Oil Price Volatility and Asymmetric Leverage Effects

Eunhee Lee and Doo Bong Han

Institute of Life Science and Natural Resources, Department of Food and Resource Economics
Korea University,

Department of Food and Resource Economics, Korea University

Authors Emails: leeeunhee@korea.ac.kr, han@korea.ac.kr

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Eunhee Lee[1] and Doo Bong Han[2]

Abstract

This study adopts a stochastic volatility (SV) model with two asymptotic regimes and a smooth transition for oil returns. We find that SV models with a smooth transition between two regimes imply an asymmetric leverage effect with different regimes. In particular, the half-life of a negative volatility shock is longer than that of a positive shock.

[1] Institute of Life Science and Natural Resources, Department of Food and Resource Economics Korea University, leeeunhee@korea.ac.kr

[2] Department of Food and Resource Economics Korea University

1. Introduction

Oil is a crucial economic resource in the commodity, manufacturing, and financial markets. Both the oil price and its volatility have significant effects on the global economy. Thus, oil price movements and shocks are closely monitored by producers, consumers, investors, and policymakers. Furthermore, the extent to which the volatility affects prices depends critically on the permanence of shocks to the variance. More generally, modeling the pricing of contingent claims relies on perceptions of how permanent the shocks to the variance are. Therefore, accurately modeling oil price volatility is meaningful. However, few studies have analyzed oil volatility. Furthermore, most studies apply ARCH-type models to estimate oil volatility and leverage effects. Recently, stochastic volatility (SV) models have been used to specify volatility as a separate random process and, thus, can have advantages over ARCH-type models when modeling the dynamics of return series.[3] However, SV models used in previous studies have not been able to explain the asymmetric leverage effect for volatility regimes and ignore the possibility that the half-life of volatility shocks could depend on the sign of the shocks.

This study adopts an SV model with two asymptotic regimes and a smooth transition between their returns, as proposed by Park (2002) and Kim et al. (2009), in order to fully capture the stylized facts of oil price dynamics. We find two distinct characteristics of oil price volatility. First, SV models with a smooth transition between two regimes imply an asymmetric leverage effect in different states of the regimes. Second, the half-life of a negative volatility shock is rather longer than that of a positive shock. This is another

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[3] See Kim et al. (1998) and Jacquier et al. (1994, 2004).

asymmetric effect on oil price volatility, because a negative shock does not have the same, but opposite effect of a positive shock with the same magnitude. Therefore, oil refiners, investors, and policymakers should consider the asymmetric leverage effects and the asymmetric speed of an adjustment in oil price volatility.

2. Model

We consider the following SV model, as proposed by Park (2002) and Kim et al. (2009). We let r_t be a demeaned return series. Then,

$$r_t = \sqrt{f_t(x_t)}\varepsilon_t \tag{1}$$

$$x_{t+1} = \alpha\, x_t + u_{t+1}, \tag{2}$$

where $\begin{pmatrix} \varepsilon_t \\ u_{t+1} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right),$ \hfill (3)

and x_t is a scalar latent volatility factor that generates the stochastic volatilities of oil and is assumed to be AR(1). If the AR(1) coefficient of latent volatility factors $\alpha \approx 1$, the volatility can be persistent. The correlation between the return and volatility is imposed in order to test the leverage effect for the oil market. Therefore, the correlation parameter, ρ, generates a leverage effect if $-1 \leq \rho < 0$.

The actual volatilities in this study are generated by the parametric logistic function, which is given by

$$f(x_t) = \mu + \frac{\beta}{1+exp(-\lambda(x_t-\kappa))}, \text{ with } \mu > 0, \beta > 0 \text{ and } \lambda > 0. \tag{4}$$

The parameters μ and $\mu + \beta$ represent the asymptotic low and high volatility regime, respectively. The parameters λ and κ specify the transition between the two regimes (i.e., the speed and the reflection point of the transition). As the transition speed increases, λ

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increases, and the actual volatilities are realized by one of the two asymptotic regimes. Given

the reflection point κ, if the value of the latent volatility factor x_t is lower than that of κ, the

volatility is closer to the asymptotic low regime, μ; otherwise x_t is greater than κ, and the

volatility is closer to the asymptotic high regime, $\mu + \beta$.

In this study, we use the Bayesian approach to estimate our model. We define some additional

notation, for convenience. Let $R = (r_1, \ldots, r_T)$ and $X = (x_1, \ldots, x_T)$ be the vector of

demeaned oil returns and the vector of latent variables, respectively. In addition, we define

$\theta = (\alpha, \rho, \mu, \beta, \lambda, \kappa)$ as the vector of unknown parameters. By Bayes' theorem, the joint

posterior is given by $p(\theta, X|R) \propto p(R, X|\theta)p(\theta)$, where $p(\theta) =$

$p(\alpha)p(\rho)p(\mu)p(\beta)p(\lambda)p(\kappa)$. We assume that $p(\alpha) \sim B(\overline{a_1}, \overline{a_2}), p(\mu) \sim G(\overline{\mu_1}, \overline{\mu_2}),\ p(\beta) \sim$

$G(\overline{\beta_1}, \overline{\beta_2}),\ p(\kappa) \sim N(\overline{\kappa_1}, \overline{\kappa_2}),\ p(\lambda) \sim G(\overline{\lambda_1}, \overline{\lambda_2})$, and $p(\rho) \sim U(-1, 1)$. [4] For the usual

Bayesian procedure, we implement a Markov chain Monte Carlo (MCMC) method to sample

the latent factors and the parameters from $p(\theta, X|R)$. The Bayesian MCMC approach is

particularly suitable, has been proven to perform well, and produces relatively accurate

results. For our MCMC procedure, we employ the Gibbs sampler and the Metropolis–

Hastings (MH) algorithm within the Gibbs sampler. In particular, to sample x_t, we use the

grid-based chain suggested by Tierney (1994).

3. Estimation Results

We use weekly oil futures prices from January 2, 1986, to October 10, 2014, obtained

from Datastream. The returns are calculated as the natural log differences of the prices. We

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[4] B, G, N, and U denote the beta, gamma, normal, and uniform distributions, respectively.

draw 200,000 samples for each parameter and latent variable using the Gibbs sampler, and

discard the first 84,000 samples as a burn-in period.

Table 1 presents the estimation results for the stochastic volatility of the oil returns and

reports the posterior means, standard deviations (SD), and the 5th and 95th quantiles. The last

column lists the convergence diagnostics (CD) by Geweke (1992). Our results indicate

relatively high convergence diagnostics for all parameters. The estimated parameters are

significant at the 5% significance level, except for κ. However, the estimated parameter for

κ is significant at the 10% significance level and converges well.

Table 1 Estimation Results

	Posterior				
Parameter	Mean	SD	5%	95%	CD
μ	0.00025	0.00008	0.00010	0.00041	1.2900
β	0.0067	0.0011	0.0045	0.0089	-0.0727
κ	0.8189	0.4659	-0.0944	1.7321	0.1597
λ	0.4882	0.0469	0.3964	0.5801	0.7793
α	0.9858	0.0050	0.9760	0.9956	0.0632
ρ	-0.4980	0.0884	-0.6712	-0.3248	0.2309

Our empirical results reveal that the asymptotic low and high levels of the stochastic

volatilities for oil are $\sqrt{\mu} = 1.59\%$ and $\sqrt{\mu + \beta} = 8.34\%$, respectively, in a given week. The

AR(1) coefficient of the latent factor, α, is 0.9858, which is highly persistent and can generate

highly autocorrelated volatility or volatility clustering. The estimate of the correlation

coefficient, ρ, is -0.498 and is significant, implying a negative relation between shocks to

returns and volatility. Several studies on the oil and commodity markets, such as Schwartz

and Trolle (2009), Vo (2009), Larsson and Nossman (2011), and Du et al. (2011), show that

the correlation coefficient is negative, but not significant. However, our finding strongly

supports the leverage effect in the oil market.
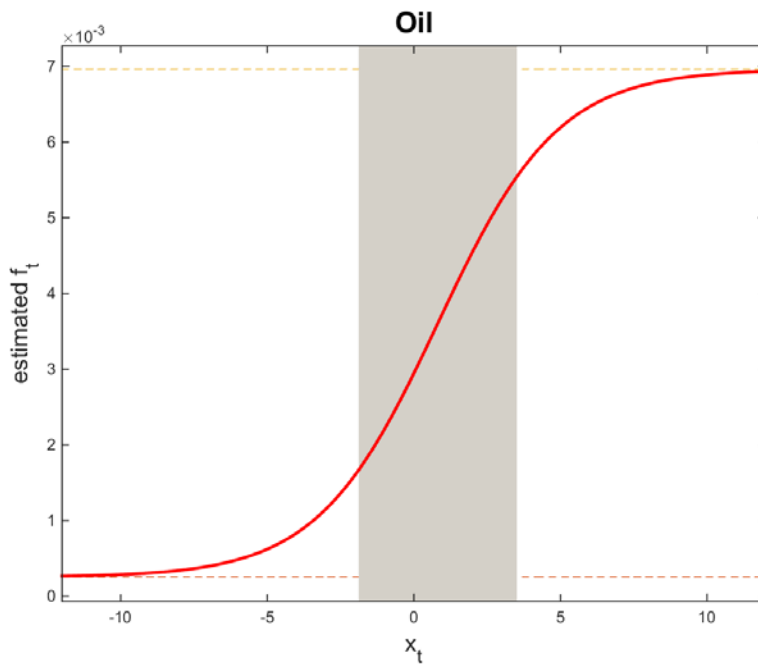
Figure 1 Estimated Volatility Function

Figure 1 shows the estimated logistic volatility function. The horizontal axis denotes the

latent factor, x_t, and the vertical line indicates the estimated conditional variance, f_t. The

dashed lines represent the asymptotic low and high regimes, respectively, and the shaded area

implies a transition period, which is the interval [-1.88, 3.52].[5] Therefore, we can regard the

area below the lower boundary of the transition period as the low volatility regime and the

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[5] Kim et al. (2009) note that the interval $[\kappa - \left(\frac{1}{\lambda}\right) log\left(2 - \sqrt{3}\right), \kappa - \left(\frac{1}{\lambda}\right) log\left(2 + \sqrt{3}\right)]$ can be regarded as the transition period, where $f'''(x) = 0$ at the endpoints of this interval.

area above the upper boundary as the high volatility regime. This model differs from the usual regime-switching model, which assumes only two regimes in the economy and an exogenous and abrupt change in switching regimes, which is unrealistic. Figure 2 displays the extracted latent factor, x_t, which generates the oil volatilities. The extracted latent factors from the SV model show that the latent factors stayed in the high state of volatility around 1986, during the Gulf War from August 1990 to February 1991, and the global financial crisis and the recession period. Vo (2009) notes that the oil volatility surges to a high level around 1986, when Saudi Arabia, the dominant member of OPEC, stopped acting as a swing producer and let oil prices plummet.

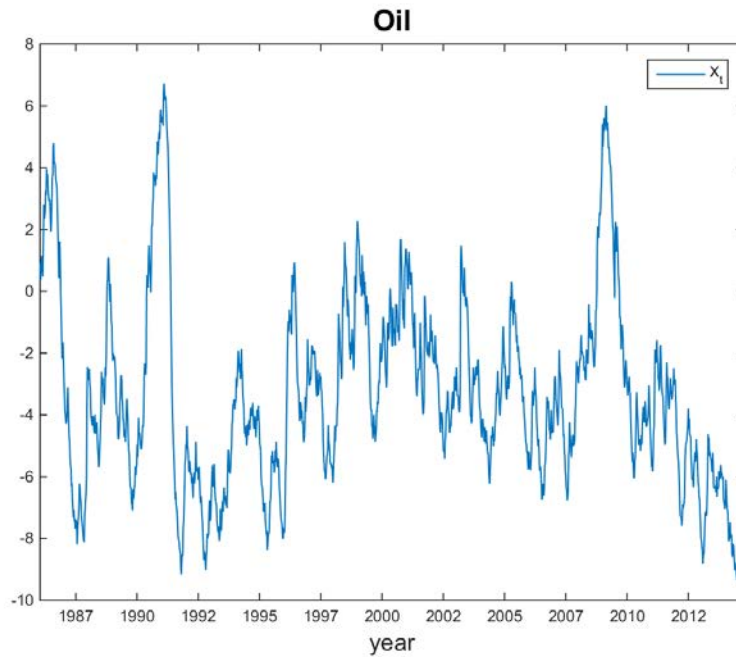**Figure 2 Estimated Latent Factor**

Figure 3 shows the estimated and realized volatility for oil. The dotted and thick lines display the absolute value of oil returns and the estimated volatilities, $\sqrt{\hat{f}_t}$, respectively. The estimated volatilities explain the realized volatilities well, particularly in light of their trend behaviors.

**Figure 3 Absolute Returns and Estimated Volatility**
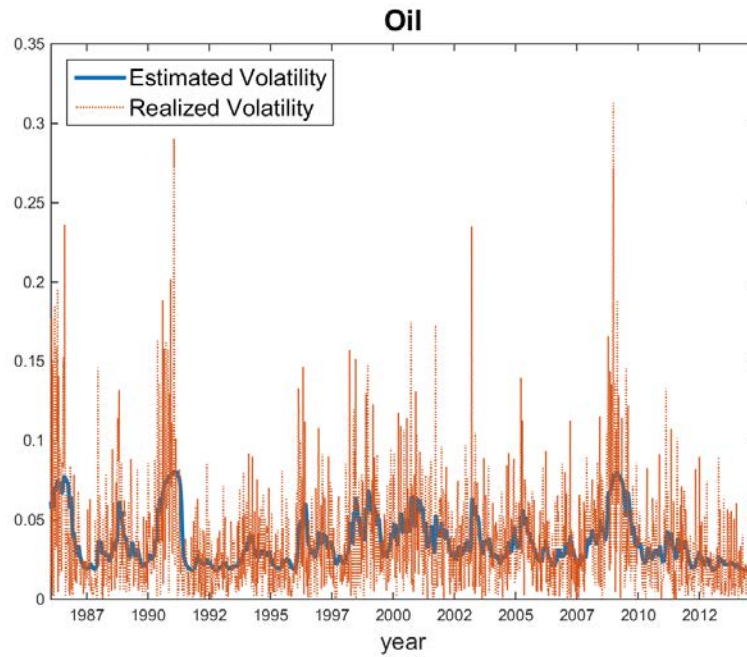
Table 2 Leverage Effects with Different Regimes

Size of shock (ε_t)	Regime	Volatility growth rate
Negative shock	Low	0.1674
	Transition	0.1332
	High	0.0246
Positive shock	Low	-0.1070
	Transition	-0.1233
	High	-0.0378

Table 2 quantifies the magnitude of the leverage effects across the states of the economy.

The empirical results show that a negative shock to the oil price return has a bigger impact on

the volatility than does a positive shock during the low-volatility regime. However, the

reverse is true during the high-volatility regime.[6]

Figure 4 Estimated Impulse Response Function: Positive and Negative Shock
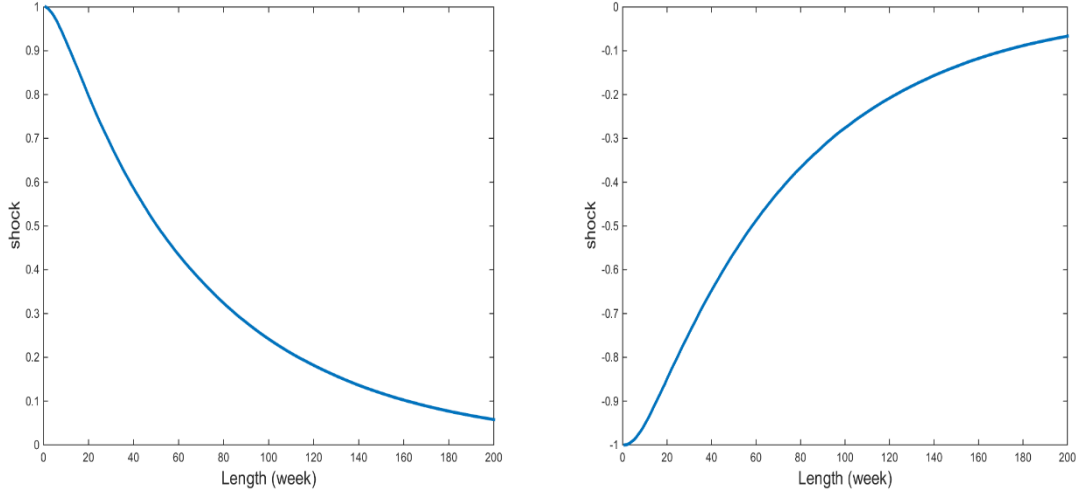
For the standard SV model, the impulse response function (IRF) is calculated as the coefficient of the moving average representation. However, estimating the IRF for our SV model is not as simple. For a given size of shock, we first simulate the IRFs conditioned on every initial condition, and then by averaging all the simulated impulse response sequences to avoid obtaining an impulse response conditioned on a specific initial condition. Moreover, using the estimated IRF, we measure the rate of mean reversion by calculating the half-life of a volatility shock. Figure 4 displays the estimated IRFs for a positive shock (left) and a negative shock (right). From our simulation result, the half-life of a positive volatility shock is 51 weeks, while that of a negative volatility shock is 61 weeks. Thus, the half-life of a negative volatility shock is markedly longer than that of a positive shock. Therefore, the

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[6] These features are evident regardless of the size of the shock.

effect of a volatility shock is not symmetric.

4. Conclusions

This study adopts an SV model with two asymptotic regimes and a smooth transition between them for the oil market. According to the empirical results of the SV model, the leverage effect is asymmetric with different states of volatilities, and the rate of the mean reversion depends on the sign of the shock. To the best of our knowledge, this is the first empirical study to examine the asymmetric leverage effect with different regimes and the estimated half-life of volatilities with a negative and a positive shock.

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