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CONSTANT VERSUS NON-CONSTANT PRODUCTION ELASTICITIES

A. I. Medani

INTRODUCTION

Various alternative production models may be proposed for analysing the same body of data for a set of farms. These models represent alternative hypotheses concerning the production elasticities. The question is which formulation is more useful for the specific purposes for which production functions are employed. This study compares, empirically, the relative performance of three general alternative regression production models in explaining the behaviour of farm firms in a sample. These three models are :

(*A*) *Model I*

Production functions fitted to the data of each farm separately.[1] Such functions may be specified as follows[2]:

$$Y'_{it} = A'_i + \sum_{j=1}^{J} \alpha_{ji} X'_{jit} + U'_{it} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Primes indicate logarithmic values.

i refers to farm (i=1, 2, ..., I)

t refers to time (t=1, 2, ..., T)

j refers to input (j=1, 2,..........................., J)

where

Y_{it} is output of farm i at time period t.

A_i is the intercept coefficient of farm i.

X_{jit} is input of factor j used by farm i at time period t.

α_{ji} is the elasticity of output with respect to input j for farm i.

U_{it} is a random disturbance term.

The specification in (1) allows different production elasticities for each farm.

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1. See L. Hurwicz, "System with Nonadditive Disturbances," in Statistical Inference in Dynamic Economic Models, T. Koopmans (Ed.), 1950.

2. Preliminary investigations of the data used in this study and subsequent analysis indicate that the Cobb-Douglas form provides a satisfactory fit to the data used in this study. Probably several forms of functions would fit a set of data reasonably well and conform to economic logic. However, the objective here is to compare the performance of alternative production models for a set of data *given* the form of the production function.

(*B*) *Model II*

A production function fitted to the pooled data of all farms in a sample. Such a function may be written as follows[3] :

$$Y'_{it} = A' + \sum_{i=1}^{I} K_i n_{it} + \sum_{=1}^{J} \propto_j X'_{jit} + U'_{it} \quad \ldots\ldots\ldots\ldots \quad (2)$$

where

A is a general constant.

n_{it} is an indicator variable that takes the value of one if farm i is present at time t, and zero otherwise.

K_i is the effect of farm i.

$\propto_j$ is the elasticity of output with respect to input j.

The rest of the terms are defined as in (1) above.

The model in (2) assumes that differences among farms are reflected in the intercept coefficients, but do not modify the elasticities.

(*C*) *Model III*

A third method of analysis of the data, where interest is in aggregate relationship, is to fit a function directly to the aggregated data. Thus, the values of each variable are aggregated over all farms at a given point in time to yield macro-variables. Then, a single function is fitted to this aggregate time series data.[4] This function, in logarithmic form, becomes Model III as follows :

$$Y'_t = A'_{\bullet} + \sum_{j=1}^{J} \propto_j X'_{jt} + U'_t \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where

$$Y'_t = \sum_{i=1}^{I} Y'_{it}, \quad X'_{jt} = \sum_{i=1}^{I} X'_{jit}.$$

The rest of the terms are defined as above.

These three formulations represent three hypotheses about the elasticities and the intercept coefficients.

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3. I. Hoch "Estimation of Production Function Parameters Combining Time Series and Cross-section Data," *Econometrica*, 1962, pp. 34-41.

To obtain determinate estimates of the parameters in (2) one must impose a constraint on one of the parameters. This can be done in a number of ways. A most convenient one is to set $K_i=0$ for some i. This is equivalent to setting $n_{it}=0$ for some i.

4. Y. Grunfeld and Z. Griliches, "Is Aggregation Necessarily Bad?," *The Review of Economics and Statistics*, February, 1960, pp. 1-13.

No particular formulation can be said to be statistically and economically "correct" or "incorrect." The question is which formulation is more useful for the specific purposes for which production functions are employed. A major use of production functions is for explanation of past variation in output from given input factors. Another use is for prediction of outputs from given inputs outside the sample and time period from which the production functions are derived. Still another purpose of production function analysis is to develop a production relationship which can be employed in making recommendations to managers regarding more economic organization of their resources.

There have not been many serious attempts to study the relative performance of these alternative production models. One such study is the one by Grunfeld and Griliches.[5] Grunfeld and Griliches attempt to determine whether by fitting production functions to observations from each individual firm separately or to the sums of the firms' variables, one obtains more information about the aggregate behaviour of the firms in a sample. Thus, Grunfeld and Griliches' study compares the performance of Model I and Model III above. They conclude that disaggregation sometimes does not improve and may even worsen the explanation of aggregates.

One may be interested in explaining or predicting output at two levels: either at the aggregate level or at the level of the individual firm. Grunfeld and Griliches were interested in explaining aggregate output. Most of the emphasis in this study, however, is on the more traditional purposes in farm management analysis of explaining and predicting output at the individual farm level. Thus, one might pose the opposite of the Grunfeld-Griliches' question : If one is interested in individual farm output behaviour, how much does one gain or lose if he uses an individual farm function instead of an aggregate function ? Specifically, Model I corresponds to a separate function fit to each individual farm; Model II specifies an alternative specification of the individual farm function utilizing separate constants for each farm. The question is : does fitting a production function to each farm separately (Model I) significantly increase the explanation and prediction of output variation from given inputs over that provided by the more usual type of production fit to a sample of relatively homogeneous farms (Model II) ?

The present study attempts to extend such analysis to all three alternative formulations and to examine the relative performance of these models in explaining and predicting the behaviour of farm firms in a sample.[6]

CRITERIA FOR SELECTION BETWEEN THE ALTERNATIVE MODELS

A relevant measure of the explanatory and predicting power of a model is the coefficient of determination. The coefficient of determination measures the amount of variation in the dependent variables that can be explained by the independent variables. A comparison of comparable R^2 from one model to that of another model indicates the relative performance of the alternative models. The model with the higher R^2 values explains the greater proportion of total variance

5. Y. Grunfeld and Z. Griliches, *op. cit.*

6. For lack of space, a comparison of the alternative models in making recommendations to managers regarding more economic organization of their resources is left to another occasion.

and hence judged to be superior. Also R^2 serves as a measure of goodness of prediction.

Grunfeld and Griliches' measures of performance was based on comparable R^2 values for the aggregate series. The R^2 for the aggregate model (Model III) is given directly. However, a "composite" R^2 is calculated for the aggregate performance of the individual functions[7] (Model I). Following Grunfeld and Griliches, the model with the higher R^2 is judged to be superior.

Similarly, appropriate measures of R^2 could provide initial indication of relative performance at the individual farm level. R^2 is computed directly for Model II. For Model I, a "comparable" R^2 has to be calculated.[8] Again, higher R^2 values may be rated as measures of superior performance.

THE SAMPLE AND THE VARIABLES

The alternative production functions specified in the first section were applied in an empirical study of six Sudanese poultry farms over a seven-year period, covering the years 1961 through 1967. The records of these farms give a complete account of each farm's monthly business expenditure and monthly receipts. In addition, actual amounts of physical outputs produced and inputs consumed within a particular month are given. Changes in capital stock inventory can be determined from the records. Also, changes over time in the value of capital items are indicated.

Monthly data[9] have been used in the computation of the production functions. In each particular production function computed, output was defined as a function of five inputs. Output consists of monthly income from egg sales plus or minus monthly changes in poultry stock inventory, all in constant pounds. The five inputs are : monthly pounds of mash, monthly labour in man-days, estimated monthly capital services in constant pounds and monthly operating cost in constant pounds. Capital services were defined as annual changes in building and equipment inventory divided by 12 after the necessary monthly adjustments were made for purchases during the year, plus cost of minor repairs. Operating cost consists of expenditures on electricity, transportations, insecticides, medicines and so on. In addition to these five inputs, seasonality and time were introduced in the functions as explanatory variables. Seasonality or monthly effect was defined as a dummy variable. Time was introduced as a continuous variable.

Various functions were computed from the sample observations. A production function has been derived for each of the six farms in the sample separa-

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7. For details see Grunfeld and Griliches, *loc. cit.* Model II might be included in this comparison. However, major interest here is in the Grunfeld-Griliches question which makes comparisons of Models I and III relevant.

8. Model III might also be evaluated in explaining or predicting output at the individual firm level. However, there have been no serious proposals in the literature that aggregate functions be applied directly to individual farms. Such use of aggregate functions appears unwarranted without extensive testing.

9. Monthly data per farm provides sufficient number of observations to allow the parameters in the functions to be estimated with a larger number of degrees of freedom. Hence, the observed parameters should be more reliable. Moreover, monthly data may reflect the production process more accurately if there are significant monthly and seasonal influences.

tely. These individual functions were computed from monthly observations for the period 1961-1966. Observations beyond 1966 were left to evaluate prediction outside the sample range. A production function for the pooled data of the six farms for the period 1961-1966 was estimated. Then, an aggregate production function was computed for the sample for the period 1961-1966.

RELATIVE PERFORMANCE OF THE ALTERNATIVE MODELS

The rest of this study evaluates the relative performance of the relevant models in explanation and prediction of output behaviour of individual farms or aggregate farms. Specifically, this study evaluates the relative performance of the alternative specifications for : (1) explanation of output variation at the individual farm level, (2) explanation of aggregate output variation, (3) prediction of output at the individual farm level from given inputs, and (4) prediction of aggregate output from given inputs.

The relative performance of the models is evaluated by the criteria developed earlier.

(*a*) *Explanation of Output Variation*

Individual farm level : Residuals from each of the six individual farm production functions (Model I) were obtained for each month and each farm separately. These were squared and summed over the six farms for the period 1961-1966. This sum of squared residuals was divided by the sum of squared deviations of the pooled dependent variable of the six farms about its mean for the period 1961-1966. The result was subtracted from one to give an R^2 in value (0.9138) comparable to the multiple correlation coefficient obtained directly from the production functions of the pooled data (0.9012). The R^2 coefficient of Model II is smaller than the individual production functions. This result implies that individual farm production functions (Model I) explain output variation of the individual farms better than does the production function of the pooled data (Model II).

The R^2 values are suggestive of the relative performance of the models. The question now arises whether Model I provides an explanation of output variation which is "better" in a statistical sense than that from Model II, that is, the multiple correlation coefficient from Model I significantly higher than from Model II ? This is equivalent to test whether the differences between the sum of squared residuals from the two models are significantly different. Hence, squared residuals were computed for each farm at each month, using both the parameters derived from the particular individual farm production functions (Model I) and those computed from the production function of the six farms pooled data (Model II). These squared residuals were summed over the sample period for each farm separately. Then, the differences between the sum of squared residuals of the two models were calculated for each farm separately. The differences were then ranked according to their absolute value. After the ranking is completed, the sign of the difference was attached to the ranks and the Wilcoxon Matched Pairs Signed Rank test[10] was used to determine whether significant differences exist between

10 A short discussion of this test is given in D. Owen: Handbook of Statistical Tables, Addison-Wesley, 1962, p. 325.

the sum of squared residuals of the two models. The sum of the positive ranks was computed and used as a test statistic. The results of these computations are given in Table I.

TABLE I—WILCOXON TEST FOR THE DIFFERENCES AMONG THE SUM OF SQUARED RESIDUALS OF MODELS I AND II

Hypothesis	S.S.R. Model I = S.S.R. Model II
Sum of Positive Ranks	21
Critical Probability	0·950
Computed Probability	1·000
Result	Reject hypothesis

The sums of squared residuals computed from the parameters derived from Model I are lower than the corresponding ones computed from Model II. These differences were found to be statistically significant at the one per cent level. This result implies that Model I, on the average, yields smaller sums of squared residuals and hence a higher multiple correlation coefficient. Thus, by the criteria suggested earlier, Model I describes the output variation of the individual farms in the sample better than does Model II.

Aggregate level : The relative performance of Models I and III in describing the aggregate behaviour of the farms in the sample was explored in a similar manner. A "composite" R^2 value of the six individual farms (Model I) in explaining aggregate output was computed in the manner of Grunfeld-Griliches. This value corresponds to the R^2 value from the aggregate production function of Model III. Residuals of each of the six individual farm production functions were obtained for each farm and each month separately, using parameters derived from Model I. These were summed over farms at each month and squared. Then these squared residuals were summed over the sample period. This sum of squared residuals was divided by the sum of squared deviation of the aggregate dependent variable about its mean and the result subtracted from one to give a "composite" R^2, (0.9108), corresponding to the one obtained from the aggregate production function (0.9004).

This R^2 coefficient for Model I is larger than that obtained for Model III. Such a result implies that the individual farm production functions (Model I) explain the aggregate behaviour of the farms in the sample better than the aggregate production function (Model III). Griliches and Grunfeld found empirical cases where Model III could be preferred to Model I.

An analysis of the differences in R^2 value between Model I and Model III was made in terms of the Wilcoxon test. The results of these computations are given in Table II.

TABLE II—WILCOXON TEXTS FOR THE DIFFERENCES AMONG THE SUM OF SQUARED RESIDUALS OF MODELS I AND III

Hypothesis	S.S.R. Model I=S.S.R. Model III
Sum of Positive Ranks	21
Critical Probability	0·950
Computed Probability	1·00
Result	Reject hypothesis

The sums of squared residuals computed from the parameters derived from Model I are lower than the corresponding sums of squared residuals computed from the parameters derived from Model III. These differences were found to be statistically significant at the one per cent level. Thus, in this case, aggregate output is explained more fully by a disaggregated model than by an aggregated one.

(*b*) *Prediction of Output*

The relative performances of the three models are evaluated by attempting to predict output for the period 1967 which lies outside the sample period used in fitting the functions, 1961-1966. The results are discussed below.

Individual farm level : Residuals for each of the six farms were computed for the year 1967 for each month and each farm separately, using both the parameters derived for the particular individual farm production function (Model I) and then computed from the production function of the pooled data of the six farms (Model II). These residuals were squared for each month and each model separately. Then, these were summed over farms and months for each model separately. The sum of squared residuals from each model was divided by the sum of squared deviations of the dependent variable about its mean. Each of these results was subtracted from one to give comparable R^2 values 0.70816 for Model I and 0.66496 for Model II.

The R^2 coefficient from Model I is larger than that obtained from Model II. Such a result suggests that individual farm production functions (Model I) are better than production functions of the pooled data (Model II) in predicting output at the farm level.

An analysis of the differences between Models I and II in predicting output at the farm level was made in terms of the Wilcoxon test. The sum of squared residuals for each of the farms was computed from each of the parameters derived from Model I and Model II for 1967. The differences between the sum of squared residuals from the two models were calculated for each farm separately. The absolute differences were ranked and then each rank was given the sign of the difference. The sum of positive ranks was computed and used as a test statistic. The results of these computations are given in Table III.

TABLE III—WILCOXON TEST FOR THE DIFFERENCES AMONG THE SUM OF SQUARED RESIDUALS OF MODELS I AND II

Hypothesis	S.S.R. Model I = S.S.R. Model II
Sum of Positive Ranks	21
Critical Probability	0·95
Computed Probability	1·00
Result	Reject hypothesis

The differences between the two sets of sum of squared residuals were found to be statistically significant at the one per cent level. This result implies that Model I yields a significantly smaller sum of squared residuals and hence a significantly higher multiple correlation coefficient. Thus, Model I appears to be better in predicting output at the farm level.

Aggregate level : The relative performance of Models I and III in predicting output at the aggregate level was similarly examined. Residuals for each of the six farms were computed for 1967 for each month separately, using the parameters derived from the particular farm production function (Model I). These residuals were summed over the farms at each month. The sum was squared and added up over months for 1967. This sum of squared residuals was divided by the sum of squared deviations of the aggregate dependent variable about its mean for 1967. The result was subtracted from one to give a "composite" R^2 for the six individual farm production functions, 0.73853 in value. Then, residuals were calculated for each month for 1967, using the parameters derived from the aggregate production function (Model III). These residuals were squared at each month and summed over the entire period. Then the sum of squared residuals was divided by the sum of squared deviations of the aggregate dependent variable about its mean for 1967. The result was subtracted from one to yield an estimate of the multiple correlation coefficient of the aggregate production functions, 0.61326 in value.

The R^2 coefficient for Model I is larger than that obtained from Model III. Such a result suggests that individual farm production functions (Model I) are better than aggregate production functions (Model III) in predicting aggregate output.

An analysis of the differences between Model I and Model III in predicting aggregate output was made in terms of Wilcoxon test. Squared residuals were computed for each month separately from the parameters derived from both Model I and Model III. These were added up for each month and each model separately. The differences between the sum of squared residuals computed from the two models were calculated for each month separately. The absolute differences were ranked and then each rank was given the sign of the difference. The sum of the positive ranks was computed and used as a test statistic. The results of these computations are given in Table IV.

TABLE IV—WILCOXON TEST FOR THE DIFFERENCES AMONG THE SUM OF SQUARED RESIDUALS OF MODELS I AND III

Hypothesis	S.S.R. Model I = S.S.R. Model III
Sum of Positive Ranks	78
Critical Probability	0·950
Computed Probability	1.000
Result	Reject hypothesis

The differences between the two sets of sum of squared residuals were found to be statistically significant at the one per cent level. This result implies that Model I yields a smaller sum of squared residuals and hence a higher multiple correlation coefficient. Thus, Model I appears to be better than Model III in predicting output at the aggregate level.

CONCLUSIONS

Data on a set of farms consists of combined cross-section and time series observations. Such cross-section time series data offer a wide range of specific model formulations. Three major alternative regression models have been tested in this study and evaluated with respect to their ability to explain output variation within the sample period, and to predict output variation outside the sample period. The three models are : Model I—separate functions fitted to time series data for each farm, allowing different production elasticities for each farm; Model II—a function fitted to the combined cross-section time series data pooled for all farms; Model III—a production function fitted to the time series variables aggregated over all farms.

One major question examined was : can explanation and prediction of output at the individual farm level be significantly improved by fitting separate functions to each farm, allowing different elasticities among farms (Model I), as compared with functions employing constant elasticities among farms (Model II) ? This question was tested initially using "comparable" measure of R^2 at the individual farm level. The overall R^2 value from Model I was 0.9138 compared with 0.9012 from Model II. Further, the test of the difference between the sum of squared residuals for the two models showed that Model I explained a significantly larger proportion of the variation in output than Model II, when tested at the one per cent level. In terms of prediction outside the sample period, Model I again provided a higher "comparable" R^2 than Model II (0.70816 *versus* 0.66496). This difference was significant at the one per cent level. In summary, explanation and prediction of output variation at the individual farm levels is significantly improved by allowing different production elasticities for each farm. While statistically significant, the difference in R^2 values is not extremely large in an absolute sense. Thus, the added cost of the more detailed farm analysis must be weighted against added improvement in estimates. Of course, frequently, insufficient data on individual farms preclude analysis along the lines of Model I.

Another major question examined was : If interest is in explanation and prediction of aggregate output from a group of farms or industry, what is the gain, if any, from explaining or predicting output for each farm with a separate function and then aggregating, versus direct explanation or prediction of output from a single function fitted directly to aggregate data ? This analysis showed that the "composite" R^2 value from the former was 0.9108 *versus* 0.9004 for the latter within the sample period. In terms of prediction outside the sample period, Model I again provided a higher "comparable" R^2 than Model III (0.73853 *versus* 0.61326). These differences were found to be significant at the one per cent level. Thus, the conclusion is that disaggregation is likely to increase the degree of explanation and prediction of aggregate output. However, the differences are not extremely great from an absolute standpoint. Also, disaggregation is likely to be extremely costly in many practical cases because of lack of published data on individual farms. Thus, for many practical purposes, the degree of refinement in explanation and prediction from disaggregation at the farm level within a fairly homogeneous industry is not likely to be warranted.
