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Models Based on Positive Mathematical Programming: State of the Art and Further Extensions

Thomas Heckeley¹, Wolfgang Britz¹

¹ University of Bonn, Institute for Agricultural Policy, Market Research and Economic Sociology, Bonn, Germany

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Thomas Heckelei, Wolfgang Britz

University of Bonn, Institute for Agricultural Policy,
Market Research and Economic Sociology, Bonn, Germany

Abstract

Mathematical programming models have received renewed interest in the area of agricultural and agri-environmental policy analysis. Their ability to explicitly represent physical constraints makes them specifically suited for connecting economic and bio-physical aspects of agricultural systems. Furthermore, they allow for a direct representation of many current agricultural policy measures related to production activity levels. The introduction of Positive Mathematical Programming (PMP) addressed problems with plausibility of simulated behaviour and lack of empirical validation of these models. This paper reviews the development of PMP in its various forms and takes a look at approaches beyond PMP contributing to the quest for empirically specified programming models.

1. Introduction

During the 1960's and 70's, aggregate linear programming models were en vogue in agricultural economics. Large scale systems, dis-aggregated by region and farm types, covering regional transport flows or including recursive-dynamic features were developed both in the US and Europe. However, few of the systems were kept alive, and, gradually, new model types like multi-commodity models, computable general equilibrium models and duality based farm/farm-household models dominated the scene. Part of the development was probably due to 'business cycles' in the research community. But equally likely, as the large-scale models based on mathematical programming were marketed as policy decision support systems, their drawbacks became obvious. Besides tremendous data and computing needs, they struggled with poor tracking records of observed behaviour and jumpy responses in simulation exercises. The introduction of Positive Mathematical Programming (PMP, Howitt 1995a) made the supply response of agricultural programming models more realistic by allowing perfect calibration to observed base year behaviour and by avoiding providing a more smooth simulation re-

sponse compared to linear programming models. Since its introduction, PMP and related methods have been applied to a rapidly growing number of models at the farm, regional, and sectoral level¹.

Coincidentally, the increasing use of PMP-type approaches during the last decade was accompanied by at least three other developments supporting the renewed interest in programming models: Firstly, the EU's switch from price support to other policy instruments tied to farms and production activities made econometric approaches based on behavioural functions more complicated or not sufficiently direct in representing policy instruments. Since the mid of eighties the CAP introduced – among others – dairy quotas, set-aside obligations in conjunction with crop specific payments per ha, voluntary set-aside, stocking density restrictions, farm specific ceilings and the nitrate directive. Most of these policy instruments are much easier to model in the context of a programming model. Secondly, the increasing interest in joint production of agricultural outputs and environmental goods (and 'bads') is elegantly handled by an activity based approach which allows for a straightforward link between economic and biophysical models. At the same time, activity based approaches ease communication and exchange in multi-disciplinary research projects. Thirdly, restrictions such as the land balance or requirement constraints for animal feed enhance credibility of simulation exercises by preventing implausible results. A feature getting more relevant as quantitative economic models are increasingly adopted as a tool in the agricultural policy decision process by many European public administrations. These developments are quickly acknowledged by the agricultural economists who often benefit from an engineering background and have access to data or other knowledge on the actual production process. This reduces the need for a duality based, indirect representation of the interplay between behaviour and technology at firm level, but instead an explicit and often more accurate representation of the production feasibility set. Hence, there is little doubt that our tool box will comprise models with a rich and explicit specification of technology for quite some time to come.

PMP may be understood as well as one cornerstone of a bridge crossing the gap between econometric models and programming models with more explicit specification of technology. Bridging the gap may be one of the main issues in agricultural economics when concepts as sustainability regarding economic, social and environmental aspects need to be analysed. Econometrics allows basing the parameterisation of economic models on observed behaviour. As agricultural economists, we have access to very rich statistical data sets such as the ten-thousands of single farm records in the Farm Accounting Data Network (FADN) or farm structure surveys. Given the arguments above regarding the advantages of a primal technology representation, its combination with econometric approaches therefore seems one of the promising avenues for research in agricultural economics in the years ahead and has the potential to render our research more credible.

¹ See for example farm level applications in Rosen and Sexton (1993), Arfini and Paris (1995), Garvey and Steele (1998), Gohin and Chantreuil (1999) and Judez et al. (2001); regional applications: Hatchett et al. (1991), Howitt (1995b), Barkaoui and Butault (1999), Barkaoui et al. (2001), Graindorge et al. (2001), Helming et al. (2001), and Paris et al. (2001); sectoral applications: House (1987), Kasnakoglu and Bauer (1988), Horner et al. (1992), Ribaud et al. (1994), CAPRI (2000) and Heckeles and Britz (2001).

The main objective of this paper is to review and evaluate PMP and its extensions as well as related methods aiming at the specification of optimization models with constraints. Special attention is given to the method's consequences for simulation behaviour and their empirical content. Given the restricted space, the review cannot be comprehensive and will instead selectively choose some methods for a more detailed consideration and subject others to a more cursory treatment. For many additional details, conceptual contributions, and different perspectives in this area among other things, we refer to a forthcoming book by Howitt (2005) on agricultural and environmental policy models. He gives a comprehensive treatment of calibration, estimation and optimisation. Furthermore, a review by Henry de Frahan et al. (2005) on PMP with a somewhat different focus and additional applications is another useful source. Further details and conceptual connections of PMP type models with other calibration methods and supply modelling approaches are given by Heckeley (2002) on which also some parts of this review are based.

The paper is organised as follows: Section 2 reviews the original PMP approach and those variations that employ dual values of calibration constraints in the process. Methodologies calibrating or estimating programming models without the use of calibration constraints are discussed in section 3. Section 4 draws conclusion and hints at possible paths of future research.

2. Using dual values of calibration constraints: the original PMP approach and variations

As the word 'positive' in PMP implies, the original motivation of PMP was to increase the reliability of a constrained optimisation model by using observed behaviour in the specification phase. Calibration constraints were introduced for observed levels of endogenous variables, and their dual values impacted on the specification of appropriate non-linear functions. The resulting model then reproduced these endogenous variables without the calibration constraints. The following paragraphs review that approach in detail.

The starting point of PMP is a profit maximising linear programming problem²:

$$\underset{x}{\text{Max}} Z = p'x - c'x$$

subject to

$$Ax \leq b \quad [\lambda]$$

$$x \geq 0$$

(1)

² The following chapter follows the argumentation in Heckeley 2002.

where:

Z = objective function value

\mathbf{p} = $(N \times 1)$ vector of product prices

\mathbf{x} = $(N \times 1)$ vector of production activity levels

\mathbf{c} = $(N \times 1)$ vector of accounting cost per unit of activity

\mathbf{A} = $(M \times N)$ matrix of coefficients in resource constraints

\mathbf{b} = $(M \times 1)$ vector of available resource quantities

$\boldsymbol{\lambda}$ = $(M \times 1)$ vector of dual variables associated with the resource constraints

The problem above might represent an optimisation problem at the farm or some aggregate level as commonly used in applied agricultural policy modelling. Given an appropriate data set based on farm accounting or sectoral (regional) averages, the solution of this problem will, in general, be overspecialised, because the number of empirically justified (or available) resource constraints is usually well below the number of observed activities. Since the number of nonzero activities in an LP framework is upper bounded by the number of resource constraints, overspecialisation must occur by design.

The problem of overspecialisation is generally more severe in aggregate models for several reasons (see also Howitt 1995a: 330):

1. The number of empirically justified constraints relative to the number of observed production activities is smaller compared to the farm level.
2. Data, time and computational restrictions oftentimes do not allow specifying relevant non-linearity in aggregate technology that would force more production activities into the solution.
3. For the same reasons, output price endogeneity and risk behaviour, which would both imply some tendency towards diversification, are often not incorporated into the objective function of the model.

Model solutions that deviate substantially from observed production quantities are certainly not appealing in the context of ‘selling’ these models to political decision-makers. Neglecting so many relevant factors determining observed supply behaviour indeed renders the usefulness of these models for policy decision support questionable.

Therefore, applied modellers invested significant efforts into calibrating linear programming models to better reproduce observed values. Calibration within the framework of LP’s was mainly done by introducing additional rotational constraints or simply by adding upper and lower bounds on certain production activities. Apart from the weak theoretical and empirical justification of the additional constraints at the aggregate level, they also (inappropriately) constrain the set of possible simulation results such that ad-hoc mechanisms were incorporated to increase model flexibility in simulation runs (flexibility constraints). An extensive overview and discussion on ‘pre-PMP’ calibration approaches is given in Hazell and Norton (1986) and Bauer and Kasnakoglu (1990).

A second line of work introduced an objective function non-linear in variables to explicitly model risk behaviour or endogenous prices. The non-linearity yields interior solutions for certain production activities – independent of the constraints – and thereby provides some relief from the overspecialisation problem. However, experiences show (e.g. Meister et al.

1978) that the problem does not fully disappear. Furthermore, even if all observed production activities are also nonzero in the optimal solution, deviations in optimal from observed levels will still occur, asking for further calibration exercises with the above mentioned negative implications for the models simulation response.

PMP had some advantages compared to these earlier solutions. Firstly, it allowed for perfect calibration without introducing artificial constraints. Secondly, the non-linear terms allowed for interior solutions overcoming the overspecialisation problem in LP and, thirdly, led to smooth reactions to exogenous shocks, which promised more realistic simulation behaviour, two properties shared with other approaches introducing a non-linear objective. And fourthly, compared to solutions introducing bounds, the impact of PMP on the simulation behaviour was seen as less severe.

Even before the approach was formalised by Howitt (1995a), the technique has been employed by a series of pragmatic, policy oriented modelling exercises (for example: Howitt and Gardner 1986; House 1987; Kasnakoglu and Bauer 1988; Bauer and Kasnakoglu 1990; Horner et al. 1992; Schmitz 1994; for more references see Howitt 1995a)³. In many of these early applications, PMP was introduced in already existing programming models which were typically rich in constraints, replacing some other mean of calibration.

There was significant interest and continued implementation of this approach after the article of Howitt (1995a) made PMP widely known (in the area of agricultural sector modelling see for example Arfini 1996; Cypri 1996, 2000, Gohin and Chantreuil 1999; Arfini and Paris 1995; Barkaoui and Butault 1999, Barkaoui et al. 2001; Graindorge et al. 2001, Helming et al. 2001). In many of the examples mentioned, new tools were developed acknowledging the properties of PMP in the design from the beginning as only such constraints were introduced that were justifiable and could be formulated based on available data, or risk behaviour was neglected when lack of time series on prices and yields made the identification of the stochastic properties of returns to production activities difficult.

The general idea of PMP is to use information contained in dual variables of calibration constraints, which bound the LP-problem to observed activity levels (Phase 1). These dual values are used to specify a non-linear objective function such that observed activity levels are reproduced by the optimal solution of the new programming problem without bounds (Phase 2).

Phase 1 of this procedure is formally described by extending model (1) in the following way:

$$\begin{aligned}
 & \max_x Z = p'x - c'x \\
 & \text{subject to} \\
 & \quad Ax \leq b \quad \begin{bmatrix} \lambda \\ \rho \end{bmatrix} \\
 & \quad x \leq (x^0 + \varepsilon) \\
 & \quad x \geq [0] \\
 & \text{where:} \\
 & \quad \mathbf{x}^0 = (N \times 1) \text{ vector of observed activity levels}
 \end{aligned} \tag{2}$$

³ Schaible (1997) and (2000) uses a very similar approach, but does not refer at all to the PMP literature.

ϵ = (N×1) vector of a small positive numbers
 ρ = dual variables associated with the calibration constraints

The addition of the calibration constraints will force the optimal solution of the linear programming model (2) to exactly reproduce the observed base year activity levels \mathbf{x}^0 , given that the specified resource constraints allow for this solution (which they should if the data are consistent, see Hazell and Norton 1986: 266f). ‘Exactly’ is accurately understood to mean within the range of the positive perturbations of the calibration constraints, ϵ , which are included to guarantee that all binding resource constraints of model (1) remain binding here and thus avoid a degenerate dual solution.

We can partition the vector \mathbf{x} into two subsets, an ((N-M)×1) vector of ‘preferable’ activities, \mathbf{x}^p , which are bounded by the calibration constraints, and a (M×1) vector of ‘marginal’ activities, \mathbf{x}^m , which are constrained solely by the resource constraints. To simplify notation, without loss of generality, we assume that all elements in \mathbf{x}^0 are nonzero and all resource constraints are binding. Then, the Kuhn-Tucker conditions imply that

$$\rho^p = p^p - c^p - A^{p'} \lambda \quad (3)$$

$$\rho^m = [0] \quad (4)$$

$$\lambda = (A^{m'})^{-1} (p^m - c^m) \quad (5)$$

where the superscripts p and m indicate subsets of original vectors and matrices corresponding to preferable and marginal activities, respectively. Whereas the dual values of the calibration constraints are zero for marginal activities (ρ^m) as shown in (1.4), they are equal to the difference of price and marginal cost for preferable activities (ρ^p) as seen in (1.5), latter being the sum of variable cost per activity unit (c) and the marginal cost of using fixed resources ($A^p \lambda$). It should be noted here, that the dual values of the resource constraints (λ) only depend on objective function entries and coefficients of marginal activities.

In *Phase 2* of the procedure, the dual values of the calibration constraints ρ^p are employed to specify a non-linear objective function such that the marginal cost of the preferable activities are equal to their respective prices at the base year activity levels \mathbf{x}^0 . Given that the implied variable cost function has the right curvature properties (convex in activity levels) the solution to the resulting programming problem will be a “boundary point, which is the combination of binding constraints and first order conditions” (Howitt 1995a: 330).

Howitt (1995a) and Paris and Howitt (1998) interpret the dual variable vector ρ associated with the calibration constraints as capturing any type of model mis-specification, data errors, aggregation bias⁴, risk behaviour and price expectations.

⁴ To deal with aggregation errors in regional or sector modelling, Önal and McCarl (1991) provide the theoretical basis of an exact aggregation procedure based on extreme point representation under the assumption of full information on every farm and suggest empirical approximation procedures using the available aggregate information on all farms.

In principle, any type of non-linear function with the required properties qualifies for Phase 2. For reasons of computational simplicity and lacking strong arguments for other type of functions, a quadratic cost function is often employed (exceptions: Paris and Howitt 1998 and 2000). The general version of this variable cost function to be specified is then

$$C^v = \mathbf{d}'\mathbf{x} + \frac{1}{2}\mathbf{x}'\mathbf{Q}\mathbf{x} \quad (6)$$

with:

\mathbf{d} = ($N \times 1$) vector of parameters associated with the linear term and
 \mathbf{Q} = ($N \times N$) symmetric, positive (semi-) definite matrix of parameters associated with the quadratic term.

The parameters are then specified such that the linear 'marginal variable cost' (\mathbf{MC}^v) functions fulfil

$$\mathbf{MC}^v = \frac{\partial C^v(\mathbf{x}^o)}{\partial \mathbf{x}} = \mathbf{d} + \mathbf{Q}\mathbf{x}^o = \mathbf{c} + \mathbf{p} \quad (7)$$

Note, however, that the derivatives (7) of this *variable* cost function do not incorporate the opportunity cost of fixed resources ($\mathbf{A}^p\boldsymbol{\lambda}$) which remain captured in the ultimate model by the dual values of the resource constraints.

Given that we have a set of parameters satisfying (7), we obtain the final non-linear programming problem that reproduces observed activity levels as

$$\begin{aligned} \max_{\mathbf{x}} Z &= \mathbf{p}'\mathbf{x} - \mathbf{d}'\mathbf{x} - \frac{1}{2}\mathbf{x}'\mathbf{Q}\mathbf{x} \\ \text{subject to} & \\ Ax &\leq 0 \quad [\boldsymbol{\lambda}] \\ x &\geq 0 \end{aligned} \quad (8)$$

It should be noted at this point that the dual values of the resource constraints in model (8) at \mathbf{x}^o do not differ from the one in model (2). They are still determined by the marginal profitability of the marginal activities at their observed levels \mathbf{x}^{om} , $(\mathbf{A}^m)^{-1} [\mathbf{p}^m - (\mathbf{d}^m + \mathbf{x}^{om}\mathbf{q}^m)]$, which remains equal to $(\mathbf{A}^m)^{-1}[\mathbf{p}^m - \mathbf{c}^m]$ in the specification step, because of (4) and (7). Consequently, the value of equation (5) remains unchanged.

The Parameter Specification Problem

Calibration of an agricultural farm, regional or sectoral programming model to observed quantities is not the distinctive property of the PMP-approach. This can be achieved by appropriate constraints – see model (2) – as well. More interesting is, whether a PMP calibrated programming model is able to capture the behavioural response of farmers to changing economic conditions, so that it is capable of evaluating impacts of political-, market-, or technical developments on agriculture. This response depends on the interplay between the constraints and the now non-linear objective function. However, the first and second order conditions – (7) and the positive (semi-) definiteness of \mathbf{Q} – still allow for almost any magnitude of response behaviour of the resulting model.

The problem of condition (7) is that it implies an underdetermined specification problem as long as we consider a flexible functional form. In the case of the second order flexible quadratic function we have $N+N(N+1)/2$ parameters which we try to specify on the basis of N pieces of information (the marginal variable cost equations (7)). There are an infinite number of parameter sets which satisfy these conditions, i.e. lead to a perfectly calibrating model, but each set implies a different response behaviour to changing economic incentives.

In order to see this, we derive the supply functions implied by the PMP calibrated model (1.8). If we start from the Lagrangian formulation

$$L(x) = p'x - d'x - 0.5x'Qx + \lambda[b - Ax] \quad (9)$$

and continue to assume that all optimal activity levels are positive we obtain the first order conditions in gradient format as

$$\frac{\partial L}{\partial x} = p - d - Qx - A'\lambda = 0 \quad (10)$$

and

$$\frac{\partial L}{\partial \lambda} = b - Ax = 0 \quad (11)$$

Solving (10) for \mathbf{x} results in

$$x = Q^{-1}(p - d - A'\lambda) \quad (12)$$

and substituting the right hand side of (12) into (11) allows to solve for

$$\lambda = (AQ^{-1}A')^{-1}(AQ^{-1}(p - d) - b) \quad (13)$$

The vector of optimal activity levels as a function of exogenous model parameter can then be expressed as

$$\mathbf{x} = \mathbf{Q}^{-1}(\mathbf{p} - \mathbf{d}) - \mathbf{Q}^{-1}\mathbf{A}'(\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}')^{-1}(\mathbf{A}\mathbf{Q}^{-1}(\mathbf{p} - \mathbf{d}) - \mathbf{b}) \quad (14)$$

The gradient of (14) with respect to the price vector is proportional to the marginal supply response in this case (since product supply is constant per activity unit) and given by

$$\frac{\partial \mathbf{x}}{\partial \mathbf{p}} = \mathbf{Q}^{-1} - \mathbf{Q}^{-1} \mathbf{A}' (\mathbf{A} \mathbf{Q}^{-1} \mathbf{A}')^{-1} \mathbf{A} \mathbf{Q}^{-1} \quad (15)$$

which finally reveals that the full \mathbf{Q} -matrix is relevant for the supply response of each single product. This is even true when \mathbf{Q} is diagonal (and consequently \mathbf{Q}^{-1} as well), because the fixed allocable inputs (resource constraints) still link all production activities with each other. The second summand in (14) which is $-\mathbf{Q}^{-1} \mathbf{A}'$ times the gradient of $\boldsymbol{\lambda}$ with respect to \mathbf{p} ensures that all elements of \mathbf{Q}^{-1} enter each element of the supply gradient.

The different methods developed to choose among the infinite number of calibrating parameter sets increasingly recognised the need to introduce additional information in order to avoid arbitrary simulation behaviour. We give here a short overview on the principles employed without an extensive discussion (see also Umstätter 1999: 30ff. or for a detailed evaluation Röhm 2001: 8ff. with respect to some of the approaches mentioned below).

Early Specification Rules

In the 'early' days of PMP the specification problem with respect to the quadratic cost function was simply solved by letting $\mathbf{d} = \mathbf{c}$ and setting all off-diagonal elements of \mathbf{Q} to 0 (e.g. Howitt and Mean 1983, Bauer and Kasnakoglu 1990, Schmitz 1994; Arfini and Paris 1995). The N diagonal elements of \mathbf{Q} , q_{ii} , were then calculated as

$$q_{ii} = \frac{p_i}{x_i^0} \text{ for all } i = 1, \dots, N \quad (16)$$

It is easily verified that the resulting variable cost function satisfies condition (7). This specification rule leads to a cost function which is linear in 'marginal' activity levels, because the elements of $\mathbf{p}^m = 0$. This in turn implies that $\boldsymbol{\lambda}$ remains constant, because it is determined by the profitability of the marginal activities alone which is constant per activity unit. Consequently, a price increase for products of the preferable production activities leads to a substitution of marginal activities, but leaves the other preferable activity levels unchanged until the first marginal activity is driven out of the basis.

This specification is purely motivated by computational simplicity in the absence of additional information. Its repeated use can only be explained by a focus on the calibration property in hope that a rich technological specification in terms of constraints would provide a realistic simulation response. In hindsight, it is easy to argue that technological constraints which are not even closely capable of reproducing base year observations are not in any way more likely to capture behavioural response to changing economic incentives. Ex-post simulations performed by Cypris (2000) with the German regionalised sector model RAUMIS show that this approach results in a very poor response behaviour of the resulting model characterised by strong overreactions to changes in economic incentives (i.e. high implied elasticities).

Paris (1988) used an alternative specification rule where the linear cost function parameters d are set to zero in addition to the off diagonal elements of \mathbf{Q} , and calculated

$$q_{ii} = \frac{c_i + \rho_i}{x_i^0} \quad \forall i = 1, \dots, N \quad (17)$$

which achieves positive diagonal elements of \mathbf{Q} also for the marginal activities. In this case, the reduction of marginal activities caused by the expansion of a preferable activity would immediately change the dual values of the resource constraints, π , which in turn alters the optimal solution for other preferable activities. Although this is a generally more realistic property of (aggregate) producer response, the quantitative specification remains just as arbitrary.

Another ad hoc solution to obtain increasing marginal cost function for the marginal activities is to retrieve some share δ of one limiting resource dual value λ_i and add it to the calibration dual vector ρ to obtain a modified calibration dual vector ρ_M (Röhm and Dabbert 2003). For the case that a prior shadow price of the limiting resource, λ_{IT} , is available, they define this share of the resource dual value as

$$\delta = (\lambda_i - \lambda_{IT}) / \lambda_i \quad (18)$$

and obtain the adjusted calibration dual vector as follows:

$$\rho_A = \rho + A_i \delta \lambda_i \quad (19)$$

where A_i is the row of \mathbf{A} corresponding to the modified shadow price.

The idea is further extended by the authors to include greater competitiveness among closely related activities whose requirements for limiting resources are more similar than with other activities implying greater substitutability between them. In order to achieve that, additional calibration constraints for linear aggregates of activities are introduced, with the non-linear cost function comprising terms for both individual and for aggregates of activities. Some modifications of (and are then introduced for this purpose.

Average Cost Approach

If one is willing to assume that the observed vector of accounting cost per activity unit, \mathbf{c} , is equal to the average cost of the crop specific variable cost function, this condition can be used in addition to the marginal cost condition (7) to determine the parameters. This approach implies that

$$q_{ii} = \frac{2\rho_i}{x_i^0} \quad \text{and} \quad d_i = c_i - \rho_i \quad \forall i = 1, \dots, N \quad (20)$$

In comparison with specification (16), the diagonal elements get larger implying a reduced price elasticity. The average cost approach excludes positive off-diagonal elements of \mathbf{Q} by

definition, because crop specific average cost is not defined in this case. In fact, the allocation of variable inputs to certain crop activities is not consistent with the joint technology underlying a multi-product cost function. This approach has been used in Heckeles and Britz (2000) in the context of an ex-post simulation exercise for comparative purposes. For further discussion see also Gohin and Chantreuil 1999.

All specifications considered so far let the second derivatives of the variable costs function with respect to activity level x_i , q_{ii} , increase with the size of the dual values and decrease with the level of the calibrated variables. Both properties have their merits: large differences between marginal accounting plus marginal resource costs and marginal revenues were typically found in speciality crops, say horticultural activities, where low supply responses were often deemed realistic by modellers. Secondly, the reciprocal relation between x_i^0 and q_{ii} made supply elasticities in simulations independent of the scaling of \mathbf{x} . These two appealing properties were probably the reason that these approaches survived for some years.

Use of Exogenous Supply Elasticities

A generally more convincing specification is the incorporation of exogenous elasticities. It reduces the role of PMP to all it really can be in the context of just one observation on activity levels: a calibration method. At the same time it provides the mean for incorporating prior information

Existing applications (for example Helming et al. 2001) are restricted to the use of exogenous own-price elasticities $\bar{\epsilon}_{ii}$. The off-diagonal elements of \mathbf{Q} are set to zero and the marginal effect of price changes on the shadow prices λ is ignored (second summand in (14) vanishes). In this case, the partial derivative $\partial x_i / \partial p_i$ is equal to q_{ii}^{-1} so that the elasticity formula evaluated at observed quantities can be solved directly for q_{ii} to obtain

$$\bar{\epsilon}_{ii} = \frac{1}{q_{ii}} \frac{p_i^0}{x_i^0} \Leftrightarrow q_{ii} = \frac{1}{\bar{\epsilon}_{ii}} \frac{p_i^0}{x_i^0} \quad \forall i = 1, \dots, N \quad (21)$$

as the appropriate value for a given $\bar{\epsilon}_{ii}$. In order to satisfy the calibration condition (7) the linear parameters of the variable cost function are then determined as

$$d_i = c_i + \rho_i - q_{ii} x_i^0 \quad \forall i = 1, \dots, N \quad (22)$$

Because of the ignored effect on shadow prices of limited resources, the actual elasticities of the resulting model will deviate from $\bar{\epsilon}_{ii}$, and are generally lower. The exact calibration to exogenous own-price elasticities is generally possible but cannot always be obtained as a closed form solution⁵.

⁵ See Heckeles 2002 for an extended discussion on this subject.

Calibration with Maximum Entropy Criterion

Paris and Howitt (1998) proposed to overcome the problem of somewhat arbitrary parameter specifications through the use of an econometric criterion, specifically by an application of the Generalized Maximum Entropy (GME) estimator (Golan et al. 1996). This path breaking contribution allowed to elegantly address the underdetermined specification problem using prior information and provided a full specification of the \mathbf{Q} -matrix. More importantly, it defined a methodological framework which principally allows for the incorporation of more than one observation on activity levels and bridged the gap between calibration and estimation exercises. However, their example still used one observation only, demonstrating an alternative way to introduce a priori information in the calibration process. The paper characterised the process as an estimator using an uninformative prior. That characterisation may have been misinterpreted by the reader as the a-priori information together with the entropy criterion allowed to identify the estimated parameters despite the underdetermined nature of their problem.

The general idea was applied to an example with multiple observations in a cross-sectional analysis by Heckelevi & Britz 2000 which borrowed the idea of elasticity priors for the \mathbf{Q} -matrix and proposed some further modifications. On the one hand, priors were defined directly on \mathbf{Q} , and not as in Paris and Howitt (1998) on the elements of a \mathbf{LDL}' decomposition of \mathbf{Q} . On the other hand, a size dependent scaling was proposed to define region specific \mathbf{Q} -matrices in the estimation step. The resulting regional models proved to be superior in an out-of-sample validation exercise compared with models specified by the average cost approach. Both the example application in Paris and Howitt (1998) and the cross-sectional estimation in Heckelevi and Britz 2000 recovered cost functions satisfying condition (7) which makes these approaches subject to the general criticism put forward in section 3.

Paris (2001) and Paris and Howitt (2001) expand the framework of the GME-based PMP methodology to overcome some of the criticisms that have been raised against the use of a linear technology in limiting resources and the zero-marginal product for one of the calibrating constraints present in the original PMP version. The first step of this new structure is now expressed as an equilibrium problem consisting of symmetric primal and dual constraints and the third step as an equilibrium problem between demand and supply functions of inputs, on the one hand, and marginal cost and marginal revenue of the output activities, on the other hand. This new framework has inspired the authors to name it a Symmetric Positive Equilibrium Problem (SPEP). The key contribution of SPEP to the PMP literature is rendering the availability of limiting inputs responsive to output levels and input price changes.

As in standard PMP, the first step of SPEP determines the levels of the marginal costs of outputs and the shadow prices of limiting inputs. Instead of the traditional dual pair of LP models, Paris and Howitt (2001) specify an equilibrium problem with the following structure and their associated duals using in addition the available information on market rental prices to avoid degeneracy in the duals of the limiting input constraints. They propose the following dual constraints as a starting point:

$$A'\lambda + \rho + \geq p \perp x \geq 0 \quad (23)$$

$$\lambda \geq \mathbf{r} \perp \mathbf{Ax} - \mathbf{b} \geq 0 \quad (24)$$

where \mathbf{r} is a vector of observed rental prices.

The additional constraints introduce lower limits on the dual values of binding constraints for which external renting prices \mathbf{r} are available. Indeed, the second constraints (24) states that farmers are able to sell resources at prices \mathbf{r} , but once the endowment \mathbf{b} is completely used for production, the marginal values $\boldsymbol{\lambda}$ may exceed \mathbf{r} . After determining the dual values for both calibration and resource constraints, Paris and Howitt (2001) now propose to use a Generalized Leontief specification for the limiting inputs and a quadratic specification for the output vector \mathbf{x} in step 2 and 3 as follows:

$$C(\mathbf{x}, \boldsymbol{\lambda}) = \mathbf{1}' \boldsymbol{\lambda} (\mathbf{d}' \mathbf{x}) + \mathbf{1}' \boldsymbol{\lambda} (\mathbf{x}' \mathbf{Q} \mathbf{x}) / 2 + \sqrt{\boldsymbol{\lambda}}' \mathbf{s} \sqrt{\boldsymbol{\lambda}} \quad (25)$$

which then serves simulation purposes after some modifications. Paris and Howitt finally claim that because the final SPEP specification neither implies nor excludes optimizing behavior, it removes the last remnant of a normative behavior from the original PMP approach. In a critique of the SPEP approach, Britz et al. (2003) point out that the employed cost function and the ultimate simulation model obtained in phase 3 of the approach cannot be derived from any optimizing behaviour and question their general interpretability. As the arguments brought forward by Britz et al. had been published as a direct comment to Paris (2001), but not been commented upon by the author in return, we refrain here from a conclusive evaluation of the SPEP framework. The most recent farm model application of the SPEP framework is Arfini et al. (2005).

Specification Based on Decreasing Marginal Yields

All the PMP specifications mentioned above specified a non-linear *cost* function. By assumption, they attribute the marginal mis-specification of the original linear model to the input side of the production problem. It is probably obvious, however, that a misrepresentation of how revenue depends on activity levels would have the same effect. In fact, the dual values on the calibration constraints can just as well be explained by decreasing marginal yields with increasing activity levels, which is not reflected by the constant yield assumption of model (2). Howitt (1995a) uses this interpretation and introduces non-linear terms into the objective function to reflect differences between marginal and observed average crop yields caused by changing land quality.

The theoretical as well as empirical validity of the pure cost function and the pure yield function approach is somewhat questionable. Any reasonable technological and behavioural assumption in agricultural production would make it highly unlikely that input application is changed but yield remains constant or vice versa. Röhm (2001, 51ff.) acknowledges this by combining the decreasing yield with the increasing cost assumption. However, his approach is also not based on a clear technological hypothesis, i.e. a well represented relationship between inputs and outputs, and again does not provide a strong empirical base for the specification of parameters and the implied simulation behaviour of the resulting model. Therefore, we do not further elaborate on the details of the yield function approach or combined yield/cost approaches.

Summarizing evaluation of PMP related calibration approaches

Before turning to approaches that do not require a phase 1 with calibration constraints, we want to close this section with a short summary of the merits and problems associated with the main body of PMP applications in the context of agricultural sector models.

1. The PMP approach provides an elegant way to calibrate programming models to observed behaviour and renders a more realistic smooth aggregate supply response relative to a linear programming model. These merits lead to a widespread application of PMP approaches in the context of aggregate agricultural programming models.
2. The dual values associated with the calibration constraints in phase 1 of PMP potentially capture any type of marginal model mis-specification of technology, data errors, aggregation bias, representation of risk behaviour, price expectations, non-linear constraints etc. For an intelligent model specification and appropriate interpretation of results, explicit assumptions used in the model specification are desirable.
3. One observation on base year allocations alone does not contain any information on how the marginal incentives change if one moves away from the observed allocation. The infinite number of calibrating sets of parameters generally implies a different simulation response of the calibrated model. Extremely unreasonable supply responses have been generated in the past with oversimplified PMP specifications.
4. The PMP literature provided a multitude of approaches to obtain sensible and interpretable approaches to the parameter specification problem. Those comprised the employment of the relationship between average and marginal costs, explicit assumptions on decreasing yield in land allocation, and prior information in terms of exogenous elasticities. With the introduction of the Maximum Entropy criterion, a tool has been suggested to flexibly incorporate general prior information. It also allows in principle to use more than one observation which – in combination with structural assumptions on the objective function and constraints – would enable to *estimate* the model parameters underlying the observed response behaviour of producers.

Following up on the last point, the subsequent section will focus now on the general problem of the PMP-approach related to phase 1 which renders its use for parameter estimation based on multiple observations problematic. After this demonstration we have a look at approaches for calibrating and estimating programming models without a phase 1, i.e. without the use of calibration constraints. Since we consider the phase 1 as a defining character of PMP, we do not talk about “PMP-methods” anymore, but rather of alternative approaches to the calibration and estimation of programming models.

3. Calibration and estimation of optimization models without dual values of calibration constraints

The phase 1 of the PMP approach can be looked upon as a mean of providing a certain shadow price value for limiting resources in the absence of other information. It was pointed

out above in the context of equation (5) that the inclusion of calibration constraints makes shadow prices of resource constraints dependent on the profitability of marginal activities only. And those shadow prices are transferred to the calibrating solution of the quadratic programming model (8). In reality – and in the context of model (8) as indicated by equation (13) – the shadow prices will rather depend on more complex interactions described by the level technology of production activities. In fact, the determination of shadow prices by phase 1 of PMP create a fundamental inconsistency between parameter specification and the resulting quadratic optimisation model which renders any estimation of parameters using multiple observations inconsistent, as pointed out in Heckelevi (2002) and Heckelevi and Wolff (2003). We will now explain this inconsistency based on these two references in more detail and then look at alternative suggested approaches of estimating and calibrating programming models avoiding phase 1 of PMP.

Inconsistency of PMP when using multiple observations

The last section pointed out the danger of specifying models based on PMP that imply arbitrary simulation behaviour. One problem is the thin information base provided by just one year of observations on activity levels. In fact, the data in this case do not provide any information on second order properties (Hessian matrix) of the objective function. If a change in economic incentives and the resulting behaviour is not observed, then the information for parameter specification must come from other sources. Even if one would be able to specify the ‘true’ model with respect to behavioural assumptions and functional form, the parameters are still not identified. The only convincing use of PMP with just one observation is the use as a calibration method in combination with elasticities or other exogenous information on technology or behavioural response with respect to changes in activity levels.

The main focus of this section, however, shall be the inclusion of additional data looking for the bridge to typical econometric models. Paris and Howitt (1998) already pointed at the potential of introducing more than one observation. However, the question we need to address first is, whether the PMP procedure itself is designed to make best use of additional data information. We show that the marginal conditions derived from the first phase of PMP are inappropriate. They represent a mis-specified model in the sense that the inclusion of additional observations will never allow to recover the underlying model which is assumed to have generated the data.

In order to see this, we will use some of the elements already introduced in the previous sections, but look at the methodology from an econometrician’s point of view. This includes the assumption that the ultimate model to be specified is the ‘true’ model structure, or at least one that is believed to be a good approximation of the true model: Apparently, many PMP modellers thought that the final model with a non-linear objective function to be optimised under linear resource constraints is a reasonable representation of the behaviour of agricultural producers, otherwise it would not have made any sense to use this structure as the ultimate specification. The PMP procedure, however, enforces shadow prices and marginal cost values that differ from the ones implied by the non-linear model.

Suppose the quadratic model (8) is the true data generating process. The derivations (9) to (13) have shown that the shadow prices of the resource constraints under the assumption that

all activity levels are positive at the optimum can be calculated as $\lambda = (AQ^{-1}A)^{-1}(AQ^{-1}(p-d)-b)$. This is clearly different from the dual values of the resource constraints obtained in the first phase of PMP (see equation (5)) which only depend on quantities related to the marginal activities and were given by $\lambda = (A^m)^{-1}(p^m - c^m)$. The second phase of PMP then uses these dual values at the observed activity levels through enforcement of the ‘marginal cost’ equations (7), thereby implicitly imposing wrong values for the marginal variable cost as well. Given this discrepancy, it is impossible to recover the true non-linear objective function no matter how many observations on activity levels are used. The use of the *biased* marginal cost equations as estimating equations in some econometric exercise with multiple observations generally leads to inconsistent estimates. The PMP approach is fundamentally flawed in the sense that it imposes first order conditions which are incompatible with the non-linear model it ultimately tries to recover.

In principle, the problems with the dual values of the resource constraints have been recognised by several authors. Gohin and Chantreuil (1999), Cypris (2000) and Röhm (2001) suggest including observed land rents into the specification step to ensure a more reasonable value. This can either be done by introducing a land rent activity at this price into the original linear model or by adjusting the conditions for specification. Wild (2000) employs simulation exercises to show the impossibility to recover a true quadratic model with more than one observation based on phase 1 of PMP. He also shows an alternative calibration approach specifically designed for the quadratic specification which does recover the true model by simultaneously calibrating parameters and shadow price of land.

Gohin and Chantreuil and Wild already observe that in the case of a simple model structure with just one resource constraint, phase 1 of PMP is not needed as the implied value of the shadow price of land can be deduced directly. Below we review a general alternative introduced by Heckeles 2002 and Heckeles and Wolff (2003) which does not require phase 1 of PMP to calibrate or estimate any programming model even for more complex constraint structures. A direct use of the first order conditions of the assumed behavioural optimisation model makes the use of distorted shadow prices and thereby the use of the PMP-approach altogether obsolete.

A General Alternative to PMP

The ‘general’ alternative to PMP with respect to calibrating or estimating a programming model is nothing but a simple methodological principle: always to directly use the first order conditions of the very optimisation model that is assumed to represent or approximate producer behaviour and is suitable for the simulation needs of the analysts. No first phase calculating dual values of calibration constraints based on a different model is necessary. We can avoid the implied methodological inconsistency altogether and generally estimate shadow prices of resource constraints simultaneously with the other parameters of the model.

The basic principle can be illustrated by writing a general programming model with an objective function $h(\mathbf{y} | \mathbf{a})$ to be optimised subject to a constraint vector $\mathbf{g}(\mathbf{y} | \mathbf{b}) = 0$ in Lagrangian form:

$$L(\mathbf{y}, \boldsymbol{\lambda} | \boldsymbol{\alpha}, \boldsymbol{\beta}) = h(\mathbf{y} | \boldsymbol{\alpha}) + \boldsymbol{\lambda}' [\mathbf{g}(\mathbf{y} | \boldsymbol{\beta})] \quad (26)$$

where \mathbf{y} , $\boldsymbol{\lambda}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$ represent column vectors of endogenous variables, unknown dual values, parameters of the objective function, and parameters of the constraints, respectively. The appropriate first order optimality conditions are the gradients with respect to \mathbf{y} and $\boldsymbol{\lambda}$ set to zero:

$$\frac{\partial L}{\partial \mathbf{y}} = \frac{\partial h(\mathbf{y} | \boldsymbol{\alpha})}{\partial \mathbf{y}} + \boldsymbol{\lambda}' \frac{\partial \mathbf{g}(\mathbf{y} | \boldsymbol{\beta})}{\partial \mathbf{y}} = \mathbf{0} \quad (27)$$

$$\frac{\partial L}{\partial \boldsymbol{\lambda}} = \mathbf{g}(\mathbf{y} | \boldsymbol{\beta}) = \mathbf{0} \quad (28)$$

For the case of inequality constraints $\mathbf{g}(\mathbf{y} | \boldsymbol{\beta}) \leq \mathbf{0}$ we need to substitute the gradient with respect to $\boldsymbol{\lambda}$ by the complementary slackness representation⁶

$$\frac{\partial L}{\partial \boldsymbol{\lambda}} = \mathbf{g}(\mathbf{y} | \boldsymbol{\beta}) \leq \mathbf{0}; \boldsymbol{\lambda} \otimes \mathbf{g}(\mathbf{y} | \boldsymbol{\beta}) = \mathbf{0}; \quad (29)$$

The unknowns $\boldsymbol{\lambda}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$ of these Kuhn Tucker conditions can be estimated with some econometric criterion directly applied to these equations. Depending on the parametric specification appropriate curvature restrictions (second order conditions) might have to be enforced as well.

Heckelevi and Wolff (2003) point out that the direct use of optimality conditions for estimation is not a new concept by itself. In the context of investment models, for example, the dynamic equivalents of Kuhn Tucker conditions, the Euler equations, have been frequently used as estimating equations to overcome analytical and empirical problems for more complex models (Chirinko 1993, 1893f). However, their employment as an alternative to PMP or to the estimation of behavioural functions in the context of multi-output agricultural supply models had not been considered before. They argue that this approach is not only useful for the estimation of typical agricultural programming models but also provides a flexible alternative for estimating parameters of duality based behavioural functions with explicit allocation of fixed factors or other physical constraints. In this context, the only difference left between programming and econometric models is the model form used for simulation purposes.

In hindsight it might be difficult to understand why this principle had not been applied earlier for the purpose of obtaining a programming model based on observed behaviour. One reason might be the often complex structure of inequality constraints employed in agricultural programming models which seemed to make the approach infeasible for larger models – and can be in fact a problem to be discussed below. However, in most PMP publications the calibrated models have very simple structures and often only binding constraints so that the complete set of first order conditions are very limited. A more likely explanation is that most programmers neither saw the need nor the possibility to include more than one observation and to actually estimate their programming models. At the same time econometricians did not look at

⁶ The symbol ' \otimes ' represents the Hadamard or element-wise product of two matrices. If a_{ij} and b_{ij} are the elements of two matrices with equal dimension, \mathbf{A} and \mathbf{B} , then $\mathbf{A} \otimes \mathbf{B} = \mathbf{C}$, where \mathbf{C} is of the same dimension as \mathbf{A} , \mathbf{B} and each element of \mathbf{C} is defined as $c_{ij} = a_{ij} \cdot b_{ij}$.

this body of literature as a potential alternative to analyse producer behaviour under maintained optimising assumptions. Therefore, nobody seemed to have asked the question of consistency, i.e. is it possible to recover more and more closely the true model if the data information is increased. One would have quickly come to realise that the PMP paradigm of using dual values from calibration constraints of a different model than the one to be recovered is neither useful nor necessary. It is important to note that Heckeleei and Wolff (2003) make this statement only in this specific context, i.e. in the context of using multiple observations for estimation purposes. By no means do they – or the authors of this review – want to imply that the PMP approach to calibration is not useful in general.

The most innovative PMP-proponents Paris and Howitt (2001) and Paris (2001) came – in some sense – very close to the alternative principle with their (SPEP) approach calibrating a multi-input multi-output model based on marginal cost constraints discussed above which relaxed some of the restrictions of the original PMP-approach. One way to look at the 'SPEP' approach is to see that the implicit determination of the shadow prices of resources through the first phase of PMP is at least lower bounded by exogenous input prices. But what can we do, if those prices are not available or if we do not want to apply a model that assumes variability for all inputs? The use of a land constraint in aggregate agricultural supply simulation models, for example, can be very appropriate and useful with respect to the reasonableness of model results. Other resources such as water for irrigation might be truly limiting as well. Also, how do we deal with policy restrictions that were already applied in the data period and for which shadow prices are not known? In these empirically very relevant cases the use of the full set of first order necessary conditions (27) and (28) or (27) and (29) as estimating (or calibrating) equations which include the constraints and the corresponding shadow prices provides an alternative. For the typical PMP model with a cost function quadratic in land allocations, those optimality conditions were given by equations (10) and (11).

Judez et al. (2001) already suggest to directly employing these first order conditions without the first phase of PMP using observed rental prices of resources and solving for the parameters under certain additional restrictions on the parameter matrix \mathbf{Q} . They use this approach to represent the economic behaviour of different farm types based on farm accounting data from the Spanish part of the European Farm Accountancy Data Network (FADN). Apart from the original source, more detailed information on this approach can be found in Henry de Frahan et al. (2005). We now turn to applications of this general estimation approach as found in the more recent literature with some consideration of related computational problems and possible avenues of solving these.

Estimation of programming models using first order conditions

Heckeleei and Wolff (2003) provide an illustration of the estimation approach using first order conditions (10) and (11) with just one resource constraint (land) based on multiple observations. With a sufficient number of observations, the specification problem becomes over-determined and an exact calibration to observed activity levels is not possible anymore. Consequently, the authors introduce error terms allowing for a deviation of estimated from observed variable values. In principle, the parameter estimation can now proceed with classical estimation techniques such as Least Squares or Generalized Methods of Moments. However, Heck-

Heckelevi and Wolff employ the maximum entropy criterion as they also investigate the impact of additional prior information on the estimation results. A sequence of Monte Carlo simulations provides evidence that the estimation setup converges to true parameter values as the number of observations increase. Even moderately precise prior information on supply elasticities and shadow prices show a significant reduction in Mean Square Errors of parameter estimates for small sample sizes – a situation which will be most relevant in real applications of the approach.

One conceptual shortcoming of the quadratic-in-activity-levels objective function typically used in PMP applications is the limited rationalisation of the non-linear terms. They are generally not founded on explicit behavioural or technological assumptions, but rather motivated with heuristic arguments related to ‘other’ determinants of supply behaviour not captured by the explicit part of the objective function and the linear constraints. The quadratic terms basically comprise all the possible interpretations for the existence of dual values of the calibration constraints discussed above as PMP subsequently maps those into the nonlinear terms of the objective function. Therefore, Heckelevi (2002) takes a deeper look into possible rationalisations and connections of this model to heterogeneous land quality, nonlinear technological constraints, and risk approaches. Furthermore, he draws connections to a competing approach of calibrating programming models: The convex combination constraints advocated by McCarl (1982) and Önal and McCarl (1989 and 1991) to mitigate aggregation errors. A certain equivalence to a quadratic objective function can be found in a calibration context, but a full rationalisation with aggregation conditions is not possible.

Because of the limited rationalisation possibilities for the quadratic model and in order to show the connection to dual econometric models, Heckelevi and Wolff present two other optimisation models with explicit land allocation illustrating the applicability and performance of the general approach. Estimating first order conditions, they allow recovering parameters of crop specific production and profit functions while satisfying the explicit technological constraints. Compared to the traditional estimation of dual behavioural functions, this approach does not require closed form solutions for the land allocations and thereby provides a more flexible approach with respect to functional form and number of technological constraints. In one application, the authors generate a data set that where some observations are characterized by a binding land constraints (positive shadow price of land), other represent situations where not all land is used for production (shadow price of land is zero). In this case, a complementary slackness formulation as in (29) enters the estimating equations. The Monte Carlo simulations hinted at consistency of the approach not only in recovering parameters but also with respect to the estimator’s ability to identify the state of the constraint as binding or not.

Despite showing the principle advantages and general properties of the approach, the main limitation of the paper by Heckelevi and Wolff is the illustrative nature of their models. They are restricted to three production activities and one constraint. This setting cannot provide much useful information for the feasibility of real world applications to differentiated programming models. However, first experiences with this approach or variations of it are presented by Gocht (2005) and Polomé et al. (2005) who estimate farm level cost functions based on FADN data. Whereas the latter contribution is probably subject to less computational problems due to avoiding explicit constraints in the first order conditions, Gocht reports considerable difficulties in finding solutions for the estimation problem using a gradient based solver. A thorough evaluation of the nature of these problems could not be done at this point.

However, the principle problem of computational feasibility is well known for the type of optimization model generated in this special context of parameter estimation. Jansson and Heckelei (2004) connect the estimation problem to the literature on “bi-level” programs which is in turn a subset of mathematical programming with equilibrium constraints (MPEG). A bi-level program is an optimisation problem, called the *outer problem*, which uses the solution of another optimisation problem, called the *inner problem*, as its domain. Using the first order conditions (27) and (29) with error terms as a representation of the inner problem, the full general estimation approach suggested by Heckelei and Wolff (2003) can be expressed as

$$\underset{\alpha, \beta}{\text{Min}} H(e) \quad \text{subject to} \quad (30)$$

$$\left. \frac{\partial h(y | \alpha)}{\partial y} \right|_{(y^0 - e)} + \lambda' \left. \frac{\partial g(y | \beta)}{\partial y} \right|_{(y^0 - e)} = 0 \quad (31)$$

$$g(y^0 - e | \beta) \leq 0; \quad \lambda \otimes g(y^0 - e | \beta) = 0 \quad (32)$$

where the function $H(e)$ is some statistical criterion depending on the error vector e , for example weighted least squares, and y^0 are the observed values of y . This bi-level program seeks the optimal parameter values with respect to the statistical criterion such that the solutions also satisfy the optimality conditions of the inner problem. For a less abstract example, the estimation problem of the parameters d and Q of the quadratic programming problem (8) is given (using first order conditions (10) and (11)) as

$$\underset{d, Q}{\text{Min}} H(e) \quad \text{subject to} \quad (33)$$

$$p - d - Q(x^0 - e) - A' \lambda = 0 \quad (34)$$

$$b - A(x^0 - e) = 0 \quad (35)$$

with x^0 being the observed activity levels. Although not illustrated above, this approach can also accommodate observations (and error terms) on shadow prices of resource constraints (see Heckelei and Wolff 2003).

There are different approaches available for solving bi-level programs, all of which have in common that they do not work equally well for all types of problems in this class. The difficulties in numerically finding a solution are especially severe for gradient solvers if the optimality conditions of the inner problems comprise complementary slackness conditions, because gradients of these restrictions are not continuously differentiable. To mitigate this problem somewhat, one could decide a-priori, for example by data inspections, which resource and non-negativity constraints are binding and which are not. For the former, we can then formulate equality restrictions with a nonzero shadow price and for the latter we can simply leave out the restrictions as they do not matter for the data generating process. However, this approach is not always applicable, as we often do not know whether constraints are binding or not, because of lacking data or variables measured with errors. Jansson and Heckelei, for example, estimate transport cost, prices and trade flows in the context of a transport cost minimisation

problem, where observations on prices and transport costs are available. Trade flows, however, are not. The estimation not only recovers a set of transport cost and prices consistent with cost minimisation, it also provides corresponding estimates of trade flows satisfying regional market balances. Whether a trade flow is positive or zero cannot be determined a-priori.

Consequently, bi-level estimation problems will often require the development of case specific solution algorithms or at least the necessity to “play” with parameters of existing solvers for this type of problems. This will probably limit for a while the use of this general methodology for estimating complex programming models to the methodologically interested analyst. This and constraints on data availability will most likely lead to continuing use of more simple methods of programming model calibration, hopefully avoiding arbitrary specification of non-linear parameters and shadow prices as in early PMP approaches. However, Jansson (2005) shows that the investment in the direction of programming model estimation may pay off under certain conditions. In this case, the estimator’s properties in the context of a transportation models are superior to the performance of previous methods calibrating these models.

4. Summary and conclusions

Mathematical programming models at the farm, regional, and sectoral level received renewed interest in the last decade, because of their ability to directly represent many of the newly introduced policy instruments of the CAP, and because the incorporation of physical constraints render these tool optimal for the connection between economic and biophysical models. PMP and subsequent approaches provide the means to base the new generation of programming models more and more on observed behaviour.

Using non-linear terms in the objective function, the original PMP approach calibrated programming models to observed base year activity levels and guaranteed a smooth supply response behaviour relative to previously used linear programming models. This feature was especially important for aggregate programming models at regional or sectoral level. However, early specification rules for the parameters in the non-linear objective function differ substantially with respect to implied simulation behaviour. All of them could not solve the fundamental problem that one observation on activity levels does not provide any information on responses to changing economic conditions. A non-arbitrary or empirically valid supply response behaviour required the incorporation of additional prior information. Consequently, in a variation of the PMP approach, the programming models have been calibrated to exogenous elasticities. Then, the use of the Maximum Entropy criterion opened the path to more flexible incorporation of prior information and also to make use of more data information, i.e. of multiple observations on activity levels.

Later, the arbitrary determination of the shadow prices of limiting resources caused by using dual values of calibration constraint in the PMP approach was at the centre of attention. This lead to calibration approaches avoiding completely phase 1 of PMP and moving instead to less ad hoc methods of specification which employed information on rental prices of limiting resources. Another strand of literature relaxed restrictions of the quadratic objective function by moving to flexible functional forms. In this context, specifications such as the Symmet-

ric Positive Equilibrium Problem evolved, which accommodated elastic supply of limiting resources and moved away from optimizing assumptions.

In the context of using multiple observations, it could be shown that phase 1 of the original PMP approach with calibration constraints leads to inconsistent parameter estimates. Fortunately, there exists a conceptually simple alternative to use first order conditions of the programming problem as estimating or calibrating equations directly. This approach allows simultaneously estimating parameters of the cost function with dual values of resource constraints while treating the decision variables as stochastic. It is certainly a straightforward idea to estimate parameters in the framework later used for simulation, and this has been successfully applied in many standard applications of profit, cost, or indirect utility functions. However, the combination of, for example, dual profit functions and a model with an explicit primal representation of parts of the technology in form of inequalities leads to a more challenging model class. The first order conditions of such mixed primal/dual models are inequalities representing Kuhn-Tucker-Conditions, which are not continuously differentiable. Indeed, such an estimator defines a so-called bi-level program, where an outer objective function is optimized under constraints representing first order conditions of an inner optimization problem. Such bi-level programs require specific algorithms to be solved. It is therefore not astonishing that so far, few consistent parameter estimations in a primal/dual framework are documented for real world application. Indeed, application to larger models just started and the development of algorithm suitable for bi-level problems for agricultural economic model promises to be a fruitful exercise.

Generally, we conclude that there is no need for the PMP approach with calibration constraints in phase 1 anymore. It has served its purpose of providing shadow prices of resource constraints as long as the method and its implication for supply response behaviour had not been fully understood. Now, calibration and estimation of programming models proceeds and should proceed using simultaneously explicit prior or data information on shadow prices and decision variables.

Coming back to some of our arguments from the introduction, primal technology representation allows directly capturing jointness of agricultural market outputs and environmental or societal goods and “bads”, and thus analysing multi-functional aspects of agriculture. Primal technology representation in models is also a way to exploit the rich knowledge of agricultural engineering, to communicate with non-economists and link up with bio-economic models, to ease the representation of current CAP policies, and to introduce justified resource constraints to obtain plausible simulation behaviour. Calibrating and estimating consistently parameters of models comprising primal with dual elements may combine these well-known advantages of programming models with simulation responses rooted in observed behaviour, and provides means to effectively use the information available in the extensive agricultural data bases available to us.

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