Corporate Hedging in Incomplete Markets: A Solution Under Price Transmission

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Background

- Firms are looking for improved methods to more efficiently hedge input and output price risk exposure, but it is often the case that all price risk cannot be eliminated through exchange traded futures contracts.
- Since futures contracts exist for a limited number of assets, some sources of price risk cannot be directly hedged and thus hedging markets are incomplete for many firms.
- If related futures contracts do not exist for both input and output price risk, the traditional approach is to employ a ***one-sided hedge***, that is, to hedge only for the risk source with related futures contracts and remain unhedged in the other.

Objectives

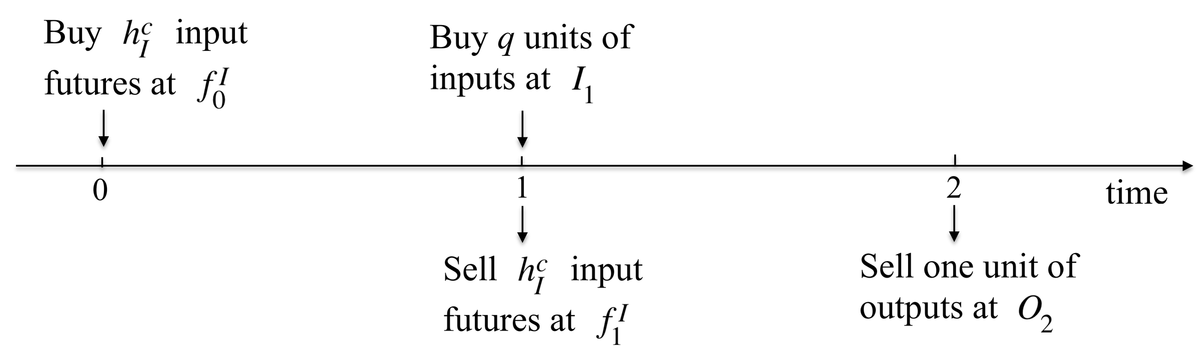
- In this paper, we present a dynamic ***two-sided hedge*** that enables firms to minimize both input and output price risk through a single tradable futures contract even in incomplete markets.
- Apply our model to a hypothetical jet fuel producer and compare the hedging effectiveness of the two-sided hedge with the classical one-sided model.

Incomplete-market Model: Overview

- The market is incomplete in that the firm cannot take exact offsetting positions to both input and output payoffs, as appropriate futures contracts exist for only one side.
- The traditional minimum-variance criterion is employed since it is reasonable for a firm to aim to avoid costly financial distress resulted from volatile profits (Fok, Carroll, and Chiou 1997).
- The two-sided model is developed by incorporating the the price transmission (PT) mechanism between input and output prices in a minimum-variance complete-market hedge model.
- The direction and magnitude of PT vary across industries. In ***cost-driving*** PT, it is the supply forces that lead to equilibrium between input and output prices; while ***demand-driving*** PT suggests that the demand side has greater bargain power.
- Consider four subcases in the model according to the direction of PT and the availability of futures contracts:
  (CO) cost-driving PT with output futures contracts;
  (CI) cost-driving PT with input futures contracts;
  (DO) demand-driving PT with output futures contracts;
  (DI) demand-driving PT with input futures contracts.
- A two-factor diffusion model with a stochastic, mean-reverting convenience yield is assumed for the underlying asset.
- The optimal dynamic two-sided hedge is the weighted average of the classic minimizing direct hedge ratio and the cross hedging ratio.

Two-sided Hedge Model: CI

- Timeline:



- Cash flow:

$$\Pi = O_2 - \left(I_1 q - \left(f_1^I - f_0^I\right)h_I^c\right) - m\left|h_I^c\right|$$

q: constant input−output ratio; m: proportional transaction costs
h_I^C: transactions in input futures market. Positive h_I^C indicates long positions.

- Cost−driving PT suggests that lagged input price positively affects output price:

$$O_2 = \theta_0 + \sum_{i=1}^{p}\theta_i O_{2-i} + \sum_{j=1}^{q} b_j I_{2-j}$$

- The objective of the firm is to minimize the variance of cash flow:

$$\min_{h_I^c}\left\{Var(\Pi)\right\}$$

$$s.t.\quad O_2 = \theta_0 + \sum_{i=1}^{p}\theta_i O_{2-i} + \sum_{j=1}^{q} b_j I_{2-j}$$
$$\Pi = O_2 - \left(I_1 q - \left(f_1^I - f_0^I\right)h_I^c\right) - m\left|h_I^c\right|$$

- The optimal hedge ratio for a CI firm, h_I^C :

$$h_I^C = \left(q - b_1\right)\beta_1 - \theta_1\beta_2$$

- Denote the price of underlying asset to be S_t , the price of the related assets to be P_t, and the price of futures to be F_t, we have the direct hedge ratio β_1 and the cross hedge ratio β_2 to be:

$$\beta_1 \equiv \frac{\mathrm{cov}(S_t, F_t)}{\mathrm{var}(F_t)} = \frac{\mathrm{cov}(I, f^I)}{\mathrm{var}(f^I)} \quad \text{and} \quad \beta_2 \equiv \frac{\mathrm{cov}(P_t, F_t)}{\mathrm{var}(F_t)} = \frac{\mathrm{cov}(O, f^I)}{\mathrm{var}(f^I)}$$

Optimal Hedge: Other Cases

- The optimal hedge in other cases is derived via a similar procedure.
- For a CO firm, the optimal hedging policy is

$$h_O^c = -\left(q - b_1\right)\beta_2 + \theta_1\beta_1$$

- For Demand−driving firms, PT suggests

$$I_1 = \chi_0 + \sum_{i=1}^{s}\chi_i I_{1-i} + \sum_{j=1}^{k} d_j O_{1-j}$$

- For a DI firm, the optimal hedging policy is

$$h_I^d = -\beta_2$$

- For a DO firm, the optimal hedging policy is

$$h_O^d = \beta_1$$

Price Dynamics, β_1 and β_2

- Underlying price S_t and convenience yield δ_t are assumed to follow a two-factor diffusion model (e.g. Schwartz 1997):

$$dS_t = \left(\mu - \delta_t\right)S_t dt + \sigma_s S_t dZ_S \qquad (1)$$
$$d\delta_t = \kappa_\delta\left(\alpha_\delta - \delta_t\right)dt + \sigma_\delta dZ_\delta$$

- The log spread c_t between the price of underlying (S_t) and price of related assets (P_t) satisfies (Bertus et al. 2009) :

$$dc_t = \kappa_c\left(\alpha_c - c_t\right)dt + \sigma_c dZ_c \qquad (2)$$

where μ, σ_i, κ_i and α_i (i=S,c) are deterministic parameters.

- From (1) and (2), we have β_1 and β_2 as function of the deterministic parameters:

$$\beta_1 = \frac{\mathrm{Cov}\left(S_t, F_t\right)}{\mathrm{Var}\left(F_t\right)} = \frac{\mathrm{Cov}\left(S_t,\ S_t A(T-t)e^{r(T-t)-H_\delta(T-t)\delta_t}\right)}{\left(A(T-t)e^{r(T-t)}\right)^2 \mathrm{Var}\left(S_t e^{-H_\delta(T-t)\delta_t}\right)};\quad \beta_2 = \frac{\mathrm{Cov}\left(P_t, F_t\right)}{\mathrm{Var}\left(F_t\right)} = \frac{\mathrm{Cov}\left(S_t e^{c_t},\ S_t A(T-t)e^{r(T-t)-H_\delta(T-t)\delta_t}\right)}{\left(A(T-t)e^{r(T-t)}\right)^2 \mathrm{Var}\left(S_t e^{-H_\delta(T-t)\delta_t}\right)}$$

Application to a Jet Fuel Producer

- The firm uses light sweet crude oil to produce jet fuel and ***only has input futures contracts***.
- Weekly data of futures prices for light sweet crude oil for delivery to Cushing, OK, and spot prices for New York Harbor jet fuel from Apr 4, 1990 to Aug, 16, 2015 is used.
- Use Kalman Filtering to estimate parameters for (1) and (2):

Parameter	Estimates
α_δ	0.0955
κ_δ	1.5142
σ_δ	0.1489
σ_s	0.1151
$\rho_{S\delta}$	0.8897
λ_δ	0.0798
α_c	0.2760
κ_c	4.0256
σ_c	0.1462
ρ_{Sc}	0.2629
$\rho_{c\delta}$	0.2414

- Use the vector autoregression (VAR) model to identify the interdependencies between input and output price series

Parameter	Estimate (Std. Error)
Equation: $I_t = \chi_0 + \chi_1 I_{t-1} + d_1 O_{t-1} + \varepsilon_{1t}$	
χ_1	1.2778*** (0.2468)
d_1	-1.3185*** (0.2346)
χ_0	-0.9171 (0.8797)
Adjusted R-squared	0.03195
Equation: $O_t = \theta_0 + b_1 I_{t-1} + \theta_1 O_{t-1} + \varepsilon_{2t}$	
$\boldsymbol{b_1}$	1.2443***(0.2609)
θ_1	-1.2565***(0.2480)
θ_0	-4.3598***(0.9300)
Adjusted R-squared	0.02201
Num. of Observations	1306

Significance codes: <0.0001 '***'

- The positive effect of lagged input price on output price (b_1) implies that the PT mechanism for the jet fuel producer is ***cost-driving***.

Comparisons of Hedging Models

- The jet fuel producer is a CI firm.
- Based on the estimated parameter values, we compute the direct and cross hedging ratio β_1 and β_2 .
- The optimal two-sided hedge policy for the CI firm is $h_I^c = \left(q - b_1\right)\beta_1 - \theta_1\beta_2$ and the one-sided hedge policy is $h_{I,one-sided}^c = q\beta_1$
- Following Ederington (1979), we calculate the effectiveness of the two-sided and one-sided model as

$$Eff_{two} = -\left(\mathrm{var}\left(\Pi_{two}\right) - \mathrm{var}\left(\Pi_{unhedged}\right)\right)$$
$$Eff_{one} = -\left(\mathrm{var}\left(\Pi_{one}\right) - \mathrm{var}\left(\Pi_{unhedged}\right)\right)$$

where Π_{one},Π_{two} and $\Pi_{unhedged}$ stand for cash flow under one-sided model, two-sided model and unhedged positions, respectively.

- Comparisons of hedging policy and hedging effectiveness:

Horizon	Hedging Policy		Effectiveness	
Panel A: Futures expiration matches hedging horizon				
	Two-sided	One-sided	Two-sided	One-sided
4 weeks	5.1402	3.9893	1.6525	1.5697
13 weeks	3.4013	2.6217	0.8210	0.7779
26 weeks	1.9172	> 1.4778	0.3133	> 0.2969
One year	0.6092	0.4696	0.0456	0.0432
Two years	0.0615	0.0474	0.0010	0.0009
Panel B: Futures expiration is two weeks longer than hedging horizon				
	Two-sided	One-sided	Two-sided	One-sided
4 weeks	4.6331	3.5718	1.3802	1.3078
13 weeks	3.1141	> 2.4004	0.7079	> 0.6707
26 weeks	1.7554	1.3531	0.2702	0.2560
One year	0.5578	0.4299	0.0394	0.0373
Two years	0.0563	0.0434	0.0008	0.0008

- Results show that the two-sided model has larger hedging ratio and results in greater hedging effectiveness for all horizons.

Conclusions

- By embodying PT in the complete-market hedging model, we present a two-sided dynamic hedging strategy for firms in incomplete markets.
- Empirical analysis using a hypothetical jet fuel producer as motivation demonstrates the effectiveness of our two-sided hedge.
- As PT is an important characteristic describing the overall operation of the market (Goodwin and Holt 1999), the two-sided hedging strategy may be practical for many firms in multiple industries.

Reference

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