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# Firm behaviour under uncertainty: a simple parametric model

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In this paper, we model production technology in a state-contingent framework. We assume that all the firms use the same stochastic technology, but they may have different risk attitudes and information sets, and *ex post* they may operate in different production environments. Firms maximise *ex ante* their preference function subject to a stochastic technology constraint; in other words, they are assumed to act rationally, thereby leaving no room for either technical or allocative inefficiency. We provide a simple parametric functional form to represent the state-contingent technology. Using simple numerical examples, we illustrate how optimal input–output choices are dramatically affected when firms have different preferences and information sets. Thus, we show that the observed disparateness of production choices among different firms can actually be attributed to the stochastic nature of the decision environment.

**Key words:** risk-averse, risk-neutral, state-contingent, uncertainty.

## 1. Introduction

There are two approaches to tackle the problem of production under uncertainty. The first of these is based on state-preference theory which can be traced to the work of Arrow (1953) and Debreu (1952). The second approach is based on stochastic production functions. The seminal analysis of the latter approach was put forward by Sandmo (1971) and Just and Pope (1978).[‡]

Arrow and Debreu realised that uncertainty could be modelled in the same way as multi-output technology. Their argument was that if uncertainty is represented by a set of possible future states of nature, then the uncertain output vectors containing state-contingent commodities are equivalent to multi-output technology. This means that uncertainty does not affect the necessary and sufficient conditions for existence and optimality of equilibrium. But uncertainty significantly reduces the empirical reasonability of the relevant necessary and sufficient conditions. This is because it would be normal to expect markets to exist for each of the commodities in the absence of uncertainty, but on the other hand it would be too optimistic to believe that markets would exist for each commodity in every possible state of nature.

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In the stochastic production approach introduced by Sandmo (1971) and Just and Pope (1978), the main idea was to derive the first-order conditions for optimisation and use the implicit function theorem to describe comparative static responses to changes in parameters such as average price level.

The basic idea behind the state-contingent approach to uncertainty is that, in addition to location in space, time and physical properties, commodities differ from each other also based on the state of nature in which they are located. Hirshleifer and Riley (1992) have applied the state-contingent approach in a general equilibrium context. More recently, Chambers and Quiggin (2000) have applied state-contingent theory to problems pertaining to decision-making under uncertainty. Chambers and Quiggin (2000) assert that 'the state contingent approach provides the best way to think about all problems involving uncertainty, including problems of consumer choice, the theory of the firm and principal-agent relationships'.

The paper is organised as follows: Section 2 defines the production technology. In Section 3, we describe how a producer's risk-neutral probabilities affect her optimal production choices. In this section, we develop a parsimonious parametric model to describe a rational producer's behaviour towards uncertainty. Further, using numerical illustrations, we show that the optimal input–output choices are dramatically affected when firms have different preferences and information sets. Finally, we offer some concluding comments in Section 4.

## 2. Production technology

We assume that all firms have access to a common stochastic production technology to produce a stochastic output designated by $\tilde{z} = (z_1, z_2)$, using deterministic input $x \in \mathbb{R}_+$. Nature resolves the uncertainty by choosing a state from a state space $\Omega = \{1, 2\}$. The production process is modelled as a two period game with nature, with periods denoted as 0 and 1, respectively. In period 0, the producer allocates input $x$ to the production process and in period 1 nature reveals the actual state of nature contained in the state space $\Omega = \{1, 2\}$ and in the process determines the realised output.

We model production using a CES specification of technology, where the relationship between the total input used across various states of nature and the *ex post* realisation of stochastic output is given by

$$x = (a_1 {z_1}^b + a_2 {z_2}^b)^{\gamma/b} \tag{1}$$

where $b$ is a transformation of elasticity of substitution and is referred to as the substitution parameter (see Arrow *et al.*, 1961), the parameter $\gamma$ represents economy of scale and $z_s$ is the amount of stochastic output produced in the state of nature $\{s\}$ in period 1 by employing $x$ amount of non-stochastic input in period 0. Here, it is important to bear in mind that only one of the two state-contingent outputs is observed *ex post* in period 1, and the unobserved output is lost in the unrealised state of nature. $a_s \geq 0$ can be either interpreted as a technology parameter related to production of output in state of nature $\{s\}$ or it can be conceived as a realisation of an unobserved random variable determined by nature *ex post*. The lowest admissible value of $b$ is one; this implies an infinite elasticity of substitution, and therefore, straight-line isoquants, meaning *ex post* output is perfectly substitutable between states of nature.

O'Donnell *et al*. (2010) model production using a state-specific state-allocable representation of technology where the input allocated to a specific state of nature $\{s\}$ is given by

$$x_s = a_s {z_s}^b, \quad s \in \Omega = \{1, 2\} \tag{2}$$

Assuming that the firms are rational and efficient, the total input used in the production process in period 0 is the sum of the inputs allocated to each state of nature, that is

$$x = x_1 + x_2 = a_1 {z_1}^b + a_2 {z_2}^b \tag{3}$$

In the O'Donnell *et al*. (2010) specification, the state-allocable technology is state-specific, that is input $x_1 = a_1 {z_1}^b$ is allocated exclusively to state of nature $\{1\}$ and input $x_2 = a_2 {z_2}^b$ is allocated specifically to state of nature $\{2\}$. For example, if this technology is used to model agricultural production, it would imply that crop yield in a 'dry' season will be zero if no input is allocated to irrigation infrastructure. Our experience shows that this is not the case, that is crop yield in a 'dry' season will be low, but not zero, if no pre-season labour is allocated to irrigation infrastructure. State-allocable technology is too simplistic and such a representation of technology is seldom observed in a real-world production process. Hence, we model production using a state-general state-contingent specification of technology. The O'Donnell *et al*. (2010) specification of technology is a special case of our CES model as (1) collapses to (3) when $\gamma = b$.

## 3. Efficient firm behaviour under uncertainty

We assume that the firms seek to maximise their utility function $W(\mathbf{y})$ where $\mathbf{y} = (y_1, y_2)$ and $y_s = z_s - wx$, $s \in \Omega$ is the *ex post* net return in the state of nature $\{s\}$. The utility function $W$ is continuously differentiable, non-decreasing and quasi-concave in its arguments. This form of utility function is quite general, and it contains the family of expected utility functions in net returns as a special case. We further assume that the firms are technically efficient, that is, they lie on the production possibility frontier. This is further ensured by the fact that the preferences are non-decreasing in net returns and that the technology proposed above is smooth. Given that the state $\{s\}$ has been realised, the variables relevant to firms' welfare in the production problem are the committed (*ex ante*) input $x$ in period 0 and realised (*ex post*) stochastic output $z_s \in \mathbb{R}_+$ in period 1. We further assume that the state-contingent utility function displays a degree of separability between input $x$ committed prior to the realisation of the state of nature and the net returns (profits) accumulated when the state of nature $\{s\}$ is realised.

For our specification of technology given by (1), the firm's optimisation problem can be written as:

$$\max_{z_1, z_2}\left\{ W(\mathbf{y}) : x = \left(\sum_{j\in\Omega} a_j z_j{}^b\right)^{\gamma/b}\right\} \quad j \in \Omega = \{1,2\} \tag{4}$$

The first-order conditions for efficient firm behaviour are:

$$\frac{\partial W(\mathbf{y})}{\partial y_s} - w\sum_{j\in\Omega}\frac{\partial W(\mathbf{y})}{\partial y_j}\frac{\partial\left(\sum_{l\in\Omega} a_l z_l{}^b\right)^{\gamma/b}}{\partial z_s} = 0 \quad j,l,s \in \Omega = \{1,2\} \tag{5}$$

Dividing both sides of (5) by $\sum_{j\in\Omega} \partial W(\mathbf{y})/\partial y_j$, we have

$$\pi_s - \gamma w a_s z_s{}^{b-1} x^{\frac{\gamma-b}{\gamma}} = 0 \quad s \in \Omega = \{1,2\} \tag{6}$$

where the risk-neutral probability $\pi_s$ of a firm in state of nature $\{s\}$ is given by

$$\pi_s \equiv \frac{\partial W(\mathbf{y})\,\partial y_s}{\sum_{j\in\Omega}\partial W(\mathbf{y})\,\partial y_j} \in (0,1) \tag{7}$$

The monotonicity of the welfare (utility) function in net returns ensures that $\sum_{s\in\Omega}\pi_s(\mathbf{y}) = 1$. $\pi_s$ is referred to as risk-neutral probability in state of nature $\{s\}$, as it represents the subjective probability that a risk-neutral firm would require in order to make the same production choices (produce the same *ex post* output using the same amount of input *ex ante*) as a rational firm with preferences $W$. Hence, the study of firms with a particular set of preferences actually boils down to analysing the behaviour of risk-neutral firms with varying subjective probabilities. This further implies that while analysing the behaviour of firms that are efficient, there is no need to explicitly take into account their risk attitudes.

Adding the risk-neutral subjective probabilities across all the states of nature gives us the *efficient frontier*[1]

$$\Xi_{\text{eff}}(w,x) = \left\{(z_1,z_2) : 1 - \gamma w x^{\frac{\gamma-b}{\gamma}}\sum_{s\in\Omega} a_s z_s^{b-1} = 0\right\} \tag{8}$$

If the firms base their risk-neutral probabilities on the technology used in the various states of nature, that is, if $\pi_s(\mathbf{y}) \propto a_s$, then they will choose to produce the same output no matter what state of nature is realised *ex post*.

[1] This is the definition given by Chambers and Quiggin (2000).

Let $\pi_j$ and $\pi_i$ be the risk-neutral probabilities in state of nature $\{j\}$ and $\{i\}$, respectively. Then based on (6), the ratio of these subjective probabilities for an efficient firm is

$$\frac{\pi_j}{\pi_i} = \frac{a_j z_j^{b-1}}{a_i z_i^{b-1}} \tag{9}$$

If $\frac{\pi_j}{\pi_i} = \frac{a_j}{a_i}$, then from (9) it must be the case that $z_j = z_i \; \forall \; i, j \in \Omega$. If $\frac{\pi_1}{\pi_2} > \frac{a_1}{a_2}$, then any rational firm will produce output $z_1 > z_2$ in period 1 by committing input $0 \leq x \leq a_1^{\frac{1}{b(1-\gamma)}}(w\gamma)^{\frac{\gamma}{1-\gamma}}$ in period 0 and if $\frac{\pi_1}{\pi_2} < \frac{a_1}{a_2}$, then it will produce $z_1 < z_2$ in period 1 by using input $0 \leq x \leq a_2^{\frac{1}{b(1-\gamma)}}(w\gamma)^{\frac{\gamma}{1-\gamma}}$ in period 0. Different firms will end up on different points on the *efficient frontier* based on their expectations (governed by their risk-neutral probabilities $\pi_s(\mathbf{y})$) about the future states of nature or their attitudes towards risk or mixture of these two factors.

For any rational firm having a general welfare (utility) function $W(\mathbf{y})$, the relationship between state-contingent output and the subjective risk-neutral probability can be derived by re-writing the first-order condition given by (6) as

$$z_s = \left(\frac{\pi_s}{\gamma w a_s}\right)^{\frac{1}{b-1}} x^{\frac{b-\gamma}{\gamma(b-1)}}, \quad s \in \Omega = \{1, 2\} \tag{10}$$

From (10), it follows that on the *efficient frontier*, the output of a rational firm in any state of nature increases with an increase in the risk-neutral probability in the corresponding state of nature, provided $b > 1$.

## 3.1. Numerical illustrations

In the state-contingent output space, for a given input $x$ having a normalised price $w$, the boundary points correspond to the producer believing that one of the two states is certain to happen. If the rational producer's risk-neutral probabilities are $(\pi_1, \pi_2) = (1, 0)$, that is if she is certain that state $\{1\}$ would be realised *ex post*, then she chooses an output bundle $(z_1, z_2) = ((\gamma w a_1)^{\frac{1}{1-b}} x^{\frac{\gamma-b}{\gamma(1-b)}}, 0)$. If she believes that state $\{2\}$ is certain to occur in period 1, that is if her risk-neutral probabilities are $(\pi_1, \pi_2) = (0, 1)$, then she chooses the point $(z_1, z_2) = (0, (\gamma w a_2)^{\frac{1}{1-b}} x^{\frac{\gamma-b}{\gamma(1-b)}})$ on the efficient frontier. For example, if technology exhibits decreasing returns to scale, that is, $\gamma = 1.25$ and a moderate degree of substitutability between the stochastic outputs, that is $b = 2$, then any firm having a risk-neutral probability vector $(\pi_1, \pi_2) = (1, 0)$ allocates (using (16)) an *ex ante* input $x = 3.8051$ to produce the output pair $(z_1, z_2) = (2.3782, 0)$. Similarly if the firm has a risk-neutral probability of $(\pi_1, \pi_2) = (0, 1)$, then it must use an *ex ante* input of $x = 59.3164$ in period 0 to produce an output combination $(z_1, z_2) = (0, 37.0726)$ in period 1.

For technology that exhibits decreasing ($\gamma = 1.25$) returns to scale, columns one to four of Tables 1–3 show the risk-neutral subjective probability in state of nature {1}, the state-contingent output combination and total input allocated to the production process for a rational firm operating on the efficient frontier with state-contingent output substitutability of $b = 2$, 11 and 1.1, respectively. In each of these tables, the risk-neutral probability in state of nature {1}, that is, $\pi_1$, increases from 0 to 1.

Firms try to strike a balance between minimising the cost of production and reducing the risk involved in the production process. Chambers and Quiggin (2000) define technology to be riskless when the firm produces the same output in every state of nature (*ex post*). In the example above, the technology is not risky if $\frac{\pi_2}{\pi_1} = \frac{a_1}{a_2} = \frac{1}{3}$, that is, if $(\pi_1, \pi_2) = (0.75, 0.25)$. Similarly, Chambers and Quiggin (2000) classify a technology to be inherently risky if cost of producing a non-stochastic output bundle is more than the cost of producing a stochastic bundle ($z_1 \neq z_2$).

Rational firms minimise cost while operating on the efficiency frontier; therefore, the optimisation problem facing rational firms can be explicitly written as

$$\underset{z_1,z_2}{\text{Min}}\, w({a_1 z_1}^b + {a_2 z_2}^b)^{\gamma/b} \text{ such that } \gamma w x^{\frac{\gamma-b}{\gamma}}({a_1 z_1}^{b-1} + {a_2 z_2}^{b-1}) = 1 \qquad (11)$$

This cost minimisation leads to non-stochastic output, that is $z_1 = z_2$. Therefore, for this technology the cost minimising output coincides with riskless output, and this can be clearly seen, for example in Table 1 where the equal output pair $z_1 = z_2 = 1.1585$ requires least input, and therefore, it is the least costly bundle. However, this may not be the case for any arbitrary

**Table 1** Production choices: moderate output substitutability and decreasing returns to scale, $(a_1, a_2) = (1.5, 0.5)$, $b = 2$, $\gamma = 1.25$, $w = 0.5$

| $\pi_1$ | $z_1$ | $z_2$ | $x$ | $p_1\|\lambda = 1$ | $p_1\|\lambda = 10$ |
|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 37.0726 | 59.3161 | 0.0000 | 0.0000 |
| 0.0500 | 0.5305 | 30.2378 | 46.0039 | 0.0000 | 0.0000 |
| 0.1000 | 0.9064 | 24.4737 | 35.3872 | 0.0000 | 0.0000 |
| 0.1500 | 1.1561 | 19.6543 | 27.0074 | 0.0000 | 0.0000 |
| 0.2000 | 1.3052 | 15.6620 | 20.4650 | 0.0000 | 0.0000 |
| 0.2500 | 1.3764 | 12.3877 | 15.4158 | 0.0000 | 0.0000 |
| 0.3000 | 1.3901 | 9.7310 | 11.5660 | 0.0001 | 0.0000 |
| 0.3500 | 1.3641 | 7.5999 | 8.6678 | 0.0011 | 0.0000 |
| 0.4000 | 1.3135 | 5.9109 | 6.5152 | 0.0067 | 0.0000 |
| 0.4500 | 1.2516 | 4.5890 | 4.9395 | 0.0282 | 0.0000 |
| 0.5000 | 1.1891 | 3.5673 | 3.8051 | 0.0848 | 0.0000 |
| 0.5500 | 1.1355 | 2.7871 | 3.0059 | 0.1899 | 0.0000 |
| 0.6000 | 1.0986 | 2.1971 | 2.4608 | 0.3333 | 0.0000 |
| 0.6500 | 1.0854 | 1.7534 | 2.1108 | 0.4878 | 0.0023 |
| 0.7000 | 1.1030 | 1.4181 | 1.9160 | 0.6300 | 0.0908 |
| 0.7500 | 1.1585 | 1.1585 | 1.8536 | 0.7500 | 0.7500 |
| 0.8000 | 1.2606 | 0.9454 | 1.9160 | 0.8457 | 0.9894 |
| 0.8500 | 1.4194 | 0.7515 | 2.1108 | 0.9170 | 0.9998 |
| 0.9000 | 1.6478 | 0.5493 | 2.4608 | 0.9643 | 1.0000 |
| 0.9500 | 1.9613 | 0.3097 | 3.0059 | 0.9900 | 1.0000 |
| 1.0000 | 2.3781 | 0.0001 | 3.8050 | 1.0000 | 1.0000 |

**Table 2** Production choices: near-zero output substitutability and decreasing returns to scale, $(a_1, a_2) = (1.5, 0.5)$, $b = 11$, $\gamma = 1.25$, $w = 0.5$

| $\pi_1$ | $z_1$ | $z_2$ | $x$ | $p_1 \vert \lambda=1$ | $p_1 \vert \lambda = 10$ |
|---|---|---|---|---|---|
| 0.0000 | 2.0211 | 8.9807 | 14.3691 | 0.0000 | 0.0000 |
| 0.0500 | 5.5180 | 8.2674 | 13.0078 | 0.0034 | 0.0000 |
| 0.1000 | 5.5658 | 7.7386 | 12.0341 | 0.0125 | 0.0000 |
| 0.1500 | 5.4969 | 7.2973 | 11.2436 | 0.0283 | 0.0000 |
| 0.2000 | 5.3957 | 6.9178 | 10.5814 | 0.0517 | 0.0000 |
| 0.2500 | 5.2872 | 6.5865 | 10.0186 | 0.0833 | 0.0000 |
| 0.3000 | 5.1816 | 6.2947 | 9.5372 | 0.1234 | 0.0000 |
| 0.3500 | 5.0837 | 6.0364 | 9.1248 | 0.1720 | 0.0000 |
| 0.4000 | 4.9963 | 5.8072 | 8.7726 | 0.2286 | 0.0002 |
| 0.4500 | 4.9209 | 5.6036 | 8.4742 | 0.2925 | 0.0009 |
| 0.5000 | 4.8586 | 5.4228 | 8.2251 | 0.3626 | 0.0035 |
| 0.5500 | 4.8104 | 5.2623 | 8.0220 | 0.4375 | 0.0131 |
| 0.6000 | 4.7773 | 5.1202 | 7.8631 | 0.5156 | 0.0464 |
| 0.6500 | 4.7605 | 4.9943 | 7.7477 | 0.5951 | 0.1519 |
| 0.7000 | 4.7615 | 4.8826 | 7.6765 | 0.6740 | 0.4099 |
| 0.7500 | 4.7824 | 4.7824 | 7.6519 | 0.7500 | 0.7500 |
| 0.8000 | 4.8265 | 4.6896 | 7.6786 | 0.8210 | 0.9402 |
| 0.8500 | 4.8984 | 4.5966 | 7.7651 | 0.8846 | 0.9914 |
| 0.9000 | 5.0065 | 4.4856 | 7.9271 | 0.9381 | 0.9994 |
| 0.9500 | 5.1675 | 4.2965 | 8.1983 | 0.9784 | 1.0000 |
| 1.0000 | 5.4504 | 1.9237 | 8.7205 | 1.0000 | 1.0000 |

**Table 3** Production choices: high output substitutability and decreasing returns to scale, $(a_1, a_2) = (1.5, 0.5)$, $b = 1.1$, $\gamma = 1.25$, $w = 0.5$

| $\pi_1$ | $z_1$ | $z_2$ | $x$ | $p_1 \vert \lambda = 1$ | $p_1 \vert \lambda = 10$ |
|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 153.0370 | 244.8589 | 0.0000 | 0.0000 |
| 0.0500 | 0.0000 | 124.6501 | 189.4681 | 0.0000 | 0.0000 |
| 0.1000 | 0.0000 | 100.4080 | 144.5875 | 0.0000 | 0.0000 |
| 0.1500 | 0.0000 | 79.8866 | 108.6457 | 0.0000 | 0.0000 |
| 0.2000 | 0.0000 | 62.6842 | 80.2358 | 0.0000 | 0.0000 |
| 0.2500 | 0.0000 | 48.4220 | 58.1065 | 0.0000 | 0.0000 |
| 0.3000 | 0.0000 | 36.7443 | 41.1536 | 0.0000 | 0.0000 |
| 0.3500 | 0.0000 | 27.3182 | 28.4109 | 0.0000 | 0.0000 |
| 0.4000 | 0.0000 | 19.8337 | 19.0403 | 0.0000 | 0.0000 |
| 0.4500 | 0.0000 | 14.0039 | 12.3234 | 0.0000 | 0.0000 |
| 0.5000 | 0.0002 | 9.5648 | 7.6519 | 0.0001 | 0.0000 |
| 0.5500 | 0.0008 | 6.2750 | 4.5187 | 0.0023 | 0.0000 |
| 0.6000 | 0.0038 | 3.9146 | 2.5090 | 0.0292 | 0.0000 |
| 0.6500 | 0.0188 | 2.2775 | 1.2950 | 0.1625 | 0.0000 |
| 0.7000 | 0.0914 | 1.1279 | 0.6437 | 0.4528 | 0.0001 |
| 0.7500 | 0.2806 | 0.2806 | 0.4490 | 0.7500 | 0.7500 |
| 0.8000 | 0.4218 | 0.0238 | 0.5475 | 0.8562 | 0.9954 |
| 0.8500 | 0.5416 | 0.0009 | 0.7368 | 0.9068 | 0.9992 |
| 0.9000 | 0.6808 | 0.0000 | 0.9804 | 0.9467 | 0.9999 |
| 0.9500 | 0.8452 | 0.0000 | 1.2847 | 0.9779 | 1.0000 |
| 1.0000 | 1.0376 | 0.0000 | 1.6602 | 1.0000 | 1.0000 |

**Table 4** Production choices: near-zero output substitutability and increasing returns to scale, $(a_1, a_2) = (1.5, 0.5)$, $b = 11$, $\gamma = 0.8$, $w = 0.5$

| $\pi_1$ | $z_1$ | $z_2$ | $x$ | $p_1\|\lambda = 1$ | $p_1\|\lambda = 10$ |
|---|---|---|---|---|---|
| 0.0000 | 0.0018 | 0.0080 | 0.0199 | 0.0000 | 0.0000 |
| 0.0500 | 0.0058 | 0.0088 | 0.0215 | 0.0499 | 0.0486 |
| 0.1000 | 0.0068 | 0.0094 | 0.0229 | 0.0998 | 0.0976 |
| 0.1500 | 0.0076 | 0.0101 | 0.0242 | 0.1497 | 0.1469 |
| 0.2000 | 0.0083 | 0.0106 | 0.0254 | 0.1996 | 0.1963 |
| 0.2500 | 0.0090 | 0.0112 | 0.0266 | 0.2496 | 0.2459 |
| 0.3000 | 0.0096 | 0.0117 | 0.0276 | 0.2996 | 0.2957 |
| 0.3500 | 0.0102 | 0.0121 | 0.0286 | 0.3496 | 0.3457 |
| 0.4000 | 0.0108 | 0.0125 | 0.0295 | 0.3996 | 0.3958 |
| 0.4500 | 0.0113 | 0.0128 | 0.0304 | 0.4496 | 0.4461 |
| 0.5000 | 0.0118 | 0.0131 | 0.0311 | 0.4997 | 0.4966 |
| 0.5500 | 0.0122 | 0.0133 | 0.0317 | 0.5497 | 0.5472 |
| 0.6000 | 0.0125 | 0.0134 | 0.0322 | 0.5998 | 0.5978 |
| 0.6500 | 0.0128 | 0.0135 | 0.0326 | 0.6499 | 0.6486 |
| 0.7000 | 0.0130 | 0.0134 | 0.0329 | 0.6999 | 0.6993 |
| 0.7500 | 0.0132 | 0.0132 | 0.0329 | 0.7500 | 0.7500 |
| 0.8000 | 0.0132 | 0.0128 | 0.0328 | 0.8001 | 0.8006 |
| 0.8500 | 0.0131 | 0.0123 | 0.0326 | 0.8501 | 0.8510 |
| 0.9000 | 0.0129 | 0.0116 | 0.0320 | 0.9001 | 0.9012 |
| 0.9500 | 0.0126 | 0.0105 | 0.0312 | 0.9501 | 0.9510 |
| 1.0000 | 0.0119 | 0.0042 | 0.0297 | 1.0000 | 1.0000 |

stochastic technology. For example, the cost minimisation for the technology having the functional form $x = a_1 {z_1}^{b_1} + a_2 {z_2}^{b_2}$ results in a purely stochastic output pair, given by $(b_2 - 1)z_1 = (b_1 - 1)z_2$, if $b_2 \neq b_1$.

Table 4 shows that for increasing returns to scale ($\gamma < 1$), riskless output choice is the most costly choice for rational and efficient firms. For example, the riskless output combination $(z_1, z_2) = (0.0132, 0.0132)$ requires a producer to use input $x = 0.0329$, which corresponds to the largest input listed in Table 4.

It is important to note that the output combination chosen by a risk-neutral firm having a certain belief (subjective probabilities) about future states of nature could have been chosen by a risk-averse (or risk loving) firm with a different set of subjective probabilities. For example, consider a producer who maximises her expected welfare (utility) and ascribes probability $p_1$ to state of nature $\{1\}$. Assuming that producer has an exponential[2] utility function, and her welfare function can be written as

$$W(y) = -p_1 \exp(-\lambda y_1) - (1 - p_1) \exp(-\lambda y_2) \tag{12}$$

where $\lambda = -\frac{W''}{W'}$ represents the coefficient of absolute risk aversion (Pratt, 1964). From the first-order condition for (12) and some algebraic manipulations, the risk-averse producer's probability in state $\{1\}$ is given by

[2] The exponential utility function allows net returns to be both negative as well as positive.

$$p_1 = \frac{\pi_1}{\pi_1 + (1 - \pi_1)\exp(-\lambda(z_1 - z_2))} \tag{13}$$

Thus if $(\lambda, \pi_1, z_1, z_2) = (1{,}0.5, 0.25, 0.75)$, then from (13), we get $p_1 = 0.5481$. This implies that any rational risk-averse firm that has unit coefficient of risk aversion, assigns a probability 0.5481 to state of nature $\{1\}$ and maximises expected exponential utility over net return will produce the same output as a risk-neutral firm that believes both states of nature are equally likely to occur. Columns 5 and 6 in Tables 1–4 report risk-averse firms' probability of state of nature $\{1\}$, having coefficient of absolute risk aversion $\lambda = 1$ (low risk) and $\lambda = 10$ (high risk), respectively.

From Tables 1–3, we observe that for any given returns to scale, the production of an output bundle gets less risky with a decrease in the degree of substitution between state-contingent outputs. For example, all the producers in Tables 1–3 face decreasing returns to scale, and we can observe that all the output combination in Table 2 are less risky compared with output combinations in Tables 1 and 3.

There is another important observation from the tables, which indicates that highly risk-averse producers choose a risky output combination only when they are certain (almost) about one of the two possible states of nature. For example, we observe that only the first and last output bundles in Table 4 are considerably risky and a risk-averse producer chooses these bundles only when she is certain about one of the two states of nature (that is, for the first bundle $p_2 = 1$ and for the second bundle $p_1 = 1$ ).

Finally, under uncertainty firms that have identical risk-neutral subjective probabilities about the future states of nature and apply the same amount of input to the production process, can produce remarkably different output *ex post*. This idea can be best illustrated by an example. We assume that there are two firms A and B that face one of the two possible states of nature. Further, we assume that the technology parameters in the two states along with the elasticity of transformation between *ex post* outputs and input price are $\gamma = 1.25$, $a_1 = 1.5$, $a_2 = 0.5$, $b = 2$ and $w = 0.5$, respectively. Because the two firms have the same beliefs about the future states of nature, we assign the subjective risk-neutral probabilities for both firms A and B to be $\pi_1 = 0.05$ and $\pi_2 = 0.95$, respectively. Therefore, using (10) both firms A and B will each choose (*ex ante*)

$$z_1 = \left(\frac{0.05}{1.5 \times 0.5 \times 1.25}\right)^{\frac{1}{2-1}} x^{\frac{2-1.25}{1.25(2-1)}} = 0.5305 \tag{14}$$

and

$$z_2 = \left(\frac{0.95}{0.5 \times 0.5 \times 1.25}\right)^{\frac{1}{2-1}} x^{\frac{2-1.25}{1.25(2-1)}} = 30.2378 \tag{15}$$

in state of nature $\{1\}$ and $\{2\}$ respectively and use the same input:

$$
\begin{aligned}
x &= \left[a_1\left(\frac{\pi_1}{a_1 w\gamma}\right)^{\frac{b}{b-1}} + a_2\left(\frac{\pi_2}{a_2 w\gamma}\right)^{\frac{b}{b-1}}\right]^{\frac{\gamma(b-1)}{b(\gamma-1)}} \\
&= \left[1.5\left(\frac{0.05}{1.5\times 0.5\times 1.25}\right)^{\frac{2}{2-1}} + 0.5\left(\frac{0.95}{0.5\times 0.5\times 1.25}\right)^{\frac{2}{2-1}}\right]^{\frac{1.25(2-1)}{2(1.25-1)}} \\
&= 46.0039
\end{aligned}
\tag{16}
$$

If firm A experiences state of nature {1} and firm B experiences state of nature {2}, then they will produce $z_A = 0.5305$ and $z_B = 30.2378$, respectively *ex post*.

To an outside observer, it may appear that firm B is more productive than firm A, but in fact this is not the case; Firm B produces more output than firm A only because it has encountered a favourable state of nature *ex post*. Discovering that a sizeable number of firms appear not to be productive, one may conclude that there are potential opportunities for beneficial policy interventions or one may alternatively interpret that existing interventions are responsible for observed loss in productivity. Analysis in the state-contingent framework therefore suggests that utmost care must be taken when drawing policy conclusions.

In most real-world data, there are various sources of behavioural and informational differences across firms. The conventional frontier model often ignores the interaction between these sources and the stochastic nature of the production environment. Therefore, an important implication of this paper is that it is necessary to reconsider all previous empirical studies of productivity, where truly uncertain nature of the production process is not taken into account.

It must be noted that it is easy to estimate flexible state-contingent models using conventional econometric techniques provided the right kind of data are available. If the realised state of nature is observed, the input is allocated in fixed proportions to different states of nature, and the input allocated to any state of nature is only a function of output in that state of nature, then unknown parameters can be estimated using standard econometric techniques such as the least square dummy variable (LSDV) model, data envelopment analysis (DEA) and stochastic frontier analysis (SFA). Also, climatic differences in crop production are often captured in a conventional production model by dummy variables, or by simply limiting the analysis to firms that encounter similar states of nature.

However, in many real-world applications, we only observe the total inputs but do not observe the inputs allocated to different states of nature. For example, many data sets in agricultural production do not have information on labour used in both 'wet' and 'dry' states of nature. These data sets only contain information on the total labour used in the production process. The model developed in this paper is applicable in this context. In these types of data samples, the risk-neutral probabilities between firms vary because of differences in information about the future states of nature available to these firms. Further, the risk-neutral probabilities of firms in the sample may also differ because of differences in their risk attitudes.

## 4. Conclusion

The current empirical modelling is mostly[3] silent about the fact that the stochastic decision environment plays a pivotal role in influencing production choices, and thus the *ex post* observed outcomes. Under uncertainty, rational producers' *ex ante* production choices are determined by the information available to them regarding the future states of nature and by their risk attitudes. In this paper, we model the production process using purely stochastic technology that not only accounts for the inherently stochastic environment in which production takes place, but also captures producer attitudes towards risk.

## References


Arrow, K.J. (1953). Le des valeurs boursiers pour la repartition la meillure des risques. Technical report, Cahiers du Seminair d'Economie, Centre Nationale de la Recherche Scientifique (CNRS), Paris.

Arrow, K.J., Chenery, H.B., Minhas, B.S., and Solow, R.M. (1961). Capital labour substitution and economic efficiency, *Review of Economics and Statistics* 43, 225–250.

Chambers, R.G. and Quiggin, J. (2000). *Uncertainty, Production, Choice and Agency: The State-Contingent Approach*. Cambridge University Press, Cambridge, UK.

Chavas, J.-P. (2008). A cost approach to economic analysis under state-contingent production uncertainty, *American Journal of Agricultural Economics* 90(2), 435–446.

Debreu, G. (1952). A social equilibrium existence theorem, *Proceedings of the National Academy of Sciences, USA* (38), 886–893.

Hirshleifer, J. and Riley, J.G. (1992). *The Analytics of Uncertainty and Information*. Cambridge University Press, Cambridge.

Just, R.E. and Pope, R.D. (1978). Stochastic specification of production functions and economic implications, *Journal of Econometrics* 7(1), 67–86.

Nauges, C., O'Donnell, C., and Quiggin, J. (2009). Uncertainty and technical efficiency in finnish agriculture. Number (53rd) Conference, Cairns, Australian Agricultural and Resource Economics Society, Australia.

O'Donnell, C.J. and Griffiths, W.E. (2006). Estimating state-contingent production frontiers, *American Journal of Agricultural Economics* 88(1), 249–266.

O'Donnell, C.J., Chambers, R.G., and Quiggin, J. (2010). Efficiency analysis in the presence of uncertainty, *Journal of Productivity Analysis* 33, 1–17.

Pratt, J. (1964). Risk aversion in the small and in the large, *Econometrica* 32, 122–136.

Sandmo, A. (1971). On the theory of the competitive firm under price uncertainty, *American Economic Review* 61(1), 65–73.


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[3] With a few exceptions such as O'Donnell and Griffiths (2006), Chavas (2008) and Nauges *et al*. (2009).