Total factor productivity for nonhomothetic, nonseparable, multiple output technologies :

a new approach and evidence for US agriculture

Robert D. WEAVER
Atsushi CHITOSE

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Robert D. WEAVER* and Atsushi CHITOSE*

Une analyse de la productivité totale des facteurs dans l'agriculture américaine

Résumé – L'estimation de la productivité totale des facteurs constitue un moyen d'analyser l'évolution de la production. Pour le secteur agricole, les études antérieures ont fourni des estimations basées sur l'approche de Jorgenson et Griliches. Elle suppose que: 1) l'agriculture est homogène, 2) la technologie agricole est caractérisée par: i) des rendements d'échelle constants, ii) la séparabilité des intrants et des produits, et iii) un progrès technique neutre au sens de Hicks. De plus, cette approche suppose que les choix de production sont tels que tous les facteurs et produits sont variables et sont à leurs niveaux optimaux. Or, certains résultats empiriques obtenus en France et aux Etats-Unis sont en contradiction avec les restrictions imposées par l'approche de Jorgenson-Griliches.

Cet article présente une mesure de la productivité totale des facteurs pour un ensemble de technologies plus vaste. Premièrement, la définition de la productivité totale des facteurs proposée par Jorgenson et Griliches (TFPJG) est rediscutée et l'on définit une notion de productivité totale des facteurs relative (RTFP). Cette notion est utile pour des technologies non-homothétiques, non-séparables en intrants et produits, ainsi qu'en présence d'un progrès technique biaisé au sens de Hicks. L'analyse théorique montre que les estimations de la productivité totale des facteurs à la Jorgenson-Griliches sont biaisées si les conditions d'application de cette approche ne sont pas remplies. Lorsque la technologie n'est pas séparable entre intrants et produits, ni caractérisée par des rendements d'échelle constants, la mesure de productivité totale de type TFPJG ne peut pas être interprétée de manière significative. Quand tous les facteurs de productions sont variables, la TFPJG sous-estime la productivité réelle si les rendements d'échelles sont décroissants. Lorsque les facteurs sont quasi-fixes et les économies d'échelles variables, le biais de la mesure du TFPJG est indéterminé. La nouvelle notion de RTFP est appliquée au cas des Etats-Unis pour la période 1948-1983 à partir de l'estimation d'une fonction de profit restreint. La séparabilité des produits et des intrants, ainsi que l'homothéticité de la technologie sont rejetées par les tests statistiques. Ensuite, les estimations de RFTP sont comparées aux estimations prenant en compte chacune des hypothèses de l'approche Jorgenson et Griliches.

Mots-clés :
productivité totale des facteurs, technologies non homothétiques, rendements d'échelle non constants, facteur quasi-fixes

Total factor productivity for non homothetic, non separable, multiple output technologies : a new approach and evidence for US agriculture

Summary – *This paper examines the measurement of total factor productivity for unrestricted technologies. The concept of* relative total factor productivity *(RTFP) is introduced for technologies which are nonhomothetic, nonseparable in inputs and outputs, and affected by nonneutral forms of disembodied technical change. Theoretical results establish estimates of total factor productivity are biased when based on technologies which are misspecified using restrictions of constant returns to scale, input-output separability, and complete factor variability. Econometric implementation allows measurement of RFTP when economic choices face quasi-fixity of factor flows. Estimates of RTFP are presented for US agriculture and compared with those based on restrictions employed by Jorgenson and Griliches using the index number approach. Results indicate that estimates based on RTFP are substantially smaller in magnitude than those based on TFP and implemented subject to the Jorgenson and Griliches restrictions.*

Key-words :
total factor productivity, non homothetic technologies, non constant return-to-scale, quasi-fixed factors

* *Department of Agricultural Economics, Pennsylvania State University, University Park, Pennsylvania 16802 Etats-Unis*

MEASUREMENT of total factor productivity presents a basis for monitoring real growth in output. For the agricultural sector, past studies have presented aggregate estimates based on a methodology that: 1) characterizes agriculture as a production process that is homogeneous across farms; 2) maintains that the production process is characterized by i) constant returns-to-scale (CRS), ii) input-output separability and iii) Hicks neutral technical change (HNTC); and 3) maintains that production choices are made under conditions in which all factors of production are variable by the decision maker. Examples of these studies include those for agriculture in the United States (Griliches, 1963 ; Ball, 1985, 1988); in the United Kingdom (Thirtle and Bottomley, 1991; and Rayner *et al.*, 1986); in Northern Ireland (Glass and McKillop, 1989); and across the European Community (Bureau *et al.*, 1991). Challenging the validity of the restrictions employed in these models has been an accumulation of empirical evidence for agriculture in the United States (Weaver, 1977, 1982, 1983, 1989; Ball, 1985; Antle, 1984; Ray, 1982; or Shumway, 1983) and in France (Guyomard, 1989).

This paper[1] examines the role of these restrictions in the measurement of total factor productivity in US agriculture and presents a new approach in which these restrictions are relaxed. The outline of the paper is as follows. In the next section, the definition of total factor productivity proposed by Jorgenson and Griliches (1967) and widely adopted by past studies is examined. Their definition maintains as prior restrictions that the underlying production technology is separable in inputs and outputs and constant returns-to-scale. Their empirical implementation of their measure of total factor productivity (TFPJG) maintains the restrictions that all inputs and outputs are variable and that economic decisions are in equilibrium. The roles of these restrictions in the Jorgenson-Griliches (JG) approach to measurement of productivity are considered. Next, a new measure of productivity is proposed which allows relaxation of these restrictions. Interpretation of past results based on the restrictions of the Jorgenson-Griliches approach is discussed within the context of the proposed new approach. It is demonstrated that estimates are biased when the restrictions associated with the Jorgenson-Griliches approach are invalid. As an illustration and to present a basis for an empirical comparison of the new approach with those used in past studies, the new approach is applied to annual data for US agriculture over the time period 1948-83.

[1] This paper was completed while R. D. Weaver was on sabbatical at INRA, Rennes.

THE JORGENSON-GRILICHES APPROACH

The origins of this approach follow from Schmookler's accounting analysis as adopted, extended, and applied by Denison (1967), Kendrick (1961) and Jorgenson and Griliches. We label the following as the Jorgenson and Griliches (J-G) restrictions on a production technology: 1) constant returns-to-scale, 2) Hicks' neutral technological change, 3) separability with respect to inputs and outputs, and 4) variability of all inputs and outputs. Define technology satisfying these restrictions as:

$$F(Y, X, \tau) = \alpha \tag{1}$$

where Y is a $m \times 1$ vector of outputs, X is a $n \times 1$ vector of inputs, τ indicates the level of technology, and α indicates the scale of technical efficiency. The J-G specification of technology is elaborated in equation (1) to include τ to allow illustration of the role of HNTC. The function F is assumed to be continuously twice differentiable with $F_y > 0$, $F_x < 0$ and $F_\tau > 0$, concave in Y, and convex in X. Total differentiation of (1) under the restriction that the level of technical efficiency remains unchanged (i.e. $\dot{\alpha} = 0$) yields[2]:

$$- F_\tau \tau \dot{\tau} = \sum_{i=1}^{m} F_i Y_i \dot{Y}_i + \sum_{h=1}^{n} F_h X_h \dot{X}_h \quad \text{where } \dot{Z} = dZ/Z \tag{2}$$

By definition, (2) presents a measure of what might be called *nominal total factor productivity*. That is, if α is defined as a scalar indicator of the level of technical efficiency (*i.e.* $F(Y,X,\tau) = \alpha$), then (2) defines the change in productivity which must have occurred if technical efficiency is to be maintained ($\dot{\alpha} = 0$), given changes in productivity as indicated by changes in the levels of outputs and inputs. This residual in productivity is attributed to technological change under the maintained hypothesis that technology is correctly characterized by (1). As defined, nominal total factor productivity is expressed in terms of units of technical efficiency. In order to translate this measure into a more intuitively interesting form, nominal total factor productivity defined by (2) must be normalized. J-G chose $\Sigma_i F_i Y_i$ as a factor of normalization. Variability of all inputs and outputs, and constant returns-to-scale implies $\Sigma F_i Y_i = - \Sigma F_h X_h$ allowing (2) to be written:

[2] Throughout this paper subscripts on products or prices will be used to indicate specific products while subscripts on functions will indicate the specific product or argument with respect to which derivatives are taken. For example, Y_i indicates the i^{th} output, P_i the i^{th} output prices, while F_i indicates the first partial derivative of the function $F(\ )$ taken with respect to Y_i.

$$\dot{TFP}_{JG} = - F_{\tau}\dot{\tau} \,/\, \Sigma F_i Y_i = \frac{\Sigma F_i Y_i \dot{Y}_i}{\Sigma F_i Y_i} - \frac{\Sigma F_h X_h \dot{X}_h}{\Sigma F_h X_h} \qquad (3)$$

J-G also maintained the hypothesis that $F(\ )$ is separable in inputs and outputs, allowing an intuitive interpretation of TFP_{JG}, (Weaver, 1977). Under these conditions, (3) may be interpreted as the difference between the rate of change of Divisia indexes of aggregate outputs (Y_I) and aggregate inputs (X_I).

The roles of the J-G restrictions can be further appreciated by considering the empirical measurement of (3). To proceed, a further maintained hypothesis is necessary to relate unobserved marginal productivities to observable data. J-G maintain as hypotheses that: 1) all factors and outputs are variable within the interval of observation (*e.g.* annually) and 2) all choices are allocatively efficient. Jointly, these two assumptions are equivalent to the assumption that all inputs and outputs are in longrun Marshallian equilibrium. By the assumption of input-output separability, an equivalent single output respecification of $F(\ )$ can be employed, and by the joint hypothesis of constant returns-to-scale and variability of all inputs, total revenues can be measured by total variable input expenditures. Under HNTC, $F_{i\tau} = F_{h\tau} = 0$ for all $i = 1, ...m$ and $h = 1, ...n$. It follows that the output and factor shares in (3) are invariant with respect to τ.

A NEW APPROACH TO MEASURING TOTAL FACTOR PRODUCTIVITY FOR THE CASE OF MULTIPLE OUTPUTS

The primary motivation for reconsideration of the measurement of $\dot{TFP}$ in agriculture follows from the inconsistency of the J-G restrictions with the characteristics of agricultural systems in most developed countries. At an intuitive level, input-output separability implies the input mix can be chosen independently of the output mix, and vice versa. An example of such separability can be found in the manufacturing process for plastic moldings where variation of output mix has little effect on input mix. A variety of manufacturing and service processes might satisfy this condition; however, the variation of input requirements across alternative crop and livestock activities suggests that input-output separability is an unlikely characteristic of US agricultural technology. Further, Weaver (1977 and 1982) reported results of statistical tests of input-output separability and found strong evidence supporting the rejection of the hypothesis. Ball (1988) also rejected the hypothesis of output separability using an aggregate US data set. Where input-output separability does not hold, the usefulness, interpretation and measurement of $T\dot{F}P_{JG}$ must be reconsidered. In this case, the right-hand side of (3) is not interpretable as the difference between the rates of growth of indexes of outputs Y_I and inputs X_I, and no useful alternative interpretation is available.

The validity of the hypothesis of constant returns-to-scale has also been empirically examined for agriculture. Weaver (1983) tested and rejected the hypothesis of homotheticity for a state level data set for the Dakotas. Weaver (1983) also presented estimates of returns-to-size given nonhomotheticity and found quantitative evidence of decreasing returns within the production period. Antle (1984) and Ray (1982) also tested and rejected homotheticity for an aggregate US data set. Since homotheticity is a necessary condition for homogeneity and constant returns-to-scale, these more restrictive hypotheses were also strongly rejected by Weaver, Antle, and Ray. Shumway (1983) studied Texas field crops though did not report evidence concerning overall functional homotheticity, while Ball (1985) assumed constant returns-to-scale. In sum, no empirical evidence from studies using flexible functional forms exists to support the restriction of CRS. At an intuitive level, uncertain priors concerning the existence of CRS strongly motivate the strategy of developing measures of technological characteristics which are free of the restriction of CRS. Where CRS does not hold, (3) would be written:

$$T\dot{F}P_{JGNC} = -F_\tau \tau\dot{\tau} / \Sigma F_i Y_i Y_i$$

$$= (\Sigma F_i Y_i \dot{Y}_i + \Sigma F_h X_h \dot{X}_h) / \Sigma F_i Y_i. \qquad (4)$$

a form which has little intuitive appeal or interpretability.

The restriction of HNTC imposed by the J-G approach has not been supported by evidence available for US agriculture (Binswanger, 1974; Weaver, 1982, 1983, 1985). Within the context of the noneconometric estimation of shares in the J-G approach, the assumption is necessary. However, so long as tractable approaches exist for empirical estimation of output and input shares that vary with τ, the restriction of HNTC is unnecessary. The variability of all products and factors of production and their adjustment to allocative equilibrium within the production or observation interval are restrictions that are also required by the J-G approach of using prices as exact measures of marginal products of technology. While convenient, this restriction is not supported by casual observation or more systematic empirical evidence. Evidence which refutes the variability of all factors of production in US agriculture has been presented for land and family labor (Weaver, 1977), for land (Weaver and Lass, 1989); and for capital and labor (Vasavada and Chambers, 1986; Vasavada and Ball, 1988; and Taylor and Monson, 1985). Quasi-fixity of products can be expected to vary across technical and economic environments (Weaver, 1982). Where those environments are heterogeneous, measurement of quasi-fixed factors allows empirical representation of the observed heterogeneity. Further, the specification of quasi-fixity of products may be posed as a testable, rather than as a maintained hypothesis (Weaver, 1980). As will be illustrated below, dual econometric approaches provide a sufficient basis for measurement of productivity in the presence of quasi-fixed factors.

To proceed in the absence of a technology restricted by the J-G assumptions, a new approach to measurement of productivity is required. The first step is to reconsider the definition of the productivity statistic. Of interest is the definition of a statistic that is proportional to nominal total factor productivity defined in equation (2). The logic adopted by J-G is appealing. That is, derive a statistic as a normalization of (2) that is denominated in the units of an output. This strategy allows productivity to be measured in terms of the growth rate of the output. As already noted the J-G choice of an aggregate product (output) would be appealing for a technology that is input-output separable. However, for the case of non-CRS, or more generally for nonhomothetic production, and where input-output separability does not hold, a different choice must be made. In this case, a natural choice for normalization is F_jY_j or F_kX_k where Y_j and X_k are any particular output or input of interest. Accordingly, for any Y_j define *relative total factor productivity* as:

$$R\dot{TFP}_j = -F_\tau\tau\dot{\tau} / F_jY_j = (\Sigma F_iY_i\dot{Y}_i + \Sigma F_hX_h\dot{X}_h) / F_jY_j \quad (5)$$

A similar measure could be defined for any X_k. Recognizing that (5) incorporates marginal rates of substitution between Y_j and other outputs and inputs, $R\dot{TFP}_j$ is interpretable as the rate of growth of Y_j not attributable to the rates of change of other outputs or inputs. Alternatively, it is the rate of growth of Y_j attributable to technical change or equivalently, the rate of technical change measured in units of Y_j.

Any product may be chosen to use for normalization, each resulting in a similar measure of relative total factor productivity. All such measures are of equal interest. A natural choice for normalization would be a dominant crop or livestock product within a homogenous production system. In any case, the resulting *RTFP* will reflect the effects of technical change ($-F_\tau\tau\dot{\tau}$) measured in the units of the output or input used in the normalization of (2). Alternative measures are related by their common base in nominal total factor productivity:

$$-F_\tau\tau\dot{\tau} = F_kX_kR\dot{TFP}_k = F_jY_jR\dot{TFP}_j \text{ for any } X_k \text{ or } Y_j \quad (6)$$

Although the proposed new approach leads to product specific measures, it should be noted that for the single output case the traditional measure TFP_{JG} is product specific as well. That is, output is arbitrarily chosen as the product of interest. As is apparent from (6), measures based on inputs may also be derived.

The implications of the constant returns-to-scale restriction can be assessed by considering $RTFP$ when production is nonhomothetic. Define a scaling function $\psi(\cdot)$ such that $F(\cdot)$ may be written:

$$G\,(\psi(Y, X, \lambda, \tau)\,Y, \lambda X, \tau) = 0 \tag{7}$$

where $\psi(\cdot)$ is a scalar and $\psi(Y,X,1,\tau) = 1$. The generalized Euler equation for this case may be written:

$$\psi_\lambda \sum G_i Y_i + \sum G_h X_h = 0 \tag{8}$$

Noting that for $\lambda=1$, $\psi_i = \psi_h = 0$, (2) may be rewritten:

$$-G_\tau \tau\dot{\tau} = \sum_i G_i Y_i \dot{Y}_i + \sum_h G_h X_h \dot{X}_h + \dot{S} \tag{9}$$

where $\dot{S} = (\sum_i G_i Y_i \psi_\lambda + \sum_h G_h X_h)\,\lambda\dot{\lambda} + \sum_i G_i Y_i \psi_\tau \tau\dot{\tau}$

However, using (8) it is clear $\dot{S} = \sum G_i Y_i \psi_\tau \tau\dot{\tau}$. Relative total factor productivity may now be defined on a product specific basis. For example, for any X_k or Y_j:

$$\begin{aligned} R\dot{TFP}_k &= -(G_\tau + \sum_i G_i Y_i \psi_\tau)\,\tau\dot{\tau} \,/\, G_k X_k \\ &= \Big[\sum_i G_i Y_i \dot{Y}_i + \sum_h G_h X_h \dot{X}_h\Big] \,/\, G_k X_k \end{aligned} \tag{10}$$

$$\begin{aligned} R\dot{TFP}_j &= -(G_\tau + \sum_i G_i Y_i \psi_\tau)\,\tau\dot{\tau} \,/\, G_j Y_j \\ &= \Big[\sum_i G_i Y_i \dot{Y}_i + \sum_h G_h X_h \dot{X}_h\Big] \,/\, G_j Y_j \end{aligned} \tag{11}$$

Expressions (10) and (11) provide product specific measures of relative total factor productivity growth which decompose productivity growth into the direct effect of technical change (G_τ) and the indirect effect of technical change that occurs as a result of economies of scale in the case of nonhomotheticity. This "nonhomotheticity" effect is measured by the response of the scale function $\psi(\cdot)$ to changes in technology. Since $F_\tau = G_\tau + \sum G_i Y_i \psi_\tau$, (10) and (11) are equivalent to (5).

The effect of homotheticity and homogeneity on $R\dot{T}FP_k$ can now be examined. If $F(\cdot)$ is homothetic in outputs, $\psi_i = 0$. However, as is apparent from (10) and (11), the measurement of $RTFP_k$ and $RTFP_j$ are not changed. If $F(\cdot)$ is homothetic in inputs, then ψ is constant and $\lambda = \lambda(Y,\psi,\tau)$ and by redefinition of (7) similar results can be obtained. If $F(\cdot)$ is homothetic in both inputs and outputs, then $F(\cdot)$ is either separable with respect to inputs and outputs, or it is homogenous of degree k in inputs (Hanoch, 1970; Lau, 1969). Where $F(\cdot)$ is homogenous of degree k in inputs, $\psi_i = \psi_h = 0$ for all i and h, $\psi_\tau = 0$, and ψ_λ is a constant λ. However, again the definitions of $RTFP_k$ and $RTFP_j$ are unchanged. The conclusion can be drawn that the proposed new measures of $RTFP$ are of general usefulness for a wide range of technical characteristics: homogeneity, homotheticity, or nonhomotheticity. The conceptual motivation for relative total factor productivity growth is to attain an intuitively interpretable measure of nominal total factor productivity. When production is nonhomothetic, the proposed measure properly incorporates changes in the numeraire product resulting from nonhomothetic scale effects induced by technical change.

Before proceeding, it is of interest to assess the implications of use of the statistic TFP_{JG} as a measure of total factor productivity when the technology and observed economic behavior do not satisfy the J-G restrictions. Expression (4) indicates that in the absence of input-output separability and CRS, a measure of nominal total factor productivity normalized by $\sum F_i Y_i$ would hold little intuitive appeal and would provide biased estimates. Nonetheless, equation (4) provides a basis for examining the nature of the error introduced by use of TFP_{JG} when the J-G restrictions do not hold. Normalizing (9) by $\sum G_i Y_i$ and using (8) to allow for nonconstant returns-to-scale, we have:

$$-G_\tau \tau\dot{\tau} \;/ \sum G_i Y_i = \frac{\sum G_i Y_i \dot{Y}_i}{\sum G_i Y_i} - \psi_\lambda \frac{\sum G_h X_h \dot{X}_h}{\sum G_h X_h} \qquad (12)$$

Using (3) and (4), it is apparent that when returns-to-scale are decreasing, use of TFP_{JG} will underestimate the left-hand side of 12), *i.e.*

$$-G_\tau \tau\dot{\tau} \;/ \sum G_i Y_i > T\dot{F}P_{JG} \qquad (13)$$

The conclusion must be drawn that where decreasing returns-to-scale exist, the usefulness of TFP_{JG} is compromised. The absence of input-output separability further compromises the usefulness of $\dot{TFP}_{JG}$ by rendering (12) uninterpretable as the difference in growth rates of Divisia indexes of aggregate output and aggregate input.

EMPIRICAL IMPLEMENTATION OF RELATIVE TOTAL FACTOR PRODUCTIVITY GROWTH

Expressions (10) - (11) involve production characteristics while observed data reflect economic choice. Any measure of total factor productivity growth using such data requires evaluation of production characteristics at points on the production surface that are consistent with economic choices. Subject to the J-G restrictions, cost minimization and revenue maximization would allow prices to be used as observable measures of marginal products and would allow (4) to be rewritten:

$$T\dot{F}P^*_{JG} = \Sigma_i \frac{P_i Y_i^*}{\Sigma P_i Y_i^*} \dot{Y}_i^* - \Sigma \frac{R_h X_h^*}{\Sigma R_h X_h} \dot{X}_h \qquad (13)$$

where * indicates the measure is based on observed economic choice. Under more general conditions of interest in this paper, we maintain the behavioral hypothesis that firms maximize short-run expected profits:

$$\max \pi = P'Y - R'X \text{ s.t. } F(Y, X, \theta, \tau) = 0 \qquad (14)$$

where P is a $m \times 1$ vector of expected prices of outputs Y, R is a $n \times 1$ vector of prices for variable inputs X, and θ is a $p \times 1$ vector of quasi-fixed inputs. The solutions (π^* Y^* X^*) to (14) define the profit function $\pi^* = \pi(P, R, \theta)$.

Where production is nonhomothetic, the expansion path is not a ray from the origin. It follows that the only means of measuring characteristics of production along the expansion path is to evaluate such measures at the firm's economic equilibrium as defined by conditions for economic efficient choice. For the technology defined in (14),

$$RT\dot{F}P_j = -\frac{F_\tau \tau}{F_j Y_j} \dot{\tau} = \frac{\Sigma F_i Y_i}{F_j Y_j} \dot{Y}_i + \frac{\Sigma F_h X_h}{F_j Y_j} \dot{X}_h + \frac{\Sigma F_r \theta_r}{F_j Y_j} \dot{\theta}_r$$

Substituting the first-order conditions, for the general case of nonhomotheticity we have:

$$RT\dot{F}P_j^* = + \sum_i \frac{P_i Y_i^*}{P_j Y_j^*} \dot{Y}_i^* - \sum_h \frac{R_h X_h^*}{P_j Y_j^*} \dot{X}_h^* - \sum_r \frac{\pi_r \theta_r}{P_j Y_j^*} \dot{\theta}_r \qquad (15)$$

and a similar expression could be derived for $RT\dot{F}P_k^*$.

Total differentiation of the profit function results in a measure which is proportional to $RT\dot{F}P_j^*$ and may be empirically derived from the estimates of the profit function. Linear homogeneity of the profit function in prices allows the definition:

$$\pi^j = \pi / P^j = \pi^j (\tilde{P}, \tilde{R}; \theta, \tau) \tag{16}$$

where $\tilde{P}$ and $\tilde{R}$ are relative prices, *i.e.* the elements of P and R are normalized by an arbitrarily chosen price, P_j. Total differentiation of (16), use of (15), and total differentiation of the profit definition provides a basis for relating the primal measure in (15) and the dual measure of the effects of technical change:

$$T\dot{F}P^*_{\pi j} = \frac{\partial \pi^j}{\partial \tau} \frac{\tau}{\pi^j} \dot{\tau} = + \frac{\tilde{P}_j Y_j}{\pi^j} (RT\dot{F}P_j^*). \tag{17}$$

This result establishes a convenient means of estimating relative total factor productivity without imposing any of the J-G restrictions.

The derivation of (17) is consistent with the requirement that estimation of productivity, or any other characteristic of technology, must be based on a maintained hypothesis concerning the economic behavior of the firm. The primal forms of *RTFP* such as (10) and (11) are meaningful to economic analysis only if evaluated at optimal choices. This could be accomplished by estimating $RTFP_j$ subject to the first-order conditions for (14). However, (17) provides a convenient alternative. Estimates of $RTFP_j$ or $RTFP_k$, are immediately available from empirical estimates of the parameters characterizing (16), or its equivalent derived for (10) and X_k. In either case, primal and dual estimates would, by design, be fully consistent with the maintained behavioral hypothesis. Analogous relationships exist for any arbitrary input, *e.g.*, for X_k:

$$T\dot{F}P_{\pi k} = - \frac{\tilde{R}_k X_k}{\pi^k} (RT\dot{F}P_{k*}) \tag{18}$$

Similar results can be derived for cost and revenue functions, or more generally, any dual function that is consistent with the observed behavior of the firm (Weaver, 1982).

INTERPRETATION OF PAST TOTAL FACTOR PRODUCTIVITY RESULTS

The proposed measure of relative total factor productivity holds strong intuitive appeal and is proposed for future research when technology and economic behavior is not consistent with the J-G restrictions. Nonetheless, it is of interest to establish the extent to which information concerning relative total factor productivity or, more importantly, the underlying nominal total factor productivity can be inferred from past estimates of TFP_{JG}. This issue is also raised by the fact that the approach for estimation proposed by (18) requires econometric estimation of the left-hand side of (18) or its equivalent under different behavioral hypotheses. Where such econometric estimates are unattainable, it is of interest to ask what could be learned from use of an index number approach to estimate TFP_{JG} (*e.g.* Ball, 1985 or Bureau *et al.*, 1992).

Each of the alternative measures (TFP_{JG}, TFP_{JGNC}, and $RTFP_j$) were presented as different normalizations of nominal total factor productivity. When all inputs and outputs are variable, comparison of (4) - (5) establishes:

$$RTFP^{v}_{j} > TFP^{v}_{JGNC} \qquad (19)$$

where the superscript v reiterates that all inputs and outputs are assumed variable. Under constant returns-to-scale, $TFP^{v}_{JGNC} = TFP^{v}_{JG}$. However, when returns-to-scale are decreasing, (13) establishes that TFP^{v}_{JG} is biased, *i.e.*

$$TFP^{v}_{JGNC} > TFP^{v}_{JG}. \qquad (20)$$

Empirical implementation of these alternative measures of productivity requires measurement of the marginal technical productivities. The approach taken depends upon the variability of inputs and outputs that is maintained. To compare TFP^{v}_{JG} with the proposed $RTFP_j$ which allows for the existence of quasi-fixed factors, partition the input vector into variable (X) and quasi-fixed inputs (θ). The proposed measure would use $\partial\pi / \partial\theta_r$ as a basis for measuring $\partial F / \partial\theta_r$, while the Jorgenson-Griliches approach would use the factor prices R_r. If the factors θ are, in fact, quasi-fixed and free disposal is assumed[3], then $\partial\pi / \partial\theta_r > R_r$.

(3) That is, free disposal would rule out the case where available service flows from the quasi-fixed factors exceeded optimal flows.

By use of this inequality as well as one implied by the generalized Euler equation, the following inequality is established:

$$\psi_\lambda \sum P_i Y_i > \sum R_h X_h + \sum R_r \theta_r \qquad (21)$$

Using this inequality and equations (13) and (15), it is clear that the difference between $RTFP_j$ and TFP^v_{JG} is indeterminant when returns are decreasing and factors θ are quasi-fixed.

RELATIVE TOTAL FACTOR PRODUCTIVITY GROWTH IN US AGRICULTURE

To illustrate the proposed approach, annual data for geographically aggregate US agriculture reported by Capalbo, Vo and Wade are used for the period 1948-83. The data set provides input and output data disaggregated into six output groups (small grain, coarse grain, field crops, fruits, vegetables, and animal products) and ten input groups (hired labor, family labor, land, energy, fertilizer, pesticides, feed and seed, other materials, structures, and other capital). Quantity aggregates for these groups were constructed using the Tornqvist approximation to the Divisia index which is a superlative index when the underlying production function has a translog form. To proceed, two issues must be resolved with respect to the specification of an econometric framework for estimation of *RTFP*: 1) further aggregation of products and 2) specification of the endogenity of products. In order to allow comparison of estimates based on *RTFP* with those presented by Capalbo and Vo, all aspects of their model specification will be retained with the exception of the J-G restrictions.

In this section, we present results based on two different specifications. In each, the ten input categories available in the data set are aggregated into three variable inputs (labor, chemicals – fertilizer and pesticides –, and materials – energy, feed and seed, and other materials), and one quasi-fixed input (capital: land, structures, and other capital). In addition, we introduce a time trend, t, to allow for systematic disembodied technical change that is not restricted to satisfy the restriction of *HNTC*. Alternative models follow from different aggregations of outputs. Aggregation of these groups was accomplished by use of the Tornqvist approximation of the Divisia index. In the first, the Capalbo and Vo specification is retained in which all outputs are aggregated under the maintained hypothesis of input-output separability. In the second, outputs are aggregated into two categories: crops and livestock.

We adopt the behavioral hypothesis that farm firms attempt to maximize profits by choice of variable outputs and inputs, given market prices and subject to available flows from quasi-fixed factors. Measures of expected output prices are unavailable from the data set and were not employed by Capalbo and Vo. For this reason and to retain comparability of results based on different approaches, these prices are presumed known at the time decisions are made. The necessity of having measures of output incentives results from the endogeneity of outputs, not the decision to estimate a profit *vs.* a cost function. Estimation of a cost function for agriculture would have to recognize the endogeneity of outputs, ultimately requiring measurement of output incentives to be used as instruments in the estimation of the cost function system (Weaver and Lass, 1989).

We specify the normalized, restricted profit function to have a translog functional form. While the exact form of the profit function is not known, use of the translog will allow a second-order approximation of the true form. To ensure consistency of the translog profit function with the behavioral hypothesis and the endogeneity of products maintained as a hypothesis, symmetry, and linear homogeneity in prices are imposed. The final estimation system is composed of the translog profit function, the output and input share equations defined by the first derivatives of the profit function taken with respect to prices, and equations for profit function elasticities for time and the quasi-fixed factor. This approach has also been used by Brown and Christensen (1981). To ensure nonsingularity of this system, the materials share equation was dropped from estimation. The systems were estimated using iterated seemingly unrelated regression. In order to provide a further comparison of estimates based on our approach with those derived under the J-G restrictions, a single output translog cost function was estimated subject to the J-G restrictions and output exogeneity as maintained in past estimations of cost functions for agriculture. Symmetry and linear homogeneity in prices was imposed and a system of the translog cost function and input cost share equations was estimated using iterated seemingly unrelated regression. The materials cost share equation was dropped to ensure nonsingularity.

The estimated restricted profit function and the J-G cost function were examined for satisfaction of implications of behavioral hypotheses that were not imposed prior to estimation. For the restricted profit function, monotonicity requires fitted variable product shares to be nonnegative and first-order parameters associated with prices to be nonnegative. This condition was satisfied at all observations for both specifications. The restricted profit function must also be quasi-convex. A necessary condition for this is that the diagonal elements of the estimated Hessian of the restricted profit function be nonpositive at all observations. Estimates for each of the output aggregation specifications were consistent with this condition. Estimates of parameters for the J-G restricted cost function were also examined and found to be consistent with monotonicity and quasi-concavity at each observation.

Of particular interest for the measurement of total factor productivity is the consistency of the estimated functions with the J-G restrictions on the production function. This was examined using a series of nested hypothesis tests. Results are reported in table 1.

Table 1. **Summary of Specification Tests**

Unrestricted model goodness-of-fit [a]	
Degrees of freedom	21
Chi-square	1355.0
Critical value (1% level)	38.9321
Input - output separability	BY1Y2 = 0 [b]
Degrees of freedom	35
t - statistic	9.37
Critical value (1% level)	2.75
Homotheticity	B1Y1=B1Y2 =B2Y1=B2Y2=0
Degrees of freedom	4
Chi-square	83.17
Critical value (1% level)	13.2767
Hicks neutral technical change [c]	B1T=B2T=0
Degrees of freedom	2
Chi-square	11.60
Critical value (1% level)	9.2103

(a) The unrestricted model was tested against the naive model in which all parameters except the constant terms were restricted to zero.

(b) Parameter labels correspond with those used in table 2.

(c) Restrictions for Hicks neutral technical change in input space were tested first. Rejection of this restriction implies rejection of Hicks neutrality in output and input space.

To begin, the validity of the specification of the two output translog profit function was examined by comparing the model with the naive model of constant shares and profit. Results in table 1 strongly reject the naive model. Conditional upon this result, further J-G restrictions were examined. Input-output separability was rejected as a restriction on the two output translog profit function. Conditional upon this result, homotheticity and Hicks neutral technical change were tested as restrictions on the multiple output restricted profit function. Results reported in table 1 strongly support rejection of these restrictions. In each case, evidence confirms these hypotheses can be rejected at the 1 % level of significance. Given these results, homogeneity and constant returns-to-scale are also rejected. To proceed, economic parameters of interest were derived from the estimated multiple output, restricted translog profit function which is consistent with nonhomothetic production and non-Hicks neutral technical change. Estimated parameters for this function are reported in table 2.

Table 2. **Parameter estimates: restricted profit function two output case**

Parameter[a]	Estimate	't' Ratio
A1	– 0.390140	– 10.49
A2	– 0.076752	– 4.21
AY1	1.108085	20.01
AY2	0.727099	9.15
AZ	5.625477	1.22
AT	0.470232	3.31
BTT	– 0.240992	– 3.19
B11	– 0.197427	– 7.89
B12	– 0.045036	– 4.64
B22	– 0.029426	– 3.10
BY1Y1	0.024309	0.43
BY1Y2	– 0.502471	– 9.37
BY2Y2	0.394997	3.41
BZZ	57.897994	1.00
B1Y1	0.185070	7.46
B1Y2	0.114566	2.50
B2Y1	0.060556	3.60
B2Y2	0.028695	1.46
B1Z	0.988672	4.12
B2Z	– 0.088764	– 0.96
BY1Z	– 0.470422	– 1.59
BY2Z	– 0.882793	– 1.72
B1T	0.126275	3.26
B2T	– 0.022899	– 1.23
BY1T	– 0.010413	– 0.18
BY2T	– 0.035886	– 0.43
BZT	2.384015	2.52

(a) Ai indicates a first-order parameter and Bij indicates a second-order parameter, where i, j = Y1 (crops), Y2 (livestock), 1 (labor), 2 (chemicals), Z (capital inputs), and T (time).

To provide further evidence concerning the validity of the specification, the elasticities of choice implied by the estimated parameters were estimated at the mean of the data. As reported in table 3, the estimated elasticities are consistent in sign with the predictions of the comparative-statics of the maintained behavioral hypothesis. The magnitudes of these estimated elasticities suggest that elasticity of choices with respect to prices is not substantial in the short-run. Following Weaver (1983), returns-to- size in the short-run, *i.e.* for given levels of fixed factors, were estimated to have a mean value of 0.430. This result confirms the existence of decreasing returns to size in the short-run.

Table 3.
Estimated elasticities of choice

	Price				
Quantity	Crops	Livestock	Labor	Chemical	Materials
Crops	0.081364	0.22013	– 0.166206	0.007453	– 0.142749
Livestock	0.335297	0.263321	– 0.176195	– 0.008467	– 0.413956
Labor	0.515772	0.358979	– 0.762209	0.082285	– 0.194828
Chemicals	– 0.158525	0.118241	0.563981	– 0.458419	– 0.065279
Materials	0.416838	0.793621	– 0.18333	– 0.008962	– 1.018167

Alternative estimates of total factor productivity are reported in table 4. These estimates indicate the percentage change in the numeraire (*e.g.* aggregate output for J-G or specific outputs for our approach) that is unexplained by predicted changes in other outputs or inputs. On an annual basis, relative total factor productivity for crops and for livestock vary within reasonable limits and they are, on average, small in magnitude. The index number approach of J-G was demonstrated above to have no proportional relationship to nominal productivity when production is not constant returns-to-scale. Similarly, no proportionality holds when other J-G restrictions do not hold. Nonetheless, it is of interest to empirically assess differences between estimates based on relative total factor productivity and those based on J-G restrictions. Table 4 reports estimates of *TFP* based on the index number (*TFP*) and econometric approaches involving the J-G restrictions[4].

By visual inspection of annual and average estimates, the estimates of relative total factor productivity are substantially smaller in magnitude than those based on J-G restrictions (*TFP*, TFP_{JGC}). We assessed the statistical properties of the differences between our measure of *RTFP* growth and those based on the J-G restrictions. Subject to the J-G restrictions, estimates were derived using the index number approach and an econometric approach using estimates of a single output cost function. To separately evaluate the empirical implications of the input-output separability restriction, estimates are also presented based on a single output translog profit function (*RTFPY*).

[4] For the index number approach, we report *TFP* constructed by our application of the Tornqvist-Theil indexing procedure using the data set reported by Capalbo, Vo and Wade. Because our constructed index series was not exactly equal to that reported by Capalbo and Vo, we assessed the hypothesis that the difference between our estimates and those of Capalbo and Vo is white noise. At a 5 % significant level, the hypothesis of white noise was not rejected for either the level of productivity, *TFP* or the growth rate, $\dot{TFP}$. To proceed, we compare the econometrically estimated *RTFP* with the index number *TFP* we estimated.

Table 4. **Estimated TFP growth rates (%) alternative approaches** [a]

Year	TFPCAP	TFP	RTFPY	TFPJGC	RTFPY1T	RTFPY2T
1949	– 2.57	– 2.55	0.67	1.10	– 1.66	– 2.25
1950	– 2.22	– 2.34	0.62	1.03	– 1.71	– 2.55
1951	2.79	2.75	0.63	1.02	– 3.71	– 5.08
1952	4.39	4.34	0.64	1.42	– 1.68	– 2.50
1953	3.39	3.39	0.75	1.12	– 0.52	– 0.84
1954	– 0.16	– 0.24	0.83	0.99	0.14	0.25
1955	– 0.94	– 0.98	0.86	1.05	0.01	0.01
1956	7.62	7.36	0.77	0.95	– 0.12	– 0.21
1957	– 2.43	– 2.45	0.78	1.13	– 0.35	– 0.54
1958	1.85	1.82	0.76	1.07	– 0.37	– 0.50
1959	– 3.54	– 3.55	0.81	1.23	– 1.46	– 2.15
1960	2.83	2.78	0.92	1.09	0.09	0.13
1961	0.83	0.86	0.87	1.13	0.05	0.08
1962	1.95	1.88	0.91	1.15	0.12	0.18
1963	1.71	1.72	0.84	1.16	– 0.24	– 0.38
1964	2.29	2.28	0.94	1.18	0.05	0.08
1965	– 0.95	– 0.97	0.85	1.24	– 1.35	– 2.06
1966	1.00	0.97	0.92	1.30	– 0.24	– 0.34
1967	2.38	2.40	0.98	1.49	0.11	0.16
1968	0.22	0.23	1.01	1.38	0.17	0.24
1969	1.07	1.06	0.99	1.43	– 0.01	– 0.01
1970	– 2.53	– 2.57	1.04	1.55	0.14	0.18
1971	5.14	5.00	1.01	1.41	0.11	0.15
1972	1.12	1.13	1.25	1.66	0.26	0.38
1973	1.63	1.58	1.88	1.46	– 0.14	– 0.22
1974	3.57	3.67	1.07	1.48	– 1.27	– 2.59
1975	2.03	2.07	1.29	1.38	0.01	0.02
1976	– 0.01	– 0.03	1.19	1.33	– 0.09	– 0.14
1977	3.83	3.74	1.00	1.46	– 0.04	– 0.07
1978	– 2.71	– 2.54	1.28	1.56	– 0.63	– 0.91
1979	5.36	5.10	1.01	1.56	– 0.17	– 0.24
1980	– 0.19	– 0.42	1.17	1.60	– 0.54	– 0.88
1981	7.15	7.04	1.01	1.93	– 0.21	– 0.31
1982	0.99	0.94	1.10	1.31	– 0.01	– 0.01
1983	– 9.57	– 9.87	1.86	1.76	– 0.48	– 0.80
Ave.	1.07	1.02	0.99	1.32	– 0.45	– 0.68

[a] TFPCAP = TFP growth rate based on TFP indexes available in Capalbo and Vo.
TFP = TFP growth rate based on our implementation of the index approach.
RTFPY = relative TFP growth rate econometrically estimated from the one-output SR profit function system. TFPJGC = TFP growth rate econometrically estimated from the one- output LR cost function system where CRS is maintained.
TFPCAP,TFP and TFPJGC are based on imposition of the J-G restrictions.
RTFPYiT = relative TFP growth rate econometrically estimated from the two-output SR profit system using the share system where shares of time and fixed factor are also included.

To evaluate the statistical properties of the differences between alternative estimates based on J-G restrictions and *RFTP* as proposed in this paper, two approaches were considered. First, a Ljung-Box test was implemented to test whether the respective differences are white noise. The null hypothesis that the differences are white noise was tested against the alternative hypothesis that the series of differences are AR(6). The test results are summarized in Table 5 and indicate the null hypothesis of white noise differences can not be rejected at a 5 % significance level for the difference between any alternative estimate versus the *RFTP* estimate. These results are consistent with the theoretical result presented above that there is no systematic relationship between any of the estimates based on J-G restrictions and nominal total factor productivity. An important implication of these results is that estimates based on the J-G restrictions fail to provide a measure that is interpretable as total factor productivity when the J-G restrictions are invalid.

Table 5. **Statistical evaluation of differences between TFP estimates**

Difference [a]	Q-Statistic [b]	RMSE
TFP *vs* TFPCAP [c]	9.25	0.1075
TFPJGC *vs* TFP	4.38	3.2688
TFP *vs* RTFPY	4.47	3.3337
TFPJGC *vs* RTFPY	6.28	0.3993
TFP *vs* RTFPY1T	3.85	3.5613
TFP *vs* RTFPY2T	3.25	3.7258
TFPJGC *vs* RTFPY1T	11.46	1.9335
RTFPY *vs* RTFPY1T	9.07	1.6182
TFPJGC *vs* RTFPY2T	10.12	2.2992
RTFPY *vs* RTFPY2T	9.41	1.9942
RTFPY1T *vs* RTFPY2T	6.46	0.4435

(a) TFPCAP - TFP growth rate based on TFP indexes available in Capalbo and Vo.
TFP=TFP growth rate that we estimated based on the index approach.
RTFPY=Relative TFP growth rate econometrically estimated from the one-output SR profit function system.
TFPJGC=TFP growth rate econometrically estimated from the one-output LR cost function system where CRS is maintained.
RTFPYiT=Relative TFP growth rate econometrically estimated from the two-output SR profit system using the share system where shares of time and fixed factor are also included, whee i=1 (crops) and 2 (livestock).

(b) Degrees of freedom = 6

(c) TFP and TFPCAP are compared to confirm the accuracy of our estimates using the Capalbo and Vo index number approach with those reported by Capalbo and Vo.

To provide a further basis for comparison, we employed the root mean square error (*RMSE*) to evaluate the magnitude of differences between measures based on J-G restrictions and those of relative total factor productivity (*RFTP*). Results are reported in table 5. The greatest differences between alternative estimates and our estimates of *RTFP* are found to follow from a comparison of the index number approach (*TFP*) versus econometric approaches (*TFPJGC*, *RTFPY1T*, or *RTFPY2T*). This follows from the smoothing implicit in econometric approaches. The *RMSE* of *RTFPY* versus *TFPJGC* is smaller than that which compares the *RTFP* measures based on the multiple output specification, *RTFPY1T* or *RTFPY2T*, versus *TFPJGC*. This suggests that the input-output separability restriction maintained in both *RTFPY* and *TFPJGC* has substantial quantitative effects on the resulting estimate. This is further confirmed by a comparison of *RTFPY1T* or *RTFPY2T* with *RTFPY*. The large magnitude of the *RMSE* for these comparisons emphasizes the impact of the input-output separability restriction. The comparison of *RTFPY1T* or *RTFPY2T* with *TFP* illustrates the substantial joint impact of the J-G restrictions and the use of an econometric *vs.* an index number approach.

CONCLUSIONS

The approach of Jorgenson and Griliches for measurement of total factor productivity was shown to yield estimates which are not interpretable as indicators of nominal total factor productivity when the Jorgenson and Griliches restrictions do not hold. These restrictions include constant returns-to-scale technology, input-output separability, variability of all factors of production within the observation period, and Hicks' neutral technical change. An intuitively interpretable alternative measure was introduced and labeled *relative total factor productivity*. Its usefulness as a measure of nominal total factor productivity change was established. In order to provide an illustration of the implementation of the measure, an empirical application was presented for US agriculture using the data set developed by Capalbo, Vo and Wade. First, the J-G restrictions were tested and rejected for the data set. Based on this result, estimates of relative total factor productivity were derived from econometric estimates of a translog restricted profit function that was free of the J-G restrictions. Estimates of relative total factor productivity were found to be smaller in magnitude than those based on the J-G restrictions. The magnitude of difference between measures based on Jorgenson and Griliches restrictions and the relative total factor productivity concept were found to be empirically substantial.

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