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MILK PRODUCTION FUNCTIONS AND OPTIMUM FEEDING SCHEDULES*

T. Jacob, V. N. Amble, M. L. Mathur and A. Subba Rao

INTRODUCTION

Formulation of economic feeding schedules for production of milk in cattle and buffaloes is one of the major problems in dairy husbandry. In our country traditionally the requirement for milk production is catered for mostly through concentrate feeds like oil cakes, grams, etc. But the present available concentrate feed in the country is not enough to meet the cattle requirement. A recent study at the Institute[1] showed that animals in the country are very much under-nourished, the amount of digestible crude protein available being only about one-third of the requirement for animals in milk and about half the requirement for dry animals, as per Morrison's standards. Moreover, feeding concentrates form an expensive item in the dairyman's bill which makes the cost of production of milk quite high. The solution to this difficulty lies in examining the possibility of substituting the concentrates by less costly and more readily available feeds. It is an accepted fact by animal nutritionists that concentrates can be substituted by protein-rich roughages without adversely affecting the milk production even though complete withholding of concentrates to dairy animals is not considered to be a sound feeding policy.[2] It is of great practical utility to know the extent to which such substitutions can be made and also the combination of feeds which will result in the most economic level of production. The results of one such study are described in the present paper.

Problems relating to the feed substitution rates and least-cost rations fall in the domain of agricultural production economics. Most of the recent developments in this field have been due to Earl O. Heady.[3] Different forms of production functions and their applications in evolving optimum feed combinations have been discussed by Heady and his co-workers in a series of research papers.[4] A beginning has been made in this line of investigations at the Institute of Agricultural Research Statistics, New Delhi, by some of the research students. K.M. Rao, K.V.N. Prasad and N.S. Murthy have undertaken studies on milk produc-

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The material for the present study came from an experiment sponsored by the Indian Council of Agricultural Research and conducted at the Southern Regional Station of the National Dairy Research Institute. The authors are grateful to Dr. G. R. Seth, Statistical Adviser, Institute of Agricultural Research Statistics for his keen interest in the problem and the sustained encouragement given in preparing the paper.

1. V. N. Amble, "Feed Requirements of Bovines and Possibilities of Meeting Them," *Indian Journal of Agricultural Economics*, Vol. XX, No. 1, January-March, 1965.

2. K. C. Sen and S. N. Ray : Nutritive Values of Indian Cattle Foods and the Feeding of Animals, I.C.A.R. Bulletin No. 25, New Delhi, 1964.

3. Earl O. Heady : Economics of Agricultural Production and Resource Use, Prentice-Hall, Inc., New York, U.S.A., 1952.

4. Earl O. Heady and J. L. Dillon : Agricultural Production Functions, Iowa State University Press, Ames, Iowa, U.S.A., 1961.

tion functions and least-cost combinations of feeds to meet the nutritional requirement (unpublished theses). However, the data used by them were obtained in the course of sample surveys on estimation of milk production and cost of milk production and as such were subject to very large variation due to a number of factors. The present study is based on data from a controlled experiment.

NATURE AND EXTENT OF THE DATA

The material utilized for the study comes from an experiment conducted at the Southern Regional Station of the National Dairy Research Institute, Bangalore, during 1959-62. The object of the experiment was to study the effect of feeding different levels of digestible crude protein (D.C.P.) for milk production and substitution of 20 per cent D.C.P. requirement of the concentrate mixture fed, by that of para grass. Thirty-two cows were taken for the experiment and four types of rations were tried.

All the cows received a maintenance ration consisting of para grass and groundnut cake in appropriate amounts to meet the requirement according to body weight and of *ragi* straw fed *ad lib.* For milk production, a concentrate mixture consisting of 40 parts of barley, 35 parts of gram, 10 parts of groundnut cake and 15 parts of wheat bran was given. The animals under ration were given a concentrate mixture for production at the rate of 1 kg. for every 2½ kg. of milk with 5 per cent fat content, thus providing 0.052 kg. of D.C.P. per kg. of milk. Under ration B the level of D.C.P. was kept the same as in ration A, but the concentrate mixture was partly replaced by a calculated amount of para grass sufficient to provide 20 per cent of the total D.C.P. Animals under ration C were given the concentrate mixture for production at the rate of 1 kg. for every 3 kg. of milk with 5 per cent fat amounting to 0.043 kg. of D.C.P. per kg. of milk. Under ration D, the D.C.P. level was maintained as that of ration C, but 20 per cent of the D.C.P. of the concentrate mixture was replaced by an equivalent amount of para grass.

The animals selected were all Thari cows and they were divided into eight blocks of four animals each so that the animals within a block were as similar as possible in respect of body weight, order of lactation and date of calving. Within a block the rations were allotted at random.

The experiment was conducted over the entire period of lactation of the animals. The quantity of feed consumed and the milk produced were recorded daily. Fat tests were conducted twice in a week. During the course of the experiment six animals died. The data pertaining to the remaining 26 animals were utilized for the study.

PROCEDURE OF ANALYSIS

The formulation of a production function relating milk yield with the feed inputs is a basic requirement for economic analysis. In a general form, a production function can be written as

$$y = f(x_1, x_2, x_3, x_4, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; x_n)$$

where y denotes the output, in this case the milk yield, and x_1, x_2, x_3, x_4, etc., refer to the various factors influencing milk production such as the quantities of different ingredients of feed given, the body weight, breed, etc. Although all these factors of input have a bearing on the production of milk, most of the nutrition studies deal only with the effects of the level of feed on milk production. This is justified also if the effects of factors like breed could be kept constant by selecting animals of the same breed. Furthermore, some of the factors could be well controlled in experimental investigations by adopting appropriate statistical designs as in the case under study. The production function can then be put in the form :

$$y = f(x_1, x_2, x_3, x_4, \ldots\ldots\ldots\ldots / x_k, \ldots x_n)$$

where x_1, x_2, x_3, x_4, etc., are the factors such as feed which were allowed to vary in the experiment and $x_k \ldots x_n$ are factors considered fixed.

One way of studying the feed-input milk-output function is to aggregate the feed given such as concentrates, greens, etc., into a single input factor like the total digestible nutrients (T.D.N.) in them. Alternatively, each of the constituent feed could be studied as an input factor. The latter approach definitely gives much more information than the former and facilitates the study as to how one input factor can be replaced by another. The second approach was followed in the present study.

The input factors going into the milk production function which were studied were the four constituents of the feed, namely, para grass (x_1), concentrate mixture (x_2), groundnut cake (x_3) and *ragi* straw (x_4). The average values of the four per cent fat-corrected milk produced per day and the average feed constituents consumed were utilized for the estimation. The entire ration given to the animal was considered for the analysis without breaking it into maintenance and production portions, since such a bifurcation would necessitate making additional assumptions regarding the applicability of Morrison's standards for maintenance requirements to Indian cattle. In order to eliminate the differences due to blocks, an analysis of dispersion was first conducted by splitting the total sum of squares and products into 'between blocks' and 'within blocks.' From the 'within blocks' figures, the functional relationship of y and x_1, x_2, x_3 and x_4 was studied.

The linear, quadratic and the Cobb-Douglas functions were fitted. Least-square method was employed for fitting the curve of each type. From the most appropriate production function, the expressions for marginal rate of productivity of concentrates and para grass when one is varying and the other fixed were obtained. The substitution coefficients of concentrates and greens were also estimated. The milk isoquants and feed isoclines were derived and the least-cost combinations of feeds for any specified level of milk production under different price ratios of concentrates and para grass were estimated.

The partial derivatives of the function with respect to concentrate feed and greens were equated to the appropriate price ratios and solving the equations simultaneously, the feeding schedules which maximize the profit under varying proportions of prices of milk, concentrate and para grass were estimated.

In all these studies, groundnut cake and *ragi* straw were considered as fixed at their respective mean values. Groundnut cake was supplied only in relatively small quantities, being meant mainly to supplement the basal ration. *Ragi* straw even though given *ad lib.* contains very little protein and is a very cheap item of feed and was intended to meet the requirements of bulk.

RESULTS AND DISCUSSIONS

The linear, quadratic and the Cobb-Douglas functions fitted are given below along with the respective multiple correlation coefficients.

Form	Fitted production function	R
Linear ..	$y = 0.687 + 0.220x_1 + 2.013x_2 - 5.732x_3 + 0.329x_4$	0.832
Quadratic ..	$y = -6.972 + 1.389x_1 + 7.772x_2 - 8.725x_3 + 0.432x_4 - 0.060x_1^2 - 1.359x_2^2 - 0.193x_1 x_2$	0.873*
Cobb-Douglas ..	$y = 1.156x_1^{0.288} \; x_2^{0.785} \; x_3^{-0.470} \; x_4^{0.032}$	0.884*

* Significant at 1 per cent level.

The percentage of the variations explained by the curve fitted was 69 in the case of the linear, 76 in that of the quadratic and 78 in that of the Cobb-Douglas functions. The regression coefficients for concentrates and greens of the linear and Cobb-Douglas functions were found to be statistically significant while those for groundnut cake and straw were not. In the case of quadratic curve, however, the regression coefficient for concentrates was just significant while the others were not. Such wide variations in results are reported by Heady in his studies also. The lack of significance of regression coefficients of the quadratic equation could be attributed to the size of the sample being small and as many as 7 degrees of freedom being lost in estimating the regression coefficients.

The linear function shows constant response at all levels of input in contrast to the quadratic and the Cobb-Douglas functions. The coefficients of the square terms in the quadratic equation are negative indicating that the marginal return diminishes as the inputs are increased. Similarly the Cobb-Douglas function also indicates a diminishing feed-into-milk transformation as the feed is increased in quantity, as the sum of the exponents is less than one.

The feed transformation ratio as well as the substitution rate are constant in the linear function. Thus whatever be the level of feeding the productivity of feed is constant. Also there is no limit to milk production in the linear case. It would thus appear not logical to assume a linear function to represent milk production at all levels of feeding. However, in limited ranges it could give reasonable estimates.

The Cobb-Douglas and the quadratic functions are very widely used for representing production phenomenon. However, the quadratic function conforms more to the physiology of milk production inasmuch as milk yield cannot be increased indefinitely with continued additional feeding unlike the Cobb-Douglas function. Moreover, the percentage of variation explained by the quadratic curve is almost as large as that accounted for by the Cobb-Douglas function and as such the former was taken as the basis for further study.

The marginal product of para grass when the variate concentrate is fixed is derived from the production function as

$$\frac{\partial y}{\partial x_1} = 1.389 - 0.120x_1 - 0.193x_2.$$

Similarly, the marginal product of concentrate is derived as

$$\frac{\partial y}{\partial x_2} = 7.772 - 0.193\, x_1 - 2.718\, x_2.$$

These expressions show the change in the milk yield for a unit change in the intake of the feed constituent. As seen, each additional unit of an input adds less to the total output than the previous unit.

The isoquants or the production contours for specified levels of milk yield are obtained from the equation

$$x_2 = [(0.193\, x_1 - 7.772) \pm \text{Sq. root}\ \{ (0.193x_1 - 7.772)^2 + 5.436\, (1.389\, x_1 - 0.060x_1^2 - 8.212 - y)\}] \ / \ (-2.718)$$

The marginal rate of substitution of concentrates by greens is given by

$$\frac{dx_2}{dx_1} = \frac{1.389 - 0.120\, x_1 - 0.193\, x_2}{7.772 - 0.193\, x_1 - 2.718\, x_2}$$

This refers to the amount of concentrates replaced by a unit addition of greens. The isoclines which trace out the paths of the feed combinations with the same marginal rates of substitution are generated from the expression for substitution rates by giving specified values. These expressions help us in the estimation of least-cost combination of feeds which will be discussed later.

Table I gives the various combinations of the two feeds, namely, concentrates and greens, for different expected levels of milk production. The corresponding substitution rates are also presented.

Values of para grass intake beyond 10 kg. have not been given in Table I because beyond this level, the least-cost combination of para grass and concentrates is not likely to occur as the price ratio of para grass to concentrate will have to be less than .04. This is not likely to occur in real world situation. The figures corresponding to 10 kg. of para grass have been given in the table merely to show the sharp manner in which the substitution rate declines.

TABLE I—COMBINATION OF PARA GRASS AND CONCENTRATES AND SUBSTITUTION RATES FOR DIFFERENT LEVELS OF MILK PRODUCTION

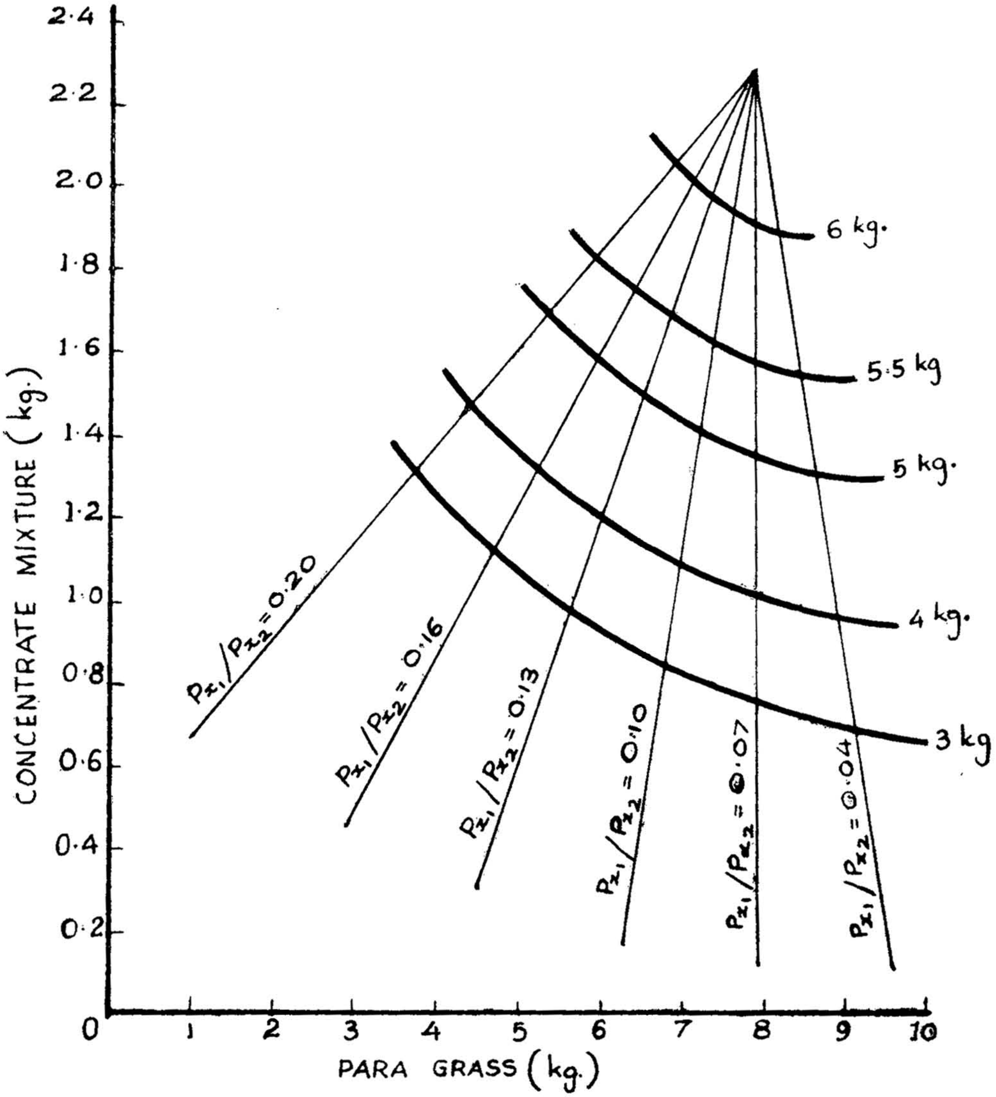
Para grass (kg.)	Level of milk production in kg. (Y)							
	y = 3		y = 4		y = 5		y = 6	
	Concentrate mixture (kg.)	Substitution rate	Concentrate mixture (kg.)	Substitution rate	Concentrate mixture (kg.)	Substitution rate	Concentrate mixture (kg.)	Substitution rate
4	1.247	0.185	1.561	0.221	2.034	0.351	—	—
5	1.080	0.150	1.367	0.170	1.758	0.222	—	—
6	0.945	0.121	1.217	0.132	1.571	0.156	2.344	0.891
7	0.838	0.094	1.102	0.099	1.459	0.108	2.022	0.173
8	0.758	0.069	1.019	0.068	1.351	0.067	1.909	0.059
9	0.701	0.043	0.967	0.036	1.305	0.024	—	—
10	0.672	0.002	0.947	0.002	—	—	—	—

As seen from Table I, for any fixed level of greens the quantity of additional concentrates required for unit increase in milk yield increases with the level of production. For the level of greens fixed at say, 6 kg., the quantity of concentrates required to raise the level of milk yield from 3 kg. to 4 kg. works out to be 0.272 kg. (1.217—0.945). The corresponding requirement of concentrates for raising the milk yield from 5 kg. to 6 kg. works out to be as high as 0.773 kg. This is in conformity with the principle of diminishing productivity of feed as the output level increases.

It can also be seen from Table I that the rate of substitution of concentrates by greens decreases as the ration includes a greater proportion of greens. This indicates that the substitution rates change when the proportion of factors in the ration changes. A constant substitution rate would have resulted if the production function were of the linear form.

From the studies on the substitution rates it is clear that substitution of concentrates by greens is feasible to a considerable extent at lower levels of milk production but the scope for such substitution reduces as the production level increases. If we consider however the economics of production it is not the extent of substitution that alone matters. It is necessary to ascertain the particular combination of feeds which results in the minimum cost for any desired level of production. The study of the isoquants and isoclines enables the estimation of such feed combination.

The map of the isoquants and the isoclines is shown in Figure 1.



The curved lines are the isoquants for the milk yields indicated.

The converging lines are the isoclines for the indicated ratios of the price of para grass (P_{x_1}) to that of concentrate mixture (P_{x_2})

The intersection of the isoquant and the isocline indicates the least-cost combination of para grass and concentrate mixture.

Figure 1—Milk Isoquants and Feed Isoclines

As seen from the graph, at higher levels of milk production the isoquants take greater curvature and shows narrower ranges of feed combinations of concentrates and greens. The maximum milk production is secured with the single combination of concentrates and greens as indicated by the point of convergence of the isoclines. In the present study, this combination of feeds worked out to be 2.29 kg. of concentrates and 7.85 kg. of para grass resulting in the maximum milk yield of 6.20 kg. The consumption of groundnut cake and *ragi* straw have been taken at their mean values, *viz.*, 0.37 kg. and 4.38 kg. respectively.

Let us now try to arrive at the least-cost combination of feeds. Consider for instance, the isoclines for the price ratio $Px_1/Px_2 = 0.10$ where Px_1 is the price of a unit of para grass and Px_2 that of concentrates. If the price ratio of para grass to concentrates is 0.1, then this isocline traces out the least-cost combination of feeds. Thus for an expected production of 5 kg. of milk and for the price ratio 0.1 the combination of feeds which results in the minimum cost works out to be 1.42 kg. of concentrates and 7.20 kg. of para grass. Other combinations can be obtained from the map by looking for the co-ordinates of the intersection of the relevant isoquant and the isocline as also from Table II.

TABLE II—LEAST-COST COMBINATION OF PARA GRASS AND CONCENTRATE MIXTURE FOR SPECIFIED LEVELS OF MILK PRODUCTION UNDER DIFFERENT PRICE RATIOS

Level of milk production (kg.)	Price ratio of para grass to concentrate mixture							
	0.04		0.07		0.10		0.20	
	Para grass (kg.)	Concentrate mixture (kg.)	Para grass (kg.)	Concentrate mixture (kg.)	Para grass (kg.)	Concentrate mixture (kg.)	Para grass (kg.)	Concentrate mixture (kg.)
3	9.11	0.68	7.93	0.76	6.82	0.85	3.70	1.32
4	8.92	0.97	7.92	1.02	6.95	1.11	4.46	1.48
5	8.67	1.32	7.91	1.36	7.20	1.42	5.41	1.70
6	8.19	1.90	7.90	1.92	7.61	1.94	6.91	2.07

As seen from Table II, if the price of para grass is one twenty-fifths of that of concentrates, then 9.1 kg. of para grass and 0.7 kg. of concentrates will form the least-cost combination to have an expected production of 3 kg. of milk, while if the para grass is one-fifth as costly as concentrates, the para grass consumption has to be reduced to two-fifths of the former amount and the concentrate feed nearly doubled. At a higher level, say, 6 kg. of milk per day, if the para grass becomes relatively costlier its consumption reduces but only by a small quantity. In fact at higher levels of milk production, the range of the least-cost combination of para grass and concentrates for different price structures gets narrower.

So far, the determination of the least-cost rations for any specified level of milk yield has been considered. It is definitely of interest to know the rations resulting in the most economic level of milk production, *i.e.*, the combination of feeds which will maximize the profit, considering the price of milk and the cost of production. From the surveys on the cost of production of milk conducted by the Institute, it has been estimated that about 60 per cent of the total cost of production of milk comes from the feed item. This percentage has been used for obtaining the fraction of the price of milk to be studied against the cost on the feed consumed by the animal.

The partial derivatives of the production function with respect to greens and concentrates when equated to the price ratios lead to the following :

$$1.389 - 0.120x_1 - 0.193x_2 = Px_1/P_y$$
$$7.772 - 0.193x_1 - 2.718x_2 = Px_2/P_y$$

where P_y, Px_1 and Px_2 are the prices per unit of milk, greens and concentrates respectively. The optimum combinations of concentrates and para grass were worked out by solving these equations for different price structures. The expected milk yield and the proportionate income minus feed cost were also calculated. As mentioned earlier, groundnut cake and *ragi* straw consumed were taken at their average values. The results are given in Table III.

TABLE III—OPTIMUM COMBINATION OF PARA GRASS AND CONCENTRATE FOR DIFFERENT PRICE STRUCTURES AND THE CORRESPONDING MILK YIELD AND RETURNS

Price in paise per kg. of			Estimated intake in kg. per animal per day		Expected milk yield in kg. per animal per day	Net return (Rs.)
Para grass	Concentrate mixture	Milk	Para grass	Concentrate mixture		
1.5	42	70	8.22	1.91	6.01	1.21
2.0	42	70	8.11	1.92	6.01	1.17
3.0	42	70	7.88	1.93	6.02	1.09
5.0	42	70	7.44	1.96	6.02	0.98
1.5	50	70	8.35	1.83	5.93	1.06
2.0	50	70	8.24	1.84	5.93	1.02
3.0	50	70	8.02	1.85	5.94	0.94
5.0	50	70	7.57	1.88	5.94	0.78
1.5	42	84	8.17	1.97	6.06	1.69
2.0	42	84	8.07	1.98	6.07	1.65
3.0	42	84	7.89	1.99	6.07	1.59
5.0	42	84	7.51	2.02	6.07	1.42

Note : The figures in the last column give [{(0.6) × (income from milk)} – feed cost], 0.6 being the assumed ratio of feed cost to total cost of production of milk.

For concentrates and milk, two prices each are given in Table III representing low and high rates. For para grass, four prices are considered including a very high rate because if substitution of concentrates by para grass goes into full swing, the demand for para grass would increase substantially and this might result in a rapid increase in the price of para grass. The results corresponding to the combination of high price for milk and high price for concentrates have not been shown in the table as these results, being based on very similar price ratio, will be almost the same as those of low prices for both concentrates and para grass.

As seen from the table if the prices per kg. of para grass, concentrate mixture and milk are 1.5, 42 and 84 paise respectively, then the optimum combination of para grass and concentrate mixture is estimated as 8.17 kg. and 1.97 kg. respectively resulting in an expected milk yield of 6.06 kg. and the net return $[\{(0.6) \times (\text{income from milk})\} - \text{feed cost}]$ would be Rs. 1.69. As the prices of para grass and concentrate mixture go up the profit decreases as one expects.