UNIVERSITY OF HOHENHEIM

Agricultural Economics Working Paper Series
Hohenheimer Agrarökonomische Arbeitsberichte

STAGE_W: An Applied General Equilibrium Model With Multiple Types of Water

Jonas Luckmann and Scott McDonald

Working Paper No. 23

Institute of Agricultural Policy and Agricultural Markets (420)
University of Hohenheim, 70593 Stuttgart, Germany

Published by
the Institute of Agricultural Policy and Markets Universität Hohenheim

ISSN 1615-0473

Editor: Institute of Agricultural Policy and Markets
Universität Hohenheim (420)
70593 Stuttgart
Phone: +49-(0)711/459-22599
Fax.: +49-(0)711/459-22601
e-mail: marktlehre@uni-hohenheim.de

Production: Institute of Agricultural Policy and Markets
Universität Hohenheim (420)
70593 Stuttgart
Germany

Veröffentlichung des Institutes für
Agrarpolitik und Landwirtschaftliche Marktlehre der Universität Hohenheim

ISSN 1615-0473

Herausgeber: Institut für Agrarpolitik und Landwirtschaftliche Marktlehre
Universität Hohenheim (420)
70593 Stuttgart
Tel.: 0711/459-22599
Fax.: 0711/459-22601
e-mail: marktlehre@uni-hohenheim.de

Gesamtherstellung: Institut für Agrarpolitik und Landwirtschaftliche Marktlehre
Universität Hohenheim (420)
70593 Stuttgart

STAGE_W: An Applied General Equilibrium Model With Multiple Types of Water

Technical Documentation

Jonas Luckmann*, Scott McDonald**

Institute of Agricultural Policy and Markets

University of Hohenheim

March 2014

STAGE_W Version 1[1]

* **Jonas Luckmann M.sc.** is working as a scientific assistant at the Department of Agricultural and Food Policy (420a), Institute of Agricultural Policy and Markets, University of Hohenheim

** **Prof Scott McDonald** is Professor of Economics at the Department of Accounting, Finance and Economics, Business School Oxford Brookes University. He is currently working as a Visiting Professor at the Department of Agricultural and Food Policy (420a), Institute of Agricultural Policy and Markets, University of Hohenheim

1 This model is subject to ongoing developments; this version of the technical document is contains details of developments up to the given date. It is a development of the STAGE model (McDonald, 2007).

Content

List of Tables

List of Figures

1 Introduction

This document provides a description of the STAGE_W computable general equilibrium (CGE) model, which is a development of the STAGE model and allows for the depiction of diverse water resources and qualities as well as the simulation of detailed water policy scenarios. STAGE_W is a member of the STAGE suite of single country computable general equilibrium models. At the core of the suite is the basic STAGE model, but the basic STAGE model is not often used in practical work rather it is customised to the setting/economic environment being explored. The guiding principle is that the basic STAGE model provides a template that can support multiple variants; indeed the expectation is that for most studies it will be necessary/desirable to make changes and/or addition to the basic STAGE model.

The basic STAGE model is characterised by several distinctive features. First, the model allows for a generalised treatment of trade relationships by incorporating provisions for non-traded exports and imports, i.e., commodities that are neither imported nor exported, competitive imports, i.e., commodities that are imported **and** domestically produced, non-competitive imports, i.e., commodities that are imported but **not** domestically produced, commodities that are exported **and** consumed domestically and commodities that are exported but **not** consumed domestically. Second, the model allows the relaxation of the small country assumption for exported commodities that do not face perfectly elastic demand on the world market. Third, the model allows for (simple) modelling of multiple product activities through an assumption of fixed proportions of commodity outputs by activities with commodities differentiated by the activities that produce them. Hence the numbers of commodity and activity accounts are not necessarily the same. Fourth, (value added) production technologies are specified as nested Constant Elasticity of Substitution (CES). And fifth, household consumption expenditure is modelled using Stone-Geary utility functions.

The main additional feature added for the STAGE_W version is the detailed description of the water sector, by allowing for the integration of various water resources as factors (e.g. groundwater, seawater, wastewater), from which specific activities (water activities e.g. pumping and purification of groundwater, desalination, water reclamation) produce water commodities (e.g. potable water, treated wastewater of different qualities). These commodities are used as inputs in the production process of other activities or are consumed by households and other agents (e.g. nature) as final users. The number and specification of water factors, activities and commodities are flexible and can be adjusted to the local conditions of the country analyzed. Also the model allows for the substitution of water commodities by water consuming activities. Besides this, the addition of two water specific tax instruments, allows for various pricing schemes, including price differentiation according to water user. All other features of the STAGE model are carried over directly to STAGE_W.

The model is designed for calibration using a reduced form of a Social Accounting Matrix (SAM) that broadly conforms to the UN System of National Accounts (SNA). Table 1 contains a macro SAM in which the active sub matrices are identified by *X* and the inactive sub matrices are identified by 0. In general the model will run for any SAM that does not contain information in the inactive sub matrices and conforms to the rules of a SAM.[2] In some cases a SAM might contain payments from and to both transacting parties, in which case recording the transactions as net payments between the parties will render the SAM consistent with the structure laid out in Table 1.

Table 1 Macro SAM for the Standard Model

	Commodities	Activities	Factors	Households	Enterprises	Government	Capital Accounts	RoW
Commodities	(X)	X	0	X	X	X	X	X
Activities	X	0	0	0	0	0	0	0
Factors	0	X	0	0	0	0	0	X
Households	0	0	X	0	X	X	0	X
Enterprises	0	0	X	0	0	X	0	X
Government	X	X	X	X	X	0	0	X
Capital Accounts	0	0	X	X	X	X	0	X
RoW	X	0	X	X	X	X	X	0
Total	X	X	X	X	X	X	X	X

The most notable differences between this SAM and one consistent with the SNA are:

1) The SAM is assumed to contain only a single 'stage' of income distribution. However, fixed proportions are used in the functional distribution of income within the model and therefore a reduced form of a SNA SAM using

[2] If users have a SAM that does not run with no information in inactive sub matrices the author would appreciate a copy of the SAM so as to further generalise the model.

apportionment (see Pyatt, 1989) will not violate the model's behavioural assumptions.

2) The trade and transport margins, referred to collectively as marketing margins, are subsumed into the values of commodities supplied to the economy.

3) A series of tax accounts are identified (see below for details), each of which relates to specific tax instruments. Thereafter a consolidated government account is used to bring together the different forms of tax revenue and to record government expenditures. These adjustments do not change the information content of the SAM, but they do simplify the modelling process. However, they do have the consequence of creating a series of reserved names that are required for the operation of the model.[3]

The model contains a section of code, immediately after the data have been read in, that resolves a number of common 'problems' encountered with SAM databases by transforming the SAM so that it is consistent with the model structure. Specifically, all transactions between an account with itself are eliminated by setting the appropriate cells in the SAM equal to zero. Second, all transfers from domestic institutions to the Rest of the World and between the Rest of the World and domestic institutions are treated net as transfers to the Rest of the World and domestic institutions, by transposing and changing the sign of the payments to the Rest of the World.[4] And third, all transfers between domestic institutions and the government are treated as net and as payments from government to the respective institution. Since these adjustments change the account totals, which are used in calibration, the account totals are recalculated within the model. An example SAM can be found in Appendix 4.

In addition to the SAM, which records transactions in value terms, three additional databases are used by the model. The first records the 'quantities' of primary inputs used by each activity. The second reports the quantities of the different water qualities consumed by each water user (activities and other agents). If such quantity data are not available then the entries in the factor use matrix are the same as those in the corresponding sub matrix of the SAM. The third series of additional data are the elasticities of substitution for imports and exports relative to domestic commodities, the elasticities of substitution for the CES production functions, the income elasticities of demand for the linear expenditure system and the Frisch (marginal utility of income) parameters for each household.

---

3 These and other reserved names are specified below as part of the description of the model.

4 Treating transfers as net can be justified on the grounds that no clear body of economic theory exists that would seem to justify the adoption of specific behavioural relationships.

All the data are accessed by the model from data recorded in Excel and GDX (GAMS data exchange) files. All the data recorded in Excel are converted into GDX format as part of the model.

2 The Computable General Equilibrium Model

The model is a member of the class of single country computable general equilibrium (CGE) models that are descendants of the approach to CGE modeling described by Dervis *et al.*, (1982). More specifically, the implementation of this model, using the GAMS (General Algebraic Modeling System) software, is a direct descendant and development of models devised in the late 1980s and early 1990s, particularly those models reported by Robinson *et al.*, (1990), Kilkenny (1991) and Devarajan *et al.*, (1994). The model is a SAM based CGE model, wherein the SAM serves to identify the agents in the economy and provides the database with which the model is calibrated. Since the model is SAM based it contains the important assumption of the law of one price, i.e., prices are common across the rows of the SAM.[5] The SAM also serves an important organisational role since the groups of agents identified by the SAM structure are also used to define sub-matrices of the SAM for which behavioural relationships need to be defined. As such the modelling approach has been influenced by Pyatt's 'SAM Approach to Modeling' (Pyatt, 1989).

The description of the model proceeds in five stages. The first stage is the identification of the behavioural relationships; these are defined by reference to the sub matrices of the SAM within which the associated transactions are recorded. The second stage is definitional, and involves the identification of the components of the transactions recorded in the SAM, while giving more substance to the behavioural relationships, especially those governing inter-institutional transactions, and in the process defining the notation. The third stage uses figures to illustrate the price and quantity systems for commodity and activity accounts that are embodied within the model. In the fourth stage an algebraic statement of the model is provided; the model's equations are summarised in a table that also provides (generic) counts of the model's equations and variables. A full listing of the parameters and variables

[5] The one apparent exception to this is for exports. However the model implicitly creates a separate set of export commodity accounts and thereby preserves the 'law of one price', hence the SAM representation in the text is actually a somewhat condensed version of the SAM used in the model (see McDonald, 2007).

contained within the model are located in Appendix 1.[6] Finally in the fifth stage there is a discussion of the default and optional macroeconomic closure and market clearing rules available within the model.

2.1 Behavioural Relationships

While the accounts of the SAM determine the agents that can be included within the model, and the transactions recorded in the SAM identify the transactions that took place, the model is defined by the behavioural relationships. The behavioural relationships in this model are a mix of non-linear and linear relationships that govern how the model's agents will respond to exogenously determined changes in the model's parameters and/or variables. Table 2 summarises these behavioural relationships by reference to the sub matrices of the SAM.

Households are assumed to choose the bundles of commodities they consume so as to maximise utility where the utility function is Stone-Geary. For a developing country a Stone-Geary function may be generally preferable since it allows for subsistence consumption expenditures, which is an arguably realistic assumption when there are substantial numbers of very poor consumers.[7] The households choose their consumption bundles from a set of 'composite' commodities that are aggregates of domestically produced and imported commodities. These 'composite' commodities are formed as Constant Elasticity of Substitution (CES) aggregates that embody the presumption that domestically produced and imported commodities are imperfect substitutes. The optimal ratios of imported and domestic commodities are determined by the relative prices of the imported and domestic commodities. This is the so-called Armington 'insight' (Armington, 1969), which allows for product differentiation via the assumption of imperfect substitution (see Devarajan *et al.*, 1994). The assumption has the advantage of rendering the model practical by avoiding the extreme specialisation and price fluctuations associated with other trade assumptions, e.g., the Salter/Swan or Australian model. In this model the country is assumed to be a price taker for all imported commodities.

6 The model includes specifications for transactions that were zero in the SAM. This is an important component of the model. It permits the implementation of policy experiments with exogenously imposed changes that impact upon transactions that were zero in the base period.

7 A Stone-Geary function reduces to a Cobb-Douglas function given appropriate specification of the parameters.

Table 2 Behavioural Relationships for the Standard Model

	Commodities	Activities	Factors	Households	Enterprises	Government	Capital	RoW	Total	Prices
Commodities	0	Leontief Input-Output Coefficients	0	Utility Functions (CD or Stone-Geary)	Fixed in Real Terms	Fixed in Real Terms and Export Taxes	Fixed Shares of Savings	Commodity Exports	Commodity Demand	Consumer Commodity Prices Prices for Exports
Activities	Domestic Production	0	0	0	0	0	0	0	Activity Supply	
Factors	0	Factor Demands (CES)	0	0	0	0	0	Factor Income from RoW	Factor Income	
Households	0	0	Fixed Shares of Factor Income	Fixed shares of income	Fixed Shares of Dividends	Fixed (Real) Transfers	0	Remittances	Household Income	
Enterprises	0	0	Fixed Shares of Factor Income	0	0	Fixed (Real) Transfers	0	Transfers	Enterprise Income	
Government	Tariff Revenue Domestic Product Taxes	Indirect Taxes on Activities	Fixed Shares of Factor Income Direct Taxes on Factor Income	Direct Taxes on Household Income	Fixed Shares of Dividends Direct Taxes on Enterprise Income	0	0	Transfers	Government Income	
Capital	0	0	Depreciation	Household Savings	Enterprise Savings	Government Savings (Residual)	0	Current Account 'Deficit'	Total Savings	
Rest of World	Commodity Imports	0	Fixed Shares of Factor Income	0	0	0	0	0	Total 'Expenditure' Abroad	
Total	Commodity Supply	Activity Input	Factor Expenditure	Household Expenditure	Enterprise Expenditure	Government Expenditure	Total Investment	Total 'Income' from Abroad		
	Producer Commodity Prices Domestic and World Prices for Imports	Value Added Prices								

Domestic production uses a four-stage production process. An example-case including four water resources and by-products as well as three water commodities is given in Figure 1 On all levels of the nesting structure, with the exception of the aggregation of intermediate inputs, the user is free to decide to apply either CES or Leontief technology (indicated respectively by the s or 0 in Figure 1). CES technology allows the proportion of inputs used to vary with their prices, while in a Leontief setup the quantity shares of inputs are fixed.

On the lowest level, different water commodities or water resources and by-products are combined to form a water-aggregate. Thereby water resources and by-products can be only used by activities which produce water commodities (water activities). Usually one water activity, e.g., desalination, pumping of groundwater, water reclamation, is linked to one specific water resource, e.g., seawater, groundwater, wastewater, and thus there is no substitution possibilities. However, as described above, the setup of the model allows for this option. That water commodities are moved from the intermediate input nest to the value added side is a special feature of this approach. This is due to the fact, that water commodities can be substituted by several activities, especially in irrigated agriculture. In this case CES-technology can be applied. In the second stage the water-composite is combined with land to form a land-water-aggregate and in the third stage the land-water composite is merged with other factors of production (labour and capital) to form a value added-water-aggregate. At the same level in a second arm all non-water-intermediate inputs are aggregated using Leontief technology, such that activities demand non-water intermediate inputs in fixed proportions relative to aggregated intermediate input of each activity. At the top-level aggregated intermediate inputs are combined with the value added-water aggregate. For activities which do not consume all inputs (e.g. no land or water), this four level nesting structure simply collapses to fewer levels.

As described above, there is usually one activity linked to each water resource or by-product. Utilising additional inputs and production factors it converts the resource or by-product to a water commodity, which is then used as an input in other activities or, in case of potable water, is consumed by households. But one water commodity can be also produced by several activities. In the example depicted in Figure 1 there are four natural resources and by-products from which three water commodities are produced, as the fresh water activity and the desalination activity both produce potable water. The fresh water activity produces potable water from fresh water, which can be ground- or surface water, while desalination requires the use of sea water as an input. Thus two activities with, typically, different cost structures produce an homogenous product. The basic STAGE model assumes that if the “same” commodity is produced by different activities it is heterogeneous – a CES aggregate. This variant is adjusted so that the option exists to define such commodities as homogenous. Given differences in costs structures it is necessary for the model to include instruments that ensure the supply price for the homogeneous is the same from each activity. This is achieved by adjusting the activity tax (TX_a) in a way to equate activity prices (PX_a) of the two activities producing potable water (compare Figure 2).

As all water activities depend on the use of a water-resources or by-product, plus other inputs that may or may not include other types of water, the user has the option to define exogenously determined extraction rates for the different water resources. Thereby one can condition the model, so that water producing activities are limited to a predefined production quantity.

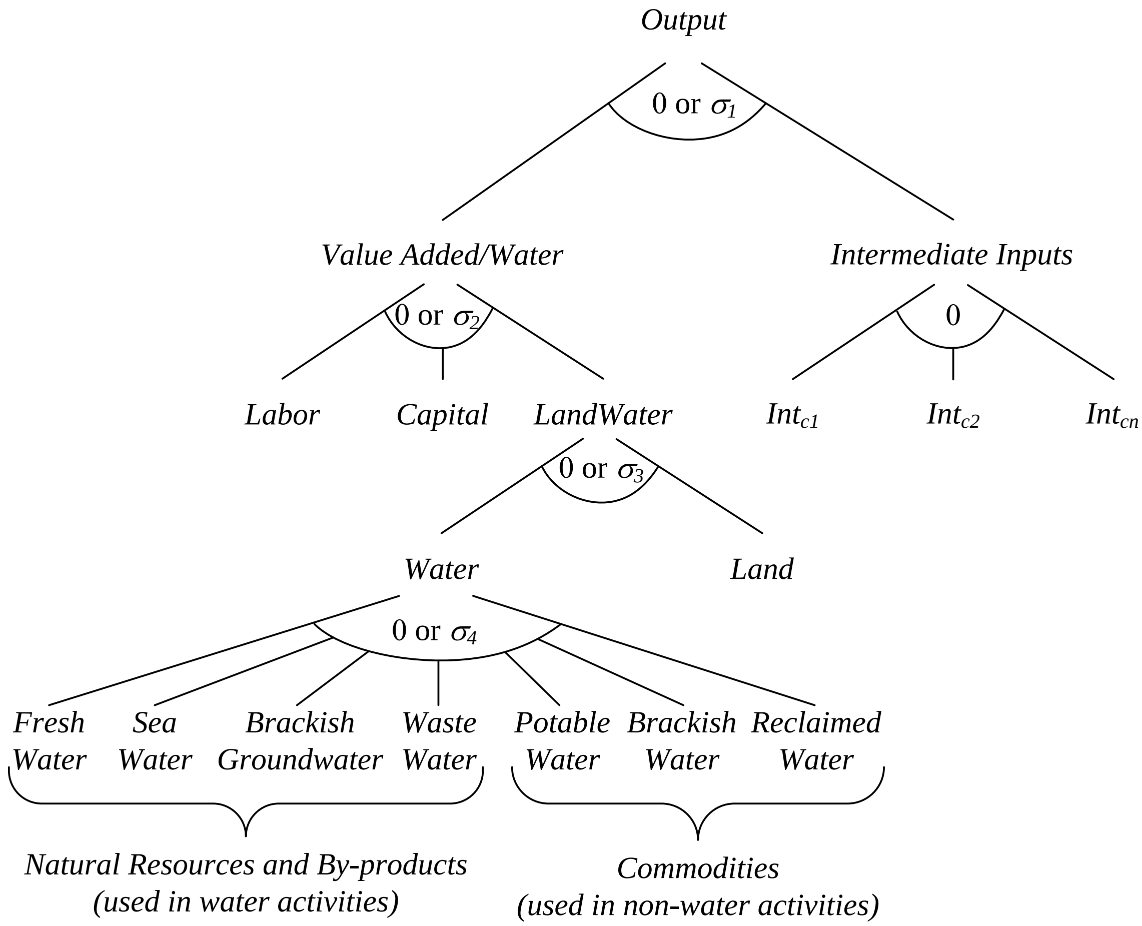
In case water leakage for the distribution network plays a role, water would be an input to its own production; if (aggregate) water is produced with Leontief technologies the implied rate of leakage is fixed while if it is produced with CES technologies it implies the rate of leakage is a function of the price of potable water. This implies a long run scenario where the water authorities respond to changes in prices by “adjusting” the rate of leakage and differs from the approach suggested by Faust *et al.* (2012).

For water consuming activities the various types of water are used according to the input structure contained in the database. Irrigated agricultural activities use (agricultural) land and one, or more, types of water. It is useful to segment these activities and commodities so as to distinguish between activities that can use the different types of water, e.g., to single out crops that are salt resistant and thus can use either brackish or potable water for irrigation or non-food crops that can be irrigated with reclaimed water. Generally non-agricultural activities do not use agricultural land but do use water of different types. In such cases the land/water aggregate collapses to the water aggregate.

Generally the nesting structure of the model is flexible and adjusts to the usage of different water commodities and factors, e.g., land, by different activities. For example, for service activities (e.g. transportation, communication), which typically do not use marginal water (reclaimed wastewater and brackish water) and land, the production structure collapses to two stages, such that potable water is combined with labor and capital in one value added nest.

Finally water resources that are reserved for environmental or other reasons, e.g., to guarantee a certain level of river flow, are not usually accorded a monetary value. Such resources are subtracted from the water resources available to the economy.

Figure 1 Production System for Activities in STAGE_W



In general, the activities are defined as multi-product activities with the assumption that the proportionate combinations of commodity outputs produced by each activity/industry remain constant; hence for any given vector of commodities demanded there is a unique vector of activity outputs that must be produced; in essence this is a strong by-product assumption.[8]

The vector of commodities demanded is determined by the domestic demand for domestically produced commodities and export demand for domestically produced commodities. Using the assumption of imperfect transformation between domestic demand

8 A variant of the model allows activities to modify their output mix in response to changes in the relative prices of the commodities produced by different activities.

and export demand, in the form of a Constant Elasticity of Transformation (CET) function, the optimal distribution of domestically produced commodities between the domestic and export markets is determined by the relative prices on the alternative markets. The model can be specified as a small country, i.e., price taker, on all export markets, or selected export commodities can be deemed to face downward sloping export demand functions, i.e., a large country assumption.

The other behavioural relationships in the model are generally linear. A few features do however justify mention. First, all the tax rates are declared as variables with various adjustments and/or scaling factors that are declared as variables or parameters according to how the user wishes to vary tax rates. If a fiscal policy constraint is imposed then one or more of the sets of tax rates can be allowed to vary equiproportionately and/or additively to define a new vector of tax rates that is consistent with the fiscal constraint. Relative tax rates can also be adjusted by the settings chosen by the user. Similar adjustment and/or scaling factors are available for a number of key parameters, e.g., household and enterprise savings rates and inter-institutional transfers. Second, technology changes can be introduced through changes in the activity specific efficiency variables – adjustment and/or scaling factors are also available for the efficiency parameters. Third, the proportions of current expenditure on commodities defined to constitute subsistence consumption can be varied. Fourth, although a substantial proportion of the sub matrices relating to transfers, especially with the rest of the world, contain zero entries, the model allows changes in such transfers, e.g., aid transfers to the government from the rest of the world may be defined equal to zero in the database but they can be made positive, or even negative, for model simulations. And fifth, the model is set up with a range of flexible macroeconomic closure rules and market clearing conditions. For convenience the default closure for the model is a standard neoclassical model closure, e.g., full employment, savings driven investment and a floating exchange rate; this is the simplest option for purposes of calibration and replication. All these closure conditions can all be readily altered; indeed it is rare for the core simulations to be implemented with the default closure.

2.2 Transaction Relationships

The transactions relationships are laid out in Table 3, which is in two parts. The prices of domestically consumed (composite) commodities are defined as PQD_c, and they are the same irrespective of which agent purchases the commodity. The quantities of commodities demanded domestically are divided between intermediate demand, $QINTD_c$, and final demand, with final demand further subdivided between demands by households, QCD_c, enterprises, $QENTD_c$, government, QGD_c, investment, $QINVD_c$, and stock changes, $dstocconst_c$. The value of total domestic demand, at purchaser prices, is therefore (PQD_c * QQ_c). Consequently the decision to represent export demand, QE_c, as an entry in the commodity row is slightly misleading, since the domestic prices of exported commodities, $PE_c = PWE_c * ER$, do not accord with the law of one price. The representation is a space saving device that removes the need to include separate rows and columns for domestic and exported commodities.[9] The price wedges between domestic and exported commodities are represented by export duties, TE_c, that are entered into the commodity columns. Commodity supplies come from domestic producers who receive the common prices, PXC_c, for outputs irrespective of which activity produces the commodity, with the total domestic production of commodities being denoted as QXC_c. Commodity imports, QM_c, are valued carriage insurance and freight (*cif*) paid, such that the domestic price of imports, PM_c, is defined as the world price, PWM_c, times the exchange rate, ER, plus an *ad valorem* adjustment for import duties, TM_c. All domestically consumed commodities are subject to a variety of product taxes, sales taxes, TS_c, and excise taxes, TEC_c. Other taxes can be readily added.[10]

Domestic production activities receive average prices for their output, PX_a, that are determined by the commodity composition of their outputs. Since activities produce multiple outputs their outputs can be represented as an index, QX_a, formed from the commodity composition of their outputs. In addition to intermediate inputs, activities also purchase primary inputs, $FD_{f,a}$, for which they pay average prices, WF_f. To create greater flexibility the model allows the price of each factor to vary according to the activity that employs the factor. Finally each activity pays production taxes, the rates, TX_a, for which are proportionate to the value of activity outputs.

[9] In this model the allocation by domestic producers of commodities between domestic and export markets is made on the supply side; implicitly there are two supply matrices – supplies to the domestic market and supplies to the export market.

[10] Various VAT systems, fuel taxes etc., have been used.

Table 3 Transactions Relationships for STAGE_W

	Commodities	Activities	Factors	Households
Commodities	0	$\left(PQD_c * QINTD_c\right)$	0	$\left(PQD_c * QCD_c\right)$
Activities	$\left(PXC_c * QXC_c\right)$ $\left(PX_a * QX_a\right)$	0	0	0
Factors	0	$\left(WF_f * FD_{f,a}\right)$	0	0
Households	0	0	$\sum_f hovash_{h,f} * YFDISP_f$	$\left(\sum_{hh} hohoconst_{hh,h}\right)$
Enterprises	0	0	$\left(\sum_f entvash_{e,f} * YFDISP_f\right)$	0
Government	$\left(TM_c * PWM_c * QM_c * ER\right)$ $\left(TE_c * PWE_c * QE_c * ER\right)$ $\left(TS_c * PQS_c * QQ_c\right)$ $(TEX_c * PQS_c * QQ_c)$ $(TWAT_c * PQS_c * QQ_c)$	$\left(TX_a * PX_a * QX_a\right)$ $(TF_{f,a} * WF_f * WFDIST_{f,a} * FD_{f,a})$ $(TWATA_c * PQD_c$ $* PQDDIST_{c,a} * QWAT2_{c,a})$	$\left(\sum_f govvash_f * YFDISP_f\right)$ $\left(TYF_f * YFDISP_f\right)$	$\left(TYH_h * YH_h\right)$
Capital	0	0	$\sum_f deprec_f$	$\left(SSH_h * YH_h\right)$
Rest of World	$\left(PWM_c * QM_c * ER\right)$	0	$\left(\sum_f worvash_f * YFDISP_f\right)$	0
Total	$\left(PQD_c * QQ_c\right)$	$\left(PX_a * QX_a\right)$	YF_f	YH_h

Table 3 (cont)Transactions Relationships for the Standard Model

	Enterprises	**Government**	**Capital**	**RoW**	**Total**
Commodities	$\left(PQD_c * QED_{c,e}\right)$	$\left(PQD_c * QGD_c\right)$	$\left(PQD_c * QINVD_c\right)$ $\left(PQD_c * dstocconst_c\right)$	$\left(PWE_c * QE_c * ER\right)$	$\left(PQD_c * QQ_c\right)$
Activities	0	0	0	0	$\left(PX_a * QX_a\right)$
Factors	0	0	0	$\left(factwor_f * ER\right)$	YF_f
Households	$HOENT_{h,e}$	$\left(hogovconst_h * HGADJ\right)$	0	$\left(howor_h * ER\right)$	YH_h
Enterprises	0	$\left(entgovconst * EGADJ\right)$	0	$\left(entwor_e * ER\right)$	VED_e
Government	$\left(TYE_e * YE_e\right)$	0	0	$\left(govwor * ER\right)$	EG
Capital	$\left(YE_e - VED_e\right)$	$\left(YG - EG\right)$	0	$\left(CAPWOR * ER\right)$	$TOTSAV$
Rest of World	0	0	0	0	Total ‘Expenditure’ Abroad
Total	YE_e	YG	$INVEST$	Total ‘Income’ from Abroad	

The model allows for the domestic use of both domestic and foreign owned factors of production, and for payments by foreign activities for the use of domestically owned factors. Factor incomes therefore accrue from payments by domestic activities and foreign activities, $factwor_f$, where payments by foreign activities are assumed exogenously determined and are denominated in foreign currencies. After allowing for depreciation, $deprec_f$, and the payment of factor taxes, TF_f, the residual factor incomes, $YFDIST_f$, are divided between domestic institutions (households, enterprises and government) and the rest of the world in fixed proportions.

Households receive incomes from factor rentals and/or sales, inter household transfers, $hohoconst_{h,h}$, transfers from enterprises, $hoentconst_h$, and government, $hogovconst_h$, and remittances from the rest of the world, $howor_h$, where remittances are defined in terms of the foreign currency. Household expenditures consist of payments of direct/income taxes, TYH_h, after which savings are deducted, where the savings rates, SHH_h, are fixed exogenously in the base configuration of the model. The residual household income is then divided between inter household transfers and consumption expenditures, with the pattern of consumption expenditures determined by the household utility functions.

The enterprise account receives income from factor sales, primarily in the form of retained profits,[11] transfers from government, $entgovconst_e$, and foreign currency denominated transfers from the rest of the world, $entwor_e$. Expenditures then consist of the payment of direct/income taxes, TYE_e, consumption, which is assumed fixed in real terms,[12] and savings, which are defined as a residual, i.e., the difference between income, YE_e, and committed expenditure, VED_e. There is an analogous treatment of government savings, i.e., the internal balance, which is defined as the difference (residual) between government income, YG, and committed government expenditure, EG. In the absence of a clearly definable set of behavioural relationships for the determination of government consumption expenditure, the quantities of commodities consumed by the government are fixed in real terms, and hence government consumption expenditure will vary with commodity prices.[13] Transfers by the government to other domestic institutions are fixed in nominal terms, although there is a

---

[11] Hence the model contains the implicit presumption that the proportions of profits retained by incorporated enterprises are constant.

[12] Hence consumption expenditure is defined as the fixed volume of consumption, $QED_{c,e}$, times the variable prices. It requires only a simple adjustment to the closure rules to fix consumption expenditures. Without a utility function, or equivalent, for enterprises it is not possible to define the quantities consumed as the result of an optimisation problem.

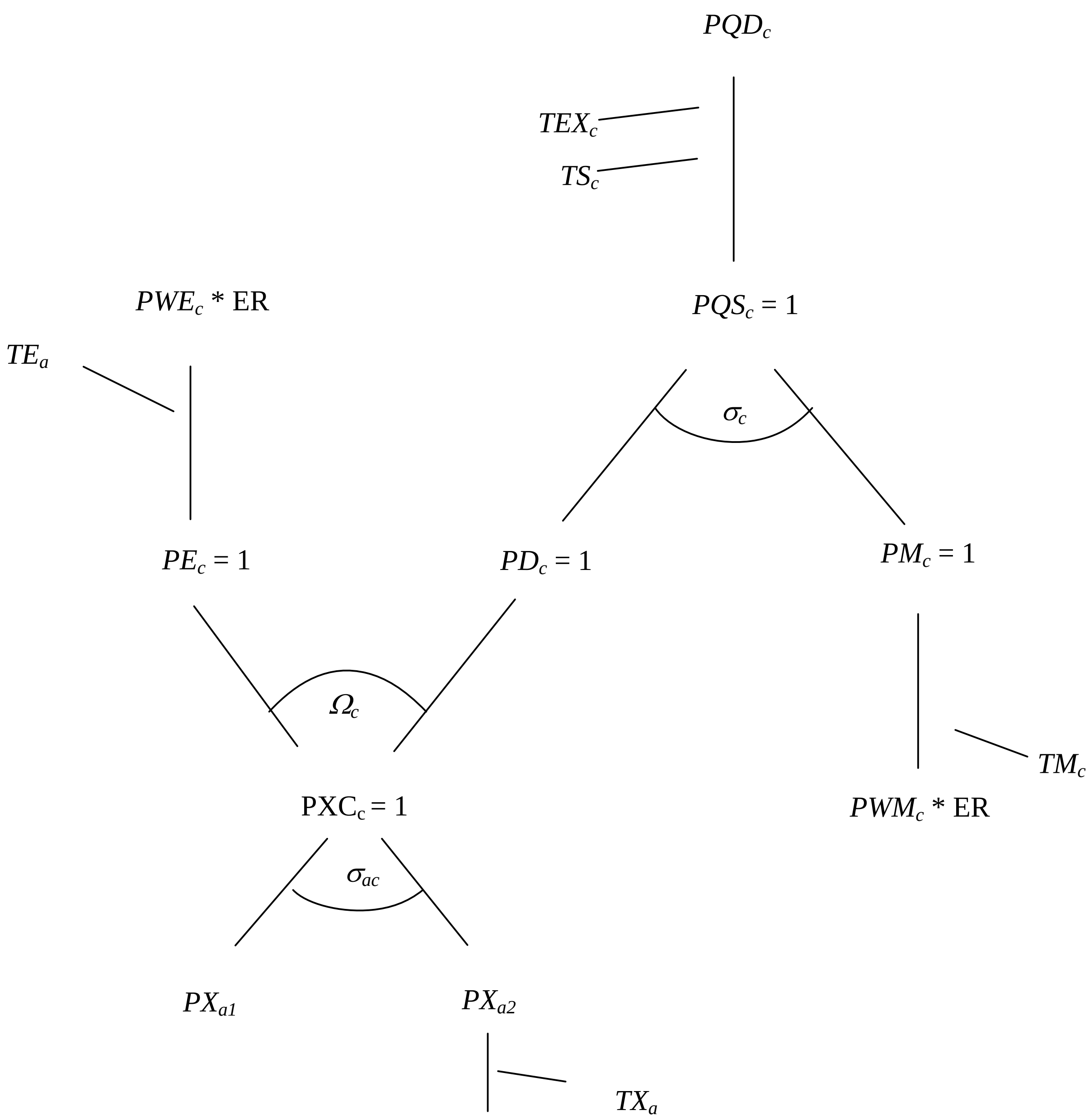
[13] The closure rules allow for the fixing of government consumption expenditure rather than real consumption.

facility to allow them to vary, e.g., with consumer prices. On the other hand government incomes can vary widely. Incomes accrue from the various tax instruments (import and export duties, sales, production and factor taxes, and direct taxes), that can all vary due to changes in the values of production, trade and consumption. The government also receives foreign currency denominated transfers from the rest of the world, *govwor*, e.g., aid transfers.

Domestic investment demand consists of fixed capital formation, $QINVD_c$, and stock changes, $dstocconst_c$. The comparative static nature of the model and the absence of a capital composition matrix underpin the assumption that the commodity composition of fixed capital formation is fixed, while a lack of information means that stock changes are assumed invariant. However the value of fixed capital formation will vary with commodity prices while the volume of fixed capital formation can vary both as a consequence of the volume of savings changing or changes in exogenously determined parameters. In the base version of the model domestic savings are made up of savings by households, enterprises, the government (internal balance) and foreign savings, i.e., the balance on the capital account or external balance, *CAPWOR*. The various closure rules available within the model allow for different assumptions about the determination of domestic savings, e.g., flexible versus fixed savings rates for households, and value of ‘foreign’ savings, e.g., a flexible or fixed exchange rate.

Incomes to the rest of the world account, i.e., expenditures by the domestic economy in the rest of the world, consist of the values of imported commodities and factor services. On the other hand expenditures by the rest of the world account, i.e., incomes to the domestic economy from the rest of the world, consist of the values of exported commodities and NET transfers by institutional accounts. All these transactions are subject to transformation by the exchange rate. In the base model the balance on the capital account is fixed at some target value, denominated in foreign currency terms, e.g., at a level deemed equal and opposite to a sustainable deficit on the current account, and the exchange rate is variable. This assumption can be reversed, where appropriate, in the model closure.

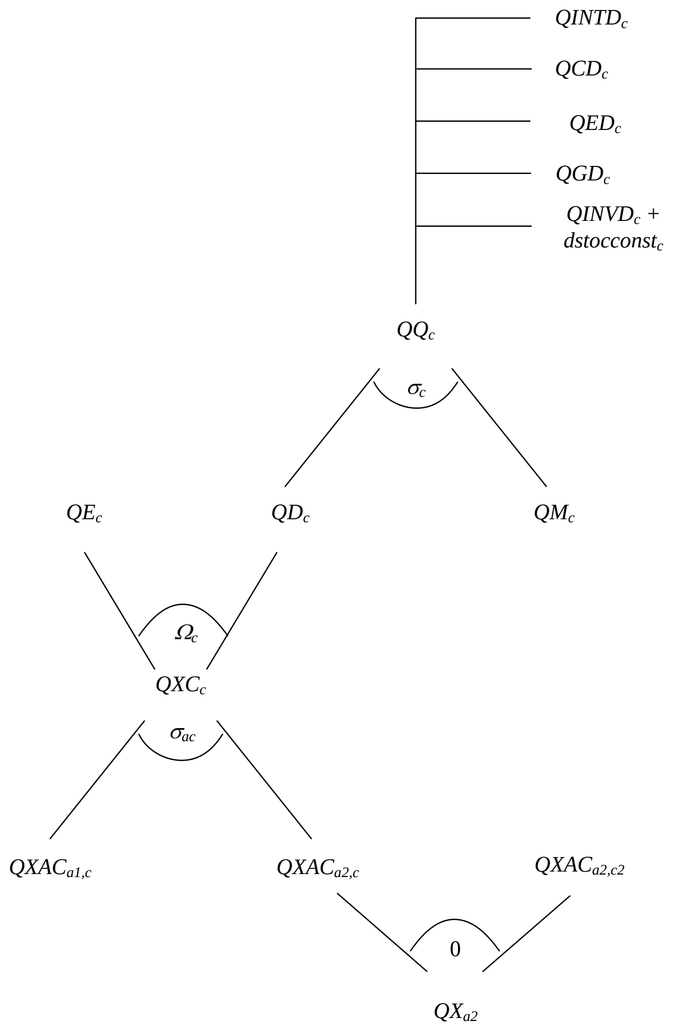
Figure 2 Price Relationships for the STAGE Model



Figures 2 and 3 provide further detail on the interrelationships between the prices and quantities for commodities and activities. The supply prices of the composite commodities (PQS_c) are defined as the weighted averages of the domestically produced commodities that are consumed domestically (PD_c) and the domestic prices of imported commodities (PM_c), which are defined as the products of the world prices of commodities (PWM_c) and the exchange rate (ER) uplifted by *ad valorem* import duties (TM_c). These weights are updated in the model through first order conditions for optima. The average prices exclude sales taxes, and hence must be uplifted by (*ad valorem*) sales taxes (TS_c) and excise taxes (TEX_c) to reflect the composite consumer price (PQD_c). The producer prices of commodities (PXC_c) are similarly defined as the weighted averages of the prices received for domestically produced commodities sold on domestic (PD_c) and export (PE_c) markets. These weights are updated in the model through first order conditions for optima. The prices received on the export market are defined as the products of the world price of exports (PWE_c) and the exchange rate (ER) less any exports duties due, which are defined by *ad valorem* export duty rates (TE_c).

The average price per unit of output received by an activity (PX_a) is defined as the weighted average of the domestic producer prices, where the weights are constant. After paying indirect/production/output taxes (TX_a), this is divided between payments to aggregate value added (PVA_a), i.e., the amount available to pay primary inputs, and aggregate intermediate inputs ($PINT_a$). Total payments for intermediate inputs per unit of aggregate intermediate input are defined as the weighted sums of the prices of the inputs (PQD_c) (Figure 5).

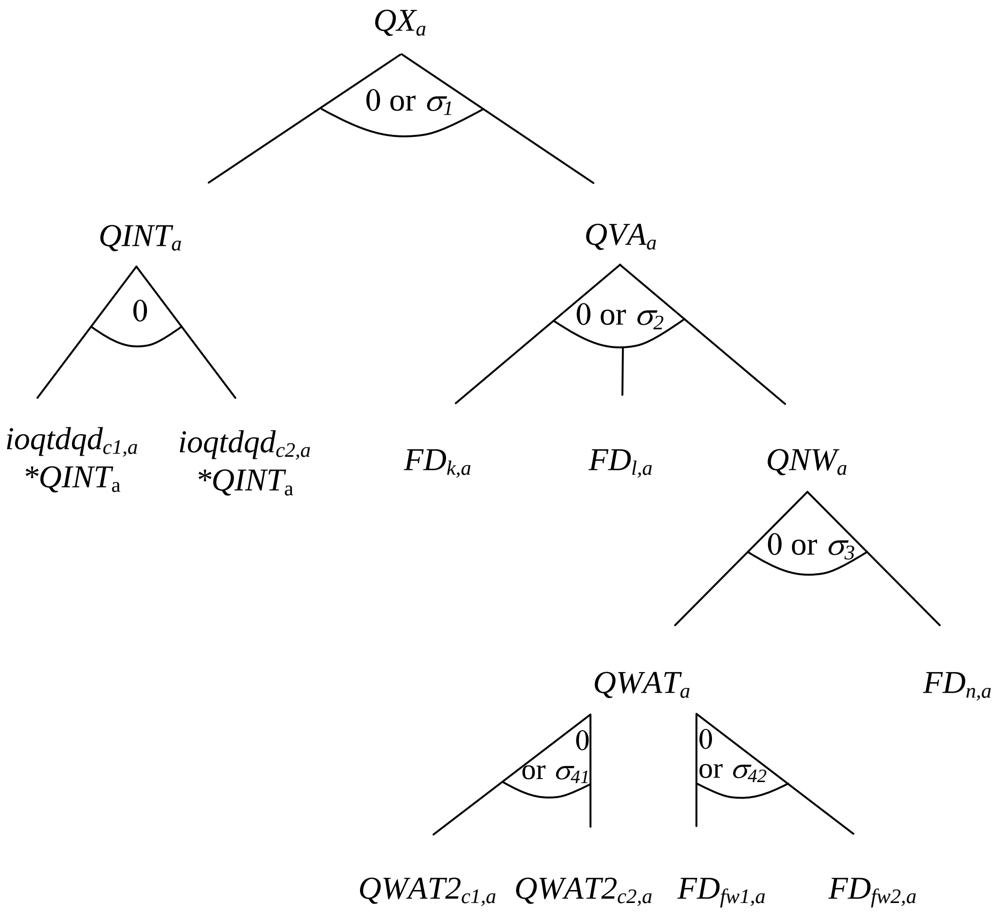
Figure 3 Quantity Relationships for the STAGE Model



Total demands for the composite commodities (QQ_c) consist of demands for intermediate inputs ($QINTD_c$), consumption by households (QCD_c), enterprises ($QENTD_c$), and government (QGD_c), gross fixed capital formation ($QINVD_c$), and stock changes ($dstocconst_c$). Supplies from domestic producers (QD_c) plus imports (QM_c) meet these demands; equilibrium conditions ensure that the total supplies and demands for all composite commodities equate. Commodities are delivered to both the domestic (QD_c) and export (QE_c) markets subject to equilibrium conditions that require all domestic commodity production (QXC_c) to be either domestically consumed or exported.

The presence of multiple product activities means that domestically produced commodities can come from multiple activities, i.e., the total production of a commodity is defined as the sum of the amount of that commodity produced by each activity. Hence the domestic production of a commodity (QXC_c) is a CES aggregate of the quantities of that commodity produced by a number of different activities ($QXAC_{a,c}$), which are produced by each activity in activity specific fixed proportions, i.e., the output of $QXAC_{a,c}$ is a Leontief (fixed proportions) aggregate of the output of each activity (QX_a).

Figure 4 Production Relationships for the STAGE_W Model: Quantities



Production relationships by activities are defined by a series of nested CES/Leontief production functions[14]. The illustration in Figure 4 shows the general four-level production structure, which holds for all activities. It is simplified, as it is based on a reduced number of inputs on most nests; however the model is flexible to include any number of inputs on all levels.

On the lowest level water activities use either their respective water resource or by-product ($FD_{fw,a}$) whereas other activities utilize water commodities ($QWAT2_{c,a}$). Both types of activities aggregate the respective inputs to $QWAT_a$. On the next level the water composit is grouped with land ($FD_{n,a}$) to form a land-water aggregate (QNW_a). This in turn is combined with other factors of production (e.g. capital $FD_{k,a}$ and labour $FD_{l,a}$) to compose value added (QVA_a). On the same level in a second nest non-water intermediate inputs are aggregated in

[14] Peroni and Rutherford (1995) demonstrate that nested CES function can approximate any flexible functional form, e.g., translog. (Perroni, C. and Rutherford, T. F.; (1995). 'Regular Flexibility of Nested CES Functions', *European Economic Review*, Vol 39 (2), pp. 335-43.)

fixed shares (Leontief technology) to form intermediate input ($QINT_a$). Finally, on the highest level QVA_a and $QINT_a$ are combined to form activity output QX_a.

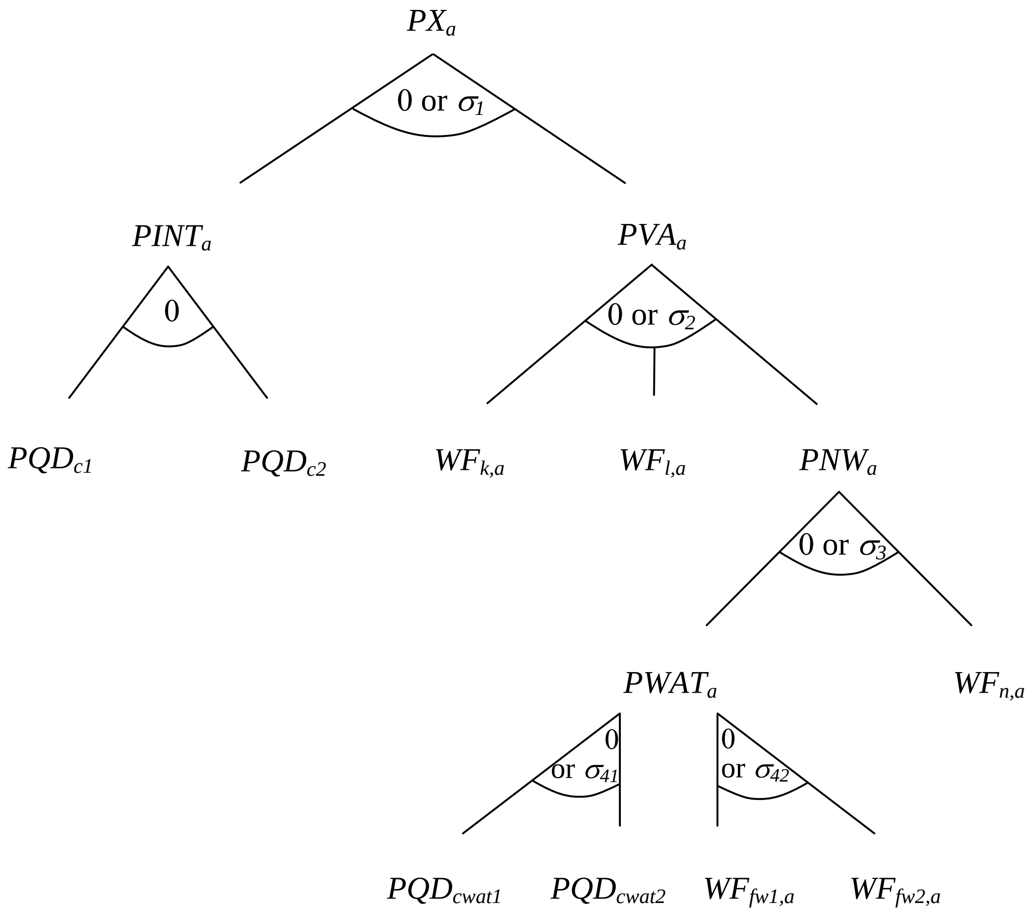
For water activities, in the basic case however, each activity is linked to one specific resource and land requirements do not play a role in the provision of water. In this case, QNW_a equals $FD_{fw,a}$ and thus the production structure collapses to a two level nest [15]. If it is desired to keep the production structure fixed, as most water providing facilities are build in a certain setup and produce for decades without many options to alter the production structure to a large extend, Leontief technology can be applied for water activities on all levels. This also guarantees that the water resource cannot be substituted by other inputs such that the quantity of water resource use directly determines the output of water commodity by the water activity. However, the model allows for a shift to CES-technology, by setting an elasticity, if required.

Water consuming activities, on the other side, usually allow for the substitution of water commodities and other factors of production, such that in the default setup on all levels CES technology is applied (σ_1 - σ_4). The optimal combinations of inputs in each CES aggregate are determined by first order conditions based on relative prices. The only exception is the aggregation of non-water intermediate inputs ($QINT_a$), for which Leontief technology is applied.

The advantage of using such a nesting structure is that it avoids making the assumption that all inputs are equally substitutable in the generation of value added. For activities which do not consume any land and/or water commodities, the nesting structure simply collapses to fewer levels.

15 Possible exceptions to this are the provision of potable water with a certain mineral level by mixing desalinated seawater (with zero mineral content) with purified freshwater. In the reclamation of wastewater land can play a role, if sewage farms are used.

Figure 5 Production Relationships for the STAGE_W Model: Prices



The price relations for the production system are illustrated in Figure 5. Note how the prices paid for intermediate inputs and water commodities (PQD_c) are the same as paid for final demands, i.e., a ‘law’ of one price relationship holds across all domestic demand. Note also that factor prices are factor and activity specific ($WF_{f,a}$), which means that the allocation of finite supplies of factors (FS_f) between competing activities depends upon relative factor prices via first order conditions for optima.

3 Algebraic Statement of the Model

3.1 Sets

The model uses a series of sets, each of which is required to be declared and have members assigned. For the majority of the sets the declaration and assignment takes place simultaneously in a single block of code.[16] However, the assignment for a number of the sets, specifically those used to control the modelling of trade relationships is carried out dynamically by reference to the data used to calibrate the model. The following are the basic sets for this model:

$$\begin{aligned}
c &= \{\text{commodities}\} \\
a &= \{\text{activities}\} \\
f &= \{\text{factors}\} \\
h &= \{\text{households}\} \\
g &= \{\text{government}\} \\
e &= \{\text{enterprises}\} \\
i &= \{\text{investment}\} \\
w &= \{\text{rest of the world}\}
\end{aligned}$$

For each set there is an alias declared that has the same membership as the corresponding basic set. The notation used involves the addition of a '*p*' suffix to the set label, e.g., the alias for *c* is *cp*.

For practical/programming purposes these basic sets are declared and assigned as subsets of a global set, *sac*,

$$sac = \{c, a, f, h, g, e, i, w, total\}.$$

All the dynamic sets relate to the modelling of the commodity and activity accounts and therefore are subsets of the sets *c* and *a*. The subsets are:

[16] For practical purposes it is often easiest if this block of code is contained in a separate file that is then called up from within the *.gms file.

$$ce_c = \{\text{export commodities}\}$$
$$cen_c = \{\text{non-export commodities}\}$$
$$ced_c = \{\text{export commodities with export demand functions}\}$$
$$cedn_c = \{\text{export commodities without export demand functions}\}$$
$$cm_c = \{\text{imported commodities}\}$$
$$cmn_c = \{\text{non-imported commodities}\}$$
$$cx_c = \{\text{commodities produced domestically}\}$$
$$cxn_c = \{\text{commodities NOT produced domestically AND imported}\}$$
$$cd_c = \{\text{commodities produced AND demanded domestically}\}$$
$$cdn_c = \{\text{commodities NOT produced AND demanded domestically}\}$$

and members are assigned using the data used for calibration. Additionally there are some sets, referring to commodities and activities, which are used to control the behavioural equations implemented in specific cases. These are:

$$cxac_c = \{\text{differentiated commodities produced domestically}\}$$
$$cxacn_c = \{\text{UNdifferentiated commodities produced domestically}\}$$
$$cwat_c = \{\text{water-commodities,which enter on the value added side of the production function}\}$$
$$cnwat_c = \{\text{non-water-commodities,which enter on the intermediate side of the production function}\}$$
$$aqx_a = \{\text{activities with CES aggregation at Level 1}\}$$
$$aqxn_a = \{\text{activities with Leontief aggregation at Level 1}\}$$
$$afx_a = \{\text{activities with CES aggregation at value added side of Level 2}\}$$
$$afxn_a = \{\text{activities with Leontief aggregation at value added side of Level 2}\}$$
$$af3x_a = \{\text{activities with CES aggregation at Level 3}\}$$
$$af3xn_a = \{\text{activities with Leontief aggregation at Level 3}\}$$
$$af4x_a = \{\text{activities with CES aggregation at Level 4}\}$$
$$af4xn_a = \{\text{activities with Leontief aggregation at Level 4}\}$$

and their memberships are set during the model calibration phase.

Finally a set is declared and assigned for a macro SAM that is used to check model calibration. This set and its members are:

$$ss = \{commdty, activity, valuad, hholds, entp, govtn, kapital, world, totals\}.$$

3.2 Reserved Names

The model also uses a number of names that are reserved, in addition to those specified in the set statements detailed above. The majority of these reserved names are components of the government set; they are reserved to ease the modelling of tax instruments. The required members of the government set, with their descriptions, are:

$$g = \left\{ \begin{array}{cc} IMPTAX & \text{Import Taxes} \\ EXPTAX & \text{Export Taxes} \\ SALTAX & \text{Sales Taxes} \\ ECTAX & \text{Excise Taxes} \\ INDTAX & \text{Indirect Taxes} \\ FACTTAX & \text{Factor Taxes} \\ DIRTAX & \text{Direct Taxes} \\ WATTAX & \text{Water Taxes} \\ WATAXA & \text{Water User Subsidies} \\ GOVT & \text{Government} \end{array} \right\}.$$

The other reserved names are for the factor account and for the capital accounts. For simplicity the factor account relating to residual payments to factors has the reserved name of *GOS* (gross operating surplus); in many SAMs this account would include payments to the factors of production land and physical capital, payments labelled mixed income and payments for entrepreneurial services. Where the factor accounts are fully articulated *GOS* would refer to payments to the residual factor, typically physical capital and entrepreneurial services.

The capital account includes provision for two expenditure accounts relating to investment. All expenditures on stock changes are registered in the account *dstoc*, while all investment expenditures are registered to the account *kap*. All incomes to the capital account accrue to the *kap* account and stock changes are funded by an expenditure levied on the *kap* account to the *dstoc* account.

3.3 Conventions

The equations for the model are set out in eleven ‘blocks’; which group the equations under the following headings ‘trade’, ‘commodity price’, ‘numéraire’, ‘production’, ‘factor’, ‘household’, ‘enterprise’, ‘government’, ‘kapital’, ‘foreign institutions’ and ‘market clearing’. This grouping of equations is intended to ease the reading of the model rather than being a requirement of the model; it also reflects the modular structure that underlies the programme and which is designed to simplify model extensions/developments.

A series of conventions are adopted for the naming of variables and parameters. These conventions are not a requirement of the modelling language; rather they are designed to ease reading of the model.

- All VARIABLES are in upper case.
- Standard prefixes for variable names are: *P* for price variables, *Q* for quantity variables, *E* for expenditure variables, *Y* for income variables, and *V* for value variables
- All variables have a matching parameter that identifies the value of the variable in the base period. These parameters are in upper case and carry a '0' suffix, and are used to initialise variables.
- A series of variables are declared that allow for the equiproportionate adjustment of groups of parameters. These variables are named using the convention ***ADJ*, where ** is the variable/parameter series they adjust.
- All parameters are in lower case, except those used to initialise variables.
- Names for parameters in CES/CET functions use standard prefixes – ρ^{**} for all elasticity parameters, *ac*** and $\delta**$ for commodity CES shift and share parameters, *ad*** and $\delta**$ for activity CES shift and share parameters and *at*** and $\gamma**$ for CET shift and share parameters;
- Other parameter names with prefixes or suffix which distinguishes their definition, e.g., *io*** is a quantity coefficient, ****sh** is a value share parameter, ****av** is an average and **const is a constant parameter, as far as possible the ** part of the name seeks to identify the component parts;
- The names for all parameters and variables are kept short.

3.4 Trade Block Equations

Trade relationships are modelled using the Armington assumption of imperfect substitutability between domestic and foreign commodities. The set of eleven equations are split across two sub-blocks – exports and imports - and provide a general structure that accommodates most eventualities found with single country CGE models. In particular these equations allow for traded and non-traded commodities while simultaneously accommodating commodities that are produced or not produced domestically and are consumed or not consumed domestically and allowing a relaxation of the small country assumption of price taking for exports.

3.4.1 Exports Block

The domestic price of exports (E1) is defined as the product of the world price of exports (*PWE*), the exchange rate (*ER*) and one minus the export tax rate[17] and are only implemented for members of the set *c* that are exported, i.e., for members of the subset *ce*. The world price of imports and exports are declared as variables to allow relaxation of the small country assumption, and are then fixed as appropriate in the model closure block.

Export Block Equations

$$PE_c = PWE_c * ER * (1 - TE_c) \qquad \forall ce \qquad \text{(E1)}$$

$$QXC_c = at_c * \left(\gamma_c * QE_c^{\rho t_c} + (1-\gamma_c) * QD_c^{\rho t_c}\right)^{\frac{1}{\rho t_c}} \qquad \forall ce \textbf{ AND } cd \qquad \text{(E2)}$$

$$\frac{QE_c}{QD_c} = \left[\frac{PE_c}{PD_c} * \frac{(1-\gamma_c)}{\gamma_c}\right]^{\frac{1}{(\rho t_c - 1)}} \qquad \forall ce \textbf{ AND } cd \qquad \text{(E3)}$$

$$QXC_c = QD_c + QE_c \qquad \forall (cen \textbf{ AND } cd) \textbf{ OR } (ce \textbf{ AND } cdn) \qquad \text{(E4)}$$

$$QE_c = econ_c * \left(\frac{PWE_c}{pwse_c}\right)^{-\eta_c} \qquad \forall ced \qquad \text{(E5)}$$

The output transformation functions (E2), and the associated first-order conditions (E3), establish the optimum allocation of domestic commodity output (*QXC*) between domestic demand (*QD*) and exports (*QE*), by way of Constant Elasticity of Transformation (CET) functions, with commodity specific share parameters (γ), elasticity parameters (ρt) and shift/efficiency parameters (*at*). The first order conditions define the optimum ratios of exports to domestic demand in relation to the relative prices of exported (*PE*) and domestically supplied (*PD*) commodities. But (E2) is only defined for commodities that are both produced and demanded domestically (*cd*) **and** exported (*ce*). Thus, although this condition might be satisfied for the majority of commodities, it is also necessary to cover

17 ALL tax rates are expressed as variables. How the tax rate variables are modeled is explained below.

those cases where commodities are produced **and** demanded domestically but **not** exported, and those cases where commodities are produced domestically **and** exported but **not** demanded domestically.

If commodities are produced domestically but **not** exported, then domestic demand for domestically produced commodities (*QD*) is, by definition (E4), equal to domestic commodity production (*QXC*), where the sets *cen* (commodities not exported) and *cd* (commodities produced and demanded domestically) control implementation. On the other hand if commodities are produced domestically but **not** demanded by the domestic output, then domestic commodity production (*QXC*) is, by definition (E4), equal to commodity exports (*QE*), where the sets *ce* (commodities exported) and *cdn* (commodities not produced or not demanded domestically) control implementation.

The equations E1 to E4 are sufficient for a general model of export relationships when combined with the small country assumption of price taking on all export markets. However, it may be appropriate to relax this assumption in some instances, most typically in cases where a country is a major supplier of a commodity to the world market, in which case it may be reasonable to expect that as exports of that commodity increase so the export price (*PE*) of that commodity might be expected to decline, i.e., the country faces a downward sloping export demand curve. The inclusion of export demand equations (E5) accommodates this feature, where export demands are defined by constant elasticity export demand functions, with constants (*econ*), elasticities of demand (η) and prices for substitutes on the world market (*pwse*).

3.4.2 Imports Block

The domestic price of competitive imports (M1) is the product of the world price of imports (*PWM*), the exchange rate (*ER*) and one plus the import tariff rate (*TM*). These equations are only implemented for members of the set *c* that are imported, i.e., for members of the subset *cm*.

The domestic supply equations are modelled using Constant Elasticity of Substitution (CES) functions and associated first order conditions to determine the optimum combination of supplies from domestic and foreign (import) producers. The domestic supplies of the composite commodities (*QQ*) are defined as CES aggregates (M2) of domestic production supplied to the domestic market (*QD*) and imports (*QM*), where aggregation is controlled by the share parameters (δ), the elasticity of substitution parameters (ρ) and the shift/efficiency parameters (*ac*). The first order conditions (M3) define the optimum ratios of imports to domestic demand in relation to the relative prices of imported (*PM*) and domestically supplied (*PDD*) commodities. But (M2) is only defined for commodities that are both produced domestically (*cx*) **and** imported (*cm*). Although this condition might be satisfied for the majority of commodities, it is also necessary to cover those cases where commodities are produced but **not** imported, and those cases where commodities are **not** produced domestically **and** are imported.

Import Block Equations

$$PM_c = PWM_c * ER * \left(1 + TM_c\right) \qquad \forall cm \qquad \text{(M1)}$$

$$QQ_c = ac_c\left(\delta_c QM_c^{-\rho_c} + \left(1-\delta_c\right)QD_c^{-\rho_c}\right)^{-\frac{1}{\rho_c}} \qquad \forall cm \textbf{ AND } cx \qquad \text{(M2)}$$

$$\frac{QM_c}{QD_c} = \left[\frac{PD_c}{PM_c} * \frac{\delta_c}{\left(1-\delta_c\right)}\right]^{\frac{1}{(1+\rho_c)}} \qquad \forall cm \textbf{ AND } cx \qquad \text{(M3)}$$

$$QQ_c = QD_c + QM_c \qquad \forall\left(cmn \textbf{ AND } cx\right) \textbf{ OR } \left(cm \textbf{ AND } cxn\right) \qquad \text{(M4)}$$

If commodities are produced domestically but **not** imported, then domestic supply of domestically produced commodities (*QD*) is, by definition (M4), equal to domestic commodity demand (*QQ*), where the sets *cmn* (commodities not imported) and *cx* (commodities produced domestically) control implementation. On the other hand if commodities are **not** produced domestically but are demanded on the domestic market, then commodity supply (*QQ*) is, by definition (M4), equal to commodity imports (*QM*), where the sets *cm* (commodities imported) and *cxn* (commodities not produced domestically) control implementation.

3.5 Commodity Price Block

The supply prices for commodities (P1) are defined as the volume share weighted sums of expenditure on domestically produced (*QD*) and imported (*QM*) commodities. These conditions derive from the first order conditions for the quantity equations for the composite commodities (*QQ*) above.[18] This equation is implemented for all commodities that are imported (*cm*) and for all commodities that are produced and consumed domestically (*cd*).

[18] Using the properties of linearly homogenous functions defined by reference to Eulers theorem.

Similarly, domestically produced commodities (*QXC*) are supplied to either or both the domestic and foreign markets (exported). The supply prices of domestically produced commodities (*PXC*) are defined as the volume share weighted sums of expenditure on domestically produced and exported (*QE*) commodities (P2). These conditions derive from the first order conditions for the quantity equations for the composite commodities (*QXC*) below.[19] This equation is implemented for all commodities that are produced domestically (*cx*), with a control to only include terms for exported commodities when there are exports (*ce*).

Commodity Price Block Equations

$$PQS_c = \frac{PD_c * QD_c + PM_c * QM_c}{QQ_c} \qquad \forall cd \textbf{ OR } cm \qquad \text{(P1)}$$

$$PXC_c = \frac{PD_c * QD_c + \left(PE_c * QE_c\right) \$ ce_c}{QXC_c} \qquad \forall cx \qquad \text{(P2)}$$

$$PQD_c = PQS_c * \left(1 + TS_c + TEX_c\right) \qquad \text{(P3)}$$

Domestic agents consume composite consumption commodities (*QQ*) that are aggregates of domestically produced and imported commodities. The prices of these composite commodities (*PQD*) are defined (P3) as the supply prices of the composite commodities plus *ad valorem* sales taxes (*TS*) and excise taxes (*TEX*). It is relatively straightforward to include additional commodity taxes.

3.6 Numéraire Price Block

The price block is completed by two price indices that can be used for price normalisation. Equation (N1) is for the consumer price index (*CPI*), which is defined as a weighted sum of composite commodity prices (*PQD*) in the current period, where the weights are the shares of each commodity in total demand (*comtotsh*). The domestic producer price index (*PPI*) is defined (N2) by reference to the supply prices for domestically produced commodities (*PD*) with weights defined as shares of the value of domestic output for the domestic market (*vddtotsh*).

19 Using the properties of linearly homogenous functions defined by reference to Eulers theorem.

Numéraire Block Equations

$$CPI = \sum_c comtotsh_c * PQD_c \tag{N1}$$

$$PPI = \sum_c vddtotsh_c * PD_c \tag{N2}$$

3.7 Production Block

3.7.1 Top level

The supply prices of domestically produced commodities are determined by purchaser prices of those commodities on the domestic and international markets. Adopting the assumption that domestic activities produce commodities in fixed proportions (*ioqxacqx*), the proportions provide a mapping (X1) between the supply prices of commodities and the (weighted) average activity prices (*PX*).[20]

In this model a four-stage production process is adopted, with the top level as a CES or Leontief function. If a CES is imposed for an activity the value of activity output can be expressed as the volume share weighted sums of the expenditures on inputs after allowing for the production taxes (*TX*), which are assumed to be applied *ad valorem* (X2). This requires the definition of aggregate prices for non-water intermediates (*PINT*); these are defined as the intermediate input-output coefficient weighted sum of the prices of non-water intermediate inputs (*PQD*), see (X3), whereby *ioqtdqd* are the intermediate input-output coefficients). A condition (*c*$*cnwat*) is imposed in the summation to guarantee, that only non-water commodities are included.

With CES technology the output by an activity, (*QX*) is determined by the aggregate quantities of factors used (*QVA*), i.e., aggregate value added, and aggregate intermediates used (*QINT*), where δ^x is the share parameter, ρ^{cx} is the substitution parameter and ADX is the efficiency variable (X5). Note how the efficiency/shift factor is defined as a variable and an adjustment mechanism is provided (X4), where *adxb* is the base value, *dabadx* is an absolute change in the base value, *ADXADJ* is an equiproportionate (multiplicative) adjustment factor, *DADX* is an additive adjustment factor and *adx*01 is a vector of zeros and

[20] In the special case of each activity producing only one commodity **and** each commodity only being produced by a single activity, which is the case in the reduced form model reported in Dervis *et al*., (1982), then the aggregation weights *ioqxacqx* correspond to an identity matrix.

non zeros used to scale the additive adjustment factor. The operation of this type of adjustment equation is explained below for the case of an import duty. The associated first order conditions defining the optimum ratios of value added to intermediate inputs can be expressed in terms of the relative prices of value added (*PVA*) and intermediate inputs (*PINT*), see (X6).

Production Block Equations: Top Level

$$PX_a = \sum_c ioqxacqx_{a,c} * PXAC_{a,c} \tag{X1}$$

$$PX_a * (1 - TX_a) * QX_a = (PVA_a * QVA_a) + (PINT_a * QINT_a) \tag{X2}$$

$$PINT_a = \sum_{c\$cnwat_c} \left(ioqtdqd_{c,a} * PQD_c \right) \tag{X3}$$

$$AD_a^X = \left[\left(adxb_a + dabadx_a \right) * ADXADJ \right] + \left(DADX * adx01_a \right) \tag{X4}$$

$$QX_a = AD_a^X \left(\delta_a^x * QVA_a^{-\rho_a^{cx}} + \left(1 - \delta_a^x\right) * QINT_a^{-\rho_a^{cx}} \right)^{-\frac{1}{\rho_a^{cx}}} \qquad \forall aqx_a \tag{X5}$$

$$\frac{QVA_a}{QINT_a} = \left[\frac{PINT_a}{PVA_a} * \frac{\delta_a^x}{\left(1 - \delta_a^x\right)} \right]^{\frac{1}{\left(1+\rho_a^{cx}\right)}} \qquad \forall aqx_a \tag{X6}$$

$$QVA_a = ioqvaqx_a * QX_a \qquad \forall aqxn_a \tag{X7a}$$

$$QINT_a = ioqintqx_a * QX_a \qquad \forall aqxn_a \tag{X7b}$$

With Leontief technology at the top level the aggregate quantities of factors used (*QVA*), i.e., aggregate value added, and intermediates used (*QINT*), are determined by simple aggregation functions, (X7a) and (X7b), where *ioqvaqx* and *ioqintqx* are the (fixed) volume shares of *QVA* and *QINT* (respectively) in *QX*. The choice of top level aggregation function is controlled by the membership of the set *aqx*, with the membership of *aqxn* being the complement of *aqx*.

3.7.2 Second level

There are two arms to the second level production nest. For aggregate value added (*QVA*) the production function can be a multi-factor CES function. For activities with CES setup on the second level (*afx*) (X9) *QVA* is based on the sum of consumed factors (*FD*) multiplied by the activity specific factor use efficiency (*ADFD*).Thereby δ^{va} is a share parameter, ρ^{va} is a substitution parameter and $ADVA_a$ is an efficiency factor. Again the efficiency/shift factor is defined as a variable with an adjustment mechanism (X8), where *advab* is the base values, *dabadva* is an absolute change in the base value, *ADVAADJ* is an equiproportionate (multiplicative) adjustment factor, *DADVA* is an additive adjustment factor and *adva*01 is a vector of zeros and non zeros used to scale the additive adjustment factor. For activities which are consuming water and/or land (conditioned by \$ $\delta^{va}_{"cnw"}$) the production function is expanded by an additional term for the land-water-composite (*QNW*) multiplied by its share parameter $\delta^{va}_{"cnw"}$.

The associated first order conditions for profit maximisation determine the wage rate of factors (*WF*) (X10) and, for those activities it applies to, the price of the land-water aggregate (*PNW*) (X11). Thereby the ratio of factor payments to factor *f* from activity *a* (*WFDIST*) is included to allow for non-homogenous factors, and is derived directly from the first order condition for profit maximisation as equality between the wage rates for each factor in each activity and the values of the marginal products of those factors in each activity. For activities with Leontief production technology on the second level (*afxn*), factor and aggregated land-water quantities (*FD* and *QNW*) as well as the price of value added (*PVA*) are determined by simple aggregation functions (X12 to X14) with the help of share parameters *io***.

On the second arm of the second level production nest (X15) intermediate input demand (*QINTD*) is defined as the product of the fixed (Leontief) input coefficients of demand for commodity *c* by activity *a* (*ioqtdqd*), multiplied by the quantity of activity intermediate input (*QINT*) to which the consumption of water commodities by activities (*QWAT2*) is added.

Production Block Equations: Second Level

$$AD_a^{VA} = \left[\left(advab_a + dabadva_a\right) * ADVAADJ\right] + \left(DADVA * adva01_a\right) \tag{X8}$$

$$QVA_a = AD_a^{VA} * \begin{bmatrix} \sum\limits_{f2\$\delta^{va}_{f2,a}} \delta^{va}_{f2,a} * AD^{FD}_{f2,a} * FD_{f2,a}^{-\rho_a^{cva}} \\ +(\delta^{va}_{"cnw",a} * QNW_a^{-\rho_a^{cva}})\$\delta^{va}_{"cnw",a} \end{bmatrix}^{-1/\rho_a^{cva}} \qquad \forall afx_a \tag{X9}$$

$$WF_{f2} * WFDIST_{f2,a} * \left(1 + TF_{f2,a}\right) = PVA_a * QVA_a$$
$$* \left[\begin{array}{l} \sum_{f2p\$\delta^{va}_{f2p,a}} \delta^{va}_{f2p,a} * AD^{FD}_{f2,a} * FD^{-\rho^{cva}_a}_{f2p,a} \\ +(\delta^{va}_{"cnw",a} * QNW_a^{-\rho^{cva}_a})\$\delta^{va}_{"cnw",a} \end{array} \right]^{-1} * \delta^{va}_{f2,a} * AD^{FD}_{f2,a}{}^{-\rho^{cva}_a} * FD_{f2,a}^{\left(-\rho^{cva}_a - 1\right)}$$

$\forall afx_a$

(X10)

$$PNW_a = PVA_a * QVA_a$$
$$* \left[\sum_{f2p\$\delta^{va}_{f2p,a}} \delta^{va}_{f2p,a} * AD^{FD}_{f2p,a} * FD^{-\rho^{cva}_a}_{f2p,a} + \delta^{va}_{"cnw",a} * QNW_a^{-\rho^{cva}_a} \right]^{-1}$$
$$* \delta^{va}_{"cnw",a} * QNW_a^{\left(-\rho^{cva}_a - 1\right)}$$

$\forall afx_a$ (X11)

$$FD_{f2,a} * AD^{FD}_{f2,a} = ioffqva_{f2,a} * QVA_a \qquad \forall afxn_a \qquad \text{(X12)}$$

$$QNW_a = \sum_{cwat} ioqnwqva_a * QVA_a \qquad \forall afxn_a \qquad \text{(X13)}$$

$$PVA_a = PNW_a * ioqnwqva_a + \sum_{f2} ioffqva_{f2,a} * WF_{f2} * WFDIST_{f2,a} * (1 + TF_{f2,a}) \qquad \forall afxn_a \qquad \text{(X14)}$$

$$QINTD_c = \sum_a ioqtdqd_{c,a} * QINT_a + \sum_a QWAT2_{c,a} \qquad \text{(X15)}$$

3.7.3 *Third level*

On the third level land enters the production function. Equation X16 is a CES production function forming the land-water aggregate (*QNW*). It holds for all activities which use land (*an*) and which allow for the substitution (*af3x*) between the water aggregate and land. It is structurally similar to equation X9 on level 2 [21] and so are the first order conditions for the prices of land (X17) and water-aggregate (X18), which resemble equations X10 and X11. For activities which do not consume water, the water term in equations X16 and X17 as well as equation X18 are dropped (conditioned by $\$\delta^{nw}_{"cwat",a}$).

Equations X19 to X21 represent the case of no substitution (Leontief production technology) (*af3xn*). Thereby equation X19 and X20 calculate factor and aggregated water

[21] The only exception is the shift parameter (atnw) which is not an endogenous variable in this case. This reflects the expectation that there will be no endogenously determined changes in this shift factor.

demand quantities (*FD* and *QWAT*) as shares of *QNW* and X21 sums factor and aggregate water prices proportionally to form *PNW*.

For activities, which do not consume land (*ann*) *iocwatqnw* is 1 and *iof3qnw* is 0. This guarantees that quantity and price of the aggregate (*QNW* and *PNW*) are equal to the quantity and price of the water aggregate (*QWAT* and *PWAT*) (X20 and X21).

Production Block Equations: Third Level

$$QNW_a = at_a^{nw} * \begin{bmatrix} \sum\limits_{f3\$\delta^{nw}_{f3p,a}} \delta^{nw}_{f3,a} * AD^{FD}_{f3,a} * FD_{f3,a}^{-\rho_a^{nw}} \\ +(\delta^{nw}_{"cwat",a} * QWAT_a^{-\rho_a^{nw}})\$\delta^{nw}_{"cwat",a} \end{bmatrix}^{-1/\rho_a^{nw}} \quad \forall af3x_a \text{ \textbf{AND} } an_a \tag{X16}$$

$$\begin{aligned} WF_{f3} * WFDIST_{f3,a} * \left(1+TF_{f3,a}\right) &= PNW_a * QNW_a \\ &* \begin{bmatrix} \sum\limits_{f3p\$\delta^{nw}_{f3p,a}} \delta^{nw}_{f3p,a} * AD^{FD}_{f3p,a} * FD_{f3p,a}^{-\rho_a^{nw}} \\ +(\delta^{nw}_{"cwat",a} * QWAT_a^{-\rho_a^{nw}})\$\delta^{nw}_{"cwat",a} \end{bmatrix}^{-1} * \delta^{nw}_{f3p,a} * AD^{FD\ -\rho_a^{nw}}_{f3,a} * FD_{f3,a}^{\left(-\rho_a^{nw}-1\right)} \\ &\forall af3x_a \end{aligned} \tag{X17}$$

$$\begin{aligned} PWAT_a &= PNW_a * QNW_a \\ &* \begin{bmatrix} \sum\limits_{f3p\$\delta^{nw}_{f3p,a}} \delta^{nw}_{f3p,a} * AD^{FD}_{f3p,a} * FD_{f3p,a}^{-\rho_a^{nw}} \\ +\delta^{nw}_{"cwat",a} * QWAT_a^{-\rho_a^{nw}} \end{bmatrix}^{-1} * \delta^{nw}_{"cwat",a} * QWAT_a^{\left(-\rho_a^{nw}-1\right)} \\ &\forall af3x_a \end{aligned} \tag{X18}$$

$$FD_{f3,a} * AD^{FD}_{f3,a} = iof3qnw_{f3,a} * QNW_a \quad \forall af3xn_a \tag{X19}$$

$$QWAT_a = iocwatqnw_a * QNW_a \quad \forall af3xn_a \text{ \textbf{OR} } ann_a \tag{X20}$$

$$\begin{aligned} PNW_a &= PWAT_a * iocwatqnw_a \\ &+ \sum_{f3} iof3qnw_{f3,a} * WF_{f3} * WFDIST_{f3,a} * (1+TF_{f3,a}) \ \forall af3xn_a \text{ \textbf{OR} } ann_a \end{aligned} \tag{X21}$$

3.7.4 Fourth Level

The lowest level of the production nest aggregates water commodities and water factors again for the two cases of CES (*af4x*) and Leontief (*af4xn*) production technology.

For activities with CES technology and which use water commodities or factors the production function is X22, with the first order conditions X23 for water factor prices and X24 for water commodity prices, analogous to X16 to X18 in the third nest. As the water commodity price (*PQD*) is defined over the set *cwat* only, the price ratio (*PQDDIST*) is included in X24 to allow for price discrimination for payments to water commodity *cwat* from activity *a*, similar to (*WFDIST*) on the factor price side.

In case Leontief technology is assumed equation X25 to X27 are applied. Thereby X25 holds for activities which consume water commodities (*acwat*) and X26 is applied for activities which use water factors (*afwat*). In both cases quantities of water commodities (*QWAT2*) or factors (*FD*) are simple shares of the water aggregate (*QWAT*). Also the price of the water aggregate is formed by a summation of the weighted shares of water commodity and/or factor input prices, including tax rates (*TWATA* and *TF*, respectively) (X27).

If additional levels of nesting are required then it is only necessary to add additional primal and first-order conditions that will have the same structures as for the third and fourth level nests but with appropriately revised set identifiers.[22]

Also the allocation of production factors on the various nesting levels can be varied. Currently this is done in an Excel workbook that contains the sets and data used to calibrate the model.

Production Block Equations: Fourth Level

$$QWAT_a = at_a^{wat} * \left[\begin{array}{l} \sum_{f4\$\delta_{f4,a}^{wat}} \delta_{f4,a}^{wat} * AD_{f4,a}^{FD} * FD_{f4,a}^{-\rho_a^{wat}} \\ + \sum_{cwat} \delta_{cwat,a}^{wat} * QWAT2_{cwat,a}^{-\rho_a^{wat}} \end{array} \right]^{-1/\rho_a^{wat}} \qquad \forall af4x_a \textbf{ AND } awat_a \qquad \text{(X22)}$$

$$\begin{aligned} WF_{f4} * WFDIST_{f4,a} * \left(1 + TF_{f4,a}\right) = PWAT_a * QWAT_a \\ * \left[\begin{array}{l} \sum_{f4p\$\delta_{f4p,a}^{wat}} \delta_{f4p,a}^{wat} * AD_{f4p,a}^{FD} * FD_{f4p,a}^{-\rho_a^{wat}} \\ + \sum_{cwatp\$\delta_{cwatp,a}^{wat}} \delta_{cwatp,a}^{wat} * QWAT2_{cwatp,a}^{-\rho_a^{wat}} \end{array} \right]^{-1} \\ * \delta_{f4p,a}^{wat} * AD_{f4,a}^{FD\ -\rho_a^{wat}} * FD_{f4,a}^{\left(-\rho_a^{wat}-1\right)} \qquad \forall af4x_a \end{aligned} \qquad \text{(X23)}$$

22 The only tricky parts are the derivation of the set mappings and the extensions to the code for calibrating the parameters.

$$PQD_{cwat} * PQDDIST_{cwat,a} * (1 + TWATA_{cwat,a}) = PWAT_a * QWAT_a$$
$$* \left[\sum_{f4p\$\delta^{wat}_{f4p,a}} \delta^{wat}_{f4p,a} * AD^{FD}_{f4p,a} * FD^{-\rho^{wat}_a}_{f4p,a} + \sum_{cwatp\$\delta^{wat}_{cwatp,a}} \delta^{wat}_{cwatp,a} * QWAT2^{-\rho^{wat}_a}_{cwatp,a} \right]^{-1} \quad \text{(X24)}$$
$$* \delta^{wat}_{cwatp,a} * QWAT2^{\left(-\rho^{wat}_a - 1\right)}_{cwat,a} \qquad \forall af4x_a$$

$$QWAT2_{c,a} = ioqwat_{c,a} * QWAT_a \qquad \forall af4xn_a \ \textbf{AND}\ acwat_a \quad \text{(X25)}$$

$$FD_{f4,a} * AD^{FD}_{f4,a} = iof4aggf4_{f4,a} * QWAT_a \qquad \forall af4xn_a \ \textbf{AND}\ afwat_a \quad \text{(X26)}$$

$$PWAT_a = \sum_{c\$cwat_c} ioqwat_{c,a} * PQD_c * PQDDIST_{c,a} * (1 + TWATA_{c,a})$$
$$+ \sum_{f4} iof4aggf4_{f4agg,f4,a} * WF_{f4} * WFDIST_{f4,a} * (1 + TF_{f4,a}) \quad \text{(X27)}$$
$$\forall af4xn_a$$

3.7.5 Commodity Outputs

Equation X28 aggregates the commodity outputs by each activity (*QXAC*) to form the composite supplies of each commodity (*QXC*). The default assumption is that when a commodity is produced by multiple activities it is differentiated by reference to the activity that produces the commodity; this is achieved by defining total production of a commodity as a CES aggregate of the quantities produced by each activity. This provides a practical/modelling solution for two typical situations; first, where there are quality differences between two commodities that are notionally the same, e.g., modern digital vv disposable cameras, and second, where the mix of commodities within an aggregate differ between activities, e.g., a cereal grain aggregate made up of wheat and maize (corn) where different activities produce wheat and maize in different ratios. This assumption of imperfect substitution is implemented by a CES aggregator function with ad^{xc} as the shift parameter, δxc as the share parameter and ρ^{xc} as the elasticity parameter.

The matching first order condition for the optimal combination of commodity outputs is therefore given by (X29), where *PXAC* are the prices of each commodity produced by each activity. Note how, as with the case of the value added production function two formulations are given for the first-order conditions and the second version is the default version used in the model. Further note that the efficiency/shift factor is in this case declare as a parameter;

this reflects the expectation that there will be no endogenously determined changes in these shift factors.

Production Block Equations: Commodity Outputs

$$QXC_c = ad_c^{xc} * \left[\sum_{a\$\delta_{a,c}^{xc}} \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}} \right]^{-1/\rho_c^{xc}} \qquad \forall cx_c \textbf{ AND } cxac_c \tag{X28}$$

$$\begin{aligned} PXAC_{a,c} &= PXC_c * ad_c^{xc} * \left[\sum_{a\$\delta_{a,c}^{xc}} \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}} \right]^{-\left(\frac{1+\rho_c^{xc}}{\rho_c^{xc}}\right)} * \delta_{a,c}^{xc} * QXAC_{a,c}^{\left(-\rho_c^{xc}-1\right)} \\ &= PXC_c * QXC_c * \left[\sum_{a\$\delta_{a,c}^{xc}} \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}} \right]^{-\left(\frac{1+\rho_c^{xc}}{\rho_c^{xc}}\right)} * \delta_{a,c}^{xc} * QXAC_{a,c}^{\left(-\rho_c^{xc}-1\right)} \end{aligned} \qquad \forall cxac_c \tag{X29}$$

$$QXC_c = \sum_a QXAC_{a,c} \qquad \forall cx_c \textbf{ AND } cxacn_c \tag{X30}$$

$$PXAC_{a,c} = PXC \qquad \forall cxacn_c . \tag{X31}$$

$$QXAC_{a,c} = ioqxacqx_{a,c} * QX_a \tag{X32}$$

However there are circumstances where perfect substitution may be a more appropriate assumption given the characteristics of either or both of the activity and commodity accounts. Thus an alternative specification for commodity aggregation is proved where commodities produced by different activities are modelled as perfect substitutes, (X30), and the matching price condition therefore requires that *PXAC* is equal to *PXC* for relevant commodity activity combinations (X31). The choice of aggregation function is controlled by the membership of the set *cxac*, with the membership of *cxacn* being the complement of *cxac*.

Finally the output to commodity supplies, where the 'weights' (*ioqxacqx*) identify the amount of each commodity produced per unit of output of each activity (X32). This equation not only captures the patterns of secondary production it also provides the market clearing conditions for equality between the supply and demand of domestic output.

3.8 Factor Block

There are two sources of income for factors. First there are payments to factor accounts for services supplied to activities, i.e., domestic value added, and second there are payments to domestic factors that are used overseas, the value of these are assumed fixed in terms of the foreign currency. Factor incomes (*YF*) are therefore defined as the sum of all income to the factors across all activities (F1).

Factor Block Equations

$$YF_f = \left(\sum_a WF_f * WFDIST_{f,a} * FD_{f,a} \right) + \left(factwor_f * ER \right) \tag{F1}$$

$$YFDISP_f = \left(YF_f * \left(1 - deprec_f \right) \right) * \left(1 - TYF_f \right) \tag{F2}$$

Before distributing factor incomes to the institutions that supply factor services allowance is made for depreciation rates (*deprec*) and factor (income) taxes (*TYF*) so that factor income for distribution (*YFDISP*) is defined (F2).

3.9 Household Block

3.9.1 Household Income

Households receive income from a variety of sources (H1). Factor incomes are distributed to households as fixed proportions (*hovash*) of the distributed factor income for all factors owned by the household, plus inter household transfers (*HOHO*), distributed payments/dividends from incorporated enterprises (*HOENT*) and real transfers from government (*hogovconst*) that are adjustable using a scaling factor (*HGADJ*) and transfers from the rest of the world (*howor*) converted into domestic currency units.

3.9.2 Household Expenditure

Inter household transfers (*HOHO*) are defined (H2) as a fixed proportions of household income (*YH*) after payment of direct taxes and savings, and then household consumption expenditure (*HEXP*) is defined as household income after tax income less savings and transfers to other households (H3).

Household Block Equations

$$YH_h = \left(\sum_f hovash_{h,f} * YFDISP_f\right) + \left(\sum_{hp} HOHO_{h,hp}\right)$$
$$+HOENT_h + \left(hogovconst_h * HGADJ * CPI\right)$$
$$+\left(howor_h * ER\right) \tag{H1}$$

$$HOHO_{h,hp} = hohosh_{h,hp} * \left(YH_h * \left(1 - TYH_h\right)\right) * \left(1 - SHH_h\right) \tag{H2}$$

$$HEXP_h = \left(\left(YH_h * \left(1 - TYH_h\right)\right) * \left(1 - SHH_h\right)\right) - \left(\sum_{hp} HOHO_{hp,h}\right) \tag{H3}$$

$$QCD_{c,h} * PQD_c = PQD_c * qcdconst_{c,h} + \sum_h beta_{c,h}$$
$$* \left(HEXP_h - \sum_{cp}\left(PQD_{cp} * qcdconst_{cp,h}\right)\right) \tag{H4}$$

Households are then assumed to maximise utility subject to Stone-Geary utility functions. In a Stone-Geary utility function household consumption demand consists of two components; ‘subsistence’ demand (*qcdconst*) and ‘discretionary’ demand, and the equation must therefore capture both elements. This can be written as (H4) where discretionary demand is defined as the marginal budget shares (*beta*) spent on each commodity out of ‘uncommitted’ income, i.e., household consumption expenditure less total expenditure on ‘subsistence’ demand. If the user wants to use Cobb-Douglas utility function this can be achieved by setting the Frisch parameters equal to minus one and all the income elasticities of demand equal to one (the model code includes documentation of the calibration steps).

3.10 Enterprise Block

3.10.1 Enterprise Income

Similarly, income to enterprises (EN1) comes from the share of distributed factor incomes accruing to enterprises (*entvash*) and real transfers from government (*entgovconst*), which are adjustable using a scaling factor (*EGADJ*) and the rest of the world (*entwor*); all converted in to domestic currency units.

Enterprise Block Equations

$$YE_e = \left(\sum_f entvash_{e,f} * YFDISP_f \right) + \left(entgovconst_e * EGADJ * CPI \right) + \left(entwor_e * ER \right) \tag{EN1}$$

$$QED_{c,e} = qedconst_{c,e} * QEADJ \tag{EN2}$$

$$HOENT_{h,e} = hoentsh_{h,e} * \left(\begin{array}{l} \left(YE_e * \left(1 - TYE_e\right)\right) * \left(1 - SEN_e\right) \\ - \sum_c \left(QED_{c,e} * PQD_c \right) \end{array} \right) \tag{EN3}$$

$$GOVENT_e = goventsh_e * \left(\begin{array}{l} \left(YE_e * \left(1 - TYE_e\right)\right) * \left(1 - SEN_e\right) \\ - \sum_c \left(QED_c * PQD_c \right) \end{array} \right) \tag{EN4}$$

$$VED_e = \left(\sum_c QED_{c,e} * PQD_c \right) \tag{EN5}$$

3.10.2 Enterprise Expenditure

The consumption of commodities by enterprises (*QED*) is defined (EN2) in terms of fixed volumes (*qedconst*), which can be varied via the volume adjuster (*QEDADJ*), and associated with any given volume of enterprise final demand there is a level of expenditure (*VED*); this is defined by (EN5) and creates an option for the macroeconomic closure conditions that distribute absorption across domestic institutions (see below).

If *QEDADJ* is made flexible, then *qedconst* ensures that the quantities of commodities demanded are varied in fixed proportions; clearly this specification of demand is not a consequence of a defined set of behavioural relationships, as was the case for households, which reflects the difficulties inherent to defining utility functions for non-household institutions.[23] If *VED* is fixed then the volume of consumption by enterprises (*QED*) must be allowed to vary, via the variable *QENTDADJ*.

The incomes to households from enterprises, which are assumed to consist primarily of distributed profits/dividends, are defined by (EN3), where *hoentsh* are defined as fixed shares

[23] Some models use some form of utility function for non-household domestic institutions. If a Cobb-Douglas function is used then value shares of demand by institution are held constant, other choices typical alter value and quantity shares.

of enterprise income after payments of direct/income taxes, savings and consumption expenditure. Similarly the income to government from enterprises, which is assumed to consist primarily of distributed profits/dividends on government owned enterprises, is defined by (EN4), where *goventsh* is defined as a fixed share of enterprise income after payments of direct/income taxes, savings and consumption expenditure.

3.11 Government Block

3.11.1 Tax Rates

All tax rates are variables in this model. The tax rates in the base solution are defined as parameters, e.g., tmb_c are the import duties by commodity c in the base solution, and the equations then allow for varying the tax rates in 5 different ways. For each tax instrument there are four methods that allow adjustments to the tax rates; two of the methods use variables that can be solved for optimum values in the model according to the choice of closure rule and two methods allow for deterministic adjustments to the structure of the tax rates. The operation of this method is discussed in detail only for the equations for import duties while the other equations are simply reported.

Import duty tax rates are defined by (GT1), where tmb_c is the vector of import duties in the base solution, $dabtm_c$ is a vector of absolute changes in the vector of import duties, *TMADJ* is a variable whose initial value is ONE, *DTM* is a variable whose initial value is ZERO and $tm01_c$ is a vector of zeros and non zeros. In the base solution the values of $tm01_c$ and $dabtm_c$ are all ZERO and *TMADJ* and *DTM* are fixed as their initial values – a closure rule decision – then the applied import duties are those from the base solution. Now the different methods of adjustment can be considered in turn:

1. If *TMADJ* is made a variable, which requires the fixing of another variable, and all other initial conditions hold then the solution value for *TMADJ* yields the optimum equiproportionate change in the import duty rates necessary to satisfy model constraints, e.g., if *TMADJ* equals 1.1 then all import duties are increased by 10%.
2. If any element of *dabtm* is non zero and all the other initial conditions hold, then an absolute change in the initial import duty for the relevant commodity can be imposed using *dabtm*, e.g., if *tmb* for one element of *c* is 0.1 (a 10% import duty) and *dabtm* for that element is 0.05, then the applied import duty is 0.15 (15%).

3. If *TMADJ* is a variable, any elements of *dabtm* are non zero and all other initial conditions hold then the solution value for *TMADJ* yields the optimum equiproportionate change in the applied import duty rates.
4. If *DTM* is made a variable, which requires the fixing of another variable, AND at least one element of *tm*01 is equal to ONE then the subset of elements of *c* identified by *tm*01 are allowed to (additively) increase by an equiproportionate amount determined by the solution value for *DTM*. Note how it is necessary to both 'free' a variable and give values to a parameter for a solution to emerge.
5. If *DTM* is made a variable AND at least one element of *tm*01 is NOT equal to ZERO then the subset of elements of *c* identified by *tm*01 are allowed to (additively) increase by an equiproportionate amount determined by the solution value for *DTM* times the values of *tm*01. Note how using different values of *tm*01 for different members of the set c will cause commodity specific changes in import duty rates, e.g., the values of *tm*01 for food commodities could be set to 0.5 and manufactured commodities set to 1, with the result that the additive increases in import duties for food commodities would be half those for other manufactured commodities.

This combination of alternative adjustment methods covers a range of common tax rate adjustment used in many applied applications while being flexible and easy to use.

Tax Rate Block Equations

$$TM_c = \left(\left(tmb_c + dabtm_c\right) * TMADJ\right) + \left(DTM * tm01_c\right) \qquad \text{(GT1)}$$

$$TE_c = \left(\left(teb_c + dabte_c\right) * TEADJ\right) + \left(DTE * te01_c\right) \qquad \text{(GT2)}$$

$$TS_c = \left(\left(tsb_c + dabts_c\right) * TSADJ\right) + \left(DTS * ts01_c\right) \qquad \forall cd_c \ \textbf{OR} \ cm_c \qquad \text{(GT3)}$$

$$TEX_c = \left(\left(texb_c + dabtex_c\right) * TEXADJ\right) + \left(DTEX * tex01_c\right) \qquad \forall cd_c \ \textbf{OR} \ cm_c \qquad \text{(GT4)}$$

$$TX_a = \left(\left(txb_a + dabtx_a\right) * TXADJ\right) + \left(DTX * tx01_a\right) \qquad \text{(GT5)}$$

$$TF_{ff,a} = \left(\left(tfb_{ff,a} + dabtf_{ff,a}\right) * TFADJ\right) + \left(DTF * tf01_{ff,a}\right) \qquad \text{(GT6)}$$

$$TYF_f = \left(\left(tyfb_f + dabtyf_f\right) * TYFADJ\right) + \left(DTYF * tyf01_f\right) \qquad \text{(GT7)}$$

$$TYH_h = \left(\left(tyhb_h + dabtyh_h\right) * TYHADJ\right) + \left(DTYH * DTY * tyh01_h\right) \qquad \text{(GT8)}$$

$$TYE_e = \left(\left(tyeb_e + dabtye_e\right) * TYEADJ\right) + \left(DTYE * DTY * tye01_e\right) . \tag{GT9}$$

$$TWAT_c = ((twatb_c + dabtwat_c) * TWATADJ) + (DTWAT * twat01_c) \tag{GT10}$$

$$TWATA_{c,a} = ((twatab_{c,a} + dabtwata_{c,a}) * TWATAADJ) + (DTWATA * twata01_{c,a}) \tag{GT11}$$

Export tax rates are defined by (GT2), where teb_c is the vector of export duties in the base solution, $dabte_c$ is a vector of absolute changes in the vector of export duties, *TEADJ* is a variable whose initial value is ONE, *DTE* is a variable whose initial value is ZERO and $te01_c$ is a vector of zeros and non zeros. Sales tax rates are defined by (GT3), where tsb_c is the vector of sales tax rates in the base solution, $dabts_c$ is a vector of absolute changes in the vector of sales taxes, *TSADJ* is a variable whose initial value is ONE, *DTS* is a variable whose initial value is ZERO and $ts01_c$ is a vector of zeros and non zeros. Excise tax rates are defined by (GT4), where $texb_c$ is the vector of excise tax rates in the base solution, $dabtex_c$ is a vector of absolute changes in the vector of import duties, *TEXADJ* is a variable whose initial value is ONE, *DTEX* is a variable whose initial value is ZERO and $tex01_c$ is a vector of zeros and non zeros.

Indirect tax rates on production are defined by (GT5), where txb_c is the vector of production taxes in the base solution, $dabtx_c$ is a vector of absolute changes in the vector of production taxes, *TXADJ* is a variable whose initial value is ONE, *DTX* is a variable whose initial value is ZERO and $tx01_c$ is a vector of zeros and non zeros.

Taxes on factor use by each factor and activity are defined by (GT6), where $tfb_{ff,a}$ is the matrix of factor use tax rates in the base solution, $dabtf_{ff,a}$ is a matrix of absolute changes in the matrix of factor use taxes, *TFADJ* is a variable whose initial value is ONE, *DTF* is a variable whose initial value is ZERO and $tf01_{ff,a}$ is a matrix of zeros and non zeros. An important feature of taxes on factor use is they enter into the first order conditions of the production functions that determine factor input choices by activities.

Factor income tax rates[24] are defined by (GT7), where $tyfb_f$ is the vector of factor income taxes in the base solution, $dabtyy_f$ is a vector of absolute changes in the vector of factor income taxes, *TYFADJ* is a variable whose initial value is ONE, *DTYF* is a variable whose initial value is ZERO and $tyf01_f$ is a vector of zeros and non zeros. Household income

24 These are defined as taxes on factor incomes that are independent of the activity that employs the factor. They could include social security type payments.

tax rates are defined by (GT8), where $tyhb_h$ is the vector of household income tax rates in the base solution, $dabtyh_h$ is a vector of absolute changes in the vector of income tax rates, *TYFADJ* is a variable whose initial value is ONE, *DTYH* and *DTY* [25]are variables whose initial values are ZERO and $tyh01_c$ is a vector of zeros and non zeros. Enterprise income tax rates are defined by (GT9), where $tyeb_e$ is the vector of enterprise income tax rates in the base solution, $dabtye_e$ is a vector of absolute changes in the income tax rates, *TYEADJ* is a variable whose initial value is ONE, *DTYE* and *DTY* are variables whose initial values are ZERO and $tye01_e$ is a vector of zeros and non zeros.

Finally two water specific tax instruments are added to the model, to allow for different pricing policy scenarios (GT10 and GT11). TWAT is a water commodity tax set up accordingly to the sales tax (GT3) and allows to drive a margin between production costs and consumer price of water commodities. Thereby $twatb_c$ is the vector of water commodity tax rates in the base solution, $dabtwat_c$ is a vector of absolute changes in the water commodity tax rates, *TWATADJ* is a variable whose initial value is ONE, *DTWAT* is a variable whose initial value is ZERO and $twat01_c$ is a vector of zeros and non zeros. *TWATA* is a water user specific tax rate, allowing to charge differentiated prices from different water users. Thereby $twatab_{c,a}$ is the matrix of water use tax rates in the base solution, $dabtwata_{c,a}$ is a matrix of absolute changes in the matrix of water use taxes, *TWATAADJ* is a variable whose initial value is ONE, *DTWATA* is a variable whose initial value is ZERO and $tf01_{ff,a}$ is a matrix of zeros and non zeros. Similar to facter use taxes, water user taxes enter into the first order conditions of the production functions and by this determine factor input choices by activities.

3.11.2 Tax Revenues

Although it is not necessary to keep the tax revenue equations separate from other equations, e.g., they can be embedded into the equation for government income (*YG*), it does aid clarity and assist with implementing fiscal policy simulations, e.g., when seeking to fix total revenues from a tax instrument. For this model there are ten tax revenue equations. The patterns of tax rates are controlled by the tax rate variable equations. In all cases the tax rates can be negative indicating a ‘transfer’ from the government.

There are six tax instruments that are dependent upon expenditure on commodities, with each expressed as an *ad valorem* tax rate. Tariff revenue (*MTAX*) is defined (GR1) as the sum of the product of tariff rates (*TM*) and the value of expenditure on imports at world prices, the

---

[25] *DTY* is also included in GT9 and thereby allows for simultaneous changes in the income tax rate of households and enterprises.

revenue from export duties (*ETAX*) is defined (GR2) as the sum of the product of export duty rates (*TE*) and the value of expenditure on exports at world prices. The sale tax revenues (*STAX*) are defined (GR3) as the sum of the product of sales tax rates (*TS*) and the value of domestic expenditure on commodities, and excise tax revenues (*EXTAX*) are defined (GR4) as the sum of the product of excise tax rates (*TEX*) and the value of domestic expenditure on commodities.

Government Tax Revenue Block Equations

$$MTAX = \sum_c \left(TM_c * PWM_c * ER * QM_c \right) \tag{GR1}$$

$$ETAX = \sum_c \left(TE_c * PWE_c * ER * QE_c \right) \tag{GR2}$$

$$\begin{aligned} STAX &= \sum_c \begin{pmatrix} TS_c * PQS_c * \\ \left(QINTD_c + QCD_c + QENTD_c + QGD_c + QINVD_c + dstocconst_c \right) \end{pmatrix} \\ &= \sum_c \left(TS_c * PQS_c * QQ_c \right) \end{aligned} \tag{GR3}$$

$$EXTAX = \sum_c \left(TEX_c * PQS_c * QQ_c \right) \tag{GR4}$$

$$ITAX = \sum_a \left(TX_a * PX_a * QX_a \right) \tag{GR5}$$

$$FTAX = \sum_{f,a} \left(TF_{f,a} * WF_f * WFDIST_{f,a} * FD_{f,a} \right) \tag{GR6}$$

$$FYTAX = \sum_f \left(TYF_f * \left(YF_f * \left(1 - deprec_f \right) \right) \right) \tag{GR7}$$

$$DTAX = \sum_h \left(TYH_h * YH_h \right) + \sum_e \left(TYE_e * YE_e \right) \tag{GR8}$$

$$WATTAX = \sum_c (TWAT_c * PQS_c * QQ_c) \tag{GR9}$$

$$WATATAX = \sum_{c,a} (TWATA_{c,a} * PQD_c * PQDDIST_{c,a} * QWAT2_{c,a}) \tag{GR10}$$

There is a single tax on production (*ITAX*). As with other taxes this is defined (GR5) as the sum of the product of indirect tax rates (*TX*) and the value of output by each activity evaluated in terms of the activity prices (*PX*). In addition activities can pay taxes based on the value of employed factors – factor use taxes (*FTAX*). The revenue from these taxes is defined (GR6) as the sum of the product of factor income tax rates and the value of the factor services employed by each activity for each factor; the sum is over both activities and factors. These two taxes are the instruments most likely to yield negative revenues through the existence of production and/or factor use subsidies.

Income taxes are collected on both factors and domestic institutions. The income tax on factors (*FYTAX*) is defined (GR7) as the product of factor tax rates (*TYF*) and factor incomes for all factors, while those on institutions (*DTAX*) are defined (GR8) as the sum of the product of household income tax rates (*TYH*) and household incomes plus the product of the direct tax rate for enterprises (*TYE*) and enterprise income.

Revenue from water commodity taxes (*WATTAX*) is defined (GR9) as the sum of the product of water commodity tax rates (*TWAT*) and the value of domestic expenditure on water commodities. The revenue from the water user tax (*WATATAX*) is negative in most cases, as usually user groups are charged reduced water prices. It is defined (GR10) as the sum of the product of water user tax rates and the value of water commodities employed by each activity; the sum is over both activities and commodities.

3.11.3 Government Income

The sources of income to the government account (G1) are more complex than for other institutions. Income accrues from ten tax instruments; tariff revenues (*MTAX*), export duties (*ETAX*), sales taxes (*STAX*), excise taxes (*EXTAX*), production taxes (*ITAX*), factor use taxes (*FTAX*), factor income taxes (*FYTAX*) direct income taxes (*DTAX*), water commodity tax (*WATTAX*), and water user tax (*WATATAX*), which are defined in the tax equation block above. In addition the government can receive income as a share (*govvash*) of distributed factor incomes, distributed payments/dividends from incorporated enterprises (*GOVENT*) and transfers from abroad (*govwor*) converted in to domestic currency units. It would be relatively easy to subsume the tax revenue equations into the equation for government income, but they are kept separate to facilitate the implementation of fiscal policy experiments. Ultimately however the choice is a matter of personal preference.

3.11.4 Government Expenditure

The demand for commodities by the government for consumption (*QGD*) is defined (G2) in terms of fixed proportions (*qgdconst*) that can be varied with a scaling adjuster (*QGDADJ*), and associated with any given volume of government final demand there is a level of expenditure defined by (G3); this creates an option for the macroeconomic closure conditions that distribute absorption across domestic institutions (see below).

Hence, total government expenditure (*EG*) can be defined (G4) as equal to the sum of expenditure by government on consumption demand at current prices, plus real transfers to households ($hogovconst_h$) that can be adjusted using a scaling factor (*HGADJ*) and real transfers to enterprises ($entgovconst_e$) that can also be adjusted by a scaling factor (*EGADJ*) As with enterprises there are difficulties inherent to defining utility functions for a government.[26] Changing *QGDADJ*, either exogenously or endogenously, by allowing it to be a variable in the closure conditions, provides a means of changing the behavioural assumption with respect to the 'volume' of commodity demand by the government. If the value of government final demand (*VGD*) is fixed then government expenditure is fixed and hence the volume of consumption by government (*QGD*) must be allowed to vary, via the *QGDADJ* variable. If it is deemed appropriate to modify the patterns of commodity demand by the government then the components of $qgdconst_c$ must be changed.

Government Income and Expenditure Block Equations

$$\begin{aligned} YG = {} & MTAX + ETAX + STAX + EXTAX + FTAX + ITAX \\ & + FYTAX + DTAX + WATTAX + WATATAX \\ & + \left(\sum_f govvash_f * YFDISP_f \right) + GOVENT + \left(govwor * ER \right) \end{aligned} \tag{G1}$$

$$QGD_c = qgdconst_c * QGDADJ \tag{G2}$$

$$VGD = \left(\sum_c QGD_c * PQD_c \right) \tag{G3}$$

$$\begin{aligned} EG = {} & \left(\sum_c QGD_c * PQD_c \right) + \left(\sum_h hogovconst_h * HGADJ * CPI \right) \\ & + \left(\sum_e entgovconst_e * EGADJ * CPI \right) \end{aligned} \tag{G4}$$

[26] Some models use some form of utility function for non-household domestic institutions. If a Cobb-Douglas function is used then value shares of demand by institution are held constant, other choices typical alter value and quantity shares.

3.12 Kapital Block

3.12.1 Savings Block

The savings rates for households (*SHH* in I1) and enterprises (*SEN* in I2) are defined as variables using the same adjustment mechanisms used for tax rates; $shhb_h$ and $senb_e$ are the savings rates in the base solution, $dabshh_h$ and $dabsen_e$ are absolute changes in the base rates, *SHADJ* and *SEADJ* are multiplicative adjustment factors, *DSHH* and *DSEN* are additive adjustment factors and $shh01_h$ and $sen01_e$ are vectors of zeros and non zeros that scale the additive adjustment factors. However, each of the savings rates equations has two additional adjustment factors – SADJ and DS. These allow the user to vary the savings rates for households and enterprises in tandem; this is useful when the macroeconomic closure conditions require increases in savings by domestic institutions and it is not deemed appropriate to force all the adjustment on a single group of institutions.

Kapital Block Equations

$$SHH_h = \left(\left(shhb_h + dabshh_h\right) * SHADJ * SADJ\right) + \left(DSHH * DS * shh01_h\right) \tag{I1}$$

$$SEN_e = \left(\left(sen_e + dabsen_e\right) * SEADJ * SADJ\right) + \left(DSEN * DS * sen01_e\right) \tag{I2}$$

$$\begin{aligned} TOTSAV = & \sum_h \left(\left(YH_h * \left(1 - TYH_h\right)\right) * SHH_h\right) \\ & + \sum_e \left(\left(YE * \left(1 - TYE_e\right)\right) * SEN_e\right) \\ & + \sum_f \left(YF_f * deprec_f\right) + KAPGOV + \left(CAPWOR * ER\right) \end{aligned} \tag{I3}$$

$$QINVD_c = IADJ * qinvdconst_c \tag{I4}$$

$$INVEST = \sum_c \left(PQD_c * \left(QINVD_c + dstocconst_c\right)\right) \tag{I5}$$

Total savings in the economy are defined (I3) as shares (*SHH*) of households' after tax income, where direct taxes (*TYH*) have first call on household income, plus the allowances for depreciation at fixed rates (*deprec*) out of factor income, the savings by enterprises at fixed rates (*SEN*) out of after tax income, the government budget deficit/surplus (*KAPGOV*) and the current account deficit/surplus (*CAPWOR*). The last two terms of I3 – *KAPGOV* and *CAPWOR* - are defined below by equations in the market clearing block.

3.12.2 Investment Block

The same structure of relationships as for enterprises and government is adopted for investment demand (I4). The volumes of commodities purchased for investment are determined by the volumes in the base period (*qinvdconst*) and can be varied using the adjuster (*IADJ*).[27] Then value of investment expenditure (*INVEST*) is equal (I5) to the sum of investment demand valued at current prices plus the current priced value of stock changes (*dstocconst*) that are defined as being fixed, usually in volume terms at the levels in the base period. If *IADJ* is made variable then the volumes of investment demand by commodity will adjust equiproportionately, in the ratios set by *qinvdconst*, such as to satisfy the closure rule defined for the capital account. Changes to the patterns of investment demand require changes in the ratios of investment demand set by *qinvdconst*.

3.13 Foreign Institutions Block

The economy also employs foreign owned factors whose services must be recompensed. It is assumed that these services receive fixed proportions of the factor incomes available for distribution, (W1).

Foreign Institutions Block Equations

$$YFWOR_f = worvash_f * YFDISP_f \tag{W1}$$

3.14 Market Clearing Block

The market clearing equations ensure the simultaneous clearing of all markets. In this model there are six relevant markets: factor and commodity markets and enterprise, government, capital and rest of world accounts. Market clearing with respect to activities has effectively been achieved by (X20), wherein the supply and demand equality for domestically produced commodities was enforced, while the demand system and the specification of expenditure relationships ensures that the household markets are cleared.

The description immediately below refers to the default set of closure rules/market clearing conditions imposed for this model; a subsequent section explores alternative closure rule configurations available with this model.

[27] Some models use some form of utility function for non-household domestic institutions. If a Cobb-Douglas function is used then value shares of demand by institution are held constant, other choices typical alter value and quantity shares.

3.14.1 Account Closures

Adopting an initial assumption of full employment, which the model closure rules will demonstrate can be easily relaxed, amounts to requiring that the factor market is cleared by equating factor supplies (*FS*) for all factors with factor demands (*FD*) (C1).

Market clearing for the composite commodity markets requires that the supplies of the composite commodity (*QQ*) are equal to total of domestic demands for composite commodities, which consists of intermediate demand (*QINTD*), household (*QCD*), enterprise (*QED*) and government (*QGD*) and investment (*QINVD*) final demands and stock changes ($dstocconst_c$) (C2). Since the markets for domestically produced commodities are also cleared (X20) this ensures a full clearing of all commodity markets.

Making savings a residual for each account clears the two institutional accounts that are not cleared elsewhere – government and rest of the world. Thus the government account clears (C3) by defining government savings (*KAPGOV*) as the difference between government income and other expenditures, i.e., a residual. The rest of world account clears (C4) by defining the balance on the capital account (*CAPWOR*) as the difference between expenditure on imports, of commodities and factor services, and total income from the rest of the world, which includes export revenues and payments for factor services, transfers from the rest of the world to the household, enterprise and government accounts, i.e., it is a residual.

Account Closure Block Equations

$$FS_f = \sum_a FD_{f,a} \tag{C1}$$

$$QQ_c = QINTD_c + \sum_h QCD_{c,h} + \sum_e QED_{c,e} + QGD_c + QINVD_c + dstocconst_c \tag{C2}$$

$$KAPGOV = YG - EG \tag{C3}$$

$$\begin{aligned} CAPWOR = {} & \left(\sum_{cm} PWM_{cm} * QM_{cm}\right) + \left(\sum_f \frac{YFWOR_f}{ER}\right) \\ & - \left(\sum_{ce} PWE_{ce} * QE_{ce}\right) - \left(\sum_f factwor_f\right) \\ & - \left(\sum_h howor_h\right) - \left(\sum_e entwor_e\right) - govwor \end{aligned} \tag{C4}$$

3.14.2 Absorption Closure

The total value of domestic final demand (absorption) (*VFDOMD*) is defined (C5) as the sum of the expenditures on final demands by households and other domestic institutions (enterprises, government and investment).

It is also useful to express the values of final demand by each non-household domestic institution as a proportion of the total value of domestic final demand; this allows the implementation of what has been called a ‘balanced macroeconomic closure’.[28] Hence the share of the value of final demand by enterprises (C6) can be defined as a proportion of total final domestic demand, and similarly for government’s value share of final demand (C7) and for investment’s value share of final demand (C8).

Absorption Closure Block Equations

$$VFDOMD = \sum_c PQD_c * \left(\sum_h QCD_{c,h} + \sum_e QED_{c,e} + QGD_c + QINVD_c + dstocconst_c \right) \qquad \text{(C5)}$$

$$VEDSH_e = {VED_e}\big/{VFDOMD} \qquad \text{(C6)}$$

$$VGDSH = {VGD}\big/{VFDOMD} \qquad \text{(C7)}$$

$$INVESTSH = {INVEST}\big/{VFDOMD} \qquad \text{(C8)}$$

If the share variables (*VEDSH*, *VGDSH* and *INVESTSH*) are fixed then the quantity adjustment variables on the associated volumes of final demand by domestic non-household institutions (*QEDADJ*, *QGDADJ* and *IADJ* or *S*ADJ*) must be free to vary. On the other hand if the volume adjusters are fixed the associated share variables must be free so as to allow the value of final demand by ‘each’ institution to vary.

3.14.3 Slack

The final account to be cleared is the capital account. Total savings (*TOTSAV*), see I3 above, is defined within the model and hence there has been an implicit presumption in the description that the total value of investment (*INVEST*) is driven by the volume of savings. This is the market clearing condition imposed by (C9). But this market clearing condition

28 The adoption of such a closure rule for this class of model has been advocated by Sherman Robinson and is a feature, albeit implemented slightly differently, of the IFPRI standard model.

includes another term, *WALRAS*, which is a slack variable that returns a zero value when the model is fully closed and all markets are cleared, and hence its inclusion provides a quick check on model specification.

SLACK Block Equation

$$TOTSAV = INVEST + WALRAS \tag{C9}$$

3.15 Model Closure Conditions or Rules

In mathematical programming terms the model closure conditions are, at their simplest, a matter of ensuring that the numbers of equations and variables are consistent. However economic theoretic dimensions of model closure rules are more complex, and, as would be expected in the context of an economic model, more important. The essence of model closure rules is that they define important and fundamental differences in perceptions of how an economic system operates (see Sen, 1963; Pyatt, 1987; Kilkenny and Robinson, 1990). The closure rules can be perceived as operating on two levels; on a general level whereby the closure rules relate to macroeconomic considerations, e.g., is investment expenditure determined by the volume of savings or exogenously, and on a specific level where the closure rules are used to capture particular features of an economic system, e.g., the degree of intersectoral capital mobility.

This model allows for a range of both general and specific closure rules. The discussion below provides details of the main options available with this formulation of the model by reference to the accounts to which the rules refer.

3.15.1 Foreign Exchange Account Closure

The closure of the rest of the world account can be achieved by fixing either the exchange rate variable (AC1a) or the balance on the current account (AC1b). Fixing the exchange rate is appropriate for countries with a fixed exchange rate regime whilst fixing the current account balance is appropriate for countries that face restrictions on the value of the current account balance, e.g., countries following structural adjustment programmes. It is a common practice to fix a variable at its initial level by using the associated parameter, i.e., ***0, but it is possible to fix the variable to any appropriate value.

The model is formulated with the world prices for traded commodities declared as variables, i.e., PWM_c and PWE_c. If a strong small country assumption is adopted, i.e., the country is assumed to be a price taker on all world commodity markets, then all world prices will be fixed. When calibrating the model the world prices will be fixed at their initial levels, (AC1c), but this does not mean they cannot be changed as parts of experiments.

Foreign Exchange Market Closure Equations

$$ER = \overline{ER} \tag{AC1a}$$

$$CAPWOR = \overline{CAPWOR} \tag{AC1b}$$

$$\begin{aligned} PWE_c &= \overline{PWE_c} \\ PWM_c &= \overline{PWM_c} \end{aligned} \tag{AC1c}$$

$$PWE_{cedn} = \overline{PWE_{cedn}} \tag{AC1d}$$

However, the model allows for a relaxation of the strong small country assumption, such that the country may face a downward sloping demand curve for one or more of its export commodities. Hence the world prices of some commodities are determined by the interaction of demand and supply on the world market, i.e., they are variables. This is achieved by limiting the range of world export prices that are fixed to those for which there are no export demand function, (AC1d), by selecting membership of the set *cedn*.[29]

3.15.2 Capital Account Closure

To ensure that aggregate savings equal aggregate investment, the determinants of either savings or investment must be fixed. There are multiple ways of achieving this result. For instance this can be achieved by fixing either the saving rates for households or the volumes of commodity investment. This involves fixing either the savings rates adjusters (AC2a) or the investment volume adjuster (AC2c). Note that fixing the investment volume adjuster (AC2c) means that the value of investment expenditure might change due to changes in the prices of investment commodities (*PQD*). Note also that only one of the savings rate adjusters should be fixed; if *SADJ* is fixed the adjustment takes place through equiproportionate changes in the savings rates of households and enterprises, if *SHADJ* is fixed the adjustment takes place through equiproportionate changes in the savings rates of households, and if *SEADJ* is fixed the adjustment takes place through equiproportionate changes in the savings

[29] Practically membership of *cedn* is set by assigning a non zero value to the export demand elasticity in the model database. The set *ced* is then defined as a complement.

rates of enterprises. Alternatively savings rates can be adjusted through the additive adjustment factors (*DS*, *DSHH*, *DSEN*) with the same relationships between the savings rates of different classes of institutions (AC2b). Note that there are other sources of savings. The magnitudes of these other savings sources can also be changed through the closure rules (see below).

Fixing savings, and thus deeming the economy to be savings-driven, could be considered a Neo-Classical approach. Closing the economy by fixing investment could be construed as making the model reflect the Keynesian investment-driven assumption for the operation of an economy.

The model includes a variable for the value of investment (*INVEST*), which can also be used to close the capital account (AC2d). If *INVEST* is fixed in an investment driven closure, then the model will need to adjust the savings rates to maintain equilibrium between the value of savings (*TOTSAV*) and the fixed value of investment. This can only be achieved by changes in the volumes of commodities demanded for investment (*QINVD*) or their prices (*PQD*). But the prices (*PQD*) depend on much more than investment, hence the main adjustment must take place through the volumes of commodities demanded, i.e., *QINVD*, and therefore the volume adjuster (*IADJ*) must be variable, as must a savings rate adjuster (*SADJ*).

Capital Account Closure Equations

$$\begin{aligned} SADJ &= \overline{SADJ} \\ SHADJ &= \overline{SHADJ} \\ SEADJ &= \overline{SEADJ} \end{aligned} \qquad \text{(AC2a)}$$

$$\begin{aligned} DS &= \overline{DS} \\ DSHH &= \overline{DSHH} \\ DSEN &= \overline{DSEN} \end{aligned} \qquad \text{(AC2b)}$$

$$IADJ = \overline{IADJ} \qquad \text{(AC2c)}$$

$$INVEST = \overline{INVEST} \qquad \text{(AC2d)}$$

$$INVESTSH = \overline{INVESTSH} \qquad \text{(AC2e)}$$

Alternatively the share of investment expenditure in the total value of domestic final demand can be fixed, (AC2e), which means that the total value of investment is fixed by reference to the value of total final demand, which requires that the investment volumes must be free to vary, i.e., IADJ must be made variable. Otherwise the adjustment mechanisms follow the same processes as for fixing *INVEST* equal to some level.

3.15.3 Enterprise Account Closure

Fixing the volumes of commodities demand by enterprises, (AC3a), closes the enterprise account. Note that this rule allows the value of commodity expenditures by the enterprise account to vary, which *ceteris paribus* means that the value of savings by enterprises (*CAPENT*) and thus total savings (*TOTSAV*) vary. If the value of this adjuster is changed, but left fixed, this imposes equiproportionate changes on the volumes of commodities demanded.

If *QEDADJ* is allowed to vary then another variable must be fixed; the most likely alternative is the value of consumption expenditures by enterprises (*VED*) (AC3b). This would impose adjustments through equiproportionate changes in the volumes of commodities demanded, and would feed through so that enterprise savings (*CAPENT*) reflecting directly the changes in the income of enterprises (*YE*). Alternatively the share of enterprise expenditure in the total value of domestic final demand can be fixed, (AC3c), which means that the total value of enterprise consumption expenditure (*VED*) is fixed by reference to the value of total final demand, but otherwise the adjustment mechanisms follow the same processes as for fixing *VED* equal to some level.

Enterprise Account Closure Equations

$$QEDADJ = \overline{QEDADJ} \tag{AC3a}$$

$$VED = \overline{VED} \tag{AC3b}$$

$$VEDSH = \overline{VEDSH} \tag{AC3c}$$

$$HEADJ = \overline{HEADJ} \tag{AC3d}$$

Finally the scaling factor for enterprise transfers to households (*HEADJ*) needs fixing (AC3d).

3.15.4 Government Account Closure

The closure rules for the government account are slightly more tricky because they are important components of the model that are used to investigate fiscal policy considerations.

The base specification uses the assumption that government savings are a residual; when the determinants of government income and expenditure are 'fixed', government savings must be free to adjust.

Thus in the base specification all the tax rates (variables) are fixed by declaring the base tax rates as parameters and then fixing all the multiplicative and additive tax rate scaling factors (AC4a – AC4u).

Consequently changes in tax revenue to the government are consequences of changes in the other variables that enter into the tax income equations (GR1 to GR10). The two other sources of income to the government are controlled by parameters, *govvash* and *govwor*, and therefore are not a source of concern for model closure.[30]

Tax Rate Adjustment Closure Equations

$$TMADJ = \overline{TMADJ} \tag{AC4a}$$

$$TEADJ = \overline{TEADJ} \tag{AC4b}$$

$$TSADJ = \overline{TSADJ} \tag{AC4c}$$

$$TEXADJ = \overline{TEXADJ} \tag{AC4d}$$

$$TXADJ = \overline{TXADJ} \tag{AC4e}$$

$$TFADJ = \overline{TFADJ} \tag{AC4f}$$

$$TYADJ = \overline{TYADJ} \tag{AC4g}$$

$$TYEADJ = \overline{TYEADJ} \tag{AC4h}$$

$$TYHADJ = \overline{TYHADJ} \tag{AC4i}$$

$$TWATADJ = \overline{TWATADJ} \tag{AC4j}$$

$$TWATAADJ = \overline{TWATAADJ} \tag{AC4k}$$

$$DTM = \overline{DTM} \tag{AC4l}$$

$$DTE = \overline{DTE} \tag{AC4m}$$

30 The values of income from non-tax sources can of course vary because each component involves a variable.

$$DTS = \overline{DTS} \tag{AC4n}$$

$$DTEX = \overline{DTEX} \tag{AC4o}$$

$$DTX = \overline{DTX} \tag{AC4p}$$

$$DTF = \overline{DTF} \tag{AC4q}$$

$$DTY = \overline{DTY} \tag{AC4r}$$

$$DTYF = \overline{DTYF} \tag{AC4s}$$

$$DTYH = \overline{DTYH} \tag{AC4t}$$

$$DTYE = \overline{DTYE} \tag{AC4u}$$

$$DTWAT = \overline{DTWAT} \tag{AC4v}$$

$$DTWATA = \overline{DTWATA} \tag{AC4w}$$

Also note that because there are equations for the revenues by each tax instrument (GR1 to GR10) it is straightforward to adjust the tax rates to achieve a given volume of revenue from each tax instrument; this type of arrangement is potentially useful in circumstances where it is argued/believed that there are binding constraints upon the revenue possibilities from specific tax instruments.

In the base specification government expenditure is controlled by fixing the volumes of commodity demand (*QGD*) through the government demand adjuster (*QGDADJ*) in (AC4x). Alternatively either the value of government consumption expenditure (*VGD*) can be fixed, (AC4y), or the share of government expenditure in the total value of domestic final demand (*VGDSH*) can be fixed, (AC4z). The scaling factor on the values of transfers to households and enterprises through the household (*HGADJ*) and enterprise (*EGADJ*) adjusters, (AC4aa and AC4ab) also need to be fixed.

Government Expenditure Closure Equations

$$QGDADJ = \overline{QGDADJ} \tag{AC4x}$$

$$VGD = \overline{VGD} \tag{AC4y}$$

$$VGDSH = \overline{VGDSH} \tag{AC4z}$$

$$HGADJ = \overline{HGADJ} \tag{AC4aa}$$

$$EGADJ = \overline{EGADJ} \tag{AC4ab}$$

$$KAPGOV = \overline{KAPGOV} \tag{AC4ac}$$

This specification ensures that all the parameters that the government can/does control are fixed and consequently that the only determinants of government income and expenditure that are free to vary are those that the government does not directly control. Hence the equilibrating condition is that government savings, the internal balance, is not fixed.

If however the model requires government savings to be fixed (AC4ac), then either government income or expenditure must be free to adjust. Such a condition might reasonably be expected in many circumstances, e.g., the government might define an acceptable level of borrowing or such a condition might be imposed externally. In its simplest form this can be achieved by allowing one of the previously fixed adjusters (AC4a to AC4ab) to vary. Thus if the sales tax adjuster (*TSADJ*) is made variable then the sales tax rates will be varied equiproportionately so as to satisfy the internal balance condition. More complex experiments might result from the imposition of multiple conditions, e.g., a halving of import duty rates coupled with a reduction in government deficit, in which case the variables *TMADJ* and *KAPGOV* would also require resetting. But these conditions might create a model that is infeasible, e.g., due to insufficient flexibility through the sales tax mechanism, or unrealistically high rates of sales taxes. In such circumstances it may be necessary to allow adjustments in multiple tax adjusters. One method then would be to fix the tax adjusters to move in parallel with each other.

However, if the adjustments only take place through the tax rate scaling factors the relative tax rates will be fixed. To change relative tax rates it is necessary to change the relevant tax parameters. Typically such changes would be implemented in policy experiment files rather than within the closure section of the model.

3.15.5 Numéraire

The model specification allows for a choice of two price normalisation equations (AC5a and AC5b), the consumer price index (CPI) and a producer price index (PPI). A *numéraire* is needed to serve as a base since the model is homogenous of degree zero in prices and hence only defines relative prices.

Numéraire Closure Equations

$$CPI = \overline{CPI} \tag{AC5a}$$

$$PPI = \overline{PPI} \tag{AC5b}$$

3.15.6 Factor Market Closure

The factor market closure rules are more difficult to implement than many of the other closure rules. Hence the discussion below proceeds in three stages; the first stage sets up a basic specification whereby all factors are deemed perfectly mobile, the second stage introduces a more general specification whereby factors can be made activity specific and allowance can be made for unemployed factors, while the third stage introduces the idea that factor market restrictions may arise from activity specific characteristics, rather than the factor inspired restrictions considered in the second stage.

<u>Full Factor Mobility and Employment Closure</u>

This factor market closure requires that the total supply (*FS*) of and total demand for factors (*FD*) equate (AC6a). The total supplies of each factor are determined exogenously and hence define the first set of factor market closure conditions. The demands for factor *f* by activity *a* and the wage rates for factors are determined endogenously. But the model specification includes the assumption that the wage rates for factors are averages, by allowing for the possibility that the payments to notionally identical factors might vary across activities through the variable that captures the ‘sectoral proportions for factor prices’. These proportions are assumed to be a consequence of the use made by activities of factors, rather than of the factors themselves, and are therefore assumed fixed, (AC6b). The same holds true for the ‘sectoral proportions for water commodity prices’ (AC6c). Finally while it may seem that factor prices must be limited to positive values the actual bounds placed upon the average factor prices, (AC6d) are plus or minus infinity. This is a consequence of the use of the PATH solver.

Basic Factor Market and Water Commodity Price Closure Equations

$$FS_f = \overline{FS}_f \tag{AC6a}$$

$$WFDIST_{f,a} = \overline{WFDIST_{f,a}} \tag{AC6b}$$

$$PQDDIST_{c,a} = \overline{PQDDIST_{c,a}} \tag{AC6c}$$

$$\begin{aligned} &\text{Min } WF_f = -\text{infinity} \\ &\text{Max } WF_f = +\text{infinity} \end{aligned} \tag{AC6d}$$

<u>Factor Immobility and/or Unemployment Closures</u>

More general factor market closures wherein factor immobility and/or factor unemployment are assumed can be achieved by determining which of the variables referring to factors are treated as variables and which of the variables are treated as parameters. If factor market closure rules are changed it is important to be careful to preserve the equation and variable counts when relaxing conditions, i.e., converting parameters into variables, and imposing conditions, i.e., converting variables into parameters, while preserving the economic logic of the model.

A convenient way to proceed is to define a block of conditions for each factor. For this model this amounts to defining the following possible equations (AC6e) where *fact* indicates the specific factor and *activ* a specific activity. This block of equations includes all the variables that were declared for the model with reference to factors plus an extra equation for *WFDIST*, i.e., $WFDIST_{fact,activ} = \overline{WFDIST_{fact,activ}}$, whose role will be defined below. The choice of which equations are binding and which are not imposed will determine the factor market closure conditions.

Factor Block Equations

$$FS_{fact} = \overline{FS_{fact}}$$
$$WFDIST_{fact,a} = \overline{WFDIST_{fact,a}}$$
$$\text{Min } WF_{fact} = -\text{infinity}$$
$$\text{Max } WF_{fact} = +\text{infinity}$$
$$FD_{fact,a} = \overline{FD_{fact,a}}$$
$$WF_{fact} = \overline{WF_{fact}}$$
$$WFDIST_{fact,activ} = \overline{WFDIST_{fact,activ}}$$
$$\text{Min } FS_{fact} = -\text{infinity}$$
$$\text{Max } FS_{fact} = +\text{infinity} \qquad \text{(AC6e)}$$

As can be seen the first four equations in the block (AC6f) are the same as those in the 'Full Factor Mobility and Employment Closure'; hence ensuring that these four equations are operating for each of the factors is a longhand method for imposing the 'Full Factor Mobility and Employment Closure'. Assume that this set of conditions represents a starting point, i.e., the first four equations are binding and the last five equations are not imposed.

Assume now that it is planned to impose a short run closure on the model, whereby a factor is assumed to be activity specific, and hence there is no inter sectoral factor mobility. Typically this would involve making capital activity specific and immobile, although it can be applied to any factor. This requires imposing the condition that factor demands are activity specific, i.e., the condition ($FD_{fact,a} = \overline{FD_{fact,a}}$) must be imposed. But the returns to this factor in different uses (activities) must now be allowed to vary, i.e., the condition (AC6b) must now be relaxed.

Factor Market Closure Equations

$$FD_{fact,a} = \overline{FD_{fact,a}} \qquad \text{(AC6f)}$$

$$WFDIST_{fact,a} = \overline{WFDIST_{fact,a}} \qquad \text{(AC6g)}$$

$$FS_{fact} = \overline{FS_{fact}} \qquad \text{(AC6h)}$$

$$WFDIST_{fact,activ} = \overline{WFDIST_{fact,activ}} \qquad \text{(AC6i)}$$

$$WF_{fact} = \overline{WF_{fact}} \tag{AC6j}$$

$$FS_{fact} = \overline{FS_{fact}} \tag{AC6k}$$

$$\begin{aligned} &\text{Min } FS_{fact} = 0 \\ &\text{Max } FS_{fact} = +\text{infinity} \end{aligned} \tag{AC6l}$$

The number of imposed conditions is equal to the number of relaxed conditions, which suggests that the model will still be consistent. But the condition fixing the total supply of the factor is redundant since if factor demands are fixed the total factor supply cannot vary. Hence the condition (AC6a) is redundant and must be relaxed. Hence at least one other condition must be imposed to restore balance between the numbers of equations and variables. This can be achieved by fixing one of the sectoral proportions for factor prices for a specific activity, i.e., (AC6b), which means that the activity specific returns to the factor will be defined relative to the return to the factor in *activ*.[31]

Start again from the closure conditions for full factor mobility and employments and then assume that there is unemployment of one or more factors in the economy; typically this would be one type or another of unskilled labour. If the supply of the unemployed factor is perfectly elastic, then activities can employ any amount of that factor at a fixed price. This requires imposing the condition that factor prices are fixed (AC6j) and relaxing the assumption that the total supply of the factor is fixed at the base level, i.e., relaxing (AC6a). It is useful however to impose some restrictions on the total supply of the factor that is unemployed. Hence the conditions (AC6l) can be imposed.[32]

This also holds for water factors in most cases: The consumption of water factors mostly depends on the demand for water commodities. Therefore extraction rates and thus *FD* need to vary. As usually one water activity is linked to one specific water factor this means also *FS* needs to be relaxed. If required an upper limit can be set by adjusting (AC6l).

[31] It can be important to ensure a sensible choice of reference activity. In particular this is important if a factor is not used, or little used, by the chosen activity.

[32] If the total demand for the unemployed factor increases unrealistically in the policy simulations then it is possible to place an upper bound of the supply of the factor and then allow the wage rate from that factor to vary.

On the other hand usually prices for the usage of water factors, if existent, are rather politically fixed and do not vary with extraction rates. In this case *WF* and also *WFDIST* should be fixed (AC6j and AC6i).

Activity Inspired Restrictions on Factor Market Closures

There are circumstances where factor use by an activity might be restricted as a consequence of activity specific characteristics. For instance it might be assumed that the volume of production by an activity might be predetermined, e.g., known mineral resources might be fixed and/or there might be an exogenously fixed restriction upon the rate of extraction of a mineral commodity. In such cases the objective might be to fix the quantities of all factors used by an activity, rather than to fix the amounts of a factor used by all activities. This is clearly a variation on the factor market closure conditions for making a factor activity specific.

Factor Market Clearing Equations

$$FD_{f,activ} = \overline{FD_{f,activ}} \qquad \text{(AC6m)}$$

$$WFDIST_{f,activ} = \overline{WFDIST_{f,activ}} \qquad \text{(AC6n)}$$

If all factors used by an activity are fixed, this requires imposing the conditions that factor demands are fixed, (AC6m), where *activ* refers to the activity of concern. But the returns to these factors in this activities must now be allowed to vary, i.e., the conditions (AC6n) must now be relaxed. In this case the condition fixing the total supply of the factor is not redundant since only the factor demands by *activ* are fixed and the factor supplies to be allocated across other activities are the total supplies unaccounted for by *activ*.

Such conditions can be imposed by extending the blocks of equations for each factor in the factor market closure section. However, it is often easier to manage the model by gathering together factor market conditions that are inspired by activity characteristics after the factor inspired equations. In this context it is useful to note that when working in GAMS that the last condition imposed, in terms of the order of the code, is binding and supersedes previous conditions.

The full set of equations included in the STAGE_W model and described in this document as well as the closure rules can be found in Appendix 2, while Appendix 3provides a translation of the equations into GAMS-code. Appendix 4 provides a

4 References

Faust, A.-K., Gonseth, C. and M. Vielle (2012). 'The economic impact of climate driven changes in water availability in Switzerland', Working paper. Research group on the Economics and Management of the Environment, École Politechnique Fédérale de Lausanne, Switzerland.

Johansen, L., (1960). *A Multi-sectoral Study of Economic Growth*. Amsterdam: North-Holland.

Kilkenny, M., (1991). 'Computable General Equilibrium Modeling of Agricultural Policies: Documentation of the 30-Sector FPGE GAMS Model of the United States', *USDA ERS Staff Report AGES* 9125.

Kilkenny, M. And Robinson, S., (1990). 'Computable General Equilibrium Analysis of Agricultural Liberalisation: Factor Mobility and Macro Closure', *Journal of Policy Modeling*, Vol 12, pp 527-556.

McDonald, S., (2007). 'Prices, Social Accounts and Economic Models', paper presented at *EUNIOS Workshop 'Agricultural Policies and Social Accounting Matrices in Developing Countries'*, University of Groningen, September, 2008.

PROVIDE (2005). 'A Standard Computable General Equilibrium Model Version 5: Technical Documentation', *PROVIDE Project Technical Paper* 2005:03. Elsenburg, RSA. http://www.elsenburg.com/provide/documents/TP2005_3

Pyatt, G., (1987). 'A SAM Approach to Modeling', *Journal of Policy Modeling*, Vol 10, pp 327-352.

Pyatt, G., (1989). 'The Method of Apportionment and Accounting Multipliers', *Journal of Policy Modeling*, Vol 11, pp 111-130.

Robinson, S., Kilkenny, M. And Hanson, K.,, (1990). 'USDA/ERS Computable General Equilibrium Model of the United States', Economic Research Service, *USDA, Staff Report AGES* 9049.

Sen, A.K., (1963). 'Neo-classical and Neo-Keynesian Theories of Distribution', *Economic Record*, Vol 39, pp 53-64.

Appendices

Appendix 1: Parameter and Variable Lists

The parameter and variable listings are in alphabetic order, and are included for reference purposes. The parameters listed below are those used in the behavioural specifications/equations of the model, in addition to these parameters there are a further set of parameters. This extra set of parameters is used in model calibrated and for deriving results; there is one such parameter for each variable and they are identified by appending a ‘0’ (zero) to the respective variable name.

Parameter List

Parameter Name	Parameter Description
ac(c)	Shift parameter for Armington CES function
actcomactsh(a,c)	Share of commodity c in output by activity a
actcomcomsh(a,c)	Share of activity a in output of commodity c
adva(a)	Shift parameter for CES production functions for QVA
adx(a)	Shift parameter for CES production functions for QX
adxc(c)	Shift parameter for commodity output CES aggregation
alphah(c,h)	Expenditure share by commodity c for household h
at(c)	Shift parameter for Armington CET function
atnw(a)	Shift parameter for CES function for land-water nest
atwat(a)	Shift parameter for CES function for water nest
beta(c,h)	Marginal budget shares
caphosh(h)	Shares of household income saved (after taxes)
comactactco(c,a)	intermediate input output coefficients
comactco(c,a)	use matrix coefficients
comentconst(c,e)	Enterprise demand volume
comgovconst(c)	Government demand volume
comhoav(c,h)	Household consumption shares
comtotsh(c)	Share of commodity c in total commodity demand
dabte(c)	Change in base export taxes on comm’y imported from region w
dabtex(c)	Change in base excise tax rate
dabtf(f)	Change in base factor use tax rate
dabtm(c)	Change in base tariff rates on comm’y imported from region w
dabts(c)	Change in base sales tax rate
dabtwat(c)	Change in base water tax rate
dabtwata(c,a)	Change in base water use tax rate
dabtx(a)	Change in base indirect tax rate
dabtye(e)	Change in base direct tax rate on enterprises
dabtyf(f)	Change in base direct tax rate on factors
dabtyh(h)	Change in base direct tax rate on households
delta(c)	Share parameter for Armington CES function
deltafd4(f4,a)	Share parameter for factors on 4. level
deltanw(f3,a)	Share parameter of land and water factors for land-water nest
deltanwva(a)	Share parameter for land-water composite in QVA
deltava(ff,a)	Share parameters for CES production functions for QVA
deltawat(a)	Share parameter for water commodity composite in land water nest
deltawat2(c,a)	Share parameter for single water commodities in water nest
deltax(a)	Share parameter for CES production functions for QX
deltaxc(a,c)	Share parameters for commodity output CES aggregation

Parameter Name	Parameter Description
deprec(f)	depreciation rate by factor f
dstocconst(c)	Stock change demand volume
econ(c)	constant for export demand equations
entgovconst(e)	Government transfers to enterprise e
entvash(e,f)	Share of income from factor f to enterprise e
entwor(e)	Transfers to enterprise e from world (constant in foreign currency)
eta(c)	export demand elasticity
factwor(f)	Factor payments from RoW (constant in foreign currency)
frisch(h)	Elasticity of the marginal utility of income
gamma(c)	Share parameter for Armington CET function
goventsh(e)	Share of entp' income after tax save and consump to govt
govvash(f)	Share of income from factor f to government
govwor	Transfers to government from world (constant in foreign currency)
hexps(h)	Subsistence consumption expenditure
hoentconst(h,e)	transfers to hhold h from enterprise e (nominal)
hoentsh(h,e)	Share of entp' income after tax save and consump to h'hold
hogovconst(h)	Transfers to hhold h from government (nominal but scalable)
hohoconst(h,hp)	interhousehold transfers
hohosh(h,hp)	Share of h'hold h after tax and saving income transferred to hp
hovash(h,f)	Share of income from factor f to household h
howor(h)	Transfers to household from world (constant in foreign currency)
invconst(c)	Investment demand volume
iocwatqnw(a)	Water commodity i-o coefficients in QNW for Level 3 Leontief agg
iocwatqva(c,a)	Water com i-o coefficients in QVA for Level 2 Leontief agg
ioffqva(ff,a)	Factor input output coefficients in QVA for Level 2 Leontief agg
iof3qnw(f3,a)	Factor input output coefficients in QNW for Level 3 Leontief agg
iof4aggf4(f4,a)	Factor input output coefficients in QWAT for Level 4 Leontief agg
ioqintqx(a)	Agg intermed quantity per unit QX for Level 1 Leontief agg
ioqvaqx(a)	Agg value added quant per unit QX for Level 1 Leontief agg
ioqwat(c,a)	Water com i-o coefficient in QWAT for Level 4 Leontief agg
kapentsh(e)	Average savings rate for enterprise e out of after tax income
predeltax(a)	dummy used to estimated deltax
pwse(c)	world price of export substitutes
qcdconst(c,h)	Volume of subsistence consumption
rhoc(c)	Elasticity parameter for Armington CES function
rhocva(a)	Elasticity parameter for CES production function for QVA
rhocx(a)	Elasticity parameter for CES production function for QX
rhocxc(c)	Elasticity parameter for commodity output CES aggregation
rhonw(a)	Elasticity parameter for CES prod function for land-water nest
rhot(c)	Elasticity parameter for Output Armington CET function
rhowat(a)	Elasticity parameter for CES production function for CWAT
sumelast(h)	Weighted sum of income elasticities
te01(c)	0-1 par for potential flexing of export taxes on comm'ies
tex01(c)	0-1 par for potential flexing of excise tax rates
tf01(ff)	0-1 par for potential flexing of factor use tax rates
tm01(c)	0-1 par for potential flexing of Tariff rates on comm'ies
ts01(c)	0-1 par for potential flexing of sales tax rates
twat01(c)	0-1 par for potential flexing of water tax rates
twata01(c,a)	0-1 par for potential flexing of water use tax rates
tx01(a)	0-1 par for potential flexing of indirect tax rates
tye01(e)	0-1 par for potential flexing of direct tax rates on e'rises
tyf01(f)	0-1 par for potential flexing of direct tax rates on factors
tyh01(h)	0-1 par for potential flexing of direct tax rates on h'holds
use(c,a)	use matrix transactions
vddtotsh(c)	Share of value of domestic output for the domestic market
worvash(f)	Share of income from factor f to RoW
yhelast(c,h)	(Normalised) household income elasticities

Variable List

Variable Name	Variable Description
ADFD(f,a)	Shift parameter for factor and activity specific efficiency
ADX(a)	Shift parameter for CES production functions for QX
ADXADJ	Scaling Factor for shift parameter on CES functions for QX
CAPWOR	Current account balance
CPI	Consumer price index
DADX	Partial scaling factor for shift parameter on CES functions for QX
DS	Partial household and enterprise savings rate scaling factor
DSEN	Partial enterprise savings rate scaling factor
DSHH	Partial household savings rate scaling factor
DTAX	Direct income tax revenue
DTE	Partial export tax rate scaling factor
DTEX	Partial excise tax rate scaling factor
DTF	Uniform adjustment to factor use tax by activity
DTM	Partial tariff rate scaling factor
DTS	Partial sales tax rate scaling factor
DTWAT	Partial water tax rate scaling factor
DTWATA	Uniform adjustment to water use tax by activity
DTX	Partial indirect tax rate scaling factor
DTY	Partial direct tax on enterprise and households rate scaling factor
DTYE	Partial direct tax on enterprise rate scaling factor
DTYF	Partial direct tax on factor rate scaling factor
DTYH	Partial direct tax on households rate scaling factor
EG	Expenditure by government
EGADJ	Transfers to enterprises by government scaling factor
ER	Exchange rate (domestic per world unit)
ETAX	Export tax revenue
EXTAX	Excise tax revenue
FD(f,a)	Demand for factor f by activity a
FS(f)	Supply of factor f
FTAX	Factor use tax revenue
FYTAX	Factor income tax revenue
GDPVA	GDP from value added
GOVENT(e)	Government income from enterprise e
HEADJ	Scaling factor for enterprise transfers to households
HEXP(h)	Household consumption expenditure
HGADJ	Scaling factor for government transfers to households
HOENT(h,e)	Household Income from enterprise e
HOHO(h,hp)	Inter household transfer
IADJ	Investment scaling factor
INVEST	Total investment expenditure
INVESTSH	Value share of investment in total final domestic demand
ITAX	Indirect tax revenue
KAPGOV	Government savings
MTAX	Tariff revenue
PD(c)	Consumer price for domestic supply of commodity c
PE(c)	Domestic price of exports by activity a
PINT(a)	Price of aggregate intermediate input
PM(c)	Domestic price of competitive imports of commodity c
PNW(a)	Water and Land composite price
PPI	Producer (domestic) price index
PQD(c)	Purchaser price of composite commodity c
PQDDIST(c,a)	Secotral proportion of water prices
PQS(c)	Supply price of composite commodity c
PVA(a)	Value added price for activity a

Variable Name	Variable Description
PWAT(sac)	Composite price for water demand
PWE(c)	World price of exports in dollars
PWM(c)	World price of imports in dollars
PX(a)	Composite price of output by activity a
PXAC(a,c)	Activity commodity prices
PXC(c)	Producer price of composite domestic output
QCD(c,h)	Household consumption by commodity c
QD(c)	Domestic demand for commodity c
QE(c)	Domestic output exported by commodity c
QENTD(c,e)	Enterprise consumption by commodity c
QENTDADJ	Enterprise demand volume scaling factor
QGD(c)	Government consumption demand by commodity c
QGDADJ	Government consumption demand scaling factor
QINT(a)	Aggregate quantity of intermediates used by activity a
QINTD(c)	Demand for intermediate inputs by commodity
QINVD(c)	Investment demand by commodity c
QM(c)	Imports of commodity c
QNW(a)	Water and Land composite quantity
QQ(c)	Supply of composite commodity c
QVA(a)	Quantity of aggregate value added for level 1 production
QWAT(sac)	Domestic demand for water composite
QWAT2(c,sac)	Domestic demand of (single) water commodities
QX(a)	Domestic production by activity a
QXAC(a,c)	Domestic commodity output by each activity
QXC(c)	Domestic production by commodity c
SADJ	Savings rate scaling factor for BOTH households and enterprises
SEADJ	Savings rate scaling factor for enterprises
SEN(e)	Enterprise savings rates
SHADJ	Savings rate scaling factor for households
SHH(h)	Household savings rates
STAX	Sales tax revenue
TE(c)	Export taxes on exported comm'y c
TEADJ	Export subsidy scaling factor
TEX(c)	Excise tax rate
TEXADJ	Excise tax rate scaling factor
TF(f)	Factor use tax rate
TFADJ	Factor use tax rate scaling factor
TM(c)	Tariff rates on imported comm’y c
TMADJ	Tariff rate scaling factor
TOTSAV	Total savings
TS(c)	Sales tax rate
TSADJ	Sales tax rate scaling factor
TWAT(c)	Water commodity tax rate
TWATADJ	Water tax rate scaling factor
TWATA(c,a)	Water user tax rate
TWATAADJ	Water user tax rate scaling factor
TX(a)	Indirect tax rate
TXADJ	Indirect tax scaling factor
TYE(e)	Direct tax rate on enterprises
TYEADJ	Enterprise income tax scaling factor
TYF(f)	Direct tax rate on factor income
TYFADJ	Factor tax scaling factor
TYH(h)	Direct tax rate on households
TYHADJ	Household income tax scaling factor
VENTD(e)	Value of enterprise e consumption expenditure
VENTDSH(e)	Value share of Ent consumption in total final domestic demand
VFDOMD	Value of final domestic demand
VGD	Value of Government consumption expenditure
VGDSH	Value share of Govt consumption in total final domestic demand

Variable Name	Variable Description
WALRAS	Slack variable for Walras's Law
WATTAX	Water commodity tax revenue
WATATAX	Water user tax revenue
WF(f)	Price of factor f
WFDIST(f,a)	Sectoral proportion for factor prices
YE(e)	Enterprise incomes
YF(f)	Income to factor f
YFDISP(f)	Factor income for distribution after depreciation
YFWOR(f)	Foreign factor income
YG	Government income
YH(h)	Income to household h

Appendix 2: Equation Listing

Table 4 Equation and Variable Counts for the Model

Name	Equation		Number of Equations	Variable	Number of Variables
	EXPORTS BLOCK				
$PEDEF_c$	$PE_c = PWE_c * ER * (1 - TE_c)$	$\forall ce$	ce	PE_c	ce
CET_c	$QXC_c = at_c * \left(\gamma_c * QE_c^{rhot_c} + (1-\gamma_c) * QD_c^{rhot_c}\right)^{\frac{1}{rhot_c}}$	$\forall ce$ **AND** cd	c	QDD_c	c
$ESUPPLY_a$	$\frac{QE_c}{QD_c} = \left[\frac{PE_c}{PD_c} * \frac{(1-\gamma_c)}{\gamma_c}\right]^{\frac{1}{(rhot_c - 1)}}$	$\forall ce$ **AND** cd	c	QE_c	c
$EDEMAND_c$	$QE_c = econ_c * \left(\frac{PWE_c}{pwse_c}\right)^{-eta_c}$	$\forall ced$			
$CETALT_c$	$QXC_c = QD_c + QE_c$	$\forall (cen$ **AND** $cd)$ **OR** $(ce$ **AND** $cdn)$			
	IMPORTS BLOCK				
$PMDEF_c$	$PM_c = PWM_c * ER * (1 + TM_c)$	$\forall cm$	cm	PM_c	cm
$ARMINGTON_c$	$QQ_c = ac_c \left(\delta_c QM_c^{-rhoc_c} + (1-\delta_c) QD_c^{-rhoc_c}\right)^{-\frac{1}{rhoc_c}}$	$\forall cm$ **AND** cx	c	QQ_c	c
$COSTMIN_c$	$\frac{QM_c}{QD_c} = \left[\frac{PD_c}{PM_c} * \frac{\delta_c}{(1-\delta_c)}\right]^{\frac{1}{(1+rhoc_c)}}$	$\forall cm$ **AND** cx	c	QM_c	c
$ARMALT$	$QQ_c = QD_c + QM_c$	$\forall (cmn$ **AND** $cx)$ **OR** $(cm$ **AND** $cxn)$			

Name	Equation	Number of Equations	Variable	Number of Variables
	COMMODITY PRICE BLOCK			
$PQDDEF_c$	$PQD_c = PQS_c * \left(1 + TS_c + TEX_c\right)$	c	PQD_c	c
$PQSDEF_c$	$PQS_c = \dfrac{PD_c * QD_c + PM_c * QM_c}{QQ_c} \quad \forall cd$ **OR** cm	c	PQS_c	c
$PXCDEF_c$	$PXC_c = \dfrac{PD_c * QD_c + \left(PE_c * QE_c\right)\$ce_c}{QXC_c} \quad \forall cx$	cx	PXC_c	cx
	NUMERAIRE BLOCK			
$CPIDEF$	$CPI = \sum_c comtotsh_c * PQD_c$	1	CPI	1
$PPIDEF$	$PPI = \sum_c vddtotsh_c * PD_c$	1	PPI	1

Name	Equation	Number of Equations	Variable	Number of Variables
	PRODUCTION BLOCK			
$PXDEF_a$	$PX_a = \sum_c ioqxacqx_{a,c} * PXC_c$	a	PX_a	a
$PVADEF_a$	$PX_a * (1 - TX_a) * QX_a = (PVA_a * QVA_a) + (PINT_a * QINT_a)$	a	PVA_a	a
$PINTDEF_a$	$PINT_a = \sum_c \left(ioqtdqd_{c,a} * PQD \right)_c$	a	$PINT_a$	a
$ADXEQ_a$	$AD_a^X = \left[(adxb_a + dabadx_a) * ADXADJ \right] + (DADX * adx01_a)$	a	AD_a^X	a
$QXPRODFN_a$	$QX_a = AD_a^X \left(\delta_a^x * QVA_a^{-\rho_a^{cx}} + (1-\delta_a^x) * QINT_a^{-\rho_a^{cx}} \right)^{-\frac{1}{\rho_a^{cx}}} \quad \forall aqx_a$	a	QX_a	a
$QXFOC_a$	$\frac{QVA_a}{QINT_a} = \left[\frac{PINT_a}{PVA_a} * \frac{\delta_a^x}{(1-\delta_a^x)} \right]^{\frac{1}{(1+\rho_a^{cx})}} \quad \forall aqx_a$	a	$QINT_a$	a
$QVADEF$	$QVA_a = ioqvaqx_a * QX_a \quad \forall aqxn_a$			
$QINTDEF$	$QINT_a = ioqintqx_a * QX_a \quad \forall aqx_a$			
$ADVAEQ$	$AD_a^{VA} = \left[(advab_a + dabadva_a) * ADVAADJ \right] + (DADVA * adva01_a)$	a	AD_a^{VA}	a
$QVAPRODFN_a$	$QVA_a = AD_a^{VA} * \left[\begin{array}{l} \sum_{f2\$\delta_{f2,a}^{va}} \delta_{f2,a}^{va} * AD_{f2,a}^{FD} * FD_{f2,a}^{-\rho_a^{cva}} \\ +(\delta_{"cnw",a}^{va} * QNW_a^{-\rho_a^{cva}})\$\delta_{"cnw",a}^{va} \end{array} \right]^{-1/\rho_a^{cva}} \quad \forall afx_a$	a	QVA_a	a

Name	Equation	Number of Equations	Variable	Number of Variables
$QVAFOC1_{f2,a}$	$WF_{f2} * WFDIST_{f2,a} * \left(1+TF_{f2,a}\right) = PVA_a * QVA_a * \left[\begin{array}{l} \sum_{f2p\$\delta^{va}_{f2p,a}} \delta^{va}_{f2p,a} * AD^{FD}_{f2,a} * FD_{f2p,a}^{-\rho^{cva}_a} \\ +(\delta^{va}_{"cnw",a} * QNW_a^{-\rho^{cva}_a})\$\delta^{va}_{"cnw",a} \end{array}\right]^{-1} * \delta^{va}_{f2,a} * AD^{FD}_{f2,a}{}^{-\rho^{cva}_a} * FD_{f2,a}^{\left(-\rho^{cva}_a - 1\right)} \quad \forall afx_a$	$f2*a$	$FD_{f2,a}$	$f2*a$
$QVAFOC2_a$	$PNW_a = PVA_a * QVA_a * \left[\sum_{f2p\$\delta^{va}_{f2p,a}} \delta^{va}_{f2p,a} * AD^{FD}_{f2p,a} * FD_{f2p,a}^{-\rho^{cva}_a} + \delta^{va}_{"cnw",a} * QNW_a^{-\rho^{cva}_a}\right]^{-1} * \delta^{va}_{"cnw",a} * QNW_a^{\left(-\rho^{cva}_a - 1\right)} \quad \forall afx_a$	a	QNW_a	a
$QVAEQ1_{f2,a}$	$FD_{f2,a} * AD^{FD}_{f2,a} = ioqnwqva_{f2,a} * QVA_a \quad \forall afxn_a$	$f2*a$	$FD_{f2,a}$	$f2*a$
$QVAEQ2_a$	$QNW_a = ioqnwqva_{"cnw",a} * QVA_a \quad \forall afxn_a$	a	QNW_a	a
$PVAEQ_a$	$PVA_a = PNW_a * ioqnwqva_{"cnw",a} + \sum_{f2} ioqnwqva_{f2,a} * WF_{f2} * WFDIST_{f2,a} * (1+TF_{f2,a}) \quad \forall afxn_a$	a	QVA_a	a
$QINTDEQ_c$	$QINTD_c = \sum_a ioqtdqd_{c,a} * QINT_a + \sum_a QWAT2_{c,a}$	c	$QINTD_c$	c

Name	Equation	Number of Equations	Variable	Number of Variables
$QNWPRODFN1_a$	$QNW_a = ad_a^{nw} * \left[\begin{array}{l} \sum_{f3\$\delta^{nw}_{f3p,a}} \delta^{nw}_{f3,a} * AD^{FD}_{f3,a} * FD^{-\rho^{nw}_a}_{f3,a} \\ +(\delta^{nw}_{"cwat",a} * QWAT_a^{-\rho^{nw}_a})\$\delta^{nw}_{"cwat",a} \end{array} \right]^{-1/\rho^{nw}_a}$ $\forall af3x_a$ **AND** an_a	a	QNW_a	a
$QNWFOC1_{f3,a}$	$WF_{f3} * WFDIST_{f3,a} * \left(1+TF_{f3,a}\right) = PNW_a * QNW_a * \left[\begin{array}{l} \sum_{f3p\$\delta^{nw}_{f3p,a}} \delta^{nw}_{f3p,a} * AD^{FD}_{f3p,a} * FD^{-\rho^{nw}_a}_{f3p,a} \\ +(\delta^{nw}_{"cwat",a} * QWAT_a^{-\rho^{nw}_a})\$\delta^{nw}_{"cwat",a} \end{array} \right]^{-1} * \delta^{nw}_{f3p,a} * AD^{FD\ -\rho^{nw}_a}_{f3,a} * FD_{f3,a}^{\left(-\rho^{nw}_a - 1\right)}$ $\forall af3x_a$	$f3*a$	$FD_{f3,a}$	$f3*a$
$QNWFOC2_a$	$PWAT_a = PNW_a * QNW_a * \left[\begin{array}{l} \sum_{f3p\$\delta^{nw}_{f3p,a}} \delta^{nw}_{f3p,a} * AD^{FD}_{f3p,a} * FD^{-\rho^{nw}_a}_{f3p,a} \\ +\delta^{nw}_{"cwat",a} * QWAT_a^{-\rho^{nw}_a} \end{array} \right]^{-1} * \delta^{nw}_{"cwat",a} * QWAT_a^{\left(-\rho^{nw}_a - 1\right)}$ $\forall af3x_a$	a	$QWAT_a$	a
$QNWEQ1_{f3,a}$	$FD_{f3,a} * AD^{FD}_{f3,a} = ioqwatqnw_{f3,a} * QNW_a$ $\forall af3xn_a$	$f3*a$	$FD_{f3,a}$	$f3*a$
$QNWEQ2_a$	$QWAT_a = ioqwatqnw_{"cwat",a} * QNW_a$ $\forall af3xn_a$ **OR** ann_a	a	$QWAT_a$	a
$PNWEQ_a$	$PNW_a = PWAT_a * ioqwatqnw_{"cwat",a} + \sum_{f3} ioqwatqnw_{f3,a} * WF_{f3} * WFDIST_{f3,a} * (1+TF_{f3,a})$ $\forall af3xn_a$ **OR** ann_a	a	QNW_a	a

Name	Equation	Number of Equations	Variable	Number of Variables
$WATDPRODFN_a$	$QWAT_a = ad_a^{wat} * \left[\begin{array}{l} \sum\limits_{f4\$\delta_{f4,a}^{wat}} \delta_{f4,a}^{wat} * AD_{f4,a}^{FD} * FD_{f4,a}^{-\rho_a^{wat}} \\ + \sum\limits_{cwat} \delta_{cwat,a}^{wat} * QWAT2_{cwat,a}^{-\rho_a^{wat}} \end{array} \right]^{-1/\rho_a^{wat}}$ $\forall af4x_a$ **AND** $awat_a$	a	$QWAT_a$	a
$FD4FOC_{f4,a}$	$WF_{f4} * WFDIST_{f4,a} * \left(1 + TF_{f4,a}\right) = PWAT_a * QWAT_a$ $* \left[\begin{array}{l} \sum\limits_{f4p\$\delta_{f4p,a}^{wat}} \delta_{f4p,a}^{wat} * AD_{f4p,a}^{FD} * FD_{f4p,a}^{-\rho_a^{wat}} \\ + \sum\limits_{cwatp\$\delta_{cwatp,a}^{wat}} \delta_{cwatp,a}^{wat} * QWAT2_{cwatp,a}^{-\rho_a^{wat}} \end{array} \right]^{-1}$ $* \delta_{f4p,a}^{wat} * AD_{f4,a}^{FD\ -\rho_a^{wat}} * FD_{f4,a}^{\left(-\rho_a^{wat}-1\right)}$ $\forall af4x_a$	$f4*a$	$FD_{f4,a}$	$f4*a$
$WATDFOC_{cwat,a}$	$PQD_{cwat} * PQDDIST_{cwat,a} * (1 + TWATA_{cwat,a}) = PWAT_a * QWAT_a$ $* \left[\begin{array}{l} \sum\limits_{f4p\$\delta_{f4p,a}^{wat}} \delta_{f4p,a}^{wat} * AD_{f4p,a}^{FD} * FD_{f4p,a}^{-\rho_a^{wat}} \\ + \sum\limits_{cwatp\$\delta_{cwatp,a}^{wat}} \delta_{cwatp,a}^{wat} * QWAT2_{cwatp,a}^{-\rho_a^{wat}} \end{array} \right]^{-1} * \delta_{cwatp,a}^{wat} * QWAT2_{cwat,a}^{\left(-\rho_a^{wat}-1\right)}$ $\forall af4x_a$	$cwat*a$	$QWAT2_{cwat,a}$	$cwat*a$
$QWATEQ1_{c,a}$	$QWAT2_{c,a} = iofcqwat_{c,a} * QWAT_a$ $\forall af4xn_a$ **AND** $acwat_a$	$cwat*a$	$QWAT2_{cwat,a}$	$cwat*a$
$QWATEQ2_{f4,a}$	$FD_{f4,a} * AD_{f4,a}^{FD} = iofcqwat_{f4,a} * QWAT_a$ $\forall af4xn_a$ **AND** $afwat_a$	$f4*a$	$FD_{f4,a}$	$f4*a$

Name	Equation	Number of Equations	Variable	Number of Variables
$PWATEQ_a$	$PWAT_a = \sum_{c\$cwat_c} iofcqwat_{c,a} * PQD_c * PQDDIST_{c,a} * (1+TWATA_{c,a})$ $+\sum_{f4} iofcqwat_{f4,a} * WF_{f4} * WFDIST_{f4,a} * (1+TF_{f4,a}) \quad \forall af4xn_a$	a	$PWAT_a$	a
$COMOUT_c$	$QXC_c = ad_c^{xc} * \left[\sum_{a\$\delta_{a,c}^{xc}} \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}}\right]^{-1/\rho_c^{xc}} \quad \forall cx_c$ **AND** $cxac_c$	c	QXC_c	c
$COMOUT2_c$	$QXC_c = \sum_a QXAC_{a,c} \quad \forall cx_c$ **AND** $cxacn_c$	c	QXC_c	c
$COMOUTFOC_{a,c}$	$PXAC_{a,c} = PXC_c * ad_c^{xc} * \left[\sum_{a\$\delta_{a,c}^{xc}} \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}}\right]^{-\left(\frac{1+\rho_c^{xc}}{\rho_c^{xc}}\right)}$ $* \delta_{a,c}^{xc} * QXAC_{a,c}^{\left(-\rho_c^{xc}-1\right)}$ $= PXC_c * QXC_c * \left[\sum_{a\$\delta_{a,c}^{xc}} \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}}\right]^{-\left(\frac{1+\rho_c^{xc}}{\rho_c^{xc}}\right)}$ $* \delta_{a,c}^{xc} * QXAC_{a,c}^{\left(-\rho_c^{xc}-1\right)} \quad \forall cxac_c$	$a*c$	$PXAC_{a,c}$	$a*c$
$COMOUTFOC2_{a,c}$	$PXAC_{a,c} = PXC \quad \forall cxacn_c$	$a*c$	$PXAC_{a,c}$	$a*c$
$ACTIVOUT_{a,c}$	$QXAC_{a,c} = ioqxacqx_{a,c} * QX_a$	$a*c$	$QXAC_{a,c}$	$a*c$

Name	Equation	Number of Equations	Variable	Number of Variables
	FACTOR BLOCK			
$YFEQ_f$	$YF_f = \left(\sum_a WF_f * WFDIST_{f,a} * FD_{f,a} \right) + \left(factwor_f * ER \right)$	f	YF_f	f
$YFDISPEQ_f$	$YFDISP_f = \left(YF_f * \left(1 - deprec_f \right) \right) * \left(1 - TYF_f \right)$	f	$YFDIST_f$	f
	HOUSEHOLD BLOCK			
$YHEQ_h$	$YH_h = \left(\sum_f hovash_{h,f} * YFDISP_f \right) + \left(\sum_{hp} HOHO_{h,hp} \right) + HOENT_h + \left(hogovconst_h * HGADJ * CPI \right) + \left(howor_h * ER \right)$	h	YH_h	h
$HOHOEQ_{h,hp}$	$HOHO_{h,hp} = hohosh_{h,hp} * \left(YH_h * \left(1 - TYH_h \right) \right) * \left(1 - SHH_h \right)$	$h*hp$	$HOHO_{h,hp}$	$h*hp$
$HEXPEQ_h$	$HEXP_h = \left(\left(YH_h * \left(1 - TYH_h \right) \right) * \left(1 - SHH_h \right) \right) - \left(\sum_{hp} HOHO_{hp,h} \right)$	h	$HEXP_h$	h
$QCDEQ_c$	$QCD_c = \left(\sum_h \left(PQD_c * qcdconst_{c,h} + \sum_h beta_{c,h} * \left(HEXP_h - \sum_c \left(PQD_c * qcdconst_{c,h} \right) \right) \right) \right) * \frac{1}{PQD_c}$	c	QCD_c	c

Name	Equation	Number of Equations	Variable	Number of Variables
	ENTERPRISE BLOCK			
$YEEQ_e$	$YE_e = \left(\sum_f entvash_{e,f} * YFDISP_f\right) + \left(entgovconst_e * EGADJ * CPI\right) + \left(entwor_e * ER\right)$	e	YE_e	e
$QENTDEQ_{c,e}$	$QED_{c,e} = qedconst_{c,e} * QEDADJ$	$c*e$	$QED_{c,e}$	$c*e$
$VEDEQ_e$	$VED_e = \left(\sum_c QED_{c,e} * PQD_c\right)$	e	VED_e	e
$HOENTEQ_{h..}$	$HOENT_{h,e} = hoentsh_{h,e} * \begin{pmatrix} \left(YE_e * \left(1 - TYE_e\right)\right) * \left(1 - SEN_e\right) \\ - \sum_c \left(QED_{c,e} * PQD_c\right) \end{pmatrix}$	$h*e$	$HOENT_{h,e}$	$h*e$
$GOVENTEQ_e$	$GOVENT_e = goventsh_e * \begin{pmatrix} \left(YE_e * \left(1 - TYE_e\right)\right) * \left(1 - SEN_e\right) \\ - \sum_c \left(QED_c * PQD_c\right) \end{pmatrix}$	e	$GOVENT_e$	e

Name	Equation	Number of Equations	Variable	Number of Variables
	TAX RATE BLOCK			
$TMDEF_c$	$TM_c = \left(\left(tmb_c + dabtm_c\right) * TMADJ\right) + \left(DTM * tm01_c\right)$	*cm*	TM_c	*cm*
$TEDEF_c$	$TE_c = \left(\left(teb_c + dabte_c\right) * TEADJ\right) + \left(DTE * te01_c\right)$	*ce*	TE_c	*ce*
$TSDEF_c$	$TS_c = \left(\left(tsb_c + dabts_c\right) * TSADJ\right) + \left(DTS * ts01_c\right)$	*c*	TS_c	*c*
$TEXDEF_c$	$TEX_c = \left(\left(texb_c + dabtex_c\right) * TEXADJ\right) + \left(DTEX * tex01_c\right)$	*c*	TEX_c	*c*
$TXDEF_a$	$TX_a = \left(\left(txb_a + dabtx_a\right) * TXADJ\right) + \left(DTX * tx01_a\right)$	*a*	TX_a	*a*
$TFDEF_{ff,a}$	$TF_{ff,a} = \left(\left(tfb_{ff,a} + dabtf_{ff,a}\right) * TFADJ\right) + \left(DTF * tf01_{ff,a}\right)$	*f*a*	$TF_{f,a}$	*f*a*
$TYFDEF_f$	$TYF_f = \left(\left(tyfb_f + dabtyf_f\right) * TYFADJ\right) + \left(DTYF * tyf01_f\right)$	*f*	TYF_f	*f*
$THYDEF_h$	$TYH_h = \left(\left(tyhb_h + dabtyh_h\right) * TYHADJ\right) + \left(DTYH * DTY * tyh01_h\right)$	*h*	TYH_h	*h*
$TYEDEF_e$	$TYE_e = \left(\left(tyeb_e + dabtye_e\right) * TYEADJ\right) + \left(DTYE * DTY * tye01_e\right)$	*e*	TYE_e	*e*
$TWATDEF_c$	$TWAT_c = ((twatb_c + dabtwat_c) * TWATADJ) + (DTWAT * twat01_c)$	*c*	$TWAT_c$	*c*
$TWATADEF_{c,a}$	$TWATA_{c,a} = ((twatab_{c,a} + dabtwata_{c,a}) * TWATAADJ)$ $+ (DTWATA * twata01_{c,a})$	*c*a*	$TWATA_{c,a}$	*c*a*

Name	Equation	Number of Equations	Variable	Number of Variables
	TAX REVENUE BLOCK			
MTAXEQ	$MTAX = \sum_{c}\left(TM_{c} * PWM_{c} * ER * QM_{c}\right)$	1	*MTAX*	1
ETAXEQ	$ETAX = \sum_{c}\left(TE_{c} * PWE_{c} * ER * QE_{c}\right)$	1	*ETAX*	1
STAXEQ	$STAX = \sum_{c}\left(\begin{array}{l} TS_{c} * PQS_{c} * \\ \left(QINTD_{c} + QCD_{c} + QED_{c} + QGD_{c} + QINVD_{c} + dstocconst_{c}\right. \end{array}\right.$ $= \sum_{c}\left(TS_{c} * PQS_{c} * QQ_{c}\right)$	1	*STAX*	1
EXTAXEQ	$EXTAX = \sum_{c}\left(TEX_{c} * PQS_{c} * QQ_{c}\right)$	1	*EXTAX*	1
ITAXEQ	$ITAX = \sum_{a}\left(TX_{a} * PX_{a} * QX_{a}\right)$	1	*ITAX*	1
FTAXEQ	$FTAX = \sum_{f,a}\left(TF_{f,a} * WF_{f} * WFDIST_{f,a} * FD_{f,a}\right)$	1	*FTAX*	1
FYTAXEQ	$FYTAX = \sum_{f}\left(TYF_{f} * \left(YF_{f} * \left(1 - deprec_{f}\right)\right)\right)$	1	*FTAX*	1
DTAXEQ	$DTAX = \sum_{h}\left(TYH_{h} * YH_{h}\right) + \sum_{e}\left(TYE_{e} * YE\right)$	1	*DTAX*	1
WATTAXEQ	$WATTAX = \sum_{c}(TWAT_{c} * PQS_{c} * QQ_{c})$	1	*WATTAX*	1
WATATAXEQ	$WATATAX = \sum_{c,a}(TWATA_{c,a} * PQD_{c} * PQDDIST_{c,a} * QWAT2_{c,a})$	1	*WATATAX*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	GOVERNMENT BLOCK			
YGEQ	$$YG = MTAX + ETAX + STAX + EXTAX + FTAX + ITAX + FYTAX + DTAX + WATTAX + WATATAX + \left(\sum_f govvash_f * YFDISP_f\right) + GOVENT + (govwor * ER)$$	1	YG	1
$QGDEQ_c$	$$QGD_c = qgdconst_c * QGDADJ$$	c	QGD_c	c
VGDEQ	$$VGD = \left(\sum_c QGD_c * PQD_c\right)$$	1	$VQGD$	1
EGEQ	$$EG = \left(\sum_c QGD_c * PQD_c\right) + \left(\sum_h hogovconst_h * HGADJ * CPI\right) + \left(\sum_e entgovconst_e * EGADJ * CPI\right)$$	1	EG	1

Name	Equation	Number of Equations	Variable	Number of Variables
	INVESTMENT BLOCK			
$SHHDEF_h$	$SHH_h = \left(\left(shhb_h + dabshh_h\right) * SHADJ * SADJ\right) + \left(DSHH * DS * shh01_h\right)$	h	SHH	H
$SENDEF_e$	$SEN_e = \left(\left(sen_e + dabsen_e\right) * SEADJ * SADJ\right) + \left(DSEN * DS * sen01_e\right)$	e	SEN	e
$TOTSAVEQ$	$TOTSAV = \sum_h \left(\left(YH_h * \left(1 - TYH_h\right)\right) * SHH_h\right) + \sum_e \left(\left(YE * \left(1 - TYE_e\right)\right) * SEN_e\right) + \sum_f \left(YF_f * deprec_f\right) + KAPGOV + \left(CAPWOR * ER\right)$	1	$TOTSAV$	1
$QINVDEQ_c$	$QINVD_c = \left(IADJ * qinvdconst_c\right)$	c	$QINVD_c$	c
$INVEST$	$INVEST = \sum_c \left(PQD_c * \left(QINVD_c + dstocconst_c\right)\right)$	1	$INVEST$	1
	FOREIGN INSTITUTIONS BLOCK			
$YFWOREQ_f$	$YFWOR_f = worvash_f * YFDISP_f$	f	$YFWOR_f$	f

Name	Equation	Number of Equations	Variable	Number of Variables
	MARKET CLEARING BLOCK			
$FMEQUIL_f$	$FS_f = \sum_a FD_{f,a}$	f	FS_f	f
$QEQUIL_c$	$QQ_c = QINTD_c + \sum_h QCD_{c,h} + \sum_e QED_{c,e} + QGD_c + QINVD_c + dstocconst_c$	c		
$CAPGOVEQ$	$KAPGOV = YG - EG$	1	$CAPGOV$	1
$CAEQUIL$	$CAPWOR = \left(\sum_c pwm_c * QM_c\right) + \left(\sum_f \frac{YFWOR_f}{ER}\right) - \left(\sum_c pwe_c * QE_c\right) - \left(\sum_f factwor_f\right) - \left(\sum_h howor_h\right) - entwor - govwor$	1	$CAPWOR$	1
$VFDOMDEQ$	$VFDOMD = \sum_c PQD_c * \left(\sum_h QCD_{c,h} + \sum_e QED_{c,e} + QGD_c + QINVD_c + dstocconst_c\right)$	1	$VFDOMD$	1
$VENTDSHEQ$	$VENTDSH_e = VENTD_e / VFDOMD$	1	$VENTDSH$	1
$VGDSHEQ$	$VGDSH = VGD / VFDOMD$	1	$VGDSH$	1
$INVESTSHEQ$	$INVESTSH = INVEST / VFDOMD$	1	$INVESTSH$	1
$WALRASEQ$	$TOTSAV = INVEST + WALRAS$	1	$WALRAS$	1

Name	Equation	Number of Equations	Variable	Number of Variables
	MODEL CLOSURE			
			$\overline{ER}$ or $\overline{CAPWOR}$	1
			$\overline{PWM_c}$ and $\overline{PWE_c}$ or $\overline{PWE_{cedn}}$	2*c*
			$\overline{PQDDIST_{c,a}}$	*c*a*
			$\overline{SADJ}$, $\overline{SHADJ}$, $\overline{SEADJ}$ or $\overline{IADJ}$ or $\overline{INVEST}$ or $\overline{INVESTSH}$	1
			$\overline{QEDADJ}$ or $\overline{VED}$ or $\overline{VEDSH}$	1
	At least **one** of		$\overline{TMADJ}$, $\overline{TEADJ}$, $\overline{TSADJ}$, $\overline{TEXADJ}$, $\overline{TYFADJ}$, $\overline{TXADJ}$, $\overline{TFADJ}$, $\overline{TYHADJ}$, $\overline{TYEADJ}$, $\overline{TYADJ}$, $\overline{TWATADJ}$, $\overline{TWATAADJ}$, $\overline{DTM}$, $\overline{DTE}$, $\overline{DTS}$, $\overline{DTEX}$, $\overline{DTYF}$, $\overline{DTX}$, $\overline{DTF}$, $\overline{DTYH}$, $\overline{DTYE}$, $\overline{DTY}$, $\overline{DTWAT}$, $\overline{DTWATA}$ and $\overline{CAPGOV}$	7
			at least **two** of $\overline{QGDADJ}$, $\overline{HGADJ}$, $\overline{EGADJ}$, $\overline{VGD}$ and $\overline{VGDSH}$	3
			$\overline{FS_f}$ and $\overline{WFDIST_{f,a}}$	(*f**(*a*+1))
			$\overline{CPI}$ or $\overline{PPI}$	1

Appendix 3: Equation Code

```
*################ 15. EQUATIONS ASSIGNMENTS ###############
* ------- TRADE BLOCK ----------------------------------------------
* #### Exports Block

* For some c there are no exports hence only implement for ce(c)

 PEDEF(c)$ce(c)..  PE(c) =E= PWE(c) * ER * (1 - TE(c)) ;

* For some c there are no exports hence only implement for ce(c)

 CET(c)$(cd(c) AND ce(c))..
           QXC(c) =E= at(c)*(gamma(c)*QE(c)**rhot(c) +
                     (1-gamma(c))*QD(c)**rhot(c))**(1/rhot(c)) ;

 ESUPPLY(c)$(cd(c) AND ce(c))..
                  QE(c) =E= QD(c)*((PE(c)/PD(c))*((1-gamma(c))
                               /gamma(c)))**(1/(rhot(c)-1)) ;

 EDEMAND(c)$ced(c)..
                  QE(c) =E= econ(c)*((PWE(c)/pwse(c))**(-eta(c))) ;

* For c with no exports OR for c with no domestic production
* domestic supply is by CETALT

 CETALT(c)$((cd(c) AND cen(c)) OR (cdn(c) AND ce(c)))..
                  QXC(c) =E= QD(c) + QE(c) ;

* #### Imports Block

* For some c there are no imports hence only implement for cm(c)

 PMDEF(c)$cm(c)..  PM(c) =E= (PWM(c) *(1 + TM(c))) * ER ;

* For some c there are no imports or domestic production
* hence only implement for cd(c) AND cm(c)

 ARMINGTON(c)$(cx(c) AND cm(c) )..
        QQ(c)  =E= ac(c)*(delta(c)*QM(c)**(-rhoc(c)) +
                  (1-delta(c))*QD(c)**(-rhoc(c)))**(-1/rhoc(c)) ;

 COSTMIN(c)$(cx(c) AND cm(c))..
                  QM(c) =E= QD(c)*((PD(c)/PM(c))*(delta(c)/
                               (1-delta(c))))**(1/(1+rhoc(c))) ;

* For c with no imports OR for c with no domestic production
* supply is from ARMALT

 ARMALT(c)$((cx(c) AND cmn(c)) OR (cxn(c) AND cm(c)))..
                   QQ(c) =E= QD(c) + QM(c) ;

* ------- COMMODITY PRICE BLOCK ------------------------------------
 PQDDEF(c)$(cd(c) OR cm(c))..
            PQD(c) =E= PQS(c) * (1 + TS(c) + TEX(c)+ TWAT(c)) ;
```

```
 PQSDEF(c)$((cd(c) OR cm(c)))..
            PQS(c)*QQ(c) =E= (PD(c)*QD(c))+(PM(c)*QM(c)) ;

 PXCDEF(c)$(cx(c))..
           PXC(c)*QXC(c) =E= (PD(c)*QD(c)) + (PE(c)*QE(c))$ce(c) ;

* ------- NUMERAIRE PRICE BLOCK -----------------------------------

 CPIDEF..            CPI =E= SUM(c,comtotsh(c)*PQD(c)) ;

 PPIDEF..            PPI =E= SUM(c,vddtotsh(c)*PD(c)) ;

* ------- PRODUCTION BLOCK ----------------------------------------

*------------- Level 1---------------------------

 PXDEF(a)$QINT0(a)..     PX(a) =E= SUM(c,ioqxacqx(a,c)*PXAC(a,c)) ;

 PVADEF(a)..  PX(a)*(1 - TX(a))*QX(a)
                        =E= (PVA(a)*QVA(a)) + (PINT(a)*QINT(a)) ;

 PINTDEF(a)..     PINT(a) =E= SUM(c$cnwat(c),ioqtdqd(c,a) * PQD(c)) ;

 ADXEQ(a)..       ADX(a) =E= ((adxb(a) + dabadx(a)) * ADXADJ)
                                    + (DADX * adx01(a)) ;

* CES aggregation functions for Level 1 of production nest
 QXPRODFN(a)$aqx(a)..
                 QX(a) =E= ADX(a)*(deltax(a)*QVA(a)**(-rhocx(a))
                            + (1-deltax(a))*QINT(a)**(-rhocx(a)))
                                   **(-1/rhocx(a)) ;

 QXFOC(a)$aqx(a)..
                 QVA(a) =E= QINT(a)*((PINT(a)/PVA(a))*(deltax(a)/
                               (1-deltax(a))))**(1/(1+rhocx(a))) ;

* Leontief aggregation functions for Level 1 of production nest

 QINTDEF(a)$aqxn(a)..
                QINT(a) =E= ioqintqx(a) * QX(a) ;

 QVADEF(a)$aqxn(a)..
                 QVA(a) =E= ioqvaqx(a) * QX(a) ;

*----------------------- Level 2---------------------------------

* CES aggregation functions for Level 2 of production nest

 ADVAEQ(a)..     ADVA(a) =E= ((advab(a) + dabadva(a)) * ADVAADJ)
                                    + (DADVA * adva01(a)) ;

* CES Prod'n func'n
 QVAPRODFN(a)$(QVA0(a) and afx(a))..
        QVA(a)  =E= ADVA(a)*(SUM(f2$deltava(f2,a), deltava(f2,a)
                     *(ADFD(f2,a)* FD(f2,a))**(-rhocva(a)))
                      +(deltava("cnw",a)*QNW(a)
                      **(-rhocva(a)))$deltava("cnw",a))
                      **(-1/rhocva(a));

* FOC for the factor composite
```

```
QVAFOC1(f2,a)$(deltava(f2,a)and afx(a) )..
        WF(f2)*WFDIST(f2,a)*(1 + TF(f2,a) )
          =E= PVA(a)*QVA(a)
             *(SUM(f2p$deltava(f2p,a),deltava(f2p,a)
             *(ADFD(f2p,a)*FD(f2p,a))**(-rhocva(a)))
             + (deltava("cnw",a)*QNW(a)
              **(-rhocva(a)))$deltava("cnw",a))**(-1)
             * deltava(f2,a)*ADFD(f2,a)**(-rhocva(a))
             * FD(f2,a)**(-rhocva(a)-1) ;

QVAFOC2(a)$(afx(a)and deltava("cnw",a) )..
     PNW(a) =E= PVA(a)*QVA(a)
               *(SUM(f2p$deltava(f2p,a),deltava(f2p,a)
               *(ADFD(f2p,a)*FD(f2p,a))**(-rhocva(a)))
               + deltava("cnw",a)*QNW(a) **(-rhocva(a)))**(-1)
               * deltava("cnw",a)*QNW(a)**(-rhocva(a)-1) ;

QVAEQ1(f2,a)$afxn(a)..
               FD(f2,a)*ADFD(f2,a) =E= ioffqva(f2,a)*QVA(a)  ;

QVAEQ2(a)$afxn(a)..
               QNW(a)    =E=   ioqnwqva(a)*QVA(a);

PVAEQ(a)$afxn(a)..
  PVA(a)  =E=  PNW(a)* ioqnwqva(a)
          +SUM(f2,ioffqva(f2,a)*WF(f2)*WFDIST(f2,a)*(1+TF(f2,a)));

* Intermediate Input Demand

QINTDEQ(c)..   QINTD(c) =E= SUM(a,ioqtdqd(c,a)*QINT(a))
                                 + SUM(a, QWAT2(c,a));

*------------- Level 3--------------------------------------------

 QNWPRODFN1(a)$(af3x(a) and an(a) )..
     QNW(a)  =E= atnw(a)*(SUM(f3$deltanw(f3,a),
                deltanw(f3,a)*(ADFD(f3,a)* FD(f3,a))**(-rhonw(a)))
                  +(deltanw("cwat",a)*QWAT(a)
                  **(-rhonw(a)))$deltanw("cwat",a))**(-1/rhonw(a));

* FOC for the factor composite

 QNWFOC1(f3,a)$(af3x(a) and deltanw(f3,a))..
      WF(f3)*WFDIST(f3,a)*(1 + TF(f3,a))
                   =E= PNW(a)*QNW(a)
                      *(SUM(f3p$deltanw(f3p,a),deltanw(f3p,a)
                      *(ADFD(f3p,a)*FD(f3p,a))**(-rhonw(a)))
                       + (deltanw("cwat",a)*QWAT(a)
                       **(-rhonw(a)))$deltanw("cwat",a))**(-1)
                       * deltanw(f3,a)*ADFD(f3,a)**(-rhonw(a))
                          * FD(f3,a)**(-rhonw(a)-1) ;

* FOC for the water-commodity composite

QNWFOC2(a)$(af3x(a) and deltanw("cwat",a) and SUM(f3,deltanw(f3,a)))
     PWAT(a) =E= PNW(a)*QNW(a)
                  *(SUM(f3p$deltanw(f3p,a),deltanw(f3p,a)
                  *(ADFD(f3p,a)*FD(f3p,a))**(-rhonw(a)))
                  + deltanw("cwat",a)*QWAT(a) **(-rhonw(a)))**(-1)
                  * deltanw("cwat",a)*QWAT(a)**(-rhonw(a)-1) ;
```

```
 QNWEQ1(f3,a)$af3xn(a)..
             FD(f3,a)*ADFD(f3,a) =E= iof3qnw(f3,a)*QNW(a) ;

 QNWEQ2(a)$(af3xn(a)or ann(a))..
              QWAT(a) =E= iocwatqnw(a)*QNW(a) ;

 PNWEQ(a)$ (af3xn(a) or ann(a))..
  PNW(a) =E= PWAT(a)*iocwatqnw(a)
     +SUM(f3,iof3qnw(f3,a)* WF(f3)*WFDIST(f3,a)*(1 + TF(f3,a)));

*------------- Level 4---------------------------

 WATDPRODFN(a)$(af4x(a) and awat(a) )..
  QWAT(a) =E= atwat(a)*(SUM(f4$deltafcwat(f4,a),deltafcwat(f4,a)
                *(ADFD(f4,a)* FD(f4,a)) **(-rhowat(a)))
               + SUM(cwat,deltafcwat(cwat,a)*QWAT2(cwat,a)
                           **(-rhowat(a))))**(-1/rhowat(a)) ;

 WATDFOC(cwat,a)$(af4x(a) and deltafcwat(cwat,a))..
    PQD(cwat)*PQDDIST(cwat,a)*(1+TWATA(cwat,a))
           =E=  PWAT(a)*QWAT(a)* (
                SUM(f4p$deltafcwat(f4p,a),deltafcwat(f4p,a)
               * ADFD(f4p,a)*FD(f4p,a)**(-rhowat(a)))+
                SUM(cwatp$deltafcwat(cwatp,a),
                deltafcwat(cwatp,a)*QWAT2(cwatp,a)
                **(-rhowat(a))))**(-1)
               *deltafcwat(cwat,a)*QWAT2(cwat,a)**(-rhowat(a)-1) ;

* FOC  for factor prices

 FD4FOC(f4,a)$(af4x(a) and  deltafcwat(f4,a))..
        WF(f4)*WFDIST(f4,a)*(1 + TF(f4,a))
                =E=  PWAT(a)*QWAT(a)*
                     (SUM(f4p$deltafcwat(f4p,a),deltafcwat(f4p,a)
                      * ADFD(f4p,a)*FD(f4p,a)**(-rhowat(a)))
                      +  SUM(cwatp$deltafcwat(cwatp,a),
                       deltafcwat(cwatp,a)*QWAT2(cwatp,a)
                       **(-rhowat(a))))**(-1)
                       * deltafcwat(f4,a)*ADFD(f4,a)**(-rhowat(a))
                       * FD(f4,a)**(-rhowat(a)-1) ;

QWATEQ1(c,a)$ (af4xn(a) and acwat(a))  ..
         QWAT2(c,a)=E= ioqwat(c,a)*QWAT(a)    ;

QWATEQ2(f4,a)$(af4xn(a) and afwat(a))..
        FD(f4,a)*ADFD(f4,a)  =E=  iof4aggf4(f4,a)*QWAT(a);

 PWATEQ(a)$(af4xn(a))  ..
        PWAT(a) =E=   SUM(c$cwat(c),ioqwat(c,a)*PQD(c)*PQDDIST(c,a)
                        *(1+TWATA(c,a)))
                    + SUM(f4, iof4aggf4(f4,a)*WF(f4)*WFDIST(f4,a)
                              *(1 + TF(f4,a)));

*-----------------------------------------------------------------
* Commodity Output

* CES aggregation of differentiated commodities

 COMOUT(c)$(cx(c) and cxac(c))..
```

```
            QXC(c) =E= adxc(c)*(SUM(a$deltaxc(a,c),deltaxc(a,c)
                      *QXAC(a,c)**(-rhocxc(c))))**(-1/rhocxc(c)) ;

 COMOUTFOC(a,c)$(deltaxc(a,c) and cxac(c)  )..
         PXAC(a,c) =E= PXC(c)*QXC(c)
                       *(SUM(ap$deltaxc(ap,c),deltaxc(ap,c)
                       *QXAC(ap,c)**(-rhocxc(c))))**(-1)
                      *deltaxc(a,c)*QXAC(a,c)**(-rhocxc(c)-1) ;

* Aggregation of homogenous commodities

 COMOUT2(c)$(cx(c) and cxacn(c))..
                 QXC(c) =E= SUM(a,QXAC(a,c)) ;

 COMOUTFOC2(a,c)$(deltaxc(a,c) and cxacn(c))..
              PXAC(a,c) =E= PXC(c) ;

* Activity Output

 ACTIVOUT(a,c)$ioqxacqx(a,c)..
              QXAC(a,c) =E= ioqxacqx(a,c) * QX(a) ;

* ######## FACTOR BLOCK

 YFEQ(f)..         YF(f) =E= SUM(a,WF(f)*WFDIST(f,a)*FD(f,a))
                           + (factwor(f)*ER) ;

 YFDISPEQ(f)..
    YFDISP(f) =E= (YF(f) * (1- deprec(f)))*(1 - TYF(f)) ;
```

```
* ######## HOUSEHOLD BLOCK
* ## Household Income

 YHEQ(h)..         YH(h) =E= SUM(f,hovash(h,f)*YFDISP(f))
                           + SUM(hp,HOHO(h,hp))
                           + SUM(e,HOENT(h,e))
                           + (HGADJ * hogovconst(h)*CPI)
                           + (howor(h)*ER) ;

* Household Expenditure

 HOHOEQ(h,hp)..
              HOHO(h,hp) =E= hohosh(h,hp)
                           *((YH(h) * (1 - TYH(h))) * (1 - SHH(h)))
;

 HEXPEQ(h)..     HEXP(h) =E= ((YH(h) * (1 - TYH(h))) * (1 - SHH(h)))
                           - SUM(hp,HOHO(hp,h)) ;

 QCDEQ(c,h) ..
    PQD(c)*QCD(c,h) =E= PQD(c)*qcdconst(c,h)
                      + beta(c,h)
                      *(HEXP(h)-SUM(cp,PQD(cp)*qcdconst(cp,h))) ;

* ------- ENTERPRISE BLOCK ------------------------------------------
* ## Enterprise Income

 YEEQ(e)..         YE(e) =E= SUM(f,entvash(e,f)*YFDISP(f))
                           + (EGADJ * entgovconst(e) * CPI)
                           + (entwor(e)*ER) ;
```

```
* ## Enterprise Expenditure

 QEDEQ(c,e)..
              QED(c,e) =E= QEDADJ*qedconst(c,e) ;

 HOENTEQ(h,e)..
              HOENT(h,e) =E= hoentsh(h,e)
                             * (((YE(e) * (1 - TYE(e))) * (1 -
SEN(e)))
                             - SUM(c,QED(c,e)*PQD(c))) ;

 GOVENTEQ(e).. GOVENT(e) =E= goventsh(e)
                         * (((YE(e) * (1 - TYE(e))) * (1 - SEN(e)))
                              - SUM(c,QED(c,e)*PQD(c))) ;

 VEDEQ(e)..       VED(e) =E= SUM(c ,QED(c,e)*PQD(c)) ;

* ------- GOVERNMENT BLOCK ------------------------------------------
* #### Government Income Block

* ## Government Tax Rates

 TMDEF(c)..        TM(c) =E= ((tmb(c) + dabtm(c)) * TMADJ)
                                  + (DTM * tm01(c)) ;

 TEDEF(c)..        TE(c) =E= ((teb(c) + dabte(c)) * TEADJ)
                                  + (DTE * te01(c)) ;

 TSDEF(c)$(cd(c) OR cm(c))..
                   TS(c) =E= ((tsb(c) + dabts(c)) * TSADJ)
                                  + (DTS * ts01(c)) ;

TEXDEF(c)$(cd(c) OR cm(c))..
                 TEX(c) =E= ((texb(c) + dabtex(c)) * TEXADJ)
                                  + (DTEX * tex01(c)) ;

TXDEF(a)..       TX(a) =E= ((txb(a) + dabtx(a)) * TXADJ)
                                  + (DTX * tx01(a)) ;

TFDEF(f,a)..     TF(f,a) =E= ((tfb(f,a) + dabtf(f,a))* TFADJ)
                                  + (DTF*tf01(f,a)) ;

TYFDEF(f)..      TYF(f) =E= ((tyfb(f) + dabtyf(f)) * TYFADJ)
                                 + (DTYF * tyf01(f)) ;

TYHDEF(h)..      TYH(h) =E= ((tyhb(h) + dabtyh(h)) * TYHADJ * TYADJ)
                                 + (DTYH * DTY * tyh01(h)) ;

TYEDEF(e)..      TYE(e) =E= ((tyeb(e) + dabtye(e)) * TYEADJ * TYADJ)
                                  + (DTYE * DTY * tye01(e)) ;

TWATDEF(c)..      TWAT(c)   =E= ((twatb(c) + dabtwat(c))* TWATADJ)
                                  + (DTWAT*twat01(c)) ;

TWATADEF(c,a)..
          TWATA(c,a)=E= ((twatab(c,a)+dabtwata(c,a))* TWATAADJ)
                                  + (DTWATA*twata01(c,a)) ;

* ## Government Tax Revenues
```

```
 MTAXEQ..            MTAX =E= SUM(c,TM(c)*PWM(c)*ER*QM(c)) ;

 ETAXEQ..            ETAX =E= SUM(c,TE(c)*PWE(c)*ER*QE(c)) ;

 STAXEQ..            STAX =E= SUM(c,TS(c)*PQS(c)*QQ(c)) ;

 EXTAXEQ..           EXTAX =E= SUM(c,TEX(c)*PQS(c)*QQ(c)) ;

 ITAXEQ..            ITAX =E= SUM(a,TX(a)*PX(a)*QX(a)) ;

 FTAXEQ..
         FTAX =E= SUM((f,a),TF(f,a)*WF(f)*WFDIST(f,a)*FD(f,a)) ;

 FYTAXEQ..       FYTAX =E= SUM(f,TYF(f)*(YF(f) * (1- deprec(f)))) ;

 DTAXEQ..            DTAX =E= SUM(h,TYH(h)*YH(h))
                              + SUM(e,TYE(e)*YE(e));

 WATTAXEQ..          WATTAX  =E= SUM(c,TWAT(c)*PQS(c)*QQ(c));

 WATATAXEQ..
   WATATAX =E= SUM((c,a),TWATA(c,a)*PQD(c)*PQDDIST(c,a)*QWAT2(c,a)) ;

* ## Government Income

 YGEQ..               YG =E= MTAX + ETAX
                             + STAX + EXTAX + ITAX + FTAX
                             + FYTAX + DTAX
                             + WATTAX + WATATAX
                             + SUM(f,govvash(f)*YFDISP(f))
                             + SUM(e,GOVENT(e)) + (govwor*ER)  ;

* #### Government Expenditure Block

 QGDEQ(c)..       QGD(c) =E= QGDADJ*comgovconst(c) ;

 EGEQ..               EG =E= SUM(c,QGD(c)*PQD(c))
                             + SUM(h,hogovconst(h)*CPI*HGADJ)
                             + SUM(e,EGADJ*entgovconst(e)*CPI) ;

 VGDEQ..             VGD =E= SUM(c,QGD(c)*PQD(c)) ;

* ------- KAPITAL BLOCK -------------------------------------------
* ## Savings Block

 SHHDEF(h)..     SHH(h) =E= ((shhb(h) + dabshh(h)) * SHADJ * SADJ)
                              + (DSHH * DS * shh01(h)) ;

 SENDEF(e)..     SEN(e) =E= ((senb(e) + dabsen(e)) * SEADJ * SADJ)
                              + (DSEN * DS * sen01(e)) ;

 TOTSAVEQ..       TOTSAV =E= SUM(f,(deprec(f)*YF(f)))
                             + SUM(h,(YH(h) * (1 - TYH(h))) * SHH(h))
                             + SUM(e,(YE(e) * (1 - TYE(e))) * SEN(e))
                             + KAPGOV
                             + (CAPWOR*ER) ;

* ## Investment Block

 QINVDEQ(c)..     QINVD(c) =E= IADJ*invconst(c);
```

```
 INVESTEQ..       INVEST =E= SUM(c,PQD(c)*(QINVD(c) + dstocconst(c)))
;

* ------- FOREIGN INSTITUTIONS BLOCK -----------------------------
 YFWOREQ(f)..   YFWOR(f) =E= worvash(f)*YFDISP(f) ;

* ------- MARKET CLEARING BLOCK ----------------------------------

* ##### Account Closure

 FMEQUIL(f)..      FS(f) =E= SUM(a,FD(f,a)) ;

 QEQUIL(c)..
    QQ(c)  =E= QINTD(c) + SUM(h,QCD(c,h)) + SUM(e,QED(c,e))
                            + QGD(c) + QINVD(c) + dstocconst(c) ;

 GOVEQUIL..       KAPGOV =E= YG - EG ;

 CAEQUIL..        CAPWOR =E= SUM(cm,PWM(cm)*QM(cm))
                           + (SUM(f,YFWOR(f))/ER)
                           - SUM(ce,PWE(ce)*QE(ce))
                           - SUM(h,howor(h))
                           - SUM(e,entwor(e))
                           - govwor
                           - SUM(f,factwor(f)) ;

* #### Absorption Closure

 VFDOMDEQ..       VFDOMD =E= SUM(c, PQD(c) *
                            (SUM(h,QCD(c,h)) + SUM(e,QED(c,e))
                             + QGD(c) + QINVD(c) +
dstocconst(c))) ;

 INVESTSHEQ..
       INVESTSH * VFDOMD =E= INVEST ;

 VGDSHEQ..
          VGDSH * VFDOMD =E= VGD ;

 VEDSHEQ(e)..
       VEDSH(e) * VFDOMD =E= VED(e) ;

* ##### GDP

 GDPVAEQ..         GDPVA =E= SUM((f,a),WF(f)*WFDIST(f,a)*FD(f,a))
                             + MTAX + ETAX + STAX + EXTAX + ITAX + FTAX
                             + WATTAX + WATATAX  ;

* ##### Slack

 WALRASEQ..       TOTSAV =E= INVEST + WALRAS ;
```

Appendix 4: Example SAM

2004 SAM for Israel in 100 Million New Israeli Shekel

	ccrops	cvegfruit	cmixfarm	cagri	cind	cser	cwatpot	cwatrec	cwatsal	acrops	avegfruit	amixfarm	aagri	aind	aser	awatpot	awatrec	awatsal	awatdsal	flab	fcap	fland	fwat	hj	hao	twat	twata	tother	gov	ent	kap	row	TOTAL
ccrops										3.17	2.68	3.08	4.36	9.55	1.18									9.42	1.39						2.74	17.61	**55.18**
cvegfruit										0.01	0.20	0.01	0.09	10.43	8.46									71.14	13.66						0.29	24.11	**128.37**
cmixfarm										0.01				0.50	13.37	0.16	0.01		0.05					6.00	1.14						1.63	0.17	**23.06**
cagri										0.08	0.25	0.32	10.30	83.15	4.09									17.77	3.70						1.06	9.49	**130.21**
cind										7.64	21.99	6.15	26.47	1498.61	917.02	9.70	1.16	0.21	2.44					1014.80	160.44						425.65	1635.01	**5727.31**
cser										11.49	57.19	0.69	25.91	1000.80	2260.66	7.61	0.64	0.20	1.24					1606.05	230.94				1495.46		555.74	660.77	**7915.40**
cwatpot										4.94	14.77	0.41	4.18	4.86	9.83									17.73	3.07								**59.80**
cwatrec										0.88	2.62	0.06																					**3.55**
cwatsal										0.48	1.44																						**1.93**
acrops	38.09				1.02																												**39.11**
avegfruit		124.55			0.44																												**124.99**
amixfarm			20.82																														**20.82**
aagri				95.98	0.92																												**96.90**
aind				6.14	3382.14																												**3388.29**
aser					10.64	7196.40																											**7207.04**
awatpot							31.34																										**31.34**
awatrec								2.70																									**2.70**
awatsal									0.68																								**0.68**
awatdsal							3.04																										**3.04**
flab										8.56	18.79	8.96	16.67	700.35	2607.80	5.87	0.41	0.15	0.33													13.35	**3381.25**
fcap										4.63	13.64	1.17	10.82	66.80	1257.59	3.61	0.45	0.10	0.84													121.20	**1480.84**
fland										0.47	1.25	0.05	0.25																				**2.02**
fwat																4.04																	**4.04**
hj																				2708.60	62.34			17.38	2.51				588.83	290.29		105.90	**3775.85**
hao																				356.09	3.97			3.92	0.33				99.78	28.63		17.73	**510.45**
twat							21.35	0.61	1.11																								**23.06**
twata										-3.71	-11.08	-0.31	-3.14	-2.46																			**-20.69**
tother	0.30	-1.89	1.20	3.81	493.86	259.65	4.07	0.23	0.14	0.46	1.24	0.25	0.99	15.71	127.02	0.35	0.02	0.01	-1.85	217.19				550.03	38.88					263.45		7.58	**1982.71**
gov																							4.04			23.06	-20.69	1982.71				149.40	**2138.53**
ent																					1214.62	2.02							40.10				**1256.73**
kap																								461.61	54.37				-85.64	674.37	53.31	-117.60	**1040.42**
row	16.79	5.71	1.03	24.28	1838.29	459.35														99.36	199.91												**2644.73**
TOTAL	**55.18**	**128.37**	**23.06**	**130.21**	**5727.31**	**7915.40**	**59.80**	**3.55**	**1.93**	**39.11**	**124.99**	**20.82**	**96.90**	**3388.29**	**7207.04**	**31.34**	**2.70**	**0.68**	**3.04**	**3381.25**	**1480.84**	**2.02**	**4.04**	**3775.85**	**510.45**	**23.06**	**-20.69**	**1982.71**	**2138.53**	**1256.73**	**1040.42**	**2644.73**	**43179.65**

Legend of accounts

Commodities		Activities		Factors and Taxes		Others	
ccrops	field crops	acrops	field crops	flab	labour	gov	government
cvegfruit	fruits and vegetables	avegfruit	fruits and vegetables	fcap	capital	ent	enterprises
cmixfarm	mixed farming, gardening, forestry	amixfarm	mixed farming, gardening, forestry	fland	land	kap	capital and stock changes
cagri	other agriculture	aagri	other agriculture	fwat	fresh water resources	row	rest of the world
cind	industrial commodities	aind	industrial activities	hj	Jewish households		
cser	service commodities	aser	services	hao	Arab and other households		
cwatpot	potable water	awatpot	fresh water purification	twat	water commodity tax		
cwatrec	reclaimed water	awatrec	wastewater reclamation	twata	water user subsidy		
cwatsal	brackish water	awatsal	pumping of brackish water	tother	other taxes		
		awatdsal	desalination				

Previously released in *Agricultural Economics Working Paper Series*:

Becker, T.; Benner, E (2000): Zur Problematik der Herkunftsangabe im regionalen Marketing
Arbeitsbericht Nr. 1

Chaipan, C. (2000): The Euro and its Impact on ASEAN Economies
Arbeitsbericht Nr. 2

Henze, A. (2000): Marktwirtschaft - Wirtschaftliche Freiheit, motiviertes Handeln und Wettbewerb: Quellen des Wohlstands
Arbeitsbericht Nr. 3

Benner, E. (2000): Zur effizienten Herkunftsangabe im europäischen Binnenmarkt
Arbeitsbericht Nr. 4

Vorgrimler, D. (2000): Wettbewerbstheorie und stagnierende Märkte
Arbeitsbericht Nr. 5

Beerbaum, S. (2001): Grundzüge einer internationalen Zusammenarbeit im Klimaschutz aus ökonomischer Sicht
Arbeitsbericht Nr. 6

Vorgrimler, D.; Wübben, D. (2001): Prognose der Entwicklung des Agrartechnikmarktes - Eine Expertenbefragung nach der Delphi-Methode
Arbeitsbericht Nr. 7

Tesch, I. (2003): Informationsbedarf und Informationsbeschaffung von Konsumenten bei Lebensmitteln pflanzlicher Herkunft - Eine empirische Untersuchung anhand von Fokus-Gruppen -
Arbeitsbericht Nr. 8

Benner, D. (2004): Quality Ambiguity and the Market Mechanism for Credence Goods
Arbeitsbericht Nr. 9

Benner, E., Kliebisch, C. (2004): Regio-Marketing-Strategien des Lebensmitteleinzelhandels
Arbeitsbericht Nr. 10

Benner, E., Heidecke, S.-J. (2005): Grundpreisaufschläge bei Groß- und Familienpackungen - eine empirische Untersuchung im deutschen und französischen Lebensmitteleinzelhandel -
Arbeitsbericht Nr. 11

Becker, T. (2006): Zur Bedeutung geschützter Herkunftsangaben.
Arbeitsbericht Nr. 12, 1. und 2. Auflage.

Elsäßer, A., Benner, E., Becker, T. (2006): Marketing auf Wochenmärkten
Arbeitsbericht Nr. 13

Becker, T. (2006): Die CMA auf dem Prüfstand
Arbeitsbericht Nr. 14

Staus, A. (2007): An Ordinal Regression Model using Dealer Satisfaction Data
Arbeitsbericht Nr. 15

Kliebisch, C., Rügge, M. (2007): Alte und neue Wege des Gemeinschaftsmarketings für Agrarprodukte und Lebensmittel
Arbeitsbericht Nr. 16

Staus, A. (2008): Standard and Shuffled Halton Sequences in a Mixed Logit Model
Arbeitsbericht Nr. 17

Staus, A., Becker, T. (2009): Die Zufriedenheit der Landmaschinenhändler mit den Herstellern
Arbeitsbericht Nr. 18

Becker, T., Heinze, K. (2011): Gesellschaftliches Management von Verbraucherbeschwerden: Funktion und Finanzierung
Arbeitsbericht Nr. 19

Khalid Siddig, Dorothee Flaig, Jonas Luckmann, Harald Grethe (2011): A 2004 Social Accounting Matrix for Israel. Documentation of an Economy-Wide Database with a Focus on Agriculture, the Labour Market, and Income Distribution
Working Paper No. 20

Bücheler, G. (2011): Biokraftstoff-Zertifizierungssysteme ISCC und REDcert: Darstellung, Vergleich und kritische Diskussion
Working Paper No. 21

Gebhardt, B. (2012): Akzeptanz und Erfolg kleinräumiger Systeme der Lebensmittelversorgung im urbanen Umfeld am Beispiel Stuttgart - Empirische Untersuchungen von Verbrauchern und Unternehmen
Working Paper No. 22

pdf download at https://marktlehre.uni-hohenheim.de (section research – working paper)