Envious Preferences in Two-sided Matching

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Abstract

We develop a model of two-sided matching problem with individual-sided envious preferences that originate from an emulative envy effect in which a more desirable state that is preferred is owned by the other individual. We assume envious preferences influence an individual's decision to enter into a two-sided network instead of being unassigned. In this paper, we show that an individual-sided envious preference leads to a stable matching under a two-sided market framework. Applying the mechanism of the model to behavioral contract theory, we show that individual-proposing envious acceptance leads to stable farmer-buyer contract matching considering buyer's time invariant preference. We further argue that individual's envious preference also contributes to herd-type acceptance that dominates individual's logical preferences in participation decision under a less risky environment.

JEL codes: D47, D81, D86, L14.
Keywords: Behavioral contract design, envious acceptance algorithm, emulative envy effect, envious preference, herd-type acceptance, market design, network effect.

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Envy consists in seeing things never in themselves, but only in their relations.
— Bertrand Russell

1 Introduction

What are the consequences when individuals exhibit envious preferences instead of acting as a rational agent? Undoubtedly, there are countless social and economic situations in which individuals are influenced by their envy of others' choices to match with expected providers under a certain market platform. The issue of individual preferences is central in any two-sided matching analysis, the stability of the matching, and the problem of related mechanism and market design. Examples subject to envious preferences include the American physician model, the marriage model, the college admission model, and the assignment model (Gale and Shapley 1962; Hatfield and Milgrom 2005; Kagel et al. 1995; Kelso and Crawford 1982; Knuth 1976; Roth and Sotomayor 1988, 1989, 1992; Shapley and Shubik 1971).

In two-sided matching, most of the theories assume non-envious preferences and Bayesian models to explain agent-proposing entry or participation decisions in the network. A preponderance of the existing literature assumes that economic and demographic factors influence entry, contract participation and matching. A two-sided market typically has a cross-sided network effect where members of two different groups show their preference ranking regarding the other group members. Behavioral economists now argue that individuals' cognitive factors play an important role in decision-making, though the literature on behavioral influences on participation in contract networks is limited. With respect to the literature, Koszegi (2015) indicates that information optimizing with non-Bayesian beliefs is a concern in revealing the interaction between emotions and information.

As relatively little theoretical attention has been given to this issue, we propose a theoretical framework considering how envious preferences enter into the decision-making process. This paper explains individuals' envious preferences as originating from emulative envy and herd-type behavior in a standard agent-proposing two-sided matching framework. Emulative envy refers to a positive motivational force for individual preferences; and herd type behavior explains how individuals act based on what others are doing rather than undertaking actions based on their own evaluation of the available information. The framework would be helpful in understanding which behavioral factors are of particular importance to develop two-sided behavioral matching problems and interventions. Since the recent frameworks are mostly case-specific, considering individuals' envious preferences within market design framework requires rigorous explanations.

The strategy of the paper is as follows: Section 2 reviews related literature on case-specific individuals' preferences in matching problems. In Section 3, we outline the basic model of standard two-sided matching considering agent-sided envious preferences. This section also explains agent-proposing envious acceptance algorithm with related assumptions, definitions, lemmas, and propositions to justify the final theorem. Then, we explain the herd-type acceptance mechanism. Section 4 discusses an example of a contract-farming problem where the farmers' envious acceptance mechanism leads to farmer-buyer stable matching. Finally, we conclude in Section 5 with policy relevance and future scopes.

2 Related Literature

Theoretical studies of individual's preference in general and their types in particular, have a long tradition in economics, especially in two-sided matching problems. There is a large theoretical and empirical body of work postulates that individuals' have strict preferences in deferred acceptance mechanism initiated by Gale and Shapley (1962). In a stable marriage problem, both agents (i.e., men and women) show a preference list, ranking all the members of the counterparts in strict order of preference. A few seminal papers on the hospital-resident problem, stable roommate problem, student assignment problem, exchange-stability problem also consider strict preferences in most cases. In real-world application, America's National Residents Matching Program (NRMP) is the largest matching scheme to construct a stable matching, e.g., matching medical graduates to their first hospital post.

Second, there are some studies that explicitly examine agents' indifferent preferences in a matching framework. While Gale and Shapley's (1962) deferred acceptance mechanism leads to a stable matching when both agent and firm rankings are strict, it fails to do so when there are indifferent preferences. Erdil and Ergin (2008) provide evidence that ties in preference ranking need to be considered carefully to obtain stable matching. Abdulkadiroglu et al. (2009) also provide a theoretical and empirical basis of optimal student matching when schools have ties in their preference ranking for students, but Pareto efficient matching is unclear in this model. On the contrary, Alcalde-Unzu and Molis (2011) and Jaramillo and Manjunath (2012) consider tied preferences of agents, and show evidence of the existence of strategy-proof and Pareto efficient mechanisms in a two-sided matching market.

Third, there is a growing literature on preferences with incomplete or partial information, restricted preferences such as dichotomous decisions, sequential decision with standard and modified two-sided matching problems (Bogomolnaia and Moulin 2004; Haeringer and Iehle 2014). Recently, preference types have also been explicitly assumed to infer a model of a particular market design. Antler (2015) considers agents' preferences that depend on endogenous actions of other agents in a two-sided market. Regarding our application, players in two-sided markets sometimes have preferences influenced by their behavioral and cognitive factors such as envy. Mui (1995) incorporates envy into standard choice framework and explains envy influenced agent's behavior for specific cases. Gershman (2014) further explains the two sides of envy - destructive and constructive - to explain its personal and social impacts. Although the majority of the literature considers destructive envy following the money burning game of Zizzo (2003), its counterpart, emulative envy, is occasionally discussed. The importance of envious preferences resulting from emulative envy in reconciling the agent-sided matching problem suggests two important concepts of two-sided matching.

Our primary contribution in this paper is to explain individual-sided envious preferences instead of Bayesian rationality where both market players follow Gale and Shapley (1962) type acceptance. This paper extends the literature on the agent-proposing two-sided matching problem in three ways. First, we define and explain how the envy effect and consequent envious preferences influence entry or participation decision and lead to stable matching in a two-sided market. Second, we explain steps of the envious acceptance mechanism and algorithm in the case of many-to-many matching, which can also be extended to many-to-one or one-to-one matching assignment. Additionally, we extend the basic model to explain herd-type acceptance when an individual agent has herd instinct rather than Bayesian rationality.

3 Model

In this section we illustrate the basic model we will consider in the rest of the paper, establish the notation, formulate underlying assumptions and define the envious preferences we will work with.

3.1 Setup and Preliminaries

Let $A = \{1, 2, \dots, |N|\}$ and $F = \{F_1, F_2, \dots, F_{|N|}\}$ be finite sets of individual agents and firms, whereas agents and firms are denoted by i and j, respectively. Consider a capacity cap of the firm $q = q(q_f)_{f \in F}$, where q_f is the capacity of $f \in F$ to match with acceptable agents. We further assume a preference profile of individual agents $\mathfrak{e}^{\succ}{}_a = (\mathfrak{e}^{\succ}{}_a)_{a \in A}$, where $\mathfrak{e}^{\succ}{}_a$ shows agent a's envious preference over firms, and being unassigned, $\emptyset$. Additionally, the preference profile of firms is $\succ_F = (\succ_f)_{f \in F}$, where $\succ_f$ is firm f's preference order over agents. For simplicity, we consider that firms are endowed with identical equivalence classes.

Now we state the related definitions that apply in the model. A matching is an assignment of agents to firms $\mu: A \cup F \to A \cup F \cup \emptyset$ such that: i) $(a_i, f_j) \in \mu$ implies that a_i and f_j find each other acceptable; ii) for each agent $a_i \in A$, $|(a_i, f_j) \in \mu: f_j \in F| \leq 1$; and iii) for each firm $f_j \in F$, $|(a_i, f_j) \in \mu: a_i \in A| \leq q_j$. Consider $\mathcal{M}$ to be the matching set, where a matching μ is considered *blocked by agent* $a \in A$ if $\emptyset \succ_a \mu(f)$, and *blocked by firm* $f \in F$ if $A \subseteq \mu(f)$ such that $A \succ_a \mu(f)$. Further, μ is blocked by agent-firm pair (a, f) if $f \succ_s \mu(a)$ and there exists $A \subseteq \mu(f) \cup \{a\}$ such that $A \succ_f \mu(f)$ and $\left|A\right| \leq q_f$. A matching μ is *pair-wise stable* if it is not blocked by any individual agent of any firm-agent pair $(a_i, f_j) \notin \mu$. This matching μ is *group stable* if it is not blocked by any coalition $\mathcal{K}$ if there exist another matching $\mu' \in \mathcal{M}$ and $\mathcal{K} \subset F \cup A$ such that $\{f\} = F \cap \mathcal{K}$ and for all $a \in A \cap \mathcal{K}$ if $\mu'(a) \in \mathcal{K}$ and $\mu'(a) \succ_a \mu(a)$ and $\mu'(f) \in \mathcal{K} \cup \mu(f)$ and $\mu'(f) \succ_f \mu(f)$. A mechanism ψ indicates a matching for each problem where the outcome is $\psi(F, A, \succ_F, \succ_A)$ and the assignment of each agent is $\psi_a(F, A, \succ_F, \succ_A)$. The mechanism is said to be (group) stable when it solves a (group) stable matching of a given matching problem.

We consider Varian's (1974) explanation of envy where each individual agent compares his own bundle to the bundle of each of the other individual agents. Under a given state, if agent a_1 prefers the bundle of agent a_2 to his own, a_1 envies agent a_2. Consider the state that is most desirable to any a_i to be a preferred state, S_p. If any agent's belongings are lower than the preferred state we assume that to be an envious state, S_e. We further consider that agents' envious preferences satisfy the following properties:

A1 Agent shows considerable cognitive bias.
A2 Agent states his-sided truthful envious preference.
A3 No justified envy from both agents' and firms' side.
A4 Agility of agent-proposing matching is undefined.
A5 Full information matching mechanism without moral hazard.
A6 Degree of envy effect is time invariant and increases with each stable matching.

Definition 1 (Envious Preference). *Individual agent a's preferences are envious, $\mathfrak{e}^{\succ_a}$, when the envious state is strictly preferred to the preferred state such as $S_e \succ_a S_p$, and there exist:*
(a) any $\mu \succ_a \emptyset$ when $b_1 \gtrsim_a b_2 \succ_a \emptyset$; or
(b) any $\mu \succ_a \emptyset$ when $b_1 \approx_a b_2 \succ_a \emptyset$.

We simply assume a_1*'s* preference, $\succ_{a_1}$ is envious if a_1 emulates a_2 to reach the *preferred state* from existing *envious state* and accepts any existing firm rather than being unassigned. In other words, envious preferences lead an agent to accept any positive offer to overcome his risk associated with envious state.

3.2 Agent-proposing Envious Acceptance

In this section, we present results related to necessary and sufficient conditions considering strict envious preferences of the agent based on Definition 1(a). First, Proposition 1 shows that envious preferences constitute a necessary condition. Second, Proposition 2 further shows stable matching with envious preferences that leads to a sufficient condition.

Proposition 1 (Stable Matching). *Under A1-A6, if there exist* $a_1, a_2 \in A$ *with envious preference, then there exist a stable matching under* $(F, A, \mathfrak{e}^{\succ_A}, \succ_F)$.

Proof: As in the envious agent model, a matching is (pairwise) stable if it is not blocked by any envious agent or pair of agents. Let $A = \{a_1, a_2\}$ with $q_f = 1$, and $F = \{f_1, f_2\}$ with $q_a = 1$. The preference over the acceptable agents and firms in matching is following:

$\mathfrak{e}^{\succ_a}$	$\succ_f$	μ
$f_1 \mathfrak{e}^{\succ_{a1}} f_2 \mathfrak{e}^{\succ_{a1}} \emptyset$	$a_1 \succ_{f1} a_2 \succ_{f1} \emptyset$	$\mu_1(a_1, f_1)$
$f_2 \mathfrak{e}^{\succ_{a2}} f_1 \mathfrak{e}^{\succ_{a2}} \emptyset$	$a_2 \succ_{f2} a_1 \succ_{f2} \emptyset$	$\mu_2(a_2, f_2)$

Figure 1. A Stable Matching with Agent-sided Envious Preferences

Under μ, all a's are assigned to acceptable f's, and all f's are assigned to acceptable a's based on their preference priority. In this case, all possible assignments have neither blocking agents nor blocking pairs. In addition, any assignment in these matchings can not be improved upon by any agent or any pair. Hence, $\mu = \{\mu_1 \ a_1, f_1 \ , \mu_2 \ a_2, f_2 \ \}$ is stable. ■

If envious agent-proposing $\mu(a_i, f_j)$ matchings are stable, the optimal stable matching of the agent side of the market can be expressed by the following proposition.

Proposition 2 (Gale and Shapley 1962). *When all envious agents have strict preferences, there always exists an A-optimal stable matching that every agent likes relative to any other stable matching.*

Proof: In the case of Definition 1 (a), we can follow Gale and Shapley (1962). The remaining case, for example, Definition 1 (b) is pretty obvious, and is subject to proof by inspection. ■

Next, we show a mechanism, algorithm 1, where an envy effect generated from previous matching influences envious agents to match. Let $D_A \ \mu \ = \{i \in N | f P_i \mu \ i \ \}$ which is the set of agents who desire f in μ. Remove from q_{b_j} those farmers who initially entered into the matching market such as $q_{b_j} = q_{b_j} \backslash \{a_i | q_{b_j} \notin q_{a_i}\}, \forall b_j \in B$. Consider, the set of agents (unmatched from the previous stage and newly entered) $P_{a_i} := \emptyset, \forall a_i \in A$ with envious preferences. We now consider an arbitrary value of the envy effect for agents, $\eta_{A,t_0} > 0$, whereas buyers envy effect is time invariant, such that $\eta_{B,t_0} = 0$.

Algorithm 1 (Envious Agent-Firm Matching). *Let there be a set of available agents* $\mathcal{A}$ *who desire to connect with a firm through a stable matching,* μ*. In Step* t_0*, assign each agent and firm to be free* $\mu \ a_i \ := \emptyset$*, and* $\mu(f_j) := \emptyset$*. Then consider the degree of envious preference* η*, and preference order of the agent, and the firm. Assign each agent to his preferred firm, if acceptable to a firm. Otherwise, assign next achievable alternatives. If match* $\mu(a_i, f_j)$ *is stable, the iteration is complete and terminated. Then remove the matched agent and firm. Again consider a higher degree of envy effect,* $\eta_{t_1} > \eta_{t_0}$*. Iinitiate another iteration to find a stable matching. This algorithm terminates at step* T *such that* $q(F_T) = 0$ *for all* $a \in A$.

This algorithm is intuitive by its nature, and proceeds step-by-step as follows. Initially consider, an agent proposed stable matching. In initial Step t_0, rational agents a_0 make their own decisions based on their rational preferences and expectations rather than being influenced by cognitive factors. Following A1-A6, this matching contract would be stable since both agent and firm maintain the matched criteria over a certain period.

We assume in Step t_1, Proposition 1 leads to the envy effect, η_{t_0}. We consider envy to emerge from network effects that consists of both informational and demonstration effects. Informational effects convey information. A demonstration effect emerges from the choice behavior of assigned agents in the previous step resulting in envious preferences among unassigned others. Both effects influence unassigned agents to be envious of existing agents $a_0 \in A(F, A, \succ_F, \succ_A)$, and the agent $a_t \in A(F, A, \mathfrak{e}^{\succ_A}, \succ_F)$ prefers to accept the matching, μ. Assigned agents are removed, and capacity remains. If any agent considers firms unacceptable assigned to $\emptyset$. Once the process terminates with successful iteration, the agent-firm pair constitutes a stable matching that establishes the following lemma.

Lemma 1 (Algorithm Output). *Algorithm 1 terminates with a stable agent-firm matching* μ*.*

Proof: Please refer to Section 3. ■

In Step t_2, envious agents who are unassigned in t_1 are now more envious of assigned agents due to an increased degree of envy such that $\eta_{t1} > \eta_{t0}$. Assume that the process will continue until Step T, and in every repeated Step after t_1 agents A_{t1} envy to assigned agents and consider F_{t1}. This algorithm terminates at Step T whenever all firms' capacity decrease to 0.

Based on results from Proposition 1 and Lemma 1, we can characterize our initial conjectures as following Proposition 2.

Proposition 3 (Stable Outcome). *If* $e^{\succ_A}$ *is envious then an envious acceptance mechanism also selects unique (group) stable outcomes in a given problem* $(F, A, e^{\succ_a}, \succ_b)$*.*

Proof: Let $\mu(a_i, b_j)$ be output of algorithm 1 of $(F, A, e^{\succ_f}, \succ_b)$ and A^{t_0} and F^{t_0} be the set of agents and firms removed in iteration t_0 of algorithm 1. Suppose agents' preferences are envious. As $f\mathfrak{e}^{\succ_a}$ in each iteration, any $a \in A$ can be assigned to $f \in F$. We also assume, there does not exist $f \in F$ and $A \subset \mu(f)$ such that $Ae^{\succ_a}\mu(f)$. Otherwise, each f may be selected A in the step it was removed. Hence, μ is not individually blocked. Suppose, there exists $f \in F$ and $A \in A$ blocking μ and $f \in F^t$ which means f is removed in every step t. Since μ is individually envious to each $f \in F$, $f \succ_a \emptyset$. Since $\succ_a$ is envious as $e^{\succ_a}$, in every iteration t in which f selects its most preferred or identical subset, agents considering buyers b as acceptable must be in $(F_{t-1}, A_{t-1}, e^{\succ_{f-1}}, e^{\succ_{a-1}})$. Following algorithm 1, f prefers to any subset of $\mu(f) \cup A$. A contradiction. Further assume, $\mu' \in \mathcal{M}$ is group stable where $\mu' \neq \mu$. Consider one student assign in the first iteration. Suppose, $a \in A'$ and $\mu'(a) \neq \mu(a)$. All agents $f \in F$ consider a firm $b \in B$ at the top. Hence, all farmers in $\mu'(b)$ rank b first. Otherwise, individual's enviousness would be violated, and then $\mu'(a) \subseteq (F, A, e^{\succ_f}, \succ_a)$. By assumption, $\mu(b)$ is the preferred subset of the agents to buyer. Hence, μ' is blocked by $\mu\ b \setminus \mu'\ b$ and for all $b \in B'$. Following an inductive process, we find that μ' can't be a group stable. ■

As Proposition 1, 2 and Lemma 1 explain agent-sided envious preferences affect stable matching in a two-sided problem, we argue the following Theorem 1.

Theorem 1 (Matching under One-sided Envious Preferences). *An envy-induced stable matching exists in two-sided matching problems if and only if individual agent's preferences are one-sided and envious considering acceptable firms are almost identical.*

Proof: Following Proposition 1 and Proposition 2 with arguments of the mechanism cited in algorithm 2. ■

3.3 Agent-sided Herd-type Acceptance

Few agents may have herd-type acceptance, following Banerjee (1992), where this individual agent with envious preferences acts based on what other agents are accepting rather than using their information (Ahamad 2015). We extend our basic model where agent-proposing herd-type acceptance considered in matching assumes considerably higher degree of enviousness under a less risky environment.

For example, consider an arbitrary Step $T-10$, where $T=100$. At this stage, non-members' decision making is based on herd-externality. Assume degree of envy effect is considerably high such as $\eta_{T-10} \gg \eta_{t_0}$. Though the network externality is common knowledge, envious agents' make their participation decision following herd-type acceptance (see Proposition 1(4) of Banerjee 1992, pp. 806).

4 Envious Farmer and Contract Matching Problem

Consider a standard two-sided contract market for a single product where fa are the farmers (or farm producers) and b are the buyers (or contract providers). A two-sided, i.e., farmer-buyer contract market provides the platform. In this contract platform, it is assumed that farmers try to maximize their reservation utility subject to envious participation along with minimizing the associated risks, while the buyers maximize profit subject to capacity and farmer's participation constraints.

In the initial phase, few rational farmers enter based on their risk and gain factors rather than being influenced by cognitive factors. We assume all farmers fulfill the basic eligibility criteria stated by the buyers; hence it is considered the equivalence class of agents. Further, we assume a full commitment contract market where both market players can identify the contract clauses and matching criteria, and they have no incentives to deviate from the existing contract before the end period.

Under A1-A6, one-to-one contract matching during initial period t_0 would be stable over the total matching period T. At pre-contract proposal phase, we assume that Proposition 1 is satisfied which leads to the envy effect. This emerges from network effects (informational and demonstration effect) motivating non-member agents to enter into the same contract network. In the contract proposal phase, informational effects influence non-members to enter into the given network due to the members' who entered at t_0 conveying information to them about what they know. Additionally, a demonstration effect emerges from choice behavior of contract participants at contract proposal phase and subsequent periods resulting in envious preferences among non-members. Both effects influence potential off-network members to be envious of existing members, and they prefer to enter into the network. We only consider the condition (a) of Definition 1 to avoid complex proofs and algorithms to keep the paper simple.

We now explain a nontrivial, tractable type envious farmer-proposing algorithm following Bolton and Dewatripont's (2005); Gale and Shapley (1962); Haeringer and Iehle (2014) and Rastegari et al. (2013) procedures. The algorithm dynamically alternates between three main components, for example, a contract dealing component, a tentative matching component and an envy effect component. algorithm 2 also consists of four consecutive phases such as i) contract proposal phase, ii) contract dealing phase, iii) contract participation request phase, and iv) tentative matching phase. The algorithm repeatedly chooses an envious farmer and instruct them to deal with all plausible buyers based on their preference order, for example, those who are in the chosen farmer's top equivalence class and achievable. An achievable buyer must have the capacity to accept at least unmatched farmers. After the dealing has been performed followed by participation request from the farmer's side, the algorithm transitions to the tentative contract-matching phase, in which the farmers initiate a sequence of contract proposal. In this stage, buyers receive entry offers from the farmers, and the decision regarding acceptance or rejection is based on their preference order.

Algorithm 2. Envious Farmer-proposing Matching

Input: $FA, q_{fa\in FA}, \eta_{fa\in FA}, B, q_{b\in B}, \eta_{b\in B}$
Output: $\mu(fa_i, b_j)$
repeat
 initialize
  $q_{b_j} = q_{b_j} \setminus \left\{ fa_i \middle| q_{b_j} \notin q_{fa_i} \right\}, \forall b_j \in B$; /*farmers who initially entered into the network removed from q_{b_j}*/
  $\varepsilon_{fa_i} = 1, \forall fa_i \in FA$; /*current equivalence class number for each farmer fa_i*/
  $\mathcal{A}_{fa_i} := \emptyset, \forall fa_i \in FA$; /*set of farmers (unmatched from previous stage and newly entered) with envious preference from fa_i */
  $\mu\ fa_i\ := \emptyset, \forall fa_i \in FA$; /*assign each farmer to be free*/
  $\mu\ b_i\ := \emptyset, \forall b_j \in B$; /*assign each buyer to be free*/
  $\eta_{FA,t_0} > 0$; /*positive envy effect of farmers*/
  $\eta_{B,t_0} = 0$; /*no envy effect of buyers*/
 repeat
  foreach $\mu_i \in M$ **do**
   compute $\eta_{FA,r_t} = n_{pr} + n_{tp}(envious\ weight\ for\ each\ \mu \times \#\mu)$ /*compute envy effects where $\mathrm{n_{pr}}$ presents "previous" iteration's envy effect and $\mathrm{n_{tr}}$ presents envy effect of "present" iteration*/
  foreach $fa_i \in FA$ **do** /*unmatched farmer propose to enter into the contract network*/
   if $\mu(fa_i) = 0$ **and** $\mathcal{A}_{fa_i} = \emptyset$ **and** $EC\ \ fa_i\ \ > 0$ /* EC stands for new equivalence classes*/
   then $\varepsilon_i := EC\ \ fa_i$; /*assign previous equivalence classes to new equivalence classes */
    $\mathcal{A}_{fa_i} := q_{fa_i,\omega_i}$; /*$fa_i$ offers a contract deal to a buyer*/
  $fa_i = Select\ FA^{um}\ = arg\,min(\ fa_{um} \in FA^{um}\ max\ \ Preferential\ rank_{b_j}\ fa_{um}\ |b_j \in P_{fa_{um}}$ /* consider FA^{um} is the nonempty set of unmatched farmers*/
  foreach $fa_i \in \mathcal{A}_{fa_i}$ **do**
   fa_i dealing with b_j
  $\mathcal{A}_{fa_i} := \emptyset$;
 repeat
  foreach $fa_i \in A$ **do**
   if $\eta_{FA,t} > 0$ /*when magnitude of envy effect is positive*/
   if $\mu\ \ fa_i\ \ = \emptyset\ and\ P_{fa_i} \neq 0$
   then envious farmer fa_i accepts a b from the achievable buyers
   each buyer who has received one or more entry request tentatively accepts the offer from the farmers he most prefers and rejects the rest;
   each tentatively matched farmer fa_i is removed from the lists of those buyers who have in the equivalence classes to $\mu\ \ fa_i$; q_{b_j} and P_{fa_i} are updated accordingly for all $fa_i \in FA$;
  until there is no unmatched envious farmers fa_i with $\mathcal{A}_{fa_i} \neq \emptyset$ who prefers to enter into a contract farming;
 until each envious farmer is either tentatively matched to any achievable buyer or has not find any achievable buyer to be a contract farmer;
return $\mu(fa_i, b_j)$; /*new farmer-buyer stable matching is returned as outcome of envious farmer-proposing matching algorithm*/

As an example, we assume a simple framework of four farmers and four buyers in a two-sided contract market depicted in figure 2. We consider that some farmers have already entered into the contract network, and generate an envy effect, which ultimately makes the remaining farmers envious to the members. Notice that the farmers are endowed with identical equivalence class. Running farmer-proposed algorithm 2 in this setting returns a stable matching in each iteration. We also assume that the envy effect increases with the number of farmers entered into the network.

In iteration 1, let $fa_i \in FA = \{fa_1, fa_2, fa_3, fa_4\}$ and $b_j \in B = \{b_1, b_2, b_3, b_4\}$ in Iteration 1, and assume the capacity cap of each buyer $b \in B$, $q_{b_j \in B} = 1$. Suppose that fa_i submit their true preference and $q_{fa_{i \in FA}} = 1$. Each associated column of farmers and buyers depicted in figure 2 lists their underlying preferences. In figure 2 the horizontal lines separate equivalence classes. In every case, matching $\mu_k\ fa_i\ = b_j$, where $k = 1, 2, 3, 4$ would find enviously stable matching and generate envy effect. Following the Gale and Shapley (1962) procedure, the algorithm selects an unmatched buyer who is most preferred among the farmers. Then, the execution of algorithm 2 will be as follows.

In Iteration 1, consider an initial envy effect η_0 generated from the initial stage. Based on *Select* FA^{um} , farmer fa_1 and farmer fa_2 both propose contract dealing to buyer b_1 and buyer b_2, whereas fa_3 proposes dealing offer to b_2 and b_3, and fa_4 proposes to b_2 and b_4. Based on ranking order depicted in figure 2, fa_1 is selected in the first iteration who offers b_1 and b_2. fa_1 requested b_1 to enter into the contract network and the buyer accepts the offer of fa_1, and they tentatively match as $\mu_1(fa_1, b_1)$. Therefore, fa_1 and b_1 are eliminated from the FA and B of the algorithm. After every stable matching, we assume that at least a few new farmers who are envious to matched farmers will be interested to join the network. For simplicity, we only explain without considering new agents or farmers after every stable matching. In Iteration 2, again consider new envy effect η_1 generated after iteration 1 that is higher than η_0. Following *Select* FA^{um} , fa_2 is now selected to offer contract deal, and s/he offers b_2, and b_2 accepts the position offered by fa_2. They tentatively match as $\mu_2(fa_2, b_2)$. After the second iteration, b_2 is removed from fa_3's and fa_4's preference list. In addition, fa_2 and b_2 also eliminates from the algorithm 2.

$e^{\succ_{fa_1}}$	$e^{\succ_{fa_2}}$	$e^{\succ_{fa_3}}$	$e^{\succ_{fa_4}}$
b_1	b_1	b_2	b_2
b_2	b_2	b_3	b_4
b_4	b_3	b_1	b_1
b_3	b_4	b_4	b_3

$\succ_{b_1}$	$\succ_{b_2}$	$\succ_{b_3}$	$\succ_{b_4}$
fa_1	fa_2	fa_3	fa_2
fa_3	fa_1	fa_2	fa_1
fa_2	fa_3	fa_1	fa_3
fa_4	fa_4	fa_4	fa_4

$PR_{b_j}(fa_i)$			
$PR_{b_1}(fa_1) = 1$	$PR_{b_1}(fa_2) = 2$	$PR_{b_2}(fa_3) = 2$	$PR_{b_2}(fa_4) = 2$
$PR_{b_2}(fa_1) = 1$	$PR_{b_2}(fa_2) = 1$	$PR_{b_3}(fa_3) = 1$	$PR_{b_4}(fa_4) = 2$

Figure 2. An Arbitrary Setting with 4 Farmers with η_0 and 4 Buyers.

In Iteration 3, a higher envy effect η_2 that is assumed such as $\eta_2 > \eta_1 > \eta_0$. As two farmers and two buyers are eliminated from the matching process, we have remaining two farmers and two buyers. Based on ranking order, fa_3 is now selected who offers a deal to b_3, and b_3 also accepts the deal offered by fa_3 that ends in a tentative matching as $\mu_3(fa_3, b_3)$. After this round, b_3 is removed from fa_1's and fa_4's preference list. As a result, fa_3 and b_3 are eliminated from the list, and we obtain another matching $\mu_3(fa_3, b_3)$. In iteration 4, a higher envy effect η_3 occurs, which is higher than previous envy effects. Thus fa_4 is selected and offers b_4, and b_4 accepts the position offered by fa_4. They get tentatively matched as $\mu_4(fa_4, b_4)$. After this round, fa_4 and b_4 are eliminated from the list, and we obtain a final matching as $\mu_4(fa_4, b_4)$. Algorithm 2 ends at iteration 4 since every farmer has been assigned to a buyer. Finally, $\mu\ fa_i\ = b_j, \forall 1 \leq i \leq 4$ is returned. We, now, obtain four stable matchings for the farmers who enviously preferred to enter into the given contract network, such as $\mu = \{(fa_1, b_1), (fa_2, b_2), (fa_3, b_3)\}$.

Though the network externality is common knowledge, envious agents' information processing based on non-Bayesian beliefs may generate herd type externality. We assume that with a degree of envy $\eta_{he} \gg \eta_{t_0}$ that is considerably high, a few highly envious farmers may have herd-type acceptance where they act based on what other agents are accepting rather than using their own information

(Ahamad 2015, Banerjee 1992). In this case, the assumption of no moral hazard from buyers' side would be violated and exploitive contracts may occur.

5 Concluding Remarks

We develop a theoretical framework of two-sided matching where agents' one-sided envious preferences and consequent envy effect explain individuals' behavioral decision-making process. We argue that individual-proposing envious preference originates from their emulative envy, influencing the entry decision in a standard two-sided matching framework. Results also suggest that agent-sided envious preferences lead to a stable matching. We further extend this framework to explain agents' herd-type acceptance, which dominates their logical preferences in participation decision in a less risky environment.

Addressing envious preferences and herd-type acceptance in a two-sided matching framework would provide vital policy relevant framework for understanding the cognitive factors influencing participation decisions, and for the development of contracts within a behavioral framework, especially contract farming for labeled products, for example, organic, eco, local, fair-trade goods where special quality attributes are preferred. Consequently, network effects for single product markets would also lead to parallel contract network for homogeneous products, which would be a potential idea for future research. Moreover, the framework of envious preferences can also be applicable in a similar way to the entitlement to universal health coverage, financial inclusion, online application developer networks, etc.

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