A Dynamic Model of U. S. Beef Cow Inventories

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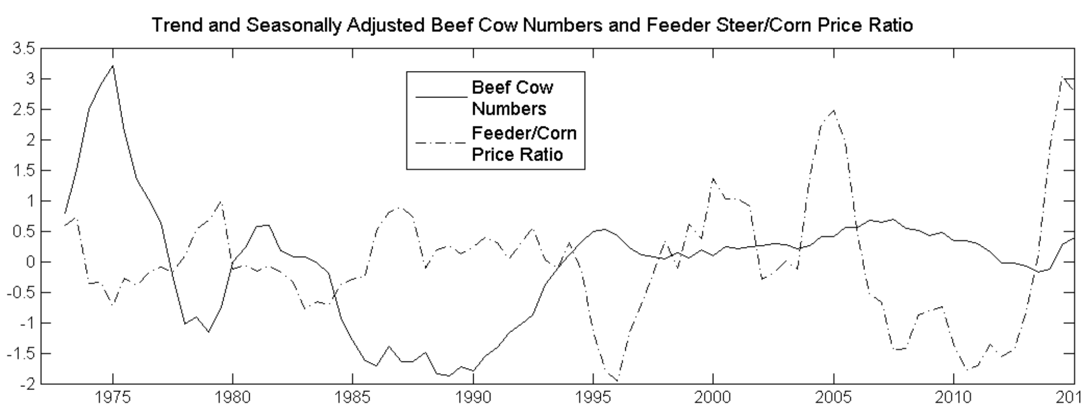
Introduction

The dynamics of beef cattle supply and the existence of cattle cycles have been widely researched topics in the last four decades. The work of Jarvis (1974) was first to treat beef cows in the context of capital goods and recognized that increasing beef prices can actually lead to reduced slaughter in the short run. This approach influenced empirical approaches to modeling the beef cattle herd such as formulated by Rucker, Burt, and LaFrance (1984) and stimulated a theoretical treatment of the dynamics of livestock production by Rosen (1987). Rosen, Murphy, and Scheinkman (1994) specifically addressed the existence of cattle cycles. More recently Aadland (2004) constructed a model to describe the putative 10 year cattle cycle by assuming that producers maximize a discounted stream of future profits subject to biological constraints and market forces.

This analysis reconsiders the dynamics of beef cow inventories in light of the shift in the structure of cattle finishing and herd management during the last fifty years. Nerlove and Fornari (1998) have criticized the Rosen et al. (1994) approach for not recognizing the structural change that occurred in the beef cattle market during the 100 plus years of their analysis. Nerlove and Fornari specifically cited changes in cattle finishing, breeding practices, and beef cattle genetics as causes for structural change. The presence of large commercial feedlots has industrialized the production of fed beef. Feedlot operators have the skill and resources to manage production and risk by taking positions in futures markets for feeder cattle, fat cattle, and feed grains. Currently, almost 90% of steers and heifers slaughtered are supplied by feedlots with over 1000 head capacity. Cow-calf operations have benefitted from increasing productivity (Marsh, 1999), research regarding optimal feeding schedules (Hennessy, 2006), and the education programs of extension specialists across the nation. Just as Holt and Craig (2006) speculated that continuous farrowing and total confinement operations may have shortened and dampened the hog cycle, the changing structure of beef production may have impacted the cattle cycle in a similar manner.

Consider the figure below which plots trend and seasonally adjusted standardized semi-annual beef cow inventories and similarly adjusted standardized semi-annual feeder steer/corn price ratios from 1973 to 2015. Since 1995, the number of beef cows (trend and seasonally adjusted) appears to have little cyclicality. This stands in stark contrast to the cyclical pattern given in Figure 1 of Aadland (2004, p.1978) using data from the 1930's to the late 1990's.



The Approach

To understand the dynamics of the U. S. beef cattle herd we examine both beef cattle inventories and feeder steer prices using semi-annual data. Building on the seminal article by Jarvis (1974), a number of studies have recognized that the value of a cow is largely determined by the value of her offspring (Rucker et al., 1984; Paarsch, 1985; Marsh, 1999; Aadland, 2004) and her salvage (slaughter) value. As Aadland has succinctly stated: "A female animal has a dual value—she is valued both as a consumable product today and simultaneously as a calf-making machine over her effective lifetime" (p.1986). The net present salvage value of a cow, of course, depends on her productive lifespan, the discounted future value of her offspring, and her discounted slaughter value. Using the decision maker of Schulz and Gunn (www.extension.iastate.edu/agdm/livestock/html/b1-74.html, March 2016) a current estimate is that the net present salvage value of a young cow is about 17% of her total discounted present value. Thus current and expected feeder cattle prices should be the primary determinant of a cow's value.

Therefore the price most relevant to the decision of herd size is that of feeder cattle and we choose to use the Oklahoma City price for 550 pound steers. This price is normalized by the price of corn (Kansas City) to reflect the value of the calf to finishers and is consistent with the approach of Holt and Craig (2006). Notice that the feeder steer/corn price ratio tends to exhibit more cyclical behavior than inventories and also exhibits some counter-cyclical tendency relative to beef cow numbers.

Our analysis of the two time series begins with unit root tests. After creating lags of cow herd (in millions of animals) and the feeder steer/corn price ratio we have 75 semi-annual observations from January 1, 1978 to July 1, 2015. The KPSS test, the augmented Dickey-Fuller test, and the Phillips-Peron test indicated that each series is integrated of order 1.

Residual based cointegration tests failed to find a cointegrating relationship between the two series under a number of alternative specifications employing polynomials in trend and the semi-annual dummy (1 for January-June, zero otherwise).

Given the lack of a contemporaneous relationship, a vector autoregression analysis was undertaken. The results for the models are reported in Table 1 and all specifications include trend and seasonal terms.

Table 1. VAR Results with time trend and semi-annual dummy

MODEL	LOG LIKELIHOOD	AIC	BIC
VAR 6:	-258.81	573.62	638.51
VAR 5:	-260.49	568.98	624.60
VAR 4:	-265.21	570.42	616.77
VAR 3:	-268.79	569.58	606.66
VAR 2:	-272.68	569.36	597.17
VAR 1:	-288.34	592.68	611.22

LR TESTS	P-VALUE
VAR 5 vs VAR 6	0.500
VAR 4 vs VAR 5	0.051
VAR 3 vs VAR 5	0.035
VAR 2 vs VAR 5	0.018

Based on the LR (likelihood ratio) tests and the AIC we select a VAR 5 to represent the system. This model revealed that the feeder steer/corn price ratio may Granger cause cow numbers, since a test of the contrary yielded a $p=0.0615$. But we found evidence that cow numbers did not Granger cause prices ($p=0.335$). This is consistent with Shonkwiler and Hinckley (1985) where current feeder calf prices are based on the economics of cattle finishing, not on the size of the cow herd.

Recalling that a VAR with deterministic components can be written as $A(L)\mathbf{y_t} = B\mathbf{x_t} + \boldsymbol{\epsilon_t}$, by the fundamental dynamic equation we have $|A(L)|\ \mathbf{y_t} = Adj(A(L))(B\mathbf{x_t} + \boldsymbol{\epsilon_t})$. Thus the series in $\mathbf{y_t}$ must share the same long run dynamics given by $|A(L)|$. A restricted VAR 5 provided four pairs of complex conjugate roots of $|A(L)|$ which implied cycles of 2.68, 2.888, 10.50, and 12.86 semi-annual periods. Analysis of the VAR 2 model (which would be selected using the BIC criterion) showed a single pair of complex conjugate roots with an implied cycle length of 17.12 semi-annual periods. Clearly there is no agreement on a cycle length. Further the irregular patterns in both series suggest that the amplitude and phase of each series has been evolving over time. To address these issues we investigate a stochastic cycle (Harvey, 1989; Parker and Shonkwiler, 2014) model which allows i) an analysis of non-stationary data; ii) direct estimation of cycle length; and iii)shifting phase and changing amplitudes.

The Stochastic Cycle Model

For a single time series, the model is specified to be a random walk (with drift) with a stochastic cycle (Harvey).

$y_t = \mu_t + \psi_t + x_t\delta + \varepsilon_t$

$\mu_t = \mu_{t-1} + \beta + \eta_t$

$\psi_t = \rho\{\cos(\lambda)\psi_{t-1} + \sin(\lambda)\psi^*_{t-1}\} + \kappa_t$

$\psi_t^* = \rho\{-\sin(\lambda)\psi_{t-1} + \cos(\lambda)\psi^*_{t-1}\} + \nu_t$

Here μ_t and ψ_t represent dynamic unobservables associated with a random walk (with drift β) and a cyclical process (with frequency λ). The coefficient ρ is termed the damping factor and ρ=1 if the cyclical process is non-stationary. The error processes ε_t, η_t, κ_t, and ν_t are assumed to be iid normal with variance-covariance matrix

$$\begin{bmatrix} \sigma_\varepsilon^2 & 0 & 0 & 0 \\ 0 & \sigma_\eta^2 & 0 & 0 \\ 0 & 0 & \sigma_\kappa^2 & 0 \\ 0 & 0 & 0 & \sigma_\nu^2 \end{bmatrix}.$$

The initial conditions ψ_0 and ψ^*_0 determine the initial amplitude and phase shift of the series and μ_0 denotes the initial level of the series which typically can be set at y_0.

Parker and Shonkwiler (2014) show that the reduced form of the model can be written in terms of the observable process y_t, the unknown parameters, and the error processes:

$$\Delta y_t = 2\rho\cos\lambda\Delta y_{t-1} - \rho^2\Delta y_{t-2} + \beta^* + \eta_t - 2\rho\cos\lambda\eta_{t-1} + \rho^2\eta_{t-2} + \Delta\kappa_t - \rho\cos\lambda\Delta\kappa_{t-1} + \rho\sin\lambda\Delta\nu_{t-1}$$
$$+ \Delta\xi_t - 2\rho\cos\lambda\Delta\xi_{t-1} + \rho^2\Delta\xi_{t-2}$$

where Δ denotes the (first) difference operator and $\xi_t = x_t\delta + \varepsilon_t$. In time series parlance the series is a type of ARIMA(2,1,2) process. This representation is valid when y_t follows a random walk, i.e. $\sigma_\eta^2>0$ and consequently the series must be first differenced to achieve stationarity. If this is not the case, the model with $\sigma_\eta^2=0$ simplifies to

$$y_t = 2\rho\cos\lambda y_{t-1} - \rho^2 y_{t-2} + \beta t + \kappa_t - \rho\cos\lambda\kappa_{t-1} + \rho\sin\lambda\nu_{t-1} + \varepsilon_t - 2\rho\cos\lambda\xi_{t-1} + \rho^2\xi_{t-2}$$

or a type of ARMA(2,2) process with constant trend. An additional simplification of the dynamic process occurs when all the noise in the system is due to the stochastic cycle. In this case σ_ε^2=0, and then

$$y_t = 2\rho\cos\lambda y_{t-1} - \rho^2 y_{t-2} + \beta t + \kappa_t - \rho\cos\lambda\kappa_{t-1} + \rho\sin\lambda\nu_{t-1} - 2\rho\cos\lambda x_{t-1}\delta + \rho^2 x_{t-2}\delta.$$

Univariate Results

Setting μ_0 equal to y_0 and imposing the customary restriction (Harvey, p.39) that $\sigma_\kappa^2 = \sigma_v^2$ for the cow herd size, we found that both σ_ε^2 and σ_η^2 converged to zero under maximum likelihood estimation of the stochastic cycle model. Imposing these restrictions and restricting ψ_0 to zero and ρ to one gave a model with 4 fewer parameters. A likelihood ratio test of the 5 restrictions yielded $X^2=3.04$ ($p\approx.551$); the approximate p-value represents the fact that 2 of the restrictions involved parameters on boundaries of the parameter space. However examination of the residual correlogram indicated temporal dependence among the residuals and this was confirmed by a Ljung-Box Q-test with 12 lags ($p=0.013$). This was addressed by adding a fourth lag of the cow herd as this yielded a Ljung-Box Q-test having a p-value of 0.628 over 12 lags. The results are reported below.

Table 2. Cow Herd: Log likelihood at convergence -30.11

Parameter/ Coefficient	Estimated Value	Robust Std. Error	z-Value
DV	0.7524	0.0443	16.998
$\sigma_\kappa=\sigma_v$	0.3131	0.0397	7.891
λ	0.1935	0.0234	8.278
β	-0.0869	0.0083	-10.53
ψ^*_0	2.182	2.4099	0.905
y_{1t-4}	-0.1238	0.0594	-2.085

The highly significant coefficient on the semi-annual dummy variable (DV) shows that the cow herd tends to be larger on July 1 than on January 1. The estimate of β can be interpreted as a decreasing trend in beef cow numbers. The estimate of λ implies a cattle cycle of $2\pi/\hat{\lambda}$, about 32.5 periods or 16.24 years. Using the delta method, we find an approximate asymptotic 95% confidence interval for the cycle of 12.4 to 20.1 years.

A similar unrestricted model is specified for the feeder steer/corn price ratio. Again both σ_{ε}^2 and σ_{η}^2 converged to zero. Imposing the same four restrictions as before ($\sigma_{\varepsilon}^2=\sigma_{\eta}^2=0$; $\psi_0=0$; $\rho=1$) resulted in a likelihood ratio test statistic of $X^2=4.94$ ($p\approx.291$). Analysis of the residual correlogram for this model showed a spike at the fourth lag, so the model was re-estimated by including y_{2t-4}. For this model of the feeder steer/corn price ratio reported in Table 3, the Ljung-Box Q-test had a p-value of 0.754 over 12 lags.

Table 3. Feeder steer/corn price ratio: Log likelihood at convergence -241.98

Parameter/ Coefficient	Estimated Value	Robust Std. Error	z-Value
DV	4.5269	0.7788	5.813
$\sigma_{\kappa}=\sigma_{\nu}$	5.3403	0.4411	12.108
λ	0.2362	0.0572	4.133
β	0.4965	0.2059	2.411
ψ^*_0	1.9118	16.2726	0.117
y_{2t-4}	-0.3913	0.1463	-2.675

The cattle cycle implied by this model is 26.6 periods or 13.3 years. The approximate asymptotic 95% confidence interval spans from 7 to 19.6 years and thus is seen to overlap much of the confidence interval obtained from the cow herd model.

The Bivariate Stochastic Cycle Model

These findings lead naturally to considering the estimation of the cattle cycle using both the beef cow herd series (y_1) and the feeder steer/corn price series (y_2). We first estimate the models simultaneously imposing the restriction that each stochastic cycle shares the same λ. Then we investigate the interrelationships between the stochastic cycles. This is accomplished by augmenting the state equation for the cow herd as follows $y_{1t} = \mu_{1t} + \psi_{1t} +$

$\alpha\psi_{2t-s} + x_{1t}\delta_1 + \varepsilon_{1t}$; where ψ_{2t-s} is the cyclical component associated with the feeder/corn price ratio.

Joint estimation of the stochastic cycle models reported in Tables 2 and 3 under the restriction of a common λ parameter generated a log likelihood of -272.41 at convergence and a corresponding likelihood ratio test statistic of X^2=0.63 with one degree of freedom. Estimated parameters and associated standard errors were largely unchanged from those reported in Tables 2 and 3. The estimate of λ was 0.2009 with robust standard error of 0.0236 which indicates a cattle cycle of 15.64 years with an associated standard error of 1.837 years. This specification indicates a longer cycle than the ten to twelve year cycle observed from the 1930's until the 1980's.

With both cycles sharing a common frequency, is possible to investigate the phase shifts between the cycles. The phase at time t for cycle i is represented by $\varphi_{it} = \tan^{-1}(\psi_{it}^*/\psi_{it})$. Then if $\varphi_{jt}>\varphi_{it}$, y_{jt} leads y_{it} by $(\varphi_{jt}-\varphi_{it})/\lambda$ time periods at time t. Generally the feeder steer/corn price ratio leads cow herd. We do see, however, that during the decade of the 1990's that the two cyclical components are largely in antiphase. Although the phase shifts have a high degree of variability, the joint model with common frequency suggested that on average the feeder steer/corn price ratio cyclical component leads the cow herd size cyclical component by 5 periods. However in subsequent estimation of the joint model with $\alpha\psi_{2t-s}$ in the state equation for cow herd, the best fit appears to be including the term $\alpha(\psi_{2t-3} + \psi_{2t-4})/2$.

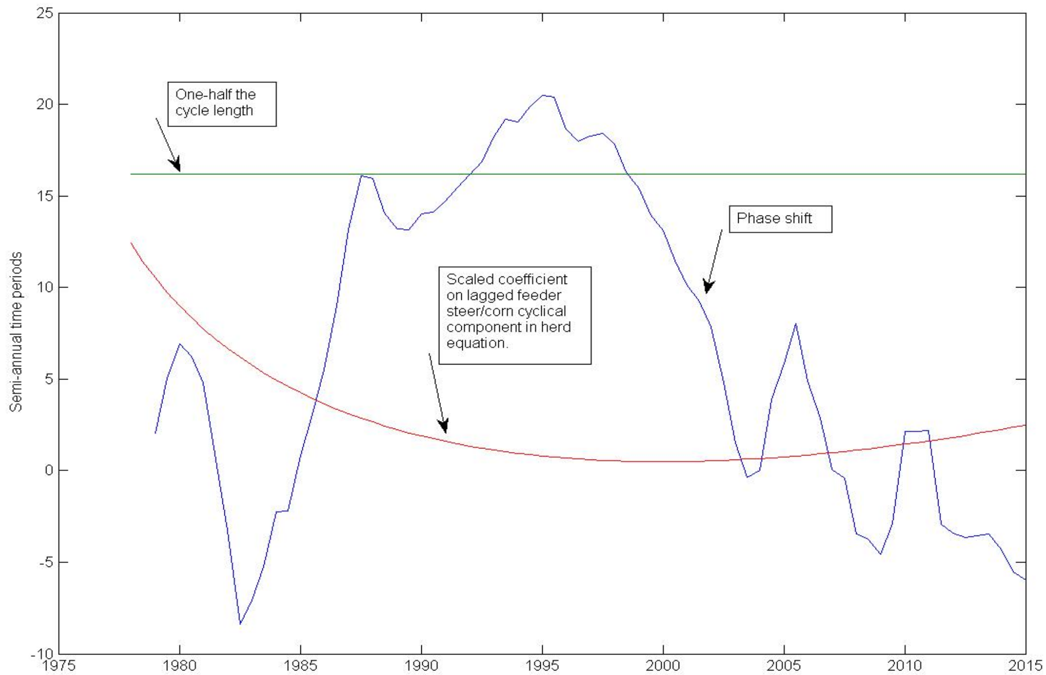
Because of the counter cyclical pattern found in the late 20th Century, a final specification was investigated. This was to allow the coefficient on the feeder steer/corn price ratio cyclical component in the herd equation to vary over time according to $(\alpha_0 + \alpha_1 t^{1/2} + \alpha_2 t)\cdot(\psi_{2t-3} + \psi_{2t-4})/2$. Estimation results for this model are reported in Table 4.

Table 4. Joint Model: Log Likelihood at Convergence -259.88

Parameter/ Coefficient	Estimated Value	Robust Std. Error	z-Value
DV_1	0.7233	0.0443	16.339
$\sigma_{1\kappa}=\sigma_{1\nu}$	0.2635	0.0347	7.6
$\lambda_1= \lambda_2$	0.1943	0.0247	7.869
β_1	-0.0681	0.0079	-8.605
ψ^*_{10}	1.12	1.828	0.613
y_{1t-4}	-0.0879	0.0444	-1.981
α_0	0.5685	0.164	3.466
α_1	-0.1582	0.0504	-3.139
α_2	0.0112	0.0038	2.941
DV_2	4.4908	0.7216	6.224
$\sigma_{2\kappa}=\sigma_{2\nu}$	5.4214	0.4202	12.903
$\lambda_1= \lambda_2$	0.1943	0.0247	7.869
β_2	0.5991	0.1844	3.249
ψ^*_{20}	19.9347	12.5686	1.586
y_{2t-4}	-0.3611	0.1317	-2.743

These joint results generate a cattle cycle of 16.17 years with an asymptotic standard deviation of 2.06 years. The phase shift in terms of the number of semi-annual periods that the cyclical component of the feeder steer/corn price ratio leads the cow herd component is illustrated in Figure 2. This phase shift is smoothed using a simple 3 period moving average. Note that when the shift exceeds one-half the cycle length, one can interpret this as the series lagging the herd size by the cycle-length minus the phase shift. Since we infer that cow herd does not Granger cause the feeder steer/corn price ratio, we do not adjust the representation. In the figure, we also plot the total scaled (by 50) coefficient on the feeder steer/corn price ratio cyclical component on the cow herd. We see that the coefficient is smallest when the two cycles are most out of phase.

Figure 2. Phase Shift in Semi-Annual Periods



Summary

Although the identification of commodity price cycles can be viewed as purely a statistical exercise, the presence of cycles has important implications. Beveridge and Nelson (1981) have pointed out that the observation of cyclical components in economic time series "has played an important role in shaping our thinking about economic phenomena" (p151.) In this case we observe a significant lengthening of the cattle cycle. Typically cycles become longer before they disappear. Whereas the popular press has already buried the cattle cycle (Speer, 2014) given the observed patterns in herd size since the 1990's, the stochastic cycle model shows its continued, albeit altered, existence.

This leads to speculation as to why in recent decades the cycle is significantly longer than the ten to twelve year cycle observed over most of the 20th Century. Since 2000 there have been a number of events which have been linked to a lengthening cattle cycle—specifically due to a long trend of decreasing number of beef cows. There were a number of droughts in major cow-calf producing areas from 2000 to 2008 and again in the southern plains from 2010 to 2013 (Petry, 2015). Higher row crop prices and the consequent expansion of crop production may also have led to a reduction in cow-calf operations. According to the 2012 *Census of Agriculture*, there was a decline of almost 175,000 cow calf operations in the previous 20 years with a bulk of these operations having less than 50 cows.

As noted previously, cattle feeding has undergone a major transformation since the 1960's. The advent of large, commercial feedlots with some being owned by meatpackers and others by more diversified firms suggests that conventional measures of feedlot returns may not reflect the rents of these operations. In 2005, the four largest processors of steers and heifers accounted for almost 80 of the market (MacDonald and McBride, 2009). It has also been noted that "Contractual relationships are becoming more complicated as backgrounders or cow-calf operations enter into joint ownership of cattle with feedlots or processors." (MacDonald and McBride, 2009).

While our bivariate stochastic cycle model cannot identify the precise causes of a lengthening cattle cycle, its value lies in showing that both the cow herd and the feeder steer/corn price ratio follow a stochastic cycle of essentially the same frequency, but with considerably different phases. It will be a matter of time to determine if the cycle continues to lengthen or if it reverts to a traditional cycle length in the 10 to 12 year range.

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