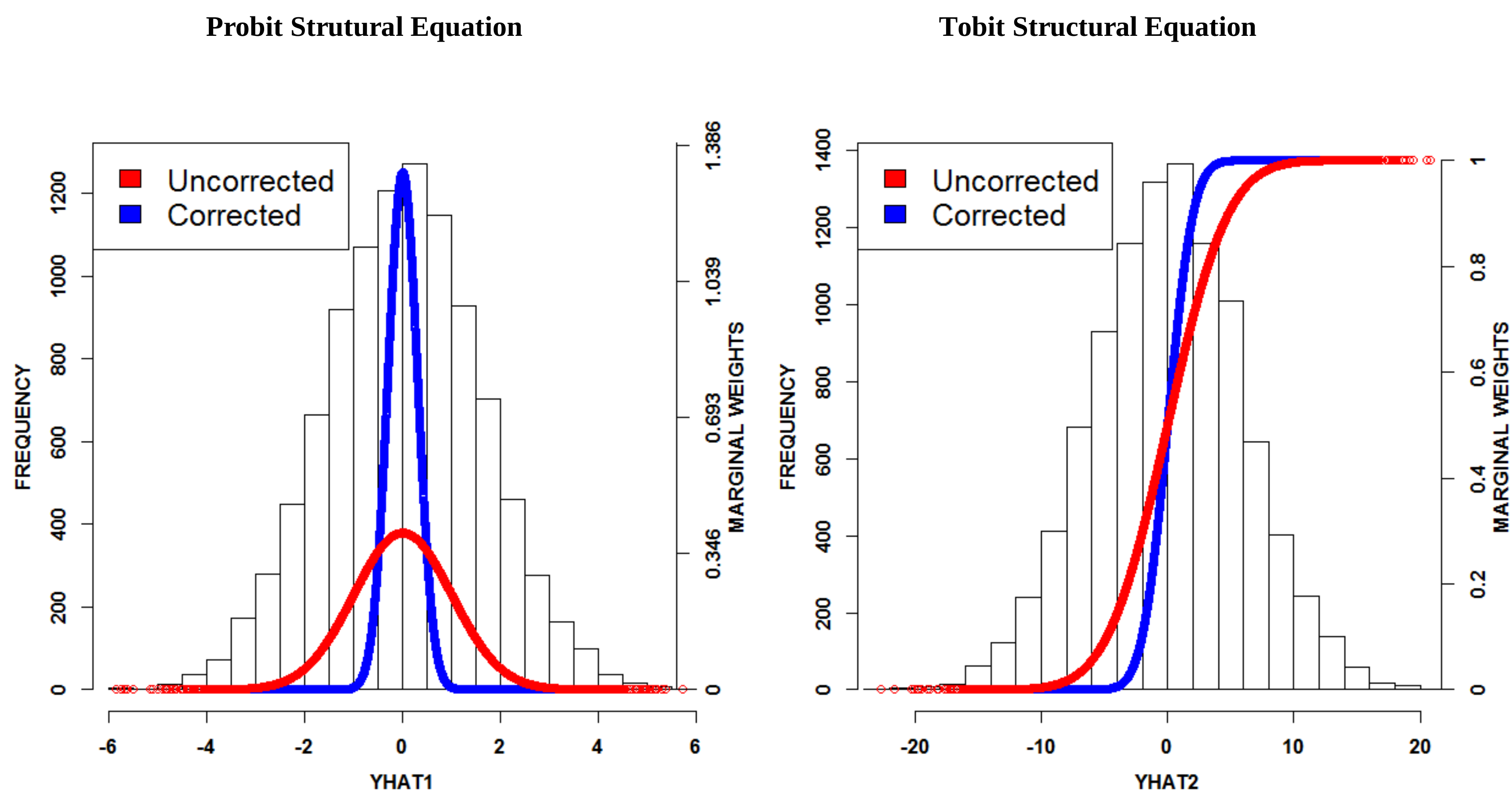

Figure 1: **Histograms of the structural linear predictors and the corresponding marginal effect weights for the Nelson-Olson Binary-Tobit example with uncorrected and corrected variance estimates**

Uncorrected and Corrected Estimated Marginal Effect Weights Plotted Against Actual Marginal Effect Weights

Histogram of Actual Marginal Effect Weights Plotted Against Uncorrected and Corrected Marginal Weights Estimates

Histogram of Uncorrected and Corrected Estimated Marginal Effect Weights Errors

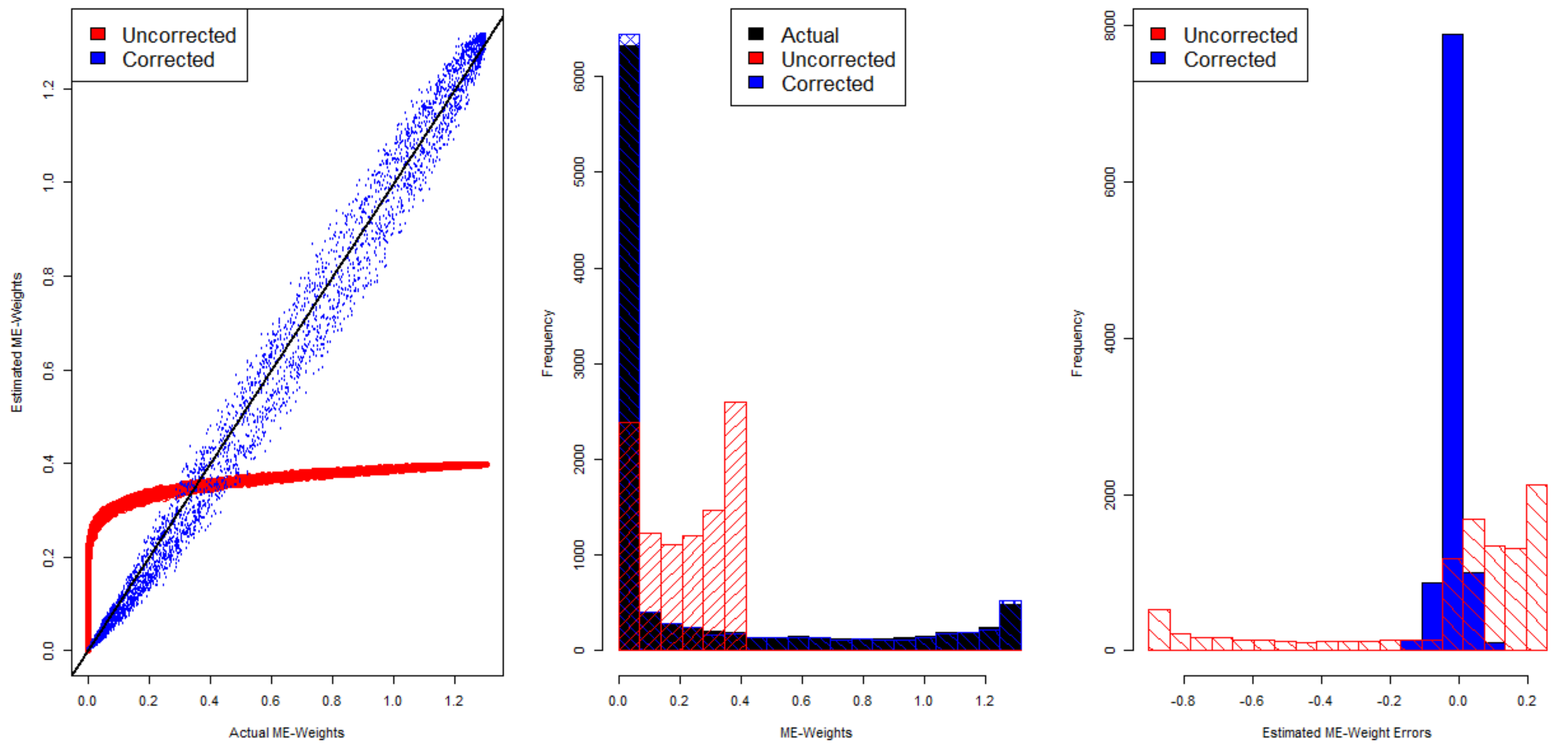

Figure 2: **Marginal effect weight plots for the binary structural equation**

Uncorrected and Corrected Estimated Marginal Effect Weights Plotted Against Actual Marginal Effect Weights

Histogram of Actual Marginal Effect Weights Plotted Against Uncorrected and Corrected Marginal Weights Estimates

Histogram of Uncorrected and Corrected Estimated Marginal Effect Weights Errors

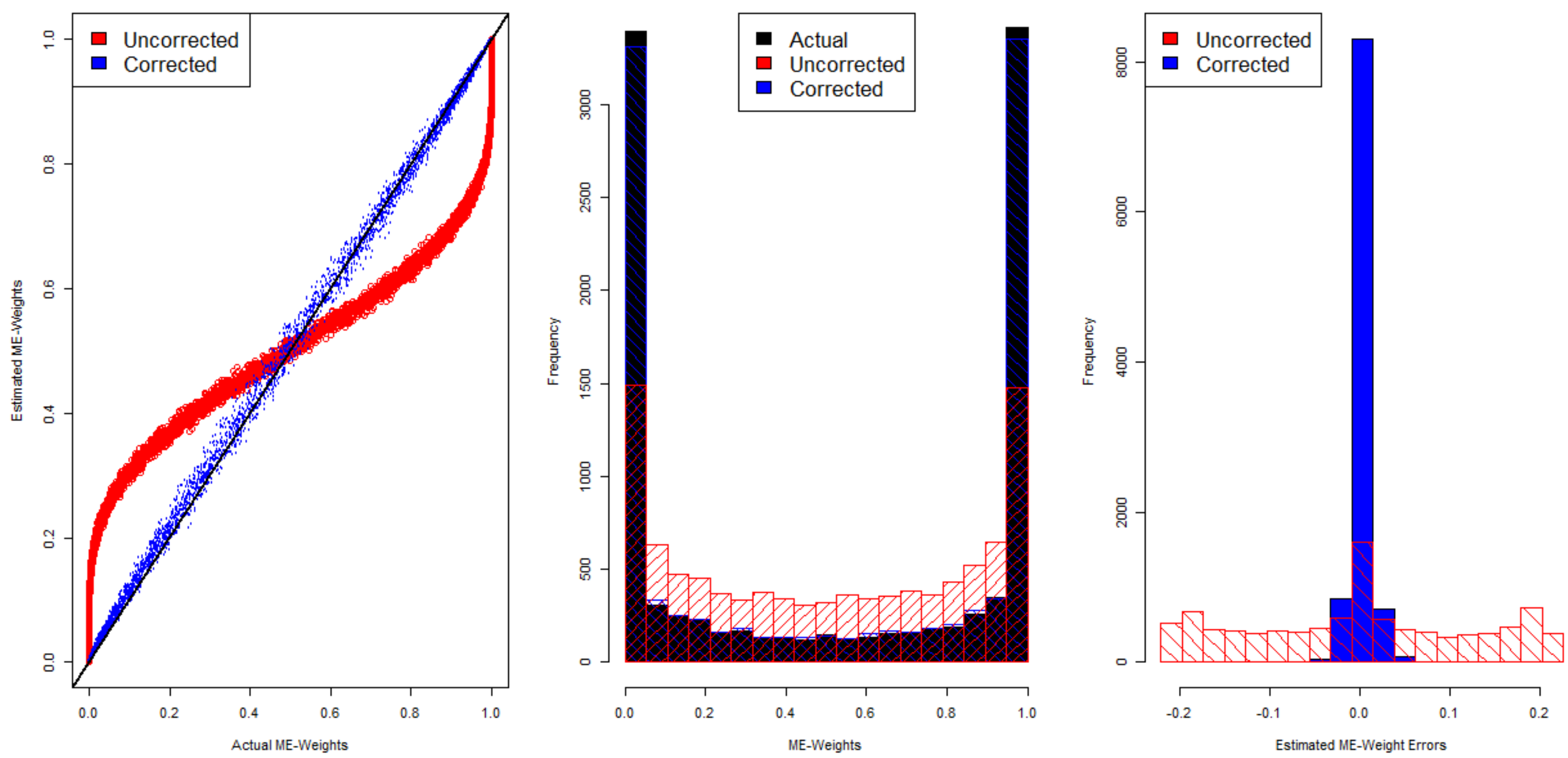

Figure 3: Marginal effect weights plots for the Tobit structural equation

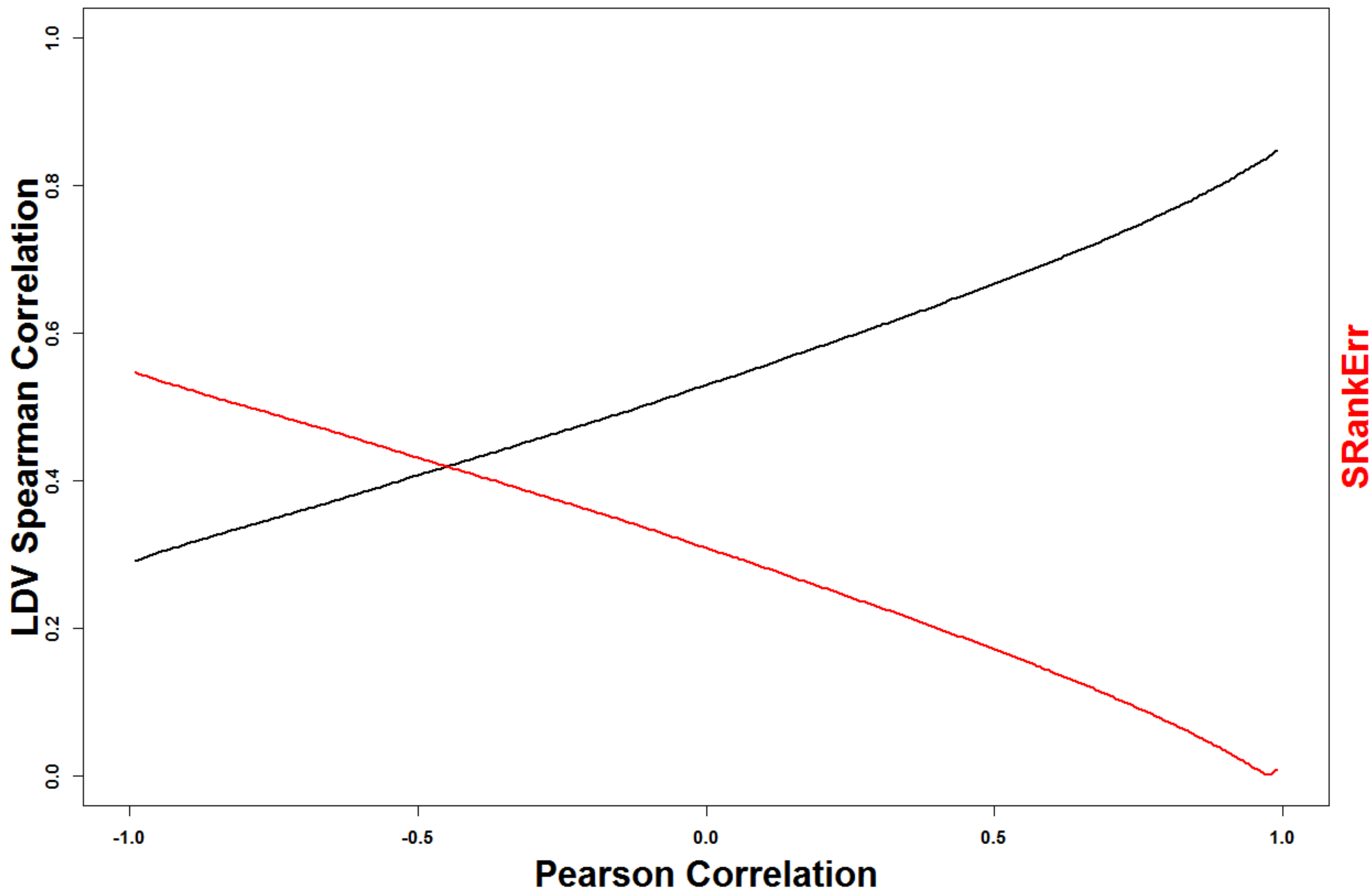

Figure 4: **Pearson to LDV Spearman correlation mapping (black) and corresponding $SRankErr$ (red)**

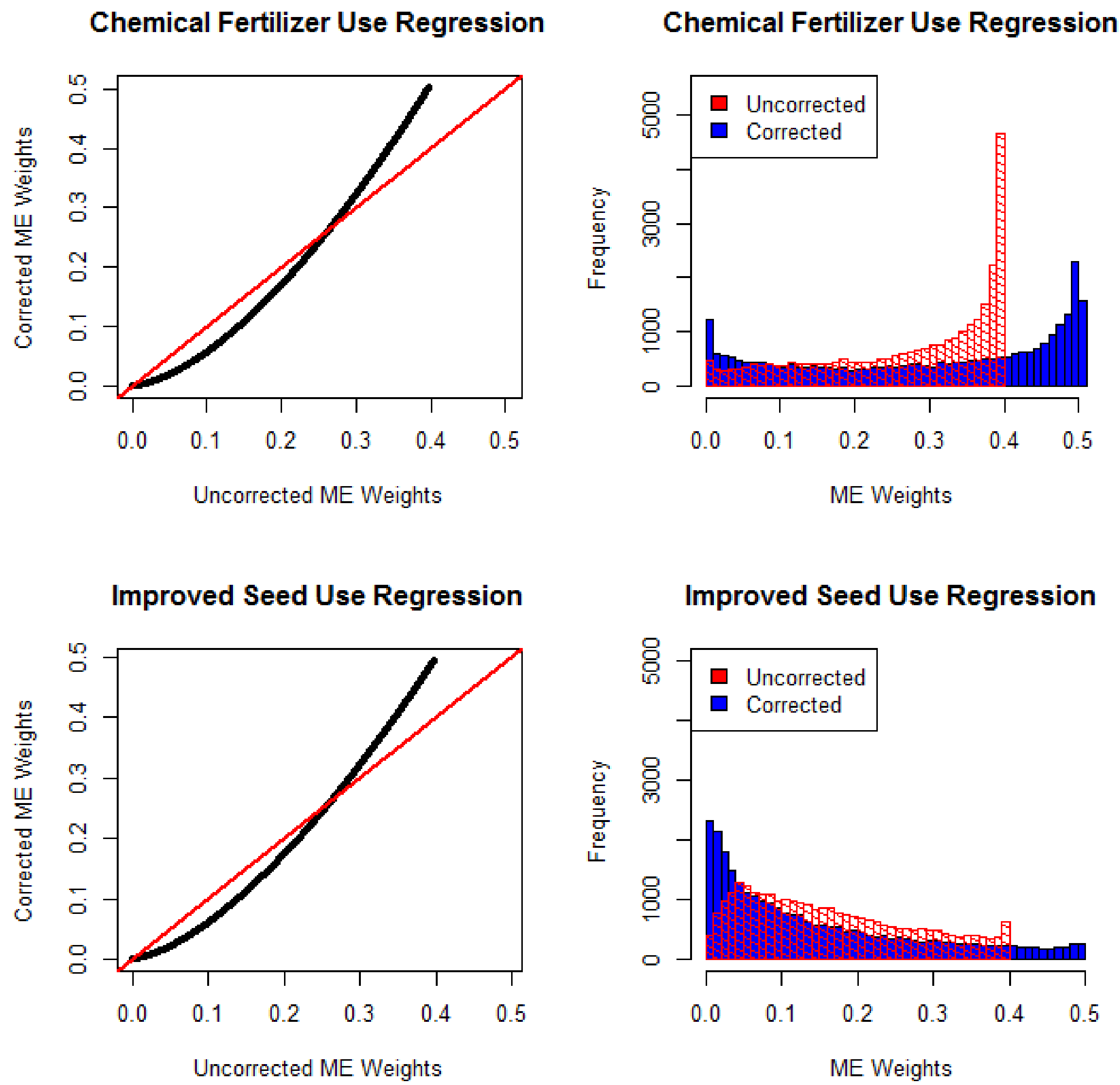

Figure 5: **A visual comparison of the uncorrected and corrected marginal effects weights for both chemical fertilizer use and improved seed use**