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Comparing the Value of Soil Test Information Using Deterministic and Stochastic Yield Response Plateau Functions

Xavier Harmon, Christopher N. Boyer, Dayton M. Lambert, James A. Larson, and C. Owen Gwathmey

We determined the value of soil test information for potassium (K) in upland cotton production using the linear response plateau (LRP) and linear response stochastic plateau (LRSP) functions. A stochastic dynamic programming model was used to determine the net present value to K fertilizer when optimal K was applied with knowledge about K carryover. Using K carryover information for K application decisions increased net present value and helped maintain steady levels of soil K. The LRSP function fit the data better than the LRP, and the value of soil testing was \$27 ha^{-1} lower over ten years using the LRSP.

Key words: cotton, dynamic programming, linear response stochastic plateau, potassium, soil test information

Introduction

Procedures to assess the levels of soil potassium (K) readily available for consumption by field crops (available K) were developed more than fifty years ago (Mehlich, 1953), and crop response to K fertilizer has been well-documented in long-term experiments (Cope, 1981). However, attention to K management in upland cotton (*Gossypium hirsutum* L.) production grew in the late 1980s and 1990s after frequent reports of late-season K deficiencies in the southeastern United States, resulting in lower yields (Maples, Thompson, and Varvil, 1988; Mullins, Burmester, and Reeves, 1997). These reports led to numerous agronomic studies in the U.S. Cotton Belt to recalibrate K fertilizer recommendations using soil test data to avoid late-season K deficiencies that produced negative impacts on cotton lint yield and fiber quality (Essington et al., 2002; Howard et al., 1998; Mullins, Schwab, and Burmester, 1999). While soil tests do not provide information on slowly available soil K or unavailable soil K, which can become available over time (Berstch and Thomas, 1985), researchers concluded that quantifying readily available K in the soil from year to year (i.e., carryover K) was important to establish K fertilizer recommendations and circumvent late-season deficiencies. Today, soil tests are commonly used to inform producers on available soil K prior to planting.

Xavier Harmon is a graduate research assistant in the Department of Agricultural and Resource Economics at the University of Tennessee, Knoxville; Christopher N. Boyer is an assistant professor in the Department of Agricultural and Resource Economics at the University of Tennessee, Knoxville; Dayton M. Lambert is an associate professor in the Department of Agricultural and Resource Economics at the University of Tennessee, Knoxville; James A. Larson is a professor in the Department of Agricultural and Resource Economics at the University of Tennessee, Knoxville; C. Owen Gwathmey is a professor emeritus in the Department of Plant Sciences at the University of Tennessee, Knoxville.
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Economists have developed models that consider soil fertilizer carryover to determine profit-maximizing K application rates over time in crop production (Heady and Dillon, 1961; Fuller, 1965; Kennedy et al., 1973; Stauber, Burt, and Linse, 1975). A carryover function is used to estimate the amount of available K in soils, given total K (applied and carryover) in previous periods. Dynamic programming is a common modeling approach to determine fertilizer rates that maximize net present value (NPV) (Kennedy, 1986). The difference between the NPV earned by a producer who considers carryover information and the NPV earned by a producer who does not consider carryover information determines the value of the carryover information (Harper et al., 2012).

Several studies have applied dynamic programming to the management of K fertilizer in crop production (Harper et al., 2012; Lanzer and Paris, 1981). Lanzer and Paris (1981) determined an economically optimal K rate over a nine-year planning horizon for double-cropped wheat and soybean in Brazil. The economically optimal K rate was 42.7 kg ha^{-1}, which was higher than the K recommendations for Brazil at that time. Harper et al. (2012) used three years of data from a cotton K fertilization experiment in Tennessee to analyze the value of soil test information under multiple information scenarios. They concluded that producers could increase NPV by \$1,613 ha^{-1} over five years when considering soil test information.

The selection of a functional form to model yield response to fertilizer is important for accurately determining optimal fertilizer rates and the value of soil test information (Ackello-Ogutu, Paris, and Williams, 1985; Kennedy, 1986). The quadratic response function has frequently been used to model cotton yield response to K fertilizer (Adeli and Varco, 2002; Bennett et al., 1965; Lombin and Mustafa, 1981; Pervez, Ashraf, and Makhdum, 2005). However, agronomists and economists alike have suggested that plateau-type response functions better describe yield response to fertilizer than the quadratic response function (Bullock and Bullock, 1994; Cerrato and Blackmer, 1990). A plateau-type function has either a linear or a polynomial relationship between crop yield, and the input until yield reaches a plateau, beyond which the input no longer limits yield; yield is limited by either another input affecting production or the plant reaches its natural maximum. Plateau functions, such as the linear response plateau (LRP) (Berck and Helfand, 1990; Paris, 1992) and quadratic-plus-plateau yield response functions, have been used to determine optimal fertilizer rates in the dynamic programming framework (Ackello-Ogutu, Paris, and Williams, 1985; Harper et al., 2012; Jomini, 1991; Lanzer and Paris, 1981).

Tembo et al. (2008) extended the LRP function by incorporating a normally-distributed year-random effect in the plateau. The plateau random effect emphasizes the impact of stochastic events such as insects, disease, and weather on crop yield response to fertilizer. The linear stochastic plateau model (LRSP) was more appropriate than deterministic functions for several crops, resulting in more accurate estimates of optimal fertilizer rates (Boyer et al., 2013; Tembo et al., 2008; Tumusiime et al., 2011). Zhou et al. (2015) used the LRSP function in a dynamic programming framework to evaluate alternative biofuel feedstock production subsidies. However, optimal fertilizer rates have never been evaluated using a stochastic plateau function, such as LRSP, in a dynamic programming framework. Using a stochastic plateau yield response function in a dynamic programming model could improve K fertilization recommendations for cotton production and offer a more accurate estimate of the value of soil test information because uncertainty around the plateau is incorporated into evaluating optimal rates.

The objective of this research was to determine the value of soil test information for available K in upland cotton production using the LRP and LRSP functions. We follow Kennedy's (1986) dynamic programming framework to solve for K fertilizer rates that maximize NPV when K carryover was and was not considered by a producer. The conceptual and econometric frameworks extend previous research by incorporating a stochastic plateau yield response function in a dynamic programming model and presenting the analytical solution to optimal fertilizer rates using a LRSP function in a dynamic programming model.

Table 1. Total Monthly Precipitation Levels for the Growing Season of Upland Cotton in Jackson, TN, from 2000 to 2008

	Precipitation Totals (cm)									
Month	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**Average**
March	9.98	7.14	33.02	9.04	6.35	10.41	4.50	2.92	24.77	12.27
April	13.26	6.30	2.79	5.94	23.06	21.69	13.77	8.26	20.90	12.89
May	8.94	12.37	14.99	–	15.82	0.91	9.14	2.18	17.42	10.22
June	10.13	12.24	6.22	15.39	7.37	17.45	12.55	6.88	7.14	10.60
July	6.25	11.96	2.16	6.15	12.04	13.87	5.38	4.47	15.95	8.69
August	7.42	11.81	13.59	8.71	12.52	18.47	8.97	1.96	6.48	9.99
September	8.31	5.79	33.25	7.09	1.75	10.03	7.34	15.95	2.01	10.17
October	2.18	18.72	16.28	10.57	20.29	0.36	6.65	22.78	8.00	11.76
Total	66.47	86.33	122.30	62.89	99.21	93.19	68.30	65.41	102.67	85.20

Source: National Oceanic and Atmospheric Administration (2015).

Data

Data on cotton yield response and soil K fertility levels were collected from a nine-year field study (2000–2008) conducted at the University of Tennessee, West Tennessee Research and Education Center at Jackson (35.63°N; 88.85°W). The soil type was Loring-Calloway silt loam (thermic Oxyaquic Fragiudal and thermic Typic Fragiaqualf). The plots were not tilled. Each year, K fertilizer (muriate of potash, 0–0–60) was broadcast by hand to individual plots prior to planting at rates of 0, 28, 56, 84, 112, 139, and 167 kg ha^{-1} of elemental K. Treatments were applied to the same plots each year, starting five years prior to the first year of this study (2000) through the last year (2008). Plots were arranged in a randomized complete block design. Fertilizer treatments were replicated five or six times.

Cotton was planted using a four-row John Deere MaxEmerge planter between April 30 and May 15 of each year. The cultivar ‘PM1218BG/RR’ was planted on all plots from 2000 to 2002. From 2003 to 2008, two contrasting cultivars were planted in a factorial arrangement relative to the K-fertility plots. The cultivars ‘PM1218BG/RR’ and ‘DP555BG/RR’ were planted from 2003 to 2005, the cultivars ‘FM960BR’ and ‘DP555BG/RR’ were planted from 2006 to 2007, and the cultivars ‘ST455B2RF’ and ‘ST5327B2RF’ were planted in 2008.[1] Plots were 9.15 by 3.86 m, containing four rows spaced 97 cm apart. Shortly before or after planting each year, nitrogen fertilizer (ammonium nitrate, 34–0–0) was uniformly drop-spread to all plots at a rate of 90 kg ha^{-1}. Ground limestone and phosphorus fertilizer were uniformly applied according to the recommendations of the University of Tennessee Extension Service (Savoy, Jr. and Joines, 2001). Supplemental irrigation was used during dry spells in all years except 2002 and 2003. Thus, all other fertilizer inputs were assumed to be non-yield limiting. Table 1 summarizes monthly growing-season rainfall by year at Jackson, Tennessee (National Oceanic and Atmospheric Administration, 2015). All other production practices followed the Savoy, Jr. and Joines (2001) guidelines for cotton production.

Seedcotton was harvested from the two interior rows of each plot twice each year using a modified John Deere 9930 spindle picker. First harvest occurred from September 7 to October 8, with a second harvest occurring fourteen to twenty-eight days later. Seedcotton weights, gin turnouts, and plot areas harvested were used to calculate lint yields. Observed lint yields from 2000 to 2008 were used to estimate yield response functions. Average annual lint yields by K rate are displayed in table 2. Improved biotechnology from the different cultivars may have increased yields over time. Therefore, cotton lint yields were tested with a deterministic quadratic time response function (Just and Weninger, 1999). Similar to cotton in Oklahoma (Boyer, Brorsen, and Tumusiime, 2015), a time trend was not present.

[1] ANOVA analysis indicated there was no difference in yield across cultivars.

Table 2. Average Annual Cotton Lint Yield by K Application Rate in Jackson, TN, from 2000 to 2008

K rate	Yield (kg ha^{-1})									
(kg ha^{-1})	2000	2001	2002	2003	2004	2005	2006	2007	2008	Average
0	986	926	532	906	1,076	976	779	1,012	669	874
28	1,224	1,354	847	1,353	1,702	1,462	1,391	1,470	1,456	1,362
56	1,252	1,392	936	1,554	2,098	1,666	1,599	1,472	1,590	1,507
84	1,334	1,533	1,124	1,572	2,030	1,551	1,578	1,427	1,787	1,548
112	1,312	1,560	1,201	1,626	2,240	1,721	1,638	1,428	1,771	1,611
139	1,314	1,530	1,198	1,535	2,081	1,557	1,468	1,265	1,621	1,508
167	1,326	1,571	1,163	1,541	2,151	1,602	1,476	1,255	1,549	1,515

Table 3. Average Pre-Planting K Carryover Levels by K Application Rate in Jackson, TN, from 2001 to 2009

K rate	Carryover K (kg ha^{-1})									
(kg ha^{-1})	2001	2002	2003	2004	2005	2006	2007	2008	2009	Average
0	133	123	156	110	99	116	104	114	101	117
28	172	165	198	168	138	146	138	159	162	161
56	199	208	229	245	198	218	211	249	249	223
84	274	249	279	333	257	267	282	304	342	287
112	364	303	337	443	344	378	334	356	417	364
139	442	389	375	525	392	454	407	375	514	430
167	514	584	421	592	493	513	523	550	597	524

Within six weeks after harvest, soil samples were collected from all plots at the 0–15 cm depth using the Mehlich I extraction method (Howard et al., 2001). The samples were tested at the University of Tennessee Soil and Forage Test Laboratory in Nashville, Tennessee. Data from the Mehlich I soil tests were used to provide information on the amount of available K in the soil. Pre-planting soil test levels from 2001 to 2009 were used to estimate the carryover function. Average soil test levels across all plots and years were in the medium soil fertility range (Savoy, Jr. and Joines, 2001) (table 3). A review of observed soil test values indicated that the variance of the carryover data may increase at higher levels of total available K. Therefore, soil test levels were tested and corrected for heteroskedasticity across years.

Average annual cotton lint and elemental K prices ($ kg^{-1}) from 1994 to 2013 were used to determine the K fertilization rates that maximized NPV over a ten-year planning horizon. Nominal prices were adjusted to reflect real prices in 2013 using the Gross Domestic Product implicit price deflator (U.S. Bureau of Economic Analysis, 2015). Real cotton prices varied from $0.82 to $2.36 kg^{-1}, and real elemental K prices varied from $0.45 to $2.02 kg^{-1} (U.S. Department of Agriculture, Economic Research Service, 2013, 2014). Real lint and K prices were not correlated over time. The total cost of soil testing included the cost of obtaining the soil sample and the chemical analysis. The cost of obtaining the soil sample was $17.19 ha^{-1} year^{-1}, which was based on University of Tennessee Custom Rate Survey (Bowling, 2013). The cost of the chemical soil analysis was $1.73 ha^{-1}year^{-1}. This cost assumes a producer soil tests on a 4 ha grid, which follows University of Tennessee recommendations for soil testing (Savoy, Jr. and Joines, 2013). A 5% discount rate was used to represent the opportunity cost of land in cotton production similar to previous dynamic programming literature (Harper et al., 2012; Kennedy et al., 1973; Park et al., 2007; Segarra and Ethridge, 1989; Watkins, Lu, and Huang, 1998).

Conceptual and Econometric Models

We find the value of the soil test when a LRP function and LRSP function was used to model yield response to K. Therefore, four scenarios are modeled: (1) NPV of returns to K fertilizer using the LRP yield response function considering K carryover; (2) NPV of returns to K fertilizer using the LRP yield response function when K carryover was not considered; (3) NPV of returns to K fertilizer using the LRSP yield response function considering K carryover; and (4) NPV of returns to K fertilizer using the LRSP yield response function when K carryover was not considered.

Dynamic Programming Model

An optimizing producer manages total K availability for continuous cotton production by applying K fertilizer at the beginning of each production year to maximize the NPV of returns to K over a planning horizon. The optimal fertilization rates are conditioned on some measure of K carryover between production periods (Kennedy, 1986; Kennedy et al., 1973):

$$\begin{aligned} \max_{A_t,\ \ldots,\ A_T} NPV \quad &= \quad \sum_{t=1}^{T} \delta^{t-1} NR_t \\ \text{Subject to:} \quad & \quad A_t, C_t \geq 0 \\ & \quad C_{t+1} = a_0 + a_1(A_t + C_t) \\ & \quad C_1 \text{ given}, \end{aligned} \tag{1}$$

where NPV is the sum of discounted returns over T years ($t = 1, \ldots, T$); A_t is applied K fertilizer (kg ha^{-1}); NR_t is the cotton lint net returns (\$ ha^{-1}) to K fertilizer; δ is a discount factor reflecting the time value of money $1/(1+r)^t$, where r is the discount rate; C_t is carryover K (kg ha^{-1}) obtained from soil testing; C_{t+1} is the carryover level (kg ha^{-1}) obtained from soil test information prior to planting in year $t+1$, which is a function of applied K fertilizer A_t (kg ha^{-1}) and carryover soil K C_t (i.e., total K available (kg ha^{-1})); a_0 and a_1 are estimated parameters for the linear carryover function; and C_1 is the soil K level before fertilizer K is applied in the first period of production.

Single-period cotton lint net returns to K fertilizer are

$$NR_t = \delta p_t^c y_t\{A_t + C_t\} - p_t^K A_t - s, \tag{2}$$

where p_t^c and p_t^K are cotton lint (\$ kg^{-1}) and K fertilizer (\$ kg^{-1}) prices, respectively; $y_t\{A_t + C_t\}$ is expected cotton lint yield (kg ha^{-1}); and s is the cost of soil testing (\$ ha^{-1}), which includes purchasing the soil test and obtaining the soil samples. Soil K from previous applications accumulates into current-period soil K levels; thus, the only relevant soil K carryover level is for the current period. Residual soil K levels were determined using a carryover function. When the producer does not consider K carryover in making current-period K fertilization decisions, the cost of the soil test was zero ($s = 0$) and the carryover level was assumed to be zero ($C_t = 0$). Thus, K carryover has no influence on current-period K fertilizer application (Harper et al., 2012; Kennedy, 1986).

We follow Kennedy's (1986) dynamic framework to determine optimal fertilizer rates that maximize NPV:

$$
\begin{aligned}
V_t\{C_t\} &= \max_{A_t}[NR_t + \delta V_{t+1}\{C_{t+1}\}] \\
\text{Subject to:} \quad & A_t, C_t \geq 0 \\
& C_{t+1} = a_0 + a_1(A_t + C_t) \\
& V_{T+1}\{C_{T+1}\} = 0 \\
& C_1 \text{ given,}
\end{aligned} \tag{3}
$$

where $V_t\{C_t\}$ is the present value of net returns (\$ ha^{-1}) from applying the profit-maximizing K application in year t and $V_{T+1}\{C_{T+1}\} = 0$ is the terminal condition stating that the producer does receive any economic value from the K remaining in the soil at the end of the planning horizon since the producer will not get to utilize the remaining soil K after the planning horizon ends (Chiang, 1992). When maximizing NPV the economic optimality principle of marginal value product (MVP) equals marginal factor cost (MFC) is complicated by intertemporal factors such as the time value of money (opportunity cost) and fertilizer carryover (Kennedy et al., 1973; Kennedy, 1986). In this framework, K fertilizer is applied at the beginning of each production year to manage total K available to the plant. The intertemporal optimization of this dynamic program determines fertilization rates through recursion using first-order conditions (Bellman, 1957). The profit-maximizing K application strategy exists when the initial state variable (C_1) is given. The optimality conditions are solved by differentiating equation (3) with respect to the decision variable A_t:

$$\frac{\partial V_t}{\partial A_t} = \delta p_t^c \frac{\partial y_t}{\partial A_t} - p_t^K + \delta \frac{dV_{t+1}}{dC_{t+1}} a_1 = 0, \tag{4}$$

which can be rearranged:

$$\delta p_t^c \frac{\partial y_t}{\partial A_t} = p_t^K - \delta \frac{dV_{t+1}}{dC_{t+1}} a_1. \tag{5}$$

By the envelope theorem (Léonard and van Long, 1992), differentiating equation (3) with respect to the state variable C_t gives

$$\frac{\partial V_t}{\partial C_t} = \delta p_t^c \frac{\partial y_t}{\partial A_t} + \delta \frac{dV_{t+1}}{dC_{t+1}} a_1. \tag{6}$$

Equation (5) can be substituted into equation (6) to get

$$\frac{\partial V_t}{\partial C_t} = p_t^K. \tag{7}$$

Equation (7) indicates that K carryover at the beginning of year t is valued at the price of K in year t. A similar result can be found for year $t+1$:

$$\frac{\partial V_{t+1}}{\partial C_{t+1}} = p_{t+1}^K, \tag{8}$$

which can be substituted into equation (5) to get

$$\delta p_t^c \frac{\partial y_t}{\partial A_t} = p_t^K - \delta p_{t+1}^K a_1. \tag{9}$$

Equation (9) is the optimal condition for the single period K application rate when a producer has information about K carryover (Kennedy, 1986). This condition indicates that the optimal K application rate in any year t is where the discounted MVP of K (left hand side of equation 9) is equal to the MFC of K, which is the current year price of K less the discounted savings from K fertilizer carried over to the next year (right hand side of equation 9).

If a producer does not consider carryover, then $C_t = 0$ and optimal condition for single-period K fertilization becomes

$$\delta p_t^c \frac{\delta y_t}{\delta A_t} = p_t^K. \tag{10}$$

In equation (10) the discounted savings of K fertilizer remaining in the soil until the next period is not considered.

K Carryover Function

A linear functional form is commonly used to model carryover in the literature (Harper et al., 2012; Jomini, 1991; Lanzer and Paris, 1981; Segarra and Ethridge, 1989). We adapt the linear carryover function by including a year random effect in the intercept. We estimated parameters for the carryover function using the actual measured total K available. The deterministic and stochastic models included identical linear carryover functions, which is

$$C_{t+1,i} = a_0 + a_1(A_{t,i} + C_{t,i}) + \tau_t + u_{t,i}, \tag{11}$$

where $\tau_t \sim N(0, \sigma_\tau^2)$ is an intercept year random effect isolating the variation in carryover across years and $u_{t,i} \sim N(0, \sigma_u^2)$ a random error term for plot i. The two stochastic terms are assumed independent. The intercept, a_0, represents some constant amount of available K that remains in the soil over the planning horizon; the slope, a_1, is the proportion of total K from the current year readily available to the next crop estimated using observed carryover K. Maximum likelihood estimates for equation (11) were obtained using the MIXED procedure in SAS 9.3 (SAS Institute, Inc., 2011).

Yield Response Functions

The LRP function is

$$y_{t,i} = \min(\beta_0 + \beta_1(A_{t,i} + C_{t,i}), \mu) + w_t + \varepsilon_{t,i}. \tag{12}$$

where β_0 and β_1 are the yield response parameters; μ is the expected plateau yield parameter (kg ha^{-1}); $w_t \sim N(0, \sigma_w^2)$ is the intercept year random effect; and $\varepsilon_{t,i} \sim N(0, \sigma_\varepsilon^2)$ is the random error term. Independence is assumed across the two stochastic terms. Similarly, the LRSP function is

$$y_{t,i} = \min(\beta_0 + \beta_1(A_{t,i} + C_{t,i}), \mu + v_t) + w_t + \varepsilon_{t,i}, \tag{13}$$

where $v_t \sim N(0, \sigma_v^2)$ is a plateau random effect. The three random effects are independent. Parameter estimates for the LRP and LRSP yield response functions were estimated using the observed K application rates and observed carryover levels from the experiment. Since the LRP and LRSP are nested response functions, Tembo et al. (2008) and Tumusiime et al. (2011) used the likelihood ratio test to determine whether the LRP or the LRSP model best describes the data. We follow this approach to determine whether the LRP or the LRSP model best describes cotton response to K. The maximum likelihood parameter estimates for equations (12) and (13) were obtained using the NLMIXED procedure in SAS 9.3 (SAS Institute, Inc., 2011).

Optimal K Fertilizer Rate

For the LRP function, the profit-maximizing K rate is the rate required to reach the plateau if the MVP below the plateau is greater than the MFC. Conversely, if the MVP of K below the plateau is less than the MFC, a profit-maximizing producer would apply zero K (Lanzer and Paris, 1981). Thus, the optimal K rate is a corner solution (zero or the plateau rate). When carryover is considered, the optimal K rate in year t is the corner solution less the carryover K in year t (C_t), given $A_t \geq 0$. However, the optimal K rate when carryover is not considered is the corner solution, assuming the carryover level or the savings associated with carryover is zero.

To solve for the optimal K rate for the LRSP considering carryover, the yield response function (equation 13) is differentiated with respect to the decision variable A_t and substituted into the optimality condition considering carryover (equation 9) to produce

$$\delta p_t^c \beta_1 (1 - \Phi) = p_t^K - \delta p_{t+1}^K a_1, \tag{14}$$

where $\Phi = \Phi[(\beta_0 + \beta_1(A_t + C_t) - \mu)/\sigma_v]$ is the standard normal cumulative distribution function and $0 \leq \Phi \leq 1$ (Tembo et al., 2008). By equation (14), the optimal K application rate was expressed as

$$A_t^* = \frac{\left[\Phi^{-1}\left(1 - \frac{p_t^K - \delta p_{t+1}^K a_1}{\delta p_t^c \beta_1}\right)\right]\sigma_v + \mu - \beta_0}{\beta_1} - C_t. \tag{15}$$

When stochastic variation is considered in the plateau, the optimal application decision will be dependent upon the ratio of K and cotton prices (Tembo et al., 2008). Similarly, when a producer considers carryover, the savings associated with K carryover must be accounted for as well. When a producer does not consider carryover, equation (15) becomes

$$A_t^* = \frac{\left[\Phi^{-1}\left(1 - \frac{p_t^K}{\delta p_t^c \beta_1}\right)\right]\sigma_v + \mu - \beta_0}{\beta_1}. \tag{16}$$

Because carryover is not considered by the producer, the saving associated with K carryover is assumed to be zero. In reality, assuming K carryover has a zero value may not be realistic of producers; however, the assumption allows us to accurately benchmark the value of soil test information for the two response functions. Derivation of the optimal application rate for the LRSP is provided in Appendix A.

Monte Carlo Simulation

The estimated response functions and analytical solutions for optimal fertilizer and yield are substituted into a simulation model to find a distribution of NPVs over a ten-year period. A Monte Carlo simulation model is developed for each of the four scenarios. Figure 1 summarizes the general process used to solve the dynamic programming model. Shaded boxes correspond to stochastic parameters in the model.

Uncertainty surrounding prices of cotton lint and K is introduced into the model through bootstrapping the real prices of cotton lint and K during each period of the ten-year planning horizon. To introduce uncertainty in the expected yield response, the yield response coefficients assumed a multivariate normal (MVN) distribution:

$$\begin{bmatrix} \beta_0^* \\ \beta_1^* \\ \mu^* \end{bmatrix} \sim MVN\left(\begin{bmatrix} \beta_0 \\ \beta_1 \\ \mu \end{bmatrix}, \begin{bmatrix} \sigma_{\beta_0}^2 & \rho_{\beta_0,\beta_1}\sigma_{\beta_0}\sigma_{\beta_1} & \rho_{\beta_0,\mu}\sigma_{\beta_0}\sigma_{\mu} \\ \rho_{\beta_1,\beta_0}\sigma_{\beta_1}\sigma_{\beta_0} & \sigma_{\beta_1}^2 & \rho_{\beta_1,\mu}\sigma_{\beta_1}\sigma_{\mu} \\ \rho_{\mu,\beta_0}\sigma_{\mu}\sigma_{\beta_0} & \rho_{\mu,\beta_1}\sigma_{\mu}\sigma_{\beta_1} & \sigma_{\mu}^2 \end{bmatrix}\right), \tag{17}$$

where the mean of the distribution is a vector of the estimated coefficients for each yield response function (equations 12 and 13); the variance of the distribution is a three by three matrix of the

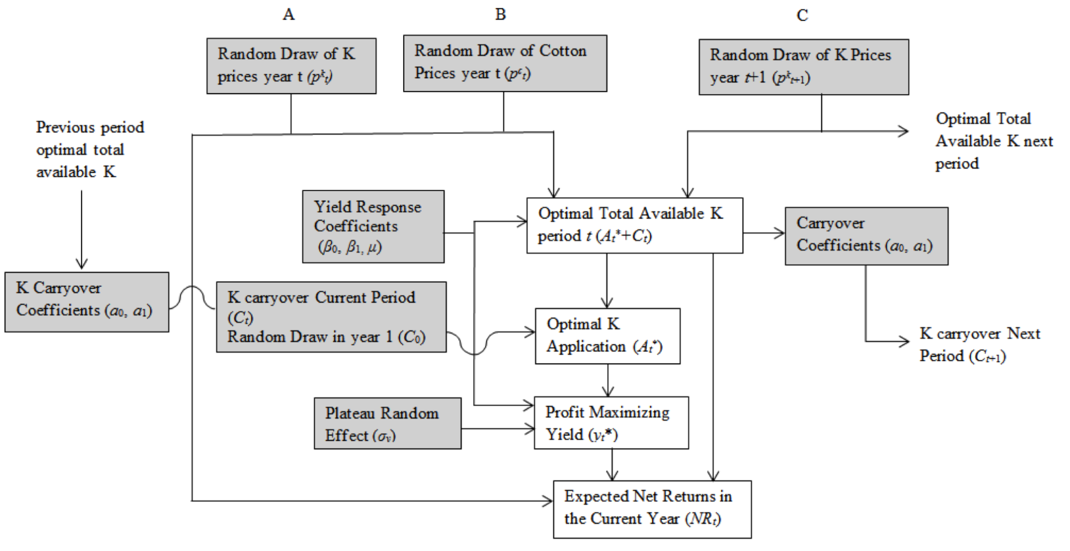

Figure 1. Flow Chart Depicting the Process of Solving the Dynamic Programming Model and Simulation for a Single Period (t)

covariance for the estimated coefficients in each yield response function (equations 12 and 13), where ρ is a correlation coefficient and an asterisk (*) indicates a randomly drawn coefficient for the simulation (Cuvaca et al., 2015). The plateau variance (v_t) was stochastic following Tembo et al.'s (2008) standard normal distribution. The coefficients of the carryover function assumed a similar multivariate normal distribution:

$$(18) \qquad \begin{bmatrix} a_0^* \\ a_1^* \end{bmatrix} \sim MVN\left(\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}, \begin{bmatrix} \sigma_{a_0}^2 & \rho_{a_0,a_1}\sigma_{a_0}\sigma_{a_1} \\ \rho_{a_1,a_0}\sigma_{a_1}\sigma_{a_0} & \sigma_{a_1}^2 \end{bmatrix} \right).$$

Making the parameter estimates in the carryover function stochastic is a unique contribution to the literature.

Uncertainty surrounding the initial carryover level (C_1) was introduced into the model by bootstrapping the observed carryover levels. The optimal K application rate in year one was found by substituting yield response and carryover parameter estimates, prices of cotton and K, and the initial carryover level into the equation for optimal K application rates. The soil K level after harvest in year t became available for use in year $t+1$ (equation 11). Therefore, after the first year decision, K applications for the remaining years ($t = 2, \ldots, 10$) were influenced by K carryover from the previous season.

One thousand iterations of the ten-year planning horizon are simulated to generate output distributions for the K application rate, K carryover level, lint yield, and NPV for each scenario. The Monte Carlo simulation is conducted using @Risk (Palisade Corporation, 2015). The expected NPVs from the simulation of the four scenarios were used to find the value of information from soil testing. The LRSP model captures the unexpected year-to-year variability in the plateau yield, thus providing a hypothesized lower estimate for the value of information from soil testing than the LRP model.

Table 4. Parameter Estimates for the Linear Response Plateau and Linear Response Stochastic Plateau Yield Response to Total Available K and the Linear Carryover Function

Parameter	Deterministic Plateau	Stochastic Plateau	Carryover
Intercept (β_0, a_0)	-14.85	-67.52	28.53^{***}
(kg ha^{-1})	(87.85)	(101.60)	(7.60)
Slope (β_1, a_1)			
(kg ha^{-1})	8.42^{***}	8.90^{***}	0.81^{***}
	(0.66)	(0.60)	(0.02)
Plateau Yield	$1,538.65^{***}$	$1,565.21^{***}$	–
(kg ha^{-1})	(16.04)	(16.08)	
Plateau Random Effect (σ_v^2)	–	$35,813.70^{***}$	–
		(4,674.91)	
Year Random Effect ($\sigma_w^2, \sigma_\tau^2$)	$54,748.99^{***}$	$37,853.80^{***}$	264.08
	(5,340.78)	(5,823.65)	
Random Error ($\sigma_\varepsilon^2, \sigma_u^2$)	$34,677.60^{***}$	$28,475.30^{***}$	3.07
	(2,549.31)	(2,138.93)	
-2 Log-Likelihood	4,977	4,923.8	

Notes: Single, double, and triple asterisks (*, **, ***) represent significance at the 10%, 5%, and 1% level. Standard errors are in parentheses. Carryover data was corrected for heteroskedasticity.

Results

Yield Response and Carryover Functions

The estimated yield response and carryover functions are presented in table 4. The parameter estimates for the LRP and LRSP functions had the expected positive signs, except the intercepts were negative. Watkins, Lu, and Huang (1998) and Stauber, Burt, and Linse (1975) also found negative intercepts for wheat, barley, and seeded grasses yield response to nitrogen when carryover was considered. Like these studies, the negative intercepts were not of concern since carryover K was always greater than zero (table 3); thus, zero total available K was not present in the data. The slope parameter estimate for the LRSP function was greater than the slope of the LRP function. Tembo et al. (2008) attributed attenuation bias to explaining the smaller slope parameter estimate for the LRP function. The expected plateau yield for the LRSP was also higher than the expected yield for the LRP, which matches previous studies (Boyer et al., 2013). The likelihood ratio statistic ($[4,977 - 4,923.8] = 53.2$) was greater than the critical value ($X^2_{1,0.05} = 3.84$), indicating the LRSP function described yield response to total available K better than the LRP function (table 4), which is similar to what Boyer et al. (2013), Tembo et al. (2008), and Tumusiime et al. (2011) observed.

The intercept for the carryover function indicated that 28.53 kg K ha^{-1} found in the soil did not come from the K application in the previous year but remained available to the plant over the planning horizon. The carryover coefficient of 0.81 implies that 81% of the total K available in the current year will be carried over to be available for use in the next year. The carryover coefficient in table 4 was similar to Harper et al.'s (2012) K carryover coefficient of 0.72.

Simulation Results

Table 5 provides simulation results for K application, K carryover, and yield for each year of the ten-year time horizon as well as the ten-year average. For the LRP function, the optimal annual K application rate when carryover was considered ranged from 9 to 29 kg ha^{-1} with an annual average rate of 22 kg ha^{-1}. When K carryover was not considered, the optimal annual average application rate was 208 kg ha^{-1}, an increase of 186 kg^{-1}ha^{-1}. Harper et al. (2012) reported optimal K application rates and carryover levels for cotton production in Tennessee higher than what we find in our study. We report results using data from a longer time-series and on a different soil type than Harper et al. (2012), which likely explains the different findings. Optimal K carryover was on average 371 kg ha^{-1}year^{-1} less when K soil test information was considered in the choice of the K fertilization rate. Fertilizer K carryover declined from an initial level of 303 kg ha^{-1} to a steady state level of 179 kg ha^{-1} when soil test information was considered, whereas soil K increased each year when K carryover was not considered in the choice of a K fertilization rate. Lint yields were the same for both K carryover scenarios. Therefore, a producer using soil K carryover information would optimize lint yields, lower K fertilization rates and costs, and consistently lower the amount of fertilizer K remaining in the soil. These results match the existing literature on the use of soil test information in making optimal fertilizer decisions (Harper et al., 2012; Kennedy et al., 1973; Park et al., 2007; Segarra and Ethridge, 1989; Watkins, Lu, and Huang, 1998).

For the LRSP function, optimal K fertilization rates ranged from 11 to 31 kg ha^{-1} in each year with a ten-year average of 25 kg ha^{-1}. A producer that did not consider soil K information applied 181 kg ha^{-1} more K fertilizer annually than a producer who considered carryover to determine the K fertilization rate. K carryover was on average 361 kg ha^{-1}year^{-1} lower when carryover was considered. Lint yields were the same for the two fertilizer carryover scenarios. Thus, a producer who considers soil test information could achieve optimal lint yields while reducing K fertilizer application each year compared to a producer who does not consider soil test information. These findings illustrate the potential of soil test information to reduce over-application of K fertilizer while maintaining optimal lint yields in cotton production.

Comparing the LRP and LRSP results, the optimal application rates and carryover levels were higher and lint yields were lower when the plateau was stochastic. The slope parameter estimate β_1 found in the LRSP results in a higher average MVP of K, which explains why the LRSP has a higher optimal application rate of K than the LRP (Tembo et al., 2008). The higher optimal K application rate also increased the optimal K carryover rate relative to the LRP. The average K carryover levels obtained from the LRP and LRSP functions when considering soil test information were classified as medium soil test ratings according to the guidelines set by the University of Tennessee Extension Service (Savoy, Jr. and Joines, 2001). However, when carryover was not considered, the K carryover levels were classified as very high soil test ratings, which may lead to nutrient imbalances (Savoy, Jr. and Joines, 2001). Maintaining a medium soil test rating would be beneficial for a producer to minimize deficiency symptoms (Savoy, Jr., 2009). At medium soil test levels, the University of Tennessee Extension recommended K fertilization rate was higher than the optimal K rate determined in this study. Finally, the optimal yield was lower with the LRSP function than the LRP. Tumusiime et al. (2011) stated that the LRP function can overestimate yield potential in years when climate conditions are not suitable for production. Thus, the LRSP function has a lower optimal yield because the variation in the yield was considered.

The Value of Soil Test Information

Table 6 shows the NPV at the optimal K rates for each of the four scenarios and the value of soil test information. Using the LRP function, the NPVs for a producer who considers and does not consider carryover were \$18,590 ha^{-1} and \$17,387 ha^{-1}, respectively, giving a value of soil test information of \$1,203 ha^{-1} or \$156 ha^{-1}year^{-1}. The respective NPVs with and without carryover information were \$17,665 ha^{-1} and \$16,489 ha^{-1} using the LRSP function, giving a value of soil test information of \$1,176 ha^{-1} or \$152 ha^{-1}year^{-1}.

Given that the LRSP function described yield response to total available K better than the LRP function, the LRP function overestimated the value of soil testing by \$27 ha^{-1} (\$4 ha^{-1}yr^{-1}). Overall, testing for K carryover and using the soil test information to make K application decisions in cotton production was profitable and helped maintain a steady level of soil K. By capturing variation in the yield plateau, the LRSP function provided a lower value of information from soil testing for K in cotton than its deterministic counterpart.

Table 5. Monte Carlo Simulation Results for Average K Application Rate, K Carryover, Yield by Year for a Ten-Year Planning Horizon

	Linear Response Plateau (LRP)		Linear Response Stochastic Plateau (LRSP)	
Year	**K Carryover Is Considered**	**K Carryover Is Not Considered**	**K Carryover Is Considered**	**K Carryover Is Not Considered**
	Optimal K Application Rate (kg ha^{-1})			
Year 1	19	208	21	206
Year 2	9	208	11	205
Year 3	13	208	16	205
Year 4	18	208	21	206
Year 5	24	208	27	207
Year 6	27	208	31	207
Year 7	29	208	30	206
Year 8	29	208	31	207
Year 9	29	208	30	205
Year 10	29	208	30	205
Average	22	208	25	206
	Pre-Planting K Carryover Level (kg ha^{-1})			
Year 1	303	303	303	303
Year 2	262	399	264	398
Year 3	225	469	228	466
Year 4	201	520	205	515
Year 5	187	557	192	552
Year 6	181	584	187	579
Year 7	180	604	187	599
Year 8	179	618	186	614
Year 9	179	628	185	624
Year 10	179	636	184	630
Average	195	566	200	561
	Optimal Cotton Lint Yield (kg ha^{-1})			
Year 1	1,539	1,539	1,469	1,469
Year 2	1,539	1,539	1,465	1,465
Year 3	1,539	1,539	1,464	1,464
Year 4	1,539	1,539	1,462	1,462
Year 5	1,539	1,539	1,467	1,467
Year 6	1,539	1,539	1,468	1,468
Year 7	1,539	1,539	1,467	1,467
Year 8	1,539	1,539	1,470	1,470
Year 9	1,539	1,539	1,456	1,456
Year 10	1,539	1,539	1,473	1,473
Average	1,539	1,539	1,466	1,466

Table 6. Net Present Value (NPV) for the Ten-Year Period when K Carryover Was and Was Not Considered Using the Linear Response Plateau and Linear Response Stochastic Plateau Yield Response Function and the Value for Soil Test Information

Value	Linear Response Plateau (LRP)	Linear Response Stochastic Plateau (LRSP)
NPV (\$ ha^{-1}) when K Carryover was Considered	\$18,590	\$17,665
NPV (\$ ha^{-1}) when K Carryover was not Considered	\$17,387	\$16,489
Value of Soil Test Information over 10 years (\$ ha^{-1})	\$1,203	\$1,176
Annual Value of Soil Test Information (\$ ha^{-1})	\$156	\$152

Conclusion

We determined the value of information from soil testing for K in upland cotton production using the LRP function and the LRSP function. Cotton yield response and soil testing data were obtained from a nine-year experiment in Jackson, Tennessee. We follow Kennedy's (1986) dynamic programming framework to find the K fertilizer rate that maximizes NPV using the LRP and LRSP functions when carryover was and was not considered. Simulation models were used to find the expected NPVs, which were compared to find the value of testing for K in cotton production.

We build on previous research by incorporating the LRSP model into a dynamic programming model to find the value of soil testing when the plateau was uncertain. The results of this study provide information on the difference in the value of soil testing when the yield plateau is deterministic and when the yield plateau is stochastic. The results provide estimates of the value of soil testing and K recommendations for cotton production under the two plateau assumptions. A limitation of this study is that the results are specific to monoculture cotton, but future research could focus on extending the model to include crop rotations.

Regardless of the response function, including carryover in the simulation model decreased the optimal K application rate and the K carryover level, while yield remained optimal in all scenarios. Producers are often thought to overapply fertilizer; however, real-life producer decisions often involve more uncertainty than what is modeled. The value of the soil test information was \$156 ha^{-1}year^{-1} when the LRP function was used and \$152 ha^{-1}year^{-1} when the LRSP function was used. The value of soil test information was greater than cost of soil testing for both response functions. This conclusion is especially true when one considers that soil test results, typically including information on other available crop nutrients, is perhaps higher than these estimates, which are based solely on the value of the K levels.

Future research could examine the optimal frequency of soil testing K. Furthermore, we assume that producers have an expected value of K carryover equal to zero when producers do not consider carryover. However, this may not be realistic since producers typically expect some level of K carryover. Future research could investigate how the value of soil testing is affected by producers having some knowledge of available soil K levels when they do not consider carryover in making K application decisions.

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Appendix A: Deriving Optimal K Rates

We follow Kennedy's (1986) dynamic programming approach to derive the optimal K application rates when producers' application decisions are conditioned on some knowledge of K carryover. When producers consider carryover, the optimality condition for a profit-maximizing producer is

$$\delta p_t^c \frac{\partial y_t}{\partial A_t} = p_t^K - \delta p_{t+1}^K a_1. \tag{A1}$$

The producer's optimal K application rate is derived by updating equation (A1) with the first-order condition of the linear response stochastic plateau (LRSP) yield response function, where the LRSP functional form is

$$y_t = (1-\Phi)(\beta_0 + \beta_1(A_t + C_t)) + \Phi\left(\mu - \frac{\sigma_v \phi}{\Phi}\right). \tag{A2}$$

where $\Phi = \Phi[(\beta_0 + \beta_1(A_t + C_t) - \mu/\sigma_v]$ is the cumulative normal distribution function and $\phi = \phi[(\beta_0 + \beta_1(A_t + C_t) - \mu)/\sigma_v]$ is the standard normal probability density function, both evaluated at the total available K level $(A_t + C_t)$ in period t. Tembo et al. (2008) derived the first-order condition of the LRSP with respect to the decision variable:

$$\frac{\partial y_t}{\partial A_t} = \beta_1(1-\Phi) = 0. \tag{A3}$$

By substituting equation (A3) into equation (A1) we obtain

$$\delta p_t^c \beta_1 (1-\Phi) = p_t^K - \delta p_{t+1}^K a_1. \tag{A4}$$

We can rearrange equation (A4) to show that

$$\Phi = 1 - \frac{p_t^K - \delta p_{t+1}^K a_1}{\delta p_t^c \beta_1}. \tag{A5}$$

If we recall that $\Phi = \Phi[(\beta_0 + \beta_1(A_t + C_t) - \mu)/\sigma_v]$, we can update equation (A5) to

$$\Phi\left[\frac{(\beta_0 + \beta_1(A_t + C_t) - \mu)}{\sigma_v}\right] = 1 - \frac{p_t^K - \delta p_{t+1}^K a_1}{\delta p_t^c \beta_1}, \tag{A6}$$

which can be rearranged to

$$\frac{(\beta_0 + \beta_1(A_t + C_t) - \mu)}{\sigma_v} = \Phi^{-1}\left(1 - \frac{p_t^K - \delta p_{t+1}^K a_1}{\delta p_t^c \beta_1}\right). \tag{A7}$$

The closed-form solution for the producer's optimal K application decision can be obtained by solving equation (A7) for the decision variable A_t:

$$A_t^* = \frac{\left[\Phi^{-1}\left(1 - \frac{p_t^K - \delta p_{t+1}^K a_1}{\delta p_t^c \beta_1}\right)\right]\sigma_v + \mu - \beta_0}{\beta_1} - C_t. \tag{A8}$$

When the producer does not consider carryover, we repeat the same process using the optimality condition for a producer who does not consider carryover (equation 10).