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**A Study on the Forest Thinning Planning Problem Considering Carbon Sequestration and  
Emission**

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## Abstract

Appropriate forest thinning is beneficial for growing forests and protecting ecological environments. To find a beneficial way for both economic and environmental aspects, carbon sequestration and emission due to forest thinning activities can be traded. However, previous works on forest planning did not consider forest thinning, nor carbon trading. Hence, this study proposes the spatial forest thinning planning problem with carbon trading, which decides forest thinning schedules over a planning period so that the total thinned timber volume over the period and the revenue from carbon trading are maximized, under some spatial constraints. This study creates a novel mathematical programming model.

*Keywords: Forest thinning, carbon trading, spatial forest planning*

## 1. Introduction

Forest harvesting is further classified into clearcutting and thinning. Forest clearcutting means that almost all trees in the area to be harvested are cut down, but it makes the soil become dried or swamped, which is harmful to saplings growing, water conservation, landscape, and animal dwelling. Forest thinning is the selective removal of trees. Appropriate forest thinning is beneficial, as it allows those uncut trees to have more space to grow, so that more timber amount could be harvested (Assmann, 1970, 1961); the ecological environment can be protected (Beck, 1983; Haveri and Carey, 2000); forest resources can be recycled. To the best of our understanding, conventional spatial forest planning problems were not designed specifically for forest thinning.

Long-run forest planning should consider not only economic aspect, but also ecological environments and social responsibility (Kangas and Kangas, 2005). Global warming due to CO<sub>2</sub> emission into atmosphere has received lots of attention recently. Some green projects have been proposed to adsorb CO<sub>2</sub> (Hassall and Associates, 1999, p. 23), and fostered an emerging carbon trading market, in which sequestration and emission of carbon are commodities that can be traded. Based on the Kyoto Protocol and the Paris Agreement, each country would have a regulated quota of CO<sub>2</sub> emission per year, and will be punished with a fine if its carbon emission amount exceeds the quota. In the carbon trading market (Marland et al., 2001), if a country (seller) reduces its emission of CO<sub>2</sub> into atmosphere, the country obtains a quota of CO<sub>2</sub> emission. When another country (buyer) exceeds its regulated quota of CO<sub>2</sub> emission, it can purchase the carbon emission quota from the country with an additional quota of CO<sub>2</sub> emission to offset its insufficient quota. Hence, the seller obtains some revenue from this trade. However, conventional forest planning problems rarely considered carbon trading.

This study proposes a spatial forest planning problem that simultaneously considers forest thinning and carbon trading. Given a forestland divided into multiple grids in which forests of each grid are assumed to be of the same age, this problem is to determine a forest thinning schedule of each year over a planning period, so that both the total harvested timber volume over the period and the revenue via carbon trading are maximized, under the adjacency constraint and the even timber flow constraint. To simplify this problem, a number of treatment schedules (TSs) are assumed by analogy of (Borges et al., 2014), in which each TS determines a timber volume thinned at each grid at the end of each year.

## 2. Problem Setting

Consider a rectangular forestland consisting of  $N$  same-size grids called *management units* (MUs), as shown in Figure 1. Consider a long-run planning period (say,  $T$  years). At the end of each year, each MU is assigned to a so-called *thinning treatment*, which defines a timber volume in this MU to be thinned. Hence, each MU is required to be assigned with  $T$  thinning treatments over the  $T$ -year planning period, which are called a *treatment schedule* (TS) for this MU. Each TS has 30 percentages, each of which represents the ratio of forests to be harvested via forest thinning at end of the corresponding year.

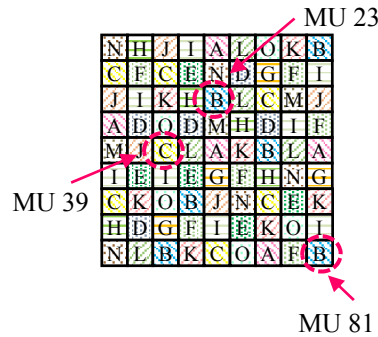


Figure 1. An example of selecting neighboring solutions for a forestland with  $9 \times 9$  MUs.

Forest harvesting generally has a rotation period to avoid excessive deforestation, e.g., the rotation period in Table 1 is 5 years in Table 1. With the rotation period  $R_a$ , all TSs can be categorized into  $R_a$  classes based on the years with forest harvesting. For example, Table 1 assumes the rotation period to be 5 years; hence, there are 5 TS classes: for  $i = 1, 2, \dots, 5$ , each TS in class  $i$  harvests at the  $i$ -th year of every 5 years. This classification is beneficial to satisfy the adjacency constraint, which can be avoided if two MUs applies TSs of different

classes, i.e., if one of the two MUs harvests at some year, then the other MU must not harvest at the same year because it applies a TS from a different TS class.

Table 1. An example with 15 TSs (divided into 5 classes) for a planning period of 30 years.

TS class	TS	Period (year)														
		1	2	3	4	5	6	7	8	9	10	11	12	13	...	30
1	A	16%	0	0	0	0	26%	0	0	0	0	6%	0	0	...	...
	B	16%	0	0	0	0	36%	0	0	0	0	16%	0	0	...	...
	C	26%	0	0	0	0	6%	0	0	0	0	6%	0	0	...	...
2	D	0	16%	0	0	0	0	26%	0	0	0	0	6%	0	...	...
	E	0	26%	0	0	0	0	6%	0	0	0	0	16%	0	...	...
	F	0	26%	0	0	0	0	6%	0	0	0	0	36%	0	...	...
3	G	0	0	16%	0	0	0	0	26%	0	0	0	0	6%	...	...
	H	0	0	6%	0	0	0	0	36%	0	0	0	0	16%	...	...
	I	0	0	26%	0	0	0	0	6%	0	0	0	0	36%	...	...
4	J	0	0	0	0.16	0	0	0	0	0.26	0	0	0	0	...	...
	K	0	0	0	0.26	0	0	0	0	0.06	0	0	0	0	...	...
	L	0	0	0	0.26	0	0	0	0	0.06	0	0	0	0	...	...
5	M	0	0	0	0	16%	0	0	0	0	26%	0	0	0	...	6%
	N	0	0	0	0	26%	0	0	0	0	6%	0	0	0	...	16%
	O	0	0	0	0	26%	0	0	0	0	6%	0	0	0	...	36%

Since environmental conditions and tree species in each MU may not be the same, e.g., plants in the MU with stable water supply grow denser; and plants in some MU may grow sparsely and are small due to its geographical location. Hence, different MUs should apply different TSs.

With the above assumptions and settings, the spatial forest thinning problem concerned in this study is to determine a TS for each MU over a  $T$ -year planning period. That is, a solution of this problem can be represented as an assignment of  $N$  TSs to  $N$  MUs, as illustrated in Figure 1, in which the letter attached to each MU is ID of the TS assigned to this MU. After an assignment of TSs to all MUs is determined, the total net present value (NPV) of the total timber amount harvested per year ( $V_1$ ), the cutting cost per year ( $V_2$ ), and the penalty cost of carbon emission due to harvesting ( $V_3$ ) can be calculated. Additionally, after forest thinning per year, the revenue of carbon sequestration ( $V_4$ ) is increased for each year when forests grow except for being harvested at the end of this year. Note that  $V_3$  and  $V_4$  can be transformed into a market value via carbon trading. Hence, the objective of the problem of finding a TS of each MU over a  $T$ -year planning period is to maximize  $V_1 + V_2 + V_3 + V_4$ , under the following two constraints: 1) *Adjacency constraint*: If two adjacent MUs share a common boundary or a common point geographically, they cannot be thinned at the same time, to preserve wildlife habitats or protect the ecology; 2) *Even timber flow constraint*: The timber amount harvested at each year must be no less than 90% and no greater than 110% of the timber amount harvested at its former year. This constraint restricts declining and rising rates of the timber volume to be harvested,

so that the value of the capacity and investment activities are forced, when the capacity cost is assumed to be very high as compared to the NPV.

### 3. The Proposed Mathematical Programming Method

By referring to the models in (Borges et al., 2014; Cacho et al., 2003), this study creates a mathematical programming model for the concerned problem. Since trees can be harvested only at the end of each of the  $T$  years, the end of the  $t$ -th year is called *key time  $t$*  throughout the rest of this paper for convenience, i.e.,  $T$  key times are considered. The notation used in this model is given in Table 2. The decision variables of this model are given as follows:

$$x_{ij} = \begin{cases} 1, & \text{if MU } i \text{ applies TS } j; \\ 0, & \text{otherwise.} \end{cases}$$

$$k_{jt} = \begin{cases} 1, & \text{if TS } j \text{ harvests a nonzero volume at key time } t; \\ 0, & \text{otherwise.} \end{cases}$$

Note that the average tree age  $\tau_{ij1}$  for each MU  $i \in \{1, 2, \dots, N\}$  and each TS  $j \in M_i$  are given. With the above notations, the problem concerned in this study is to find a TS for each MU such that the NPV of the total revenue (including the profit of the harvested timber volume and the carbon revenue or penalty cost from carbon trading) of the forest thinning scheduling is maximized, under constraints of adjacency and even timber flow. The mathematical programming model for this problem is given as follows:

$$\text{Maximize } Z = \sum_{i=1}^N \sum_{j=1}^{M_i} NPV_{ij} \cdot x_{ij} \quad (1)$$

$$\text{s.t. } \sum_{j=1}^{M_i} x_{ij} = 1, \quad \forall i \in \{1, 2, \dots, N\} \quad (2)$$

$$VH_t = \sum_{i=1}^N \sum_{j=1}^{M_i} v h_{ijt} \cdot x_{ij}, \quad \forall t = 1, 2, \dots, T \quad (3)$$

$$0.9 \cdot VH_{t-1} \leq VH_t \leq 1.1 \cdot VH_{t-1}, \quad \forall t = 2, 3, \dots, T \quad (4)$$

$$v h_{ijt} = V(\tau_{ij(t-1)} + 1) \cdot \lambda_{jt}, \quad \forall i \in \{1, 2, \dots, N\}, \quad \forall j \in M_i, \quad \forall t = 2, \dots, T \quad (5)$$

$$\tau_{ijt} = (V^{-1}(V(\tau_{ij(t-1)} + 1) - v h_{ijt})) \cdot k_{jt} + (\tau_{ij(t-1)} + 1) \cdot (1 - k_{jt}),$$

$$\forall i \in \{1, 2, \dots, N\}, \forall j \in M_i, \forall t = 2, \dots, T \quad (6)$$

$$k_{jt}x_{ij} + k_{j't'}x_{i'j'} \leq 1, \forall (i, i') \in I, \forall (j, j') \in J, \forall t = 1, 2, \dots, T \quad (7)$$

$$x_{ij} \in \{0, 1\}, \forall i \in \{1, 2, \dots, N\}, \forall j \in M_i \quad (8)$$

$$k_{jt} \in \{0, 1\}, \forall j \in M_i, \forall t = 1, 2, \dots, T \quad (9)$$

In the above model, Objective (1) is to maximize the NPV of the total revenue of the forest thinning scheduling (i.e., sum of  $NPV_{ij}$  when each MU  $i$  applies some TS  $j$ ). Constraints (2) and (8) enforce that only one  $x_{ij}$  is equal to 1, i.e., each MU must apply only one TS. Constraint (3) is used to calculate the total timber volume harvested at key time  $t$  to be the sum of the total timber volume when MU  $i$  applies some TS  $j$  at key time  $t$ . Constraint (4) is the even timber flow constraint, i.e., the timber amount  $VH_t$  harvested at each key time must be no less than 90% and no greater than 110% of the timber amount  $VH_{t-1}$  harvested at its former key time. Constraint (5) is used to compute the total timber volume  $vh_{ijt}$  when MU  $i$  applies TS  $j$  at key time  $t$ , which is nonzero only when the percentage  $\lambda_{jt}$  is nonzero. Constraints (6) is used to compute the average tree age  $\tau_{ijt}$  projected after MU  $i$  applies TS  $j$  at key time  $t$ , which is either the projected age after harvesting or the age of the last year plus 1. Constraints (7) is the adjacency constraint, i.e., adjacent MUs cannot be harvested at the same key time. Constraints (8) and (9) enforces decision variables  $x_{ij}$  and  $k_{jt}$  to be binary.

In Objective (1), we modifies (Cacho et al., 2003) to compute  $NPV_{ij}$  as follows:

$$\begin{aligned} NPV_{ij} = & \sum_{t=1}^T vh_{ijt} \cdot p_v(d(\tau_{ijt})) \cdot (1+r)^{-t} - \sum_{t=1}^T c_E(vh_{ijt}) \cdot (1+r)^{-t} \\ & + \sum_{t=1}^T (b(\tau_{ijt}) - b(\tau_{ijt-1})) \cdot v \cdot p_b \cdot (1+r)^{-t} - \sum_{t=1}^T b(V^{-1}(vh_{ijt})) \cdot v \cdot p_b \cdot (1+r)^{-t} \end{aligned} \quad (10)$$

On the right side of the above equation, the first term is the discounted market value of the timber volume harvested over  $T$  years, computed as follows: when MU  $i$  applies TS  $j$  at key time  $t$ , each sub-term in the sum is the harvested timber volume  $vh_{ijt}$ , times its timber price  $p_v(d(\tau_{ijt}))$  (depending on the average diameter of the harvested trees at age  $\tau_{ijt}$ ), and times its discount factor  $(1+r)^{-t}$ . The second term is the discounted timber cutting cost, in which the harvested timber volume  $vh_{ijt}$  leads to a cutting cost  $c_E(vh_{ijt})$ . The third term is the discounted carbon sequestration revenue, in which each sub-term in the sum is the difference between carbon stocks at  $\tau_{ijt}$  and  $\tau_{ijt-1}$  (i.e.,  $b(\tau_{ijt}) - b(\tau_{ijt-1})$ ), times the C-to-CO<sub>2</sub> transformation rate  $v$ , times the CO<sub>2</sub> price  $p_b$ , and times its discount factor  $(1+r)^{-t}$ . The last term is the discounted carbon emission penalty cost, in which the projected age when the timber volume is

$vh_{ijt}$  is calculated as  $V^{-1}(vh_{ijt})$  years, then is substituted into  $b(\cdot)$  function to obtain the amount of carbon emission, and then is transformed into a market value similarly.

In the first term in Equation (10), the timber price  $p_v$  for the harvested trees at age  $\tau$  linearly depends on the average diameter  $d(\tau)$  of the trees (Venn et al., 2000; Cacho et al., 2003), and is computed as follows:

$$p_v(d(\tau)) = \gamma_0 + \gamma_1 \cdot d(\tau); \quad (11)$$

$$d(\tau) = 200 \cdot \sqrt{\frac{a(\tau)}{\pi \cdot D}}; \quad (12)$$

where  $\gamma_0$  and  $\gamma_1$  are intercept and slope of the linear relation;  $D$  is number of trees per hectare;  $a(\tau)$  is the area of the wood at age  $\tau$ , and is computed as follows:

$$a(\tau) = \theta_a (1 - \exp(-\alpha_a \cdot \tau))^{\beta_a}; \quad (13)$$

where parameters  $\theta_a$ ,  $\beta_a$ , and  $\alpha_a$  are decided by tree species, environmental conditions, and forest management.

In the second term in Equation (10),  $b(\tau)$  is the total carbon stock for an MU in which each tree is at age  $\tau$ , which is computed as follows:

$$b(\tau) = \phi \cdot [(\rho \cdot \theta_v)^\mu \cdot w(\tau)]^{\frac{1}{1+\mu}} \quad (14)$$

where parameters  $\phi$ ,  $\rho$ ,  $\theta_v$ , and  $\mu$  are decided by tree species, environmental conditions, and forest management; and  $w(\tau)$  is the carbon stock in stemwood biomass for an MU in which each tree is at age  $\tau$ , which is computed as follows:

$$w(\tau) = \rho \cdot V(\tau) \quad (15)$$

where parameter  $\rho$  is decided by tree species; and  $V(\tau)$  is the timber volume of a forest all at age  $\tau$ , which is computed as follows:

$$V(\tau) = \theta_v [1 - \exp(-\alpha_v \cdot \tau)]^{\beta_v} \quad (16)$$

where parameters  $\theta_v$ ,  $\beta_v$ , and  $\alpha_v$  are decided by tree species, environmental conditions, and forest management.

In the third term in Equation (10), the forest cutting cost  $c_E(vh_{ijt})$  is computed as follows:



$$c_E(vh_{ijt}) = c_f + c_v \cdot vh_{ijt} \quad (17)$$

where  $c_f$  is the fixed cost; and  $c_v$  is the variable cost for each cubic meter of timber harvested at each key time.

In the fourth term in Equation (10), given a timber volume  $v$ , the projected tree age is computed as follows:

$$\tau = V^{-1}(v) = \frac{-\ln(1 - \exp((\ln v - \ln \theta_v) / \beta_v))}{\alpha_v} \quad (18)$$

## 5 Conclusion

This study has proposed a spatial forest planning problem that considers forest thinning and carbon trading (in which carbon sequestration and emission can be traded), to find a balance between economic and environmental concerns and achieve sustainability in forest planning. This study models this problem as a mathematical programming model. A comprehensive simulation analysis will be conducted in the future.

## References

- Assmann, E., 1970. The Principles of Forest Yield Study: Studies in the Organic Production, Structure, Increment and Yield of Forest Stands. Pergamonn Press, Oxford, p. 506.
- Assmann, E., 1961. Waldertragskunde: Organische Produktion, Struktur, Zuwachs und Ertrag von Waldbeständen. BLV Verlagsgesellschaft, München, p. 490.
- Beck, D.E., 1983. Thinning increases forage production in Southern Appalachian Cove Hardwoods. Southern Journal of Applied Forestry 7, 53–57.
- Borges, P., Eid, T., Bergseng, E., 2014. Applying simulated annealing using different methods for the neighborhood search in forest planning problems. European Journal of Operational Research 233, 700–710.
- Cacho, O.J., Hean, R.L., Wise, R.M., 2003. Carbon-accounting methods and reforestation incentives. Australian Journal of Agricultural and Resource Economics 47(2), 153–179.
- Haveri, B.A., Carey, A.B., 2000. Forest management strategy, spatial heterogeneity, and winter birds in Washington. Wildlife Society Bulletin 28, 643–652.

- Hassall and Associates, 1999. Greenhouse, carbon trading and land management, LWRRDC Occasional Paper 23/99, Land and Water Resources Research and Development Corporation, Canberra.
- Kangas, J., Kangas, A., 2005. Multiple criteria decision support in forest management: the approach, methods applied, and experiences gained. *Forest Ecology and Management* 207(1–2), 133–143.
- Kirkpatrick, S., Gelatt, C. D., Vecchi, M. P., 1983. Optimization by simulated annealing. *Science*, 220(4598), 671–680.
- Kirschbaum, M.U.F., 2000, What contribution can tree plantations make towards meeting Australia's commitments under the Kyoto Protocol?. *Environmental Science and Policy* 3, 83–90.
- Liu, W.-Y., Lin, C.-C., 2015. Spatial forest resource planning using a cultural algorithm with problem-specific information. *Environmental Modelling & Software* 71, 126–137.
- Marland, G., Fruit, K., Sedjo, R., 2001. Accounting for sequestered carbon: the question of permanence. *Environmental Science and Policy* 4, 259–268.
- Wong, J., Baker, T., Duncan, M., McGuire, D., Bulman, P., 2000. Forecasting Growth of Key Agroforestry Species in South-Eastern Australia: A Report for the RIRDC/LWRRDC/FWPRDC Joint Venture Agroforestry Program. Barton, A.C.T., Australia: RIRDC.