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Incorporating undesirable outputs into models of production:

an application to US agriculture

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La prise en compte des nuisances dans les modèles de production : une application à l'agriculture américaine

Résumé – Les méthodes utilisées habituellement pour calculer la croissance de la productivité globale des facteurs ne tiennent pas compte des produits joints non désirés, qui sont souvent des sous-produits apparaissant au cours du processus de fabrication des produits que l'on cherche à obtenir. Cela est dû en grande partie à la difficulté d'établir un système de prix pour les produits non désirés, qui ne sont en général pas écoulés sur les marchés. Cet article montre comment prendre en compte ces produits dans un modèle de production ; à la différence des produits désirés, qui sont facilement disponibles à un prix de marché, les produits non désirés sont peu disponibles et n'ont pas de prix de marché. Nous montrons comment les méthodes habituelles de calcul de la productivité totale des facteurs peuvent être adaptées à la prise en compte des produits non désirés. Il est nécessaire, pour cela, de disposer de leurs prix fictifs ou des coûts entraînés par la réduction de leur volume. Plutôt que de calculer ces derniers de façon exogène, nous les obtenons comme solution de programmes mathématiques décrivant la technologie de production utilisée. Celle-ci est caractérisée par le taux de baisse de la production nécessaire pour obtenir une réduction donnée de la quantité de produits non désirés. La connaissance de la valeur du taux de « compromis » permet de calculer le coût de cette réduction.

Nous appliquons cette méthode à des séries temporelles portant sur l'agriculture américaine entre 1961 et 1988. Notre base de données comprend les indices relatifs à trois intrants (le capital, le travail et les équipements), deux produits (animaux et végétaux) et un sous-produit non désiré (l'excès d'azote). La production de ce sous-produit n'est pas souhaitée, car en s'infiltrant dans la nappe phréatique il peut être nocif pour la santé. L'éliminer revient cher car, en contrepartie, il faut soit réduire le volume de la production, soit augmenter celui des intrants : la dépollution conduit alors vraisemblablement à freiner la croissance de la productivité des facteurs. D'après nos calculs, la prise en compte de l'excès d'azote dans un modèle de production agricole fait baisser la productivité totale des facteurs de 12 à 28 %.

Mots-clés :
productivité globale des facteurs, produits désirés, produits non désirés, modèle de production, agriculture américaine.

Incorporating undesirable outputs into models of production : an application to US agriculture

Summary – *Conventional total factor productivity growth calculations do not account for the undesirable outputs that are generated as byproducts of the production process that transforms inputs into desirable outputs. This omission is due to the fact that it is extremely difficult to obtain prices with which to value the undesirable outputs. In this paper we show how to use mathematical programming techniques to obtain shadow elasticities, or abatement cost elasticities, of the undesirable outputs. We then use these techniques to adjust total factor productivity growth calculations in US agriculture, which generates as an undesirable output excess nitrogen, which causes groundwater contamination. Results suggest a downward adjustment of between 12 % and 28 % of conventional total factor productivity growth calculations.*

Key-words:
models of production, US agriculture, undesirable outputs, desirable outputs, total factor productivity

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In this paper we develop an analytical framework for incorporating undesirable outputs, such as pollutants, into models of production. The model extends earlier work of Färe, Grosskopf and Lovell (1985), and Färe, Grosskopf, Lovell and Pasurka (1989). Following these authors, we use mathematical programming techniques to construct a production possibilities frontier, to measure productive efficiency, and to calculate shadow values (a dual measure of abatement costs) of undesirable outputs. We then follow Pittman (1983) by using these shadow values to adjust a conventional measure of total factor productivity growth for the generation of undesirable byproducts in the production process. Our application is to an aggregate time series, 1961-1988, of US agricultural production, where conventionally measured total factor productivity has grown at an annual rate of 1.38%. We find a substantial portion of conventionally measured total factor productivity growth to be eliminated by the incorporation of one undesirable output, an index of excess nitrogen generated as a byproduct of the application of chemical fertilizer in the production of animal and crop outputs. This excess nitrogen has grown at an annual rate of 5.04%, and is a primary source of groundwater contamination in the farm belt. Incorporating excess nitrogen into the model leads to a downward adjustment to total factor productivity growth on the order of 12% – 28%, depending on how shadow prices of excess nitrogen are calculated.

Our analysis demonstrates the feasibility of the techniques, and also demonstrates the magnitude of the problem of environmental degradation in US agriculture. Nonetheless our findings are preliminary in three important respects. First, we have not accounted for technical change in the model, although it is not clear what bias this omission introduces into our shadow price calculations. Second, although our mathematical programming models have the virtue of being nonparametric, they have the drawback of being deterministic. Consequently, they are unable to account for the effects of statistical noise due to measurement error and other causes. The undesirable output index is particularly susceptible to measurement error, and this may bias our calculated shadow prices, although in an unknown direction. We are currently experimenting with a chance-constrained programming formulation of the model in an effort to introduce a stochastic element into the analysis. Third, our data set consists of a single aggregate time series. We have only 28 observations from which to disentangle the effects of six variables, and so we have very few degrees of freedom. Consequently our adjustments to a conventional measure of total factor productivity growth are based on a very small number of reliable calculations of the shadow prices of the undesirable output. The Economic Research Service at the US Department of Agriculture is in the process of constructing a state-by-year panel data set, which will substantially enhance our ability to calculate shadow values for undesirable outputs in US agriculture. Nonetheless, given these three limitations, our preliminary findings are suggestive of the magnitude of the problem.

In the first section we develop our analytical model of production and productivity growth when undesirable byproducts are generated in the process of producing desirable outputs. Our mathematical programming models are presented in the second section. Data are described and empirical results are presented in the final section.

PRODUCTION AND PRODUCTIVITY IN THE PRESENCE OF UNDESIRABLE OUTPUTS

Suppose a producer uses inputs $x = (x_1, ..., x_n) \in R^n_+$ to produce desirable outputs $y = (y_1, ..., y_m) \in R^m_+$ and, for simplicity, a single undesirable output $b \in R_+$. If all $m + 1$ outputs are strongly (freely) disposable, then feasiblity of the production activity (y, b, x) implies feasibility of the production activity (y', b', x), for all $0 \leq y' \leq y$ and $0 \leq b' \leq b$. If all $m + 1$ outputs are only weakly disposable, then feasibility of the production activity (y, b, x) implies feasibility of the production activity $(\gamma y, \gamma b, x)$, $0 \leq \gamma \leq 1$. Strong disposability of all $m + 1$ outputs implies feasibility of any componentwise output reduction, including the undesirable output. This is obviously inappropriate in the case of an undesirable output, whose disposal is apt to be costly, whether or not disposal is constrained by regulation. However, weak disposability of all $m + 1$ outputs is also inappropriate, since it is desirable to allow for componentwise reduction of the m desirable outputs. What is needed is a model that allows for strong disposability of the m desirable outputs and only weak, costly, disposability of the undesirable output. Such a model would satisfy the conditions (i) $(\gamma y, \gamma b, x)$ is feasible for all $0 \leq \gamma \leq 1$, and (ii) (y', b, x) is feasible for all $0 \leq y' \leq y$.

A production technology incorporating strong disposability of desirable outputs and only weak disposability of the undesirable byproduct can be developed as follows. The output correspondence $P(x) = \{(y,b) : (y,b,x)$ is feasible$\}$ defines the set of all desirable and undesirable output combinations that can be produced with input vector x. A nonparametric representation of $P(x)$ satisfying the desired disposability properties is given by :

$$P(x) = \{(y,b): y_i \leq \sum_{t=1}^{T} \lambda_t y_{it}, \quad i = 1, ..., m, b = \sum_{t=1}^{T} \lambda_t b_t,$$

$$x_i \geq \sum_{t=1}^{T} \lambda_t y_{it}, \quad i = 1, ..., n, \lambda_t \geq 0, t = 1, ..., T, \sum_{t=1}^{T} \lambda_t = 1\}, \qquad (1)$$

where $t = 1,...,T$ indexes time. This technology is convex, allows for variable returns to scale and strong disposability of the m desirable outputs, and permits only weak disposability of the undesirable output. The extent of the departure from strong disposability of the undesirable output, *i.e.*, the cost of disposability, is determined by the structure of the technology constructed from the data. In particular, shadow costs of abatement are determined by the slopes of the facets defining the surface of $P(x)$ in the y_i/b dimensions. These slopes are allowed to vary from one observation to another, *i.e.*, from year to year.

We are interested in measuring observed performance relative to production possibilities as characterized by $P(x)$. We are also interested in characterizing the structure of best-practice technology, which will reveal the nature of the trade-offs between undesirable and desirable outputs, or the costs of disposing of the undesirable output in terms of the amounts of desirable outputs that must be sacrificed. Techniques for solving both of these problems are developed in the next section. We now turn our attention to an investigation of the measurement of total factor productivity growth in the presence of an undesirable output. As we will demonstrate, this requires information on the trade-offs between the undesirable output and each desirable output.

Using notation developed above, a conventional Törnqvist index of total factor productivity growth, which ignores the undesirable output, is given by:

$$TFPG^C = \sum_{i=1}^{m} \bar{r}_i \dot{y}_i - \sum_{i=1}^{n} \bar{w}_i \dot{x}_i, \tag{2}$$

where $\bar{r}_i$ and $\bar{w}_i$ are adjacent-year mean revenue shares of desirable outputs and cost shares of inputs, and $\dot{y}_i$ and $\dot{x}_i$ are growth rates of desirable outputs and inputs. Caves, Christensen and Diewert (1982) demonstrated that the Törnqvist index is a superlative index, being exact for a technology characterized by adjacent-year distance functions that are translog, which are allowed to have different first-order coefficients and which allow for variable returns to scale. It was Pittman (1983) who first demonstrated that a properly adjusted measure of total factor productivity growth, which accounts for the costly disposability of the undesirable output, is given by:

$$TFPG^A = \sum_{i=1}^{m+1} \bar{s}_i \dot{y}_i - \sum_{i=1}^{n} \bar{w}_i \dot{x}_i \tag{3}$$

where

$$\bar{s}_i = \begin{cases} \dfrac{1}{2}\left(\dfrac{p_{it}\, y_{it}}{\sum_{i=1}^{m} p_{it}\, y_{it} - s_{bt}\, b_t} + \dfrac{p_{it+1}\, y_{it+1}}{\sum_{i=1}^{m} p_{it+1}\, y_{it+1} - s_{bt+1}\, b_{t+1}} \right), & i = 1, \ldots, m \\[2ex] \dfrac{1}{2}\left(\dfrac{-s_{bt}\, b_t}{\sum_{i=1}^{m} p_{it}\, y_{it} - s_{bt}\, b_t} + \dfrac{-s_{bt+1}\, b_{t+1}}{\sum_{i=1}^{m} p_{it+1}\, y_{it+1} - s_{bt+1}\, b_{t+1}} \right), & i = m+1\ (=b) \end{cases}$$

Thus the general form of the Törnqvist index is unchanged. However there are now $m + 1$ outputs whose growth rates are to be calculated, and the mean revenue shares used to weight the rates of growth of each output are adjusted to account for the conversion of gross revenue to net revenue. If $s_{bt} = 0$ for all $t = 1, \ldots, T$, then $\bar{s}_i = \bar{r}_i$ and $TFPG^C = TFPG^A$. Otherwise this adjustment increases the weights applied to growth rates of desirable outputs, and so:

$$\sum_{i=1}^{m} \bar{s}_i \dot{y}_i > \sum_{i=1}^{m} \bar{r}_i \dot{y}_i$$

However this effect is generally more than offset by the weighted growth rate of the undesirable output, $\bar{s}_{m+1} \; \dot{y}_{m+1} = \bar{s}_b \dot{b} < 0$ Consequently, we anticipate that $TFPG^A < TFPG^C$.

All but one piece of information required to calculate the adjusted mean revenue shares is readily available; the only missing data are the annual values of s_{bt}, the «price» of the undesirable output.

It would be exceedingly difficult to calculate nominal annual shadow prices; fortunately all that is required is the calculation of annual shadow elasticities of each desirable output with respect to the undesirable output, and these are relatively easy to calculate. Sufficiency is demonstrated by noting that the adjusted revenue shares in any year can be rewritten as:

$$s_{it} = \begin{cases} \left(1 + \sum_{j \neq i}^{m} \dfrac{p_{jt}\, y_{jt}}{p_{it}\, y_{it}} - \in^{t}_{ib}\right)^{-1}, & i = 1, \ldots, m, \\[2ex] \left(1 - \sum_{i=1}^{m} (\in^{t}_{ib})^{-1}\right)^{-1} & \end{cases} \tag{5}$$

where $\in^{t}_{ib} = (s_{bt} b_t \,/\, p_{it}\, y_{it}) = (\delta y_{it} \,/\, \delta b_t)(b_t \,/\, y_{it})$, $i = 1, \ldots, m$. In this formulation the missing information is the elasticities of each desirable output with respect to the undesirable output.

It is natural to think of these elasticities as abatement cost elasticities, for they measure the proportionate reduction in each desirable output required to achieve a given proportionate reduction in the undesirable output. In the next section we show how to calculate annual values of these abatement cost elasticities.

MATHEMATICAL PROGRAMMING MODELS OF PRODUCTION INCORPORATING UNDESIRABLE OUTPUTS

In this section we present a series of four mathematical programming models of production. Each is intended to construct the output correspondence $P(x)$ described in equation (1). They differ only in their orientation, in the way they measure performance. The purpose of having four different orientations is not so much to measure performance four different ways as to have four separate sets of shadow price calculations at four different points on the boundary of production technology. In anticipation of the empirical application, each model assumes $m = 2$, $n = 3$.

The first model is a linear program which measures performance radially, as the ability of a producer to expand all outputs, desirable and undesirable, equiproportionately, given its inputs. Shadow prices of the undesirable output are then calculated at the optimal radial projection.

Problem I

$$\max_{\theta,\lambda} \theta : \theta y_i^0 \le \sum_{t=1}^{T} \lambda_t y_{it}, \qquad i = 1, 2$$

$$x_i^0 \ge \sum_{t=1}^{T} \lambda_t \qquad i = 1, 2, 3$$

$$\theta b^0 = \sum_{t=1}^{T} \lambda_t b_t$$

$$\lambda_t \ge 0, \qquad \sum_{t=1}^{T} \lambda_t = 1$$

The second model is a linear program which measures performance in terms of the ability of a producer to expand all desirable outputs equiproportionately, given its undesirable output and its inputs. Shadow prices of the undesirable output are then calculated at the optimal projection.

Problem II

$$\max_{\theta,\lambda} \theta : \theta y_i^0 \le \sum_{t=1}^{T} \lambda_t y_{it}, \qquad i = 1, 2$$

$$x_i^0 \ge \sum_{t=1}^{T} \lambda_t x_{it}, \qquad i = 1, 2, 3$$

$$b^0 = \sum_{t=1}^{T} \lambda_t b_t$$

$$\lambda_t \geq 0, \qquad \sum_{t=1}^{T} \lambda_t = 1$$

The third model is a nonlinear program which measures performance hyperbolically, as the ability of a producer to expand all desirable outputs, and contract its undesirable output, equiproportionately, given its inputs. Shadow prices of the undesirable output are then calculated at the optimal hyperbolic projection.

Problem III

$$\max_{\theta, \lambda} \theta : \theta y_i^0 \leq \sum_{t=1}^{T} \lambda_t y_{it}, \qquad i = 1, 2$$

$$x_i^0 \geq \sum_{t=1}^{T} \lambda_t x_{it}, \qquad i = 1, 2, 3$$

$$\theta^{-1} b^0 = \sum_{t=1}^{T} \lambda_t b_t$$

$$\lambda_t \geq 0, \qquad \sum_{t=1}^{T} \lambda_t = 1$$

The fourth model is a nonlinear program which measures performance in terms of the ability of a producer to contract its undesirable output, given its desirable outputs and its inputs. Shadow prices of the undesirable output are then calculated at the optimal projection.

Problem IV

$$\max_{\theta, \lambda} \theta : \theta y_i^0 \leq \sum_{t=1}^{T} \lambda_t y_{it}, \qquad i = 1, 2$$

$$x_i^0 \geq \sum_{t=1}^{T} \lambda_t x_{it}, \qquad i = 1, 2, 3$$

$$\theta^{-1} b^0 = \sum_{t=1}^{T} \lambda_t b_t$$

$$\lambda_t \geq 0, \qquad \sum_{t=1}^{T} \lambda_t = 1$$

The output of each mathematical programming problem provides four types of information for each observation in the sample:

(i) An annual scalar θ^t, which measures the performance, in terms of technical efficiency, of each annual observation.

(ii) An annual vector λ^t, which identifies and weights the efficient annual observations relative to which each annual observation is compared.

(iii) A set of annual dual variables, which measure the effect on performance, as measured by the optimal value of the objective, of a perturbation of each constraint in the mathematical programming problem.

(iv) An annual pair of abatement cost elasticities, of each desirable output with respect to the undesirable output, which are calculated from the annual values of the dual variables by means of:

$$\epsilon^t_{1b} = (s^t_b / s^t_1)\,(b^t / y^t_1)^*,$$
$$\epsilon^t_{2b} = (s^t_b / s^t_2)\,(b^t / y^t_2)^*,$$

where $(b^t / y^t_1)^*$ and $(b^t / y^t_2)^*$ are optimal projections.

Although we are interested in measuring annual performance, in terms of technical efficiency, our main interest lies in using values of dual variables to calculate annual values of the abatement cost elasticities for the undesirable output. These abatement cost elasticities are used in equation (5) to calculate annual values of adjusted revenue shares, which are then used in equation (3) to obtain an adjusted measure of total factor productivity growth.

AN APPLICATION TO US AGRICULTURE

We use annual time series data on US agricultural production over the period 1961-1988; these data update a series compiled by Ball (1985, 1988). We use indexes, normalized to 1982, of three inputs (capital, labor and materials), two desirable outputs (animals and crops) and one undesirable output (excess nitrogen). Excess nitrogen is defined as the difference between the amount of nitrogen applied from all sources (primarily chemical fertilizers and livestock manure) and the amount of nitrogen removed in the crop production process. Construction of the input and the animal and crop indexes is described in Ball (1985, 1988). The excess nitrogen index is constructed in a manner similar to that used in the Netherlands by Winteringham (1985) and Hoogervorst (1990). Briefly, excess nitrogen is computed from survey data, and is defined as chemical nitrogen applied, plus soybean and legume credits, plus livestock credits. This nitrogen accounting approach is used to estimate residual nitrogen for corn as a proxy for potential contamination of groundwater at the national level. During most of the 1961-1988 period corn accounted for close to 75% of residual nitrogen resulting form animal and crop production in the US The addition of manure to chemical sources adds a total amount of nitrogen that exceeds what can be absorbed by crops in many areas. The impact of weather and soil type also influences the amount of excess nitrogen and its potential for groundwater contamination.

We treat excess nitrogen as an undesirable output because a mounting body of evidence suggests that it enters water supplies in many regions of the United States at potentially harmful levels that may adversely affect human and animal health (Kellog, Maizel and Goss (1991)). Our objective here is not to evaluate the harm caused by excess nitrogen, but rather to attempt to measure the cost to the agricultural sector of abatement of excess nitrogen. Excess nitrogen can of course be reduced by applying less chemical fertilizer, or by altering the livestock or crop mixes. At an extreme, eliminating the use of fertilizer altogether would greatly reduce excess nitrogen, as would altering livestock mixes to eliminate populations of large manure producing animals. Relative price changes could move the sector in these directions. However any policies designed to reduce residual nitrogen loadings would impose significant costs on the sector. Adjustments to normal farm management practices aimed at reducing residual nitrogen, assuming some reasonable use of nitrogenous fertilizer and some reasonable livestock facility, would impose significant abatement costs. Adjustments to cropping patterns and to the timing of fertilizer application are costly, and produce uncertain outcomes, both in terms of profitability and in terms of the amount of groundwater contamination resulting from excess nitrogen. Adjustments to livestock herds and to the way manure is applied, stored and disposed are also costly. The objective of our empirical analysis is to measure the costs of reducing the amount of excess nitrogen generated as a byproduct of US 3production of animal and crop outputs.

Table 1.
Annual growth rates of variables used in the analysis

Variable	Annual growth rate (%)
Animal output index	+ 1.23
Crop output index	+ 2.21
Capital input index	+ 0.09
Labor input index	- 2.62
Materials input index	+ 1.28
Output index	+ 1.81
Input index	+ 0.43
Conventional total factor Productivity growth	+ 1.38
Excess nitrogen index	+ 5.04

Summary statistics on rates of growth of the variables used in this study appear in table 1. Annual values of all six indexes are given in the appendix. Conventionally measured total factor productivity growth has occurred at an annual rate of 1.38%. However the generation of excess nitrogen has grown at 5.04 % per year, and this will lead to a downward adjustment to the conventional measure, the magnitude of which will depend on the magnitudes of the two abatement cost elasticities. Our problem is to calculate these abatement cost elasticities, so that we can calculate the adjustment to the conventional total factor productivity growth measure.

Empirical results of running the four mathematical programming models are summarized in tables 2 and 3. All four models were specified in GAMS, and two nonlinear optimizers were used, MINOS and CONOPT. Table 2 contains annual measures of technical efficiency as calculated by each of the four programming models. Table 3 contains annual abatement cost elasticities.

Table 2. **Annual efficiency scores**

Year/Model	I	II	III	IV
1961	1.000	1.000	1.000	1.000
1962	1.000	1.000	1.000	1.000
1963	1.000	1.000	1.000	1.000
1964	1.000	1.000	1.000	1.000
1965	1.000	1.000	1.000	1.000
1966	1.000	1.000	1.000	1.000
1967	1.063	1.055	1.049	1.188
1968	1.042	1.092	1.080	1.857
1969	1.106	1.128	1.121	1.799
1970	1.014	1.091	1.059	2.190
1971	1.071	1.085	1.081	1.878
1972	1.048	1.045	1.042	1.457
1973	1.000	1.000	1.000	1.000
1974	1.081	1.129	1.126	2.269
1975	1.107	1.131	1.129	1.904
1976	1.000	1.179	1.082	2.976
1977	1.000	1.097	1.000	2.942
1978	1.125	1.147	1.144	2.025
1979	1.076	1.103	1.094	1.893
1980	1.000	1.084	1.000	2.565
1981	1.000	1.036	1.000	2.073
1982	1.042	1.044	1.043	1.368
1983	1.055	1.053	1.051	1.679
1984	1.013	1.037	1.026	1.615
1985	1.000	1.000	1.000	1.000
1986	1.006	1.010	1.007	1.276
1987	1.000	1.000	1.000	1.000
1988	1.000	1.000	1.000	1.000
Means of positive scores	1.061	1.086	1.076	1.942

The efficiency scores calculated from models I to III show a small but pervasive amount of inefficiency in the middle of the period, relative to the early and late years. The results of model IV suggest much higher rates of inefficiency during the middle of the period. Noting the orientation of model IV, it also suggests that in the middle years US agriculture generated a substantial amount of excess nitrogen in relation to the amounts of desirable outputs produced. This is generally confirmed by the data given in the appendix.

Table 3. **Elasticities of outputs with respect to pollution abatement**

Year/Model	Animals				Crops			
	I	II	III	IV	I	II	III	IV
1961	0	0.364			0			
1962	0	0	0.037	0.073	0	0		0.249
1963	0.055	0.055	1.081	0.075	0.085	0.588		0.249
1964	0.036	0.037		0.373				
1965	0.041	0.041			0.055	0.055		1.799
1966	0	0			0	0		
1967	0.133	0.119	0.121		0	0		
1968	0	0	0		0	0	0	
1969	0	0	0.061		0	0		
1970	0	0	0		0	0	0	
1971	0	0	0.068		0	0		
1972	0.061	0.056	0.056					
1973	0	0		0.121	0	0	0.123	0.241
1974	0	0	0.077		0	0		
1975	0	0	0		0	0	0	
1976	0	0	0		0	0	0	
1977	0	0	0		0	0	0	
1978	0	0	0		0	0	0	
1979	0	0	0		0	0	0	1.057
1980	0	0	0	0.498	0	0	0	
1981	0	0	0		0	0	0	1.053
1982	0	0	0		0	0	0	1.054
1983	0.043	0.038	0.039	0.510				
1984	0	0	0		0	0	0	0.144
1985	0	0			0	0	0.190	0.190
1986	0	0	0		0	0	0	1.053
1987	0	0			0	0	0	1.052
1988	0	0.003	0.208	0.086	0			
Means of positive elasticities	0.062	0.089	0.194	0.248	0.070	0.322	0.157	0.740

Abatement cost elasticities are reported in table 3. For models I to III the elasticities are zero in several years because the projections of observed production to the boundary of $P(x)$ occur on the horizontal segment of the frontier. At these projections so much excess nitrogen is generated that at least some amount of disposal is free. For model IV the elasticities are undefined in many years because the projections of observed production to the boundary of $P(x)$ occur on the vertical segment of the frontier. At these projections so little excess nitrogen is generated that disposal is infinitely costly. We consider both of these outcomes to be uninformative, an unfortunate but occasionally unavoidable consequence of modelling techniques we have employed and the limited number of observations currently at our disposal. Instead we concentrate on the positive elasticities that result from projections of observed production to positively shaped segments of the production frontier.

Conventional and four adjusted total factor productivity growth calculations are summarized in table 4. Since we have so few positive and finite abatement cost elasticities to work with, we assumed in the calculations that all annual values of the abatement cost elasticities are equal to the mean value of the positive elasticities reported at the bottom of each column of table 3. All remaining calculations are based on annual values of all variables.

Table 4. **Conventional and adjusted TFPG measures**

	Conventional model	Adjusted models I	II	III	IV
$\sum_{i=1}^{m} \bar{s}_i \dot{y}_i$	1.81	1.82	2.01	1.92	2.57
$\bar{s}_b \dot{b}$		-0.17	-0.38	-0.48	-1.15
$\sum_{i=1}^{m+1} \bar{s}_i \dot{y}_i$	1.81	1.65	1.63	1.44	1.42
$\sum_{i=1}^{n} \bar{w}_i \dot{x}_i$	-0.43	-0.43	-0.43	-0.43	-0.43
TFPG	1.38	1.22	1.20	1.01	0.99

The most conservative abatement cost elasticities are generated by model I, because its projection to the boundary of $P(x)$ occurs at the largest amount of excess nitrogen. In this model desirable output grows at a rate of 1.82 % per year, marginally higher than in the conventional model. However the growth of excess nitrogen reduces overall output growth by 0.17 % per year, to 1.65 % per year. Consequently measured total factor productivity growth is adjusted downward by almost 12 %, from 1.38 % per year to 1.22 % per year. Models II to IV generate larger downward adjustments since their projections to the boundary of $P(x)$ occur at smaller amounts, and so higher abatement costs, of excess nitrogen. The downward adjustments to conventionally measured total factor productivity growth in these models are 13 %, 27 % and 28 %, respectively.

CONCLUSIONS

In this paper we have developed a set of four mathematical programming models of production that incorporate the generation of undesirable byproducts in the production process. The key element in each model is the characterization of desirable outputs as being strongly disposable and the characterization of undesirable outputs as being weakly disposable. The solutions to the mathematical programs provide efficiency scores and tradeoffs between undesirable and desirable outputs at the efficient projection. These tradeoffs are shadow price ratios which can be converted to abatement cost elasticities. These elasticities are then used to correct a conventional total factor productivity growth measure for the generation of the undesirable outputs.

Application of these techniques on US agriculture illustrates the magnitude of the problem. Measured total factor productivity growth decreases by at least 12 %, and by at most 28 %, when the production of excess nitrogen is incorporated into the model.

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APPENDIX

ANNUAL VALUES OF VARIABLES USED IN THE STUDY

	Animals	Crops	Capital	Labor	Materials	Excess nitrogen
1961	0.7837	0.4942	0.7763	1.6301	0.6793	0.3801
1962	0.7930	0.5033	0.7777	1.6170	0.6825	0.4458
1963	0.8168	0.5327	0.7827	1.5587	0.7042	0.4731
1964	0.8386	0.5025	0.7931	1.4705	0.7114	0.4564
1965	0.8193	0.5720	0.7945	1.4342	0.7293	0.4876
1966	0.8368	0.5311	0.8107	1.3001	0.7522	0.7067
1967	0.8648	0.5826	0.8239	1.2523	0.7938	0.7276
1968	0.8620	0.5790	0.8441	1.2555	0.8132	1.0281
1969	0.8643	0.6199	0.8515	1.2261	0.8463	1.0184
1970	0.9025	0.5565	0.8564	1.1930	0.8559	1.2735
1971	0.9147	0.6697	0.8655	1.1799	0.8639	1.1034
1972	0.9234	0.6697	0.8727	1.1692	0.8452	0.8643
1973	0.9321	0.7370	0.8811	1.1691	0.8247	1.0857
1974	0.9211	0.6689	0.9064	1.1489	0.9111	1.3656
1975	0.8703	0.7800	0.9170	1.1418	0.8995	1.1521
1976	0.9095	0.7589	0.9317	1.1074	0.9524	1.8475
1977	0.9259	0.8492	0.9382	1.0744	0.9238	1.8712
1978	0.9274	0.8458	0.9545	1.0708	1.0588	1.2915
1979	0.9483	0.9427	0.9734	1.0709	1.1093	1.3087
1980	0.9917	0.8388	1.0013	1.0078	1.0910	1.6852
1981	1.0073	1.0103	0.9965	1.0534	1.0477	1.5301
1982	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1983	1.0177	0.6868	0.9730	0.9544	0.9782	1.1613
1984	1.0013	1.0218	0.9361	0.9678	1.0404	1.2456
1985	1.0298	1.0801	0.9234	0.8888	0.9951	1.0774
1986	1.0402	1.0092	0.8900	0.8693	0.9773	0.9405
1987	1.0557	1.0163	0.8569	0.8442	0.9776	0.7422
1988	1.0818	0.8749	0.8324	0.8304	0.9525	1.4314