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COMPLEMENTARITY BETWEEN IRRIGATION AND FERTILIZERS IN INDIAN AGRICULTURE

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This study attempts to outline the approaches to the measurement of possible relationships between irrigation and fertilizers. Notationally, complementarity[1] may be defined as

$$\left(\frac{\delta x_1}{\delta p_2}\right)_{u=0} < 0 \text{ or } \left(\frac{\delta x_2}{\delta p_1}\right)_{u=0} < 0$$

That is the cross partial derivatives of demand functions for irrigation and chemical fertilizers with respect to price of chemical fertilizers and price of water respectively are negative. In other words, if the compensating variations in income keep the farmers on the same indifference curve, an increase in the price of water will induce the farmer to demand less of fertilizers and vice versa. If the substitution effect is symmetric this will hold true for demand functions for irrigation. However, in agriculture, the use of water precedes the use of chemical fertilizers and hence if the price of chemical fertilizers increases, it is not very likely that farmers will demand less of irrigation water in an under-developed agriculture where demand for water as such is not saturated Hence, we may believe that substitution effect may or may not be symmetric. As complementarity is defined in static term, the lack of symmetry is explained by a time or a dynamic effect.

In this paper, three approaches have been adopted to analyse the complementarity aspects of these two inputs in agriculture. Firstly, simple analyses of variance on experimental data have been conducted. Thereafter, crop response functions are fitted and since demand functions for water and chemical fertilizers are derived demand functions from crop response functions, marginal analysis may be used to determine the optimum levels of chemical fertilizers and frequency and levels of irrigation. This approach, however, assumes perfect competition and neither price of chemical fertilizers (fixed by Government of India) nor water rates have been fixed up on this basis. Hence, the entire demand functions have not been analysed but the interaction term and its role has been analysed. Secondly, consumption functions for chemical fertilizers[2] are fitted directly to the data with water rate as one of the explanatory variables. Thirdly, the substitu-

1. Hicks and Allen, Slutsky and Schultz have suggested invariant measures of complementarity which are the properties of the indifference curve and demand function. Perhaps, the simplest measure of complementarity between the two goods x_i and x_j, is the sign of

$$k_{ij} = \frac{x\delta_i}{\delta p_j} + x_j \frac{\delta x_i}{\delta I} = k_{ij} \qquad \text{(I — Budget constraint)}$$

If there are two commodities, this must always be positive in sign which means that the two commodities are substitutes while in the many commodity case at least one must be of positive sign in order to satisfy the relationship.

$$\Sigma P_j K_{ij} = 0 \ (i = i, \ldots n)$$
$$K_{ij} < 0.$$

2. There are more than two prices affecting the demand for chemical fertilizers and hence our condition of $\sum P_j K_{ij} = 0$ will be satisfied even when fertilizers and irrigation are complementary. An error-term will account for other prices not explicitly considered in demand equation.

tion elasticity between chemical fertilizers and irrigation is computed under the assumption of constant returns to scale. Such an elasticity of substitution will ascertain the choice of production functions which may be fitted to explain the behaviour of producers or to break down the componentwise growth in agriculture. Perhaps, derived demand functions from such a production function will make more sense because if the elasticity of substitution departs from unity and if production functions where elasticity of substitution being unity is used, a specification bias is introduced.

In section I, the results of analyses of variance on experimental data and crop response functions are given and analysed. In section II the consumption functions for chemical fertilizers with an explanatory variable such as water rate are fitted. These demand functions are fitted to the data pertaining to the States rather than to a very micro level data because of the non-availability of data on water rate at a district or taluk level. In section III elasticity of substitution is computed by adopting SMAC production function and also by using the general approach of incorporating two side-conditions under perfect competition. Furthermore, the variable elasticity of substitution production functions are also used.

I

The Indian Council of Agricultural Research[3] carried out several experiments for crops like wheat and paddy. These experiments are carried out under controlled conditions and in each case yield without any treatment, that is, controlled yield is observed. The experiment was carried out by using various levels of nitrogen under various combinations of levels of irrigation and frequencies of irrigation. Three levels of irrigation were used (2″, 3″ and 4″) while three frequencies (2, 3 and 4) of irrigation were more probable. Against each of these combinations, three levels of nitrogen (0, 20, 40) were used and average yield was noted for various crops and districts and these data were used to analyse the interaction term. Data for two years 1957-58 and 1958-59 have been utilized. Nitrogen and phosphorus are not used in combination and in the case of non-acidic soils sometimes they are complementary inputs. In another study, this aspect has been studied.[4] Data for one district is given for illustration and operations with these data are explained.

Lakhmapur district : Wheat crop :		$F_0 = 2$			$F_1 = 3$			$F_2 = 4$		
		2″	3″	4″	2″	3″	4″	2″	3″	4″
		I_0	I_1	I_2	I_0	I_1	I_2	I_0	I_1	I_2
0	N_0	18.5	23.5	12.5	20.5	17.1	21.1	25.1	18.3	15.9
20	N_1	19.0	22.0	19.7	21.8	18.5	22.6	25.6	18.7	19.5
40	N_2	18.9	23.6	19.8	22.8	21.1	25.5	23.2	21.1	21.1

3. Fourth, Fifth and Sixth Report of Model Agronomic Experiments, *Rabi* 1957-58, *Kharif* 1958 and *Rabi* 1958-59, Indian Council of Agricultural Research, New Delhi.

4. Ashok K. Parikh, "Production Response and Parametric Programming," *Indian Journal of Agricultural Economics*, Vol. XX, No. 2, April-June, 1965.

On such sets of data, analysis of variance with linear term such as N, I and F independently and with interaction terms have been carried out. Non-linearity is studied with the help of the subsequent crop response functions. The analysis of variance is given in Table I for illustration.

TABLE I—ANALYSIS OF VARIANCE

(LAKHMAPUR DISTRICT)

Source of variation	Degrees of freedom	Sums of squares	Mean sums of squares	F
Variation due to				
N	2	34.3790	17.1895	9.9497*
I	2	17.7160	8.8580	5.1273*
F	2	10.4010	5.2005	3.0102
NI	4	26.8060	6.7015	3.8790*
IF	4	127.6310	31.9077	18.4691*
NF	4	3.6740	.9185	.5316
NIF	8	13.8210	1.7276	—
Total	26	234.4280	9.0164	

Note : With degrees of freedom $V_1 = 2$ and $V_2 = 8$, if $F > 4.46$ (5 per cent level of significance) we reject the hypothesis of no significance in variance while with $V_1 = 4$ and $V_2 = 8$ if $F > 3.84$ (5 per cent level of significance) we reject the hypothesis of no significance in variance.

The above results indicate that effect of nitrogen, and levels of irrigation are alone significant and also the joint effects of nitrogen and irrigation levels and of irrigation levels and frequency of irrigation are statistically significant. This reveals that nitrogen and irrigation are not additive as far as physical input-mix is concerned. If we get irrigation water more frequent and with respective levels of irrigation, we get a significant effect of such a combination. This reveals that frequencies and levels of irrigation are interdependent. Such effects with nitrogen have been analysed for eight districts for the years 1957-58 and 1958-59 and similarly the analysis of variance has been used on the data with respect to phosphorus.

For the years 1957-58 and 1958-59, in the case of Obedullahganj district independent effects of nitrogen, phosphorus, irrigation and frequency of irrigation are statistically significant. For Sriganganagar, all linear effects of nitrogen, irrigation and frequencies of irrigation are statistically significant in the case of years 1957-58 and 1958-59 and for phosphorus additionally, irrigation and frequencies of irrigation produce a significant joint effect. For Bichpuri, the effect of P_2O_5 is significant for the year 1958-59 while linear term F only provides significant effect. For Lakhmapur, 1957-58, nitrogen and irrigation and irrigation and frequency are joint inputs and N and I alone produce a significant effect. On the phosphorus experiment, again irrigation and frequency are interdependent while phosphorus, irrigation and frequency produces a significant variation in yield per acre. For 1958-59, N and F alone are significant while phosphorus alone turns out to be important in other experiments. For Reura Farm, nitrogen experiment does not indicate any significant effect while phosphorus effect turns out to be significant for both the years. For Nasirpur, only nitrogen effect is statistically significant for 1957-58 while frequency of irrigation alone is only significant for 1958-59 nitrogen experiment. For Powerkheda, 1957-58, nitrogen and frequency of irrigation (as independent effects) are significant while in phosphorus experiment, phosphorus, irrigation and frequency and also joint effects of levels of irrigation and frequency of irrigation are statistically significant. For 1958-59, none of these effects are significant. These experimental results when analysed by the analysis of variance indicate that interaction effects are significant only in very few cases. This may be probably because $N \times I \times F$ or $P \times I \times F$ may be highly significant which is again a term of interaction effects but has been treated as a residual in the analysis of variance and again the analysis of variance does not have quadratic terms or higher than linear terms which may have over-emphasized the role of independent effects. Consequently, it has been decided to use crop response functions for 1957-58 and 1958-59 with nitrogen and phosphorus being used in separate experiments.

Two types of crop response[5] functions are fitted (*a*) Quadratic and (*b*) Square-root. These are surface functions because production surface can be derived in three or more dimensional space. The regression coefficients, their standard errors and the coefficient of determination are shown in Tables II, III, IV and V. Quadratic and square-root response functions are one of the simpler tools to analyse the diminishing or increasing marginal productivity. In most of the cases, our results are extremely similar for both types of crop response functions. The coefficient of determination, however, does not show any such systematic tendency of being upward for quadratic or downward for square-root crop response functions, though the coefficient of determination is not apparently significantly different between two types of crop response functions. In the case of quadratic function, the detailed analysis of physical maximum maximorum (for a well-behaved function) can be shown as simply the condition that bordered Hessian determinant of the second-order derivatives are negative. Notationally,

$$\begin{bmatrix} \frac{\partial^2 y}{\partial N^2} & \frac{\partial^2 y}{\partial N \partial I} & \frac{\partial^2 y}{\partial N \partial F} \\ \frac{\partial^2 y}{\partial N \partial I} & \frac{\partial^2 y}{\partial I^2} & \frac{\partial^2 y}{\partial I \partial F} \\ \frac{\partial^2 y}{\partial F \partial N} & \frac{\partial^2 y}{\partial I \partial F} & \frac{\partial^2 y}{\partial F^2} \end{bmatrix}$$

— this matrix must be negative definite for a maximum.[6]

5. E. O. Heady and J. L. Dillon : Agricultural Production Functions, Iowa State University, Ames, Iowa, U.S.A., 1961.

6. This is equivalent to $d^2y < 0$ subject to $dy = 0$ in terms of total differential. Hicks-Allen used total differential for writing second-order conditions and they wrongly stated these second-order conditions for maxima as stability conditions.

TABLE II—EXPERIMENTAL RESULTS WITH NITROGENOUS FERTILIZERS 1957-58
REGRESSION COEFFICIENTS, THEIR STANDARD ERRORS AND R^2

me of the district	Model	a	b	c	d	e	f	g	h	i	j	k	R^2
edullahganj	Quadratic	18·7777* (3·2283)		—6·7250* (2·1428)						1·7000* (0·4586)	0·9500* (0·4586)		0·928
	Square-root	46·1691* (11·8888)		—34·2907* (11·1106)						12·6127* (3·4994)	7·9437* (3·4994)		0·927
pali	Quadratic	12·9518 (8·4439)	—0·3281* (0·1515)			0·1105* (0·0474)	0·1078 (0·0474)					·0342* (·0152)	0·754
	Square-root		—10·9701* (3·7982)			6·2201* (2·1859)	15·0256* (6·9126)	6·1485 (2·1859)				—3·5306* (1·2620)	0·784
anganagar	Quadratic	40·1370 (9·0971)			—10·9055* (3·5273)		1·2083* (0·4521)			—1·1111* (0·4044)	1·0555* (0·4044)		0·935
	Square-root	103·5721* (36·7316)			—95·1392* (28·1713)		16·4202 (6·6129)			—13·2800* (4·8265)	17·9431* (6·4947)		0·933
ipuri	Quadratic / Square-root	None of the coefficients is statistically significant.											0·761 / 0·755
hmapur	Quadratic / Square-root	None of the coefficients is statistically significant.											0·648 / 0·652
ra Farm	Quadratic	0·7724* (0·3320)				—0·2196* (·1038)						·0712* (·0334)	0·730
	Square-root	None of the coefficients is statistically significant.											0·722
rpur	Quadratic	None of the coefficients is statistically significant.											0·829
	Square-root		17·6936* (8·6628)			—10·5108* (4·9856)		—11·3323* (4·9856)				6·5429* (2·8784)	0·841
rkheda	Quadratic / Square-root	None of the coefficients is statistically significant.											0·933 / 0·936

Quadratic : $Y = a + bN + cI + dF + eNI + fIF + gNF + hN^2 + iI^2 + jF^2 + kNIF.$

Square-root : $Y = a + b\sqrt{N} + c\sqrt{I} + d\sqrt{F} + e\sqrt{NI} + f\sqrt{IF} + g\sqrt{NF} + hN + iI + jF + k\sqrt{NIF}.$

* Statistically significant.

TABLE III—EXPERIMENTAL RESULTS WITH NITROGENOUS FERTILIZERS 1958-59
REGRESSION COEFFICIENT, THEIR STANDARD ERRORS AND R^2

ne of the district	Model	a	b	c	d	e	f	g	h	i	j	k	R^2
edullahganj	Quadratic Square-root	} None of the results is significant (multicollinearity)											0·772 0·772
pali	Experiment not conducted.												
ganganagar	Quadratic Square-root	} None of the coefficients is statistically significant.											0·827 0·828
hpuri	Quadratic Square-root	} None of the coefficients is statistically significant.											low R^2 low R^2
khmapur	Quadratic Square-root	} Multicollinearity—no result significant.											0·909 0·909
ura Farm	Quadratic	46·3600* (11·6900)		—17·8600* (7·1025)			2·2891* (1·0381)						0·825
	Square-root						8·5349* (4·0231)					5·1078* (2·2648)	0·768
sirpur	Quadratic Square-root	} None of the coefficients is statistically significant. } (low R^2)											0·607 0·628
werkheda	Quadratic	None of the coefficients is statistically significant.							—·0031* (·0007)				0·964
	Square-root						21·1215* (9·8629)						0·964

Quadratic : $Y = a + bN + cI + dF + eNI + fIF + gNF + hN^2 + iI^2 + jF^2 + kNIF.$

Square-root : $Y = a + b\sqrt{N} + c\sqrt{I} + d\sqrt{F} + e\sqrt{NI} + f\sqrt{IF} + g\sqrt{NF} + hN + iI + jF + k\sqrt{NIF}.$

* Statistically significant.

TABLE IV—EXPERIMENTAL RESULTS WITH PHOSPHATIC FERTILIZERS 1957-58
REGRESSION COEFFICIENTS, THEIR STANDARD ERRORS AND R^2

Name of the district	Model	a	b	c	d	e	f	g	h	i	j	k	R^2
bedullahganj	Quadratic	20·5204* (2·9296)		—6·6361* (1·9446)						1·7055* (·4162)	0·9055*		0·842
	Square-root	46·4583* (11·1573)		—33·6984* (10·4226)						12·6494* (3·2841)	7·6061* (3·2841)		0·829
arpali	Quadratic Square-root	None of the coefficients is statistically significant.											0·552 0·546
riganganagar	Quadratic	41·5963* (8·3964)			—10·5666* (3·2556)	·0878* (·0439)	1·1833* (0·4173)			—1·1055* (0·3732)	1·0494* (0·3732)		0·915
	Square-root	105·5106* (33·9424)			—93·4565* (26·0321)		16·5970* (6·1107)			—13·2314* (4·4599)	17·7674* (6·0016)		0·912
ichpuri	Quadratic Square-root	None of the coefficients is statistically significant.											0·760 0·764
akhmapur	Quadratic Square-root	None of the coefficients is statistically significant.											0·689 0·702
eura Farm	Quadratic							—0·0316* (·00136)					0·906
	Square-root	None of the coefficients is statistically significant.											0·902
asirpur	Quadratic Square-root	None of the coefficients is statistically significant.											0·567 0·549
owerkheda	Quadratic								—·0033* (·0007)				0·956
	Square-root								—·1155* (·0525)				0·871

Quadratic : $Y = a + bP + cI + dF + ePI + fIF + gPF + hP^2 + iI^2 + jF^2 + kPIF$.

Square-root $Y = a + b\sqrt{P} + c\sqrt{I} + d\sqrt{F} + e\sqrt{PI} + f\sqrt{IF} + g\sqrt{PF} + hP + iI + jF + k\sqrt{PIF}$.

* Statistically significant.

TABLE V—EXPERIMENTAL RESULTS WITH PHOSPHATIC FERTILIZERS 1958-59
REGRESSION COEFFICIENTS, THEIR STANDARD ERRORS AND R^2

ame of the district	Model	a	b	c	d	e	f	g	h	i	j	k	R^2
bedullahganj	Quadratic	21·4185*											0·643
		(8·0129)											
	Square-root	None of the coefficients is statistically significant.											0·604
arpali	Experiment not conducted												
riganganagar	Quadratic	42·7260*	—0·8187*			0·3178*	2.4458*	0·2262*				—0·0787*	0·702
		(24·2260)	(·4010)			(0·1269)	(1·2041)	(0·0966)				(0·0310)	
	Square-root		—24·1955*			14·6653*	12·3765*					—7·3758*	0·678
			(11·3374)			(6·5341)	(5·6586)					(3·2671)	
ichpuri	Quadratic								—·0047*				0·683
									(·0016)				
	Square-root								—0·3571*				0·689
									(0·1208)				
akhmapur	Quadratic					·1428*		·1065*				—·0345*	0·834
						(·0656)		(·0499)				(·0161)	
	Square-root	None of the coefficients is statistically significant.											0·693
eura Farm	Quadratic		0·9481*			—0·1958*			—·0035*		—1·9388*		0·953
			(0·2761)			(0·0864)			(·0011)		(0·9624)		
	Square-root	None of the coefficients is statistically significant.											0·702
asirpur	Quadratic	35·0065											0·693
		(15·9887)											
	Square-root	None of the coefficients is statistically significant.											0·704
owerkheda	Quadratic	0·3055*							—·0029*				0·967
		(0·1224)							(·0006)				
	Square-root						11·3060*						0·960
							(5·3935)						

Quadratic : $Y = a + bP + cI + dF + ePI + fIF + gPF + hP^2 + iI^2 + jF^2 + k(PIF)$.

Square-root $Y = a + b\sqrt{P} + c\sqrt{I} + d\sqrt{F} + e\sqrt{PI} + f\sqrt{IF} + g\sqrt{PF} + hP + iI + jF + k\sqrt{PIF}$.

* Statistically significant.

This can be expressed in terms of Hessian determinants (under unconstrained maxima) as

$$\begin{vmatrix} 2h \end{vmatrix} < 0 \quad \begin{vmatrix} 2h & e + kF \\ e + kF & 2i \end{vmatrix} > 0, \quad \begin{vmatrix} 2h & e + kF & g + kI \\ e + kF & 2i & f + kN \\ g + kI & f + kN & 2j \end{vmatrix} < 0$$ if we take only positive levels of N, I and F.

If e and g are statistically significant, we can show that fertilizer application and availability of water (frequency or levels of irrigation) are not additive. If h, i and j are negative, it indicates diminishing marginal physical productivity with respect to each of the inputs in agricultural experiments. If the coefficients of interaction term $N \times I \times F$ is statistically significant, we have shown interaction between three inputs or between water and fertilizers. The results for 1957-58 and 1958-59 with the use of nigrogenous fertilizers suggested that there is a complementarity between the use of nitrogenous chemical fertilizers and water in the case of two districts in 1957-58. While in the case of phosphatic application for 1957-58, the interaction term is negative which suggests that either the initial controlled level of water is sufficient for a reasonable response to phosphate and any additional supply of water with phosphate decreases the yield or, alternatively, the phosphatic fertilizers react adversely to wheat crop because the soil is acidic or phosphatic fertilizers are inherently enough in the soil where this experiment has been conducted and, consequently, the yield decreases with any treatment of phosphate. This explanation seems to be likely as the coefficient of linear term in these two cases is negative.

Analysing the results districtwise, there is a strong evidence of diminishing marginal productivity with increased availability of water for 1957-58 (nitrogen) and 1957-58 (phosphorus) in the case of Obedullahganj. Similarly, in the case of Sriganganagar for 1957-58, there is a diminishing marginal productivity with respect to levels of irrigation while there is an increasing marginal productivity with frequency of irrigation and hence if water is available more often, the yield can be increased significantly. This will be relevant for a policy criterion because in under-developed agriculture, availability of water during a season at a frequent interval ensures farmers against risk and may work as an insurance against uncertainty of weather in a non-controlled situation. In a controlled experiment, there is a clear demonstration that small amounts of water available on a higher frequency results in significant increase in yield for a single crop. Perhaps this may be facilitating crop rotation and multiple cropping which may be responsible for increasing wheat yield in a rotational cycle. In the case of Powerkheda, there is a diminishing marginal productivity with respect to phosphorus for the year 1957-58.

II

In this section, the demand functions for chemical fertilizers are fitted. The demand for chemical fertilizers is a derived demand and it may be possible to derive the optimum inputs required to achieve the maximum crop output from technical crop response relationships. This will indicate total technological requirements of chemical fertilizers without considering the prevailing market constraints on the behaviour of farmers. Farmers may use chemical fertilizers according to the technologically recommended doses (price of chemical fertilizers does not enter) provided he is sure to get a return higher than his additional cost. Hence, the price of chemical fertilizers which should indicate scarcity of an input ordinarily, will ration nitrogenous or phosphatic fertilizers and will indicate an economic optimum.[7] We have not tried to use derived demand relationships from economic optimum because perfect competition may not be prevalent and even if it is so, the prices are fixed by the Government of India not on the basis of market rule but on the basis of increasing the use of chemical fertilizers in agriculture in general. Hence, the farmers are subsidized.

We have collected the data on Statewise nitrogenous fertilizers from the Fertiliser Statistics in India for the year 1959-60. These data are available Statewise for each of the types of nitrogenous chemical fertilizers.[8] Nutrient contents are used to derive the Statewise consumption in terms of nitrogen units. The prices of nitrogenous fertilizers are obtained from the Fertiliser Statistics in India. These prices are controlled by the Government of India. Price per ton of nitrogen is derived by making use of the conversion factors. Between States, prices for each product do not vary considerably. However, the consumption of types of nitrogenous fertilizers varies significantly from one State to another. Statewise consumption of each product converted to nitrogen tons is used as weight for 1959-60 and weighted average price per ton of nitrogen is derived. In order to consider further variations in prices between States, prices received by the farmers for the crops on which nitrogenous fertilizers are used extensively, is used as a deflator and this ratio is denoted as relative price of nitrogenous fertilizers. Six crops are considered for computing the farm harvest price index number. Data on water rates are obtained from the Planning Commission.

Statewise consumption of nitrogenous fertilizers is treated as the demand for nitrogen during 1959-60 and we have related this Statewise consumption with the relative price of chemical fertilizers and water charge. Data on water charge are available for paddy, sugarcane and others and it is not possible to obtain an index of water charge from such a classification. Hence, three to four different measures of water charge variable are used. R_t—is the maximum water rate which is generally applicable to sugarcane. R_t^1 is the minimum water rate which is applicable to paddy in some States and to dry crops in others.

Double-logarithmic and simple demand functions are fitted for 1959-60 (Table VI). All coefficients are tested for significance and we find that the simple linear relationship between consumption of nitrogenous chemical fertilizers and maximum water rate variable turns out to be statistically significant.

7. Economic optimum is derived from the marginal substitution of each factor under perfect competition.

$$\frac{\partial y}{\partial N} \Big/ P_N = \frac{\partial y}{\partial I} \Big/ P_I = \frac{1}{P_y}.$$

8. Four types of nitrogenous fertilizers are commonly used : Ammonium Sulphate (20.6 per cent N), Calcium Ammonium Nitrate (20.6 per cent N), Ammonium Sulphate Nitrate (26 per cent N) and Urea (44 per cent N).

TABLE VI—CONSUMPTION FUNCTIONS FOR NITROGENOUS FERTILIZERS
REGRESSION COEFFICIENTS, THEIR STANDARD ERRORS AND R^2

Dependent variable	Constant	Relative price of nitrogenous fertilizers		Water rate (maximum)		Water rate (minimum)		R^2	No. of observations
		$\log_e P_{it}$	P_{it}	$\log_e R_t$	R_t	$\log_e R_t^1$	R_t^1		
$\log_e C_{it}$	—19.3299 (23.4708)	2.7396 (3.1616)						.0770	11
$\log_e C_{it}$	—13.3429 (23.2171)	1.6720 (3.1761)		.6067 (.4800)				.2306	11
$\log_e C_{it}$	—39.0445 (29.9406)	5.1921 (3.9144)				.6918 (.6582)		.1889	11
C_{it}	—5.5705 (7.6894)		.0053 (.0045)					.1330	11
C_{it}	—1.6921 (6.1675)		.0018 (.0038)		.0729* (.0278)			.5328	11
C_{it}	—14.0729 (11.1386)		.0090 (.0057)				.2417 (.2302)	.2379	11

* Significant at 5 per cent level.

The elasticity of demand with respect to water charge turns out to be extremely high. This indicates the role of shortage of water and its effect on the demand for nitrogenous chemical fertilizers. The total correlation coefficient with two variables turns out to be .5328 and the relative price of chemical fertilizers does not add anything significant to the explanation of variation in consumption of nitrogenous fertilizers. If the slope coefficient of the demand for chemical fertilizers with respect to price charged for water is analysed, we find it of a positive sign. This will imply that as the water charge increases, the demand for chemical fertilizers increases. In other words, fertilizers and irrigation turn out to be substitutes. Data on water charge at a district level are not available and hence our results are not conclusive. It can only be stated that fertilizers and irrigation do not appear to be complementary inputs according to the derived demand approach.

III

In this section, the elasticity of substitution between irrigation and chemical fertilizers is computed with the help of various approaches. The main approaches[9] used are : (*a*) SMAC production function or CES production function and

9. K. J. Arrow, H. Chenery, B. S. Minhas and R. Solow, "Capital-Labour Substitution and Economic Efficiency," *Review of Economics and Statistics*, Vol. XLIII, August, 1961, pp. 225-250.

(*b*) Variable Elasticity of Substitution (VES) production function. The elasticity of substitution (σ) is defined as the ease at which the varying factor can be substituted for others. In the generalised Cobb-Douglas or Constant Returns to Scale Cobb-Douglas form, such an elasticity of substitution is always unity and if this substitution elasticity departs significantly from unity, empirically, Cobb-Douglas function cannot be fitted as it introduces specification bias. CES function meets this criticism but the elasticity of substitution is assumed to be constant over various fertilizer/irrigation ratios. However, when the fertilizer-use per acre of irrigated area varies, due to changes in input-price ratio, it is possible that the elasticity of substitution will vary as the fertilizer-use per unit of irrigated acre varies. The variable elasticity of substitution[10] production function meets this criticism.

We have adopted the following three methods with respect to the estimation of the elasticity of substitution. The first two methods refer to the assumption of CES function and the last one to the VES function. In all these approaches, we have made the assumption of constant returns to scale and perfect competition.

(A) C E S APPROACH

Method (1)

The CES function is

$$V = \gamma \left[\delta_1 F^{-\rho_1} + (1-\delta_1) I^{-\rho_1} \right]^{-\frac{1}{\rho_1}}$$

where γ stands for the efficiency parameter,

δ_1 stands for the distribution parameter,

ρ_1 stands as a transform of the elasticity of substitution σ ($\rho_1 = \frac{1}{\sigma} - 1$),

V is the agricultural output,

F = Total amount of nitrogenous fertilizers,

I = Irrigated area under a crop.

The following marginal products of irrigation and nitrogenous fertilizers are obtained by differentiating the function with respect to each of the inputs.

$$V_F = \frac{\delta_1}{\gamma^{\rho_1}} \left(\frac{V}{F} \right)^{\rho_1+1} \qquad (2)$$

$$V_I = \frac{1-\delta_1}{\gamma^{\rho_1}} \left(\frac{V}{I} \right)^{\rho_1+1} \qquad (3)$$

10. Yao-chi Lu and L. B. Fletcher, "A Generalisation of the CES Production Function," *Review of Economics and Statistics*, 1968.

The parameter ϱ_1 appears in the function in a non-linear way which makes the direct fitting of the function to the data extremely difficult. Hence, we use side-conditions and equate marginal rate of substitution to the input-price ratio.

$$\frac{V_I}{V_F} = \frac{1-\delta_1}{\delta_1}\left(\frac{F}{I}\right)^{\varrho_1+1} = \frac{1-\delta_1}{\delta_1}\left(\frac{F}{I}\right)^{1/\sigma} = \frac{W}{P} \qquad (4)$$

where W is the price of water and P is the price of fertilizers.

The regression from (4) is computed as $\log\frac{W}{P} = \log\frac{\delta_1}{1-\delta_1} + \frac{1}{\sigma}\log\left(\frac{F}{I}\right)$.. (4a)

The elasticity of substitution is defined as

$\sigma = \dfrac{\delta \log \dfrac{V_I}{V_F}}{\delta \log \dfrac{F}{I}}$ and by fitting a logarithmic function to (4a) we obtain

estimates for $\log\left(\frac{1-\delta_1}{\delta_1}\right)$ and $\frac{1}{\sigma}$, the reciprocal of the elasticity of substitution.

Method (2)

Another relation can be obtained from the marginal product equation alone, *i.e.*, either from (2) or from (3) ;

$$V_F = \frac{\delta_1}{\gamma^{\varrho_1}}\left(\frac{V}{F}\right)^{\frac{1}{\sigma}}$$

$$V_I = \frac{1-\delta_1}{\gamma^{\varrho_1}}\left(\frac{V}{I}\right)^{\frac{1}{\sigma}}$$

If we use (3), we can equate marginal productivity of irrigation to the water rate. Hence,

$$\log\frac{V}{I} = \log\alpha + \sigma\log W \qquad (5)$$

$$\text{where } \alpha = \left(\frac{\gamma^{\delta_1}}{1-\delta_1}\right)^{\sigma} = \frac{\gamma^{1-\sigma}}{(1-\delta_1)^{\sigma}}$$

From this relation with the data on water rate and yield per acre of irrigated area, we can estimate σ the elasticity of substitution. Similarly, from (2) equated to price of chemical fertilizers we get

$$\log \frac{V}{F} = \log \beta + \sigma \log p \tag{6}$$

$$\text{where } \beta = \left(\frac{\gamma^{\delta_1}}{\delta_1}\right)^{\sigma} = \frac{\gamma^{1-\sigma}}{\delta_2 \sigma}$$

and (6) will again provide an estimate of the elasticity of substitution. (5) and (6) both use one side-condition while (4a) uses both the side-conditions.

We have used all these various methods to estimate the elasticity of substitution from a cross-section data where each State is treated as an observation. Output per acre of irrigated area and output per acre of unirrigated area is not known separately for each of the crops. Consequently, we have used yield per acre data for the left-hand side variable of (5) and it is assumed that most of the area is irrigated under that crop and there is very little difference between irrigated area and unirrigated area. It is, however, to be expected that our yield per acre for paddy and sugarcane may be referring to irrigated acreage because both these crops are mainly grown under irrigated conditions. Data on water rate is extremely difficult to tackle with and as there are special rates for sugarcane crop as different from paddy and other crops, we have used special perennial crop water rate for sugarcane crop. It is not possible to form an index of water rate for paddy from the available data and hence we have used four different water rates for paddy. W_1 is the maximum price of water per acre farmers will be asked to pay and W_2 is the minimum price per acre. W_3 is the most likely price farmers will be paying for irrigated paddy and W_4 is the special price for paddy-growing farmers. W_4 is mixed up with W_3 to a great extent. In the absence of any other supplementary data, we have taken recourse to this crude data to measure the elasticity of substitution. The substitution elasticity cannot be computed from (6) in this study as data on chemical fertilizers are not available for its cropwise use. Consequently, data for the left-hand side variable of (6), *viz.*, output per pound of chemical fertilizers is not available.

Using two side-conditions as in (4), our equation yields an estimate of the elasticity of substitution. Rewriting for the estimation purposes, we have

$$\log\left(\frac{W}{P}\right) = \log\left(\frac{1-\delta_1}{\delta_1}\right) + \frac{1}{\sigma}\log\left(\frac{F}{I}\right) \tag{7}$$

and this requires data on fertilizers per acre of irrigated area and input-price ratio. We have two different (maximum and minimum) measures of water rate and these ratios have been used on the left-hand side. The price of nitrogenous chemical fertilizers has not varied from one State to another as it had been fixed up by the Government of India. However, in relative terms, the price of chemical fertilizers had been different. This has been obtained by dividing the price of nitrogenous fertilizers with the index number of prices received by the farmers[11] for the crops where chemical fertilizers are used.

11. It has been compiled in an article : A. Parikh, "Consumption of Nitrogenous Fertilizers : A Continuous Cross-Section Study and Covariance Analysis," *The Econometric Annual of the Indian Economic Journal*, Vol. XIV, No. 2, 1966.

Results

For sugarcane and paddy, yield per acre is related with water rate for each of the State observations for the year 1959-60. (5) is used as the basis for estimating. (6) cannot be used because yield per unit of chemical fertilizers cannot be computed from the data as fertilizers cannot be allocated cropwise. Logarithmic and simple functions have been fitted to the data. We find that the simple correlation coefficient between water rate and yield per acre for sugarcane turns out to be .39, and the marginal coefficient with respect to water rate is statistically significant (Table VII). The elasticity of substitution worked out from marginal coefficient when yield per acre and water rate over States are held at mean values in 1959-60 turns out to be .335. This is significantly different from unity and zero and this will indicate that chemical fertilizers and water are complementary inputs

TABLE VII—YIELD PER ACRE FOR SUGARCANE RELATED TO WATER RATE REGRESSION COEFFICIENTS, THEIR STANDARD ERRORS AND R^2

Dependent variable	Constant	Water rate		Nitrogen per acre		R^2	No. of observations
		$\log_e W_t$	W_t	$\log_e \frac{(N)}{(I)}$	$\left(\frac{N}{I}\right)$		
$\log_e \frac{(V)}{(I)}$	7.2475* (0.5457)	.2996 (.1724)				.2514	11
$\frac{V}{I}$	2551.6508* (663.3504)		46.2148* (18.9418)			.3981	11
$\log_e \frac{(V)}{(I)}$	7.3625* (.7214)	.1938 (.2460)		.2096 (.1758)		.3809	11
$\frac{V}{I}$	2181.2384* (888.0356)		23.0153 (35.0022)		310.7505 (364.6233)	.4374	11

* Significant at 5 per cent level. (Elasticity .3353).

to a larger extent. As far as paddy is concerned, the water charge coefficient is statistically insignificant (Table VIII) for both the models and hence, we cannot state any numerical measure of elasticity of substitution between nitrogenous fertilizers and water rate.

TABLE VIII—YIELD PER ACRE FOR PADDY RELATED WITH WATER RATE AND NITROGEN PER ACRE

Dependent variable	Constant	Water rate		R^2	No. of observations
		log R_t	R_t		
$\log_e \frac{(V)}{(I)}$	3.0157* (0.3410)	.0822 (.1577)		.0264	11
$\frac{(V)}{(I)}$	22.0500* (5.4413)		.3540 (.5369)	.0417	11

* All other models have also low R^2.

If (4) is used, then we are using two-side conditions. According to various forms tried, only (1) in Table IX indicates that the elasticity of substitution[12] is 1.24 which is significantly different from both zero and unity. This is not comparable with the substitution elasticity worked out from one side-condition. Again the logarithmic relationship does not yield high R^2 and low standard error of the estimate and the coefficient does not turn out to be statistically significant.

TABLE IX—RATIO OF WATER CHARGE TO PRICE OF CHEMICAL FERTILIZERS REGRESSED ON NITROGEN PER IRRIGATED ACREAGE
REGRESSION COEFFICIENTS, THEIR STANDARD ERRORS AND R^2

No.	No. of observations	Dependent variable	Constant	$\left(\frac{N}{I}\right)$	$\log_e\left(\frac{N}{I}\right)$	R^2
1	11	$\frac{W_1}{P}$	0.00329 (0.00508)	0.00405* (0.00128)		0.52592
2	11	$\frac{W_2}{P}$	0.00359* (0.00100)	0.00001 (0.00025)		0.00008
3	11	$\frac{W_3}{P}$	0.00377* (0.00155)	0.00074 (0.00039)		0.28358
4	11	$\log_e\left(\frac{W_1}{P}\right)$	—4.51109* (0.25871)		0.29391 (0.20292)	0.18904
5	11	$\log_e\left(\frac{W_2}{P}\right)$	—5.77392* (0.33180)		0.01848 (0.26125)	0.00056
6	11	$\log_e\left(\frac{W_3}{P}\right)$	—5.59766* (0.31470)		0.37722 (0.24684)	0.20603

* Significant at 5 per cent level.
Note : Elasticity of substitution worked out from (1) at mean values is 1.24.

The limitation of these approaches is that each of them uses data on water rates and as this is not reliable, our estimates are crude.

(B) V E S APPROACH

Lu and Fletcher[13] derived a VES production function in which they first considered three variable relationship $\left(\frac{V}{I}, \frac{F}{I} \text{ and } W\right)$ and derived a differential equation by making use of the competitive conditions of factor and product market. This production function has the same form as the CES function except that $I-\varrho$ is multiplied by $\left(\frac{F}{I}\right)^{-c\,(1\,+\,\varrho)}$

12. This is obtained as the reciprocal of $\frac{d\left(\frac{W_1}{P}\right)}{d\left(\frac{N}{I}\right)} \cdot \frac{\left(\frac{N}{I}\right)}{\left(\frac{W_1}{P}\right)} = \frac{1}{\sigma}$.

13. Yao-chi Lu and L. B. Fletcher, *op cit.*

Writing VES function of Lu and Fletcher, we get

$$V = \gamma \left[F^{-\varrho} + (1-\delta)\,\eta \left(\frac{F}{I}\right)^{-c(1+\varrho)} I^{-\varrho} \right]^{-\frac{1}{\varrho}} \quad (8)$$[14]

The elasticity of substitution is defined as

$\sigma = \frac{V_F V_I}{V.V_{FI}}$ where V_F and V_I are the first-order partial derivatives, while V_{FI} is a cross-partial derivative with respect to the subscripted variable.[15]

By substituting the partial derivatives in (8) we obtain

$$\sigma = \frac{b}{1 - c\left(1 + \frac{R}{X}\right)} \quad \text{where } X = \left(\frac{F}{I}\right) \qquad (9)$$

$$\text{and } R = -\left(\frac{dF}{dI}\right) = \frac{V_I}{V_F}.$$

Since R is a function of X, moving along an isoquant, the elasticity of substitution varies with the fertilizer-use per acre of irrigated area. Therefore, the elasticity of substitution is not a constant but a funtion of the fertilizer/irrigation ratio. The first-order condition for minimum cost under pure competition is $R = \frac{W}{P}$. Substituting the value of R into (9) yields

$$\sigma = \frac{b}{1 - c\left(1 + \frac{WI}{PF}\right)} \qquad (10)$$

and this can be used to estimate the elasticity of substitution. Our main task is to get the estimate of b and c and these can be obtained by using the side-condition of perfect competitive profit maximization. CES assumes that c is equal to zero while VES indicates that c may or may not be zero. We can test the null hypothesis of c being equal to zero by using the same logarithmic functions with the addition of one variable $\left(\frac{F}{I}\right)$ on the right-land side of (5) and correct the elasticity of substitution for varying levels of $\left(\frac{F}{I}\right)$. We now fit

$$\log\left(\frac{V}{I}\right) = \log a + b \log W + c \log\left(\frac{F}{I}\right) \qquad (11)$$

and test the null hypothesis $c = 0$ against the alternative hypothesis of $c \neq 0$. In the regular CES function $\log\left(\frac{F}{I}\right)$ has a zero coefficient.

14. b is the elasticity of output per acre of irrigated area with respect to water charge and c is the elasticity of output with respect to fertilizers per acre of irrigated acre while $\eta = 1 - b/1 - b - c$.

15. When the production function is homogeneous of degree one, σ can be written as above: R. G. D. Allen : Mathematical Analysis for Economists, Macmillan & Company London, 1938, pp. 341-343.

The empirical results on VES approach are mentioned in Table VII. None of the coefficients with respect to $\frac{F}{I}$ are statistically significant. R^2 does not improve considerably with the inclusion of this variable. Hence the hypothesis that the elasticity of substitution is not a constant but varies with the variations in fertilizer-use per acre, does not seem to be supported by the empirical results obtained in Tables VII and VIII.

Conclusion

It is attempted to measure the complementarity relationship between irrigation and chemical fertilizers in Indian agriculture. In the absence of data on water charge at the district level and millions of gallons of water supplied through irrigation to the agricultural sector, our results are inconclusive. The controlled experimental data show that fertilizers and availability of water are not additive inputs in most of the cases. It is also shown that the elasticity of substitution (indirectly near zero for strictly complementary goods) turns out to be at variance from one method of statistical estimation to another.