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Consumer Behavior Models: Time Allocation,
Consumer Assembly, and Outdoor Recreation

by

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CONSUMER BEHAVIOR MODELS: TIME ALLOCATION, CONSUMER ASSEMBLY, AND OUTDOOR RECREATION**

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I. INTRODUCTION

The concept of demand has evolved through the centuries, enriched by the natural-value, exchange-value controversy and the diamond, water paradox of the classicists (17), the intuitive insights of Marshall, (11) and the mathematical rigor of Slutsky (16) and Hicks and Allen (8). These developments led to the conceptualization of a demand function as a solution function to a constrained extremum problem (7, 8, 14, 16).

This contemporary theory seems to apply rather well to many textbook examples. Commodities to which it seems particularly inapplicable include those that require a high degree of consumer assembly (i.e., may not be purchased in a simple package) and those that entail the expenditure of blocks of time. Leisure activity is a class of commodities possessing such difficulties.

The famous letter of Professor Hotelling (9) and the "Clawson Model" (3) were apparent attempts at applying contemporary theory to commodities with a high degree of consumer assembly and signifi-

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cant time requirements. Both suggested the use of travel distance or distance of the facility from the residence of the consumer as a surrogate for recreation prices.

Burt and Brewer (2) have carried forth this suggestion by generating a method of empirically computing direct recreational benefits. Burt and Brewer computed consumer's surplus by using distance to the recreational site from the residence as a surrogate for the price of a visit.

Pearse (13) has made use of time expenditure in the form of a time travel cost. Milam and Pasour (12) have investigated the influence of opportunity time costs on the demand for recreation. It is not obvious that contemporary theory suggests any such measures even though they may seem plausible.

This paper presents a comparative summary of several extensions of contemporary theory that investigate the restrictions imposed by available time and the assembly of commodities from time and goods.

II. NAIVE MODELS

A. Contemporary Theory

Contemporary consumer theory assumes the maximization of a strictly quasi-concave utility function subject to a linear budget constraint (7, 8, 14,16) and that goods are purchased with prices and income determined exogeneously. In symbols,

$$\text{Maximize} \qquad U = U(x_1, \ldots, x_n)$$

$$\text{Subject to} \qquad p_1x_1 + p_2x_2 + \ldots + p_nx_n = I$$

The $x_1, \ldots, x_n$ are regarded as positive flows of commodities[1/] and the prices $p_1, \ldots, p_n$ and income I are non-negative. In case certain mathematical conditions (14) hold[2/], the results presented are that the demand functions implied by the first order conditions for utility maximization are single valued, differentiable and homogenous of order zero in all prices and income. In addition, the change in each good with respect to a compensated change in its own price (substitution effect), is negative for all (compensated) price changes in a neighborhood of the price-income point under consideration. It is apparent that implied hypotheses about time allocation and consumer assembly do not arise from such a model.

B. Adam in Eden

The Judeo-Christian tradition has provided us with a description of sorts of the compleat outdoor recreationist. It seems that Adam was surrounded by vast abundance of "fruits of nature" in the Garden of Eden and was commissioned to utilize them as he saw fit, with one well known exception. Since there was no scarcity in the Garden of Eden and he was alone there was no exchange problem.

It is apparent that Adam's days were of limited length and that he as with most of us today could experience only a limited number of the "fruits of nature" at a time. If we suppose that Adam had a strictly quasi-concave utility function with arguments as quantities of "fruits of nature"; that he could enjoy "fruits of nature" one at a time; and that he maximized utility each day subject to the exhaustion of available time, we could express Adam's choice problem as follows:

$$\text{Maximize} \qquad U = U(x_1,\ldots,x_n)$$
$$\text{Subject to} \qquad t_1x_1 + t_2x_2 + \ldots + t_nx_n = \tau.$$

the $x_1,\ldots,x_n$ in this case are non-negative quantities of "fruits of nature" and the per unit time requirements $t_1,\ldots,t_n$ and length of day τ are non-negative and given.[3/] The implications of such a model in terms of demands for "fruits of nature" are identical to those for goods in the contemporary model except that time parameters have assumed the allocative role of money parameters.

C. Time and Money Allocation with Fixed Proportions

After the creation of Eve, barter arrangements came about, increasing in incidence as commerce developed after the expulsion from Eden. The descendents of Adam and Eve, however, engage in the allocation of time.

By defining activities as combinations of time and goods for consumption as a unit and assuming that participation in all activities could be obtained for a fee, the choice problem of a typical individual could be specified as

$$\text{Maximize} \qquad U = U(x_1,\ldots,x_n)$$

Subject to $\sum_{i=1}^{n} p_i x_i = I$

$$\sum_{i=1}^{n} t_i x_i = \tau$$

where $x_1,\ldots,x_n$ are positive quantities of consumption activities, the p_i are prices or fees paid to participate or wages received for participating in activity i, I is a residual wealth parameter, t_i is a parameter representing the units of time required to produce one unit of x_i, τ is the length of planning period. Note that each $t_i > 0$ because x_i is an activity. Note also that a switch has been made in the definition of τ. τ as the length of planning period can vary as the needs of the planner vary. τ as the length of a calendar period cannot be treated as an ordinary parameter and can never enter into an empirical model because it can never be altered, but will only be reflected in constants. On the other hand, τ as a planning period length can enter as a variable under some conditions.[4/] Finally, note that the definition of τ, whether a fixed or changeable parameter may be altered at the whim of the modeler to suit his needs

The behavioral theorems from the model with a time and money constraint were first deduced by Graaf (5), Samuelson (14), and Scitovski (15) in the context of a theory of point rationing and applied to time allocation by Wilson (18). The theorems deducible from this model include both those obtained from contemporary theory and those obtained from the model of Adam in Eden. An additional result is that cross substitution effects in terms of time have the same algebraic sign as those in terms of money and vice versa. Finally, the substitution term for an activity with respect to its own money or time price cannot be absolutely greater when both constraints are effective than when only one constraint is effective and the other is relaxed.[5]

III. MORE REFLECTIVE MODELS

A review of "naive" consumer models has been useful in focusing upon time allocation. However, it will be useful to pursue more comprehensive models that may be more reflective of the decision processes of a consumer. In the present section refinement of the naive models will be made through a generalization of the constraints.

A. Time and Money Allocation with Variable Proportions

The linear time constraint with fixed coefficients in previous models may be altered to allow both fixed and variable time proportions in the production of activities.[6] Furthermore, relationships associated with certain parameters in the implicit production function may be derived and interpreted. The specification is as follows:

$$\text{Maximize} \qquad U = U(x_1, \ldots, x_n) \qquad 1)$$

Subject to $$F(z_1,\ldots,z_n,y_1,\ldots,y_m,v_1,\ldots,v_n,s_1,\ldots,s_r) = 0 \qquad 2)$$

$$x_i = w_i + z_i,\ i = 1,\ldots,n \qquad 3)$$

$$\sum_{i=1}^{n} q_i w_i + \sum_{i=1}^{m} p_i y_i = I \qquad 4)$$

$$\sum_{i=1}^{n} t_i w_i + \sum_{i=1} v_i = \tau \qquad 5)$$

where $x_1,\ldots,x_n$ are work and consumption activities with $w_1,\ldots,w_n$ purchased and $z_1,\ldots,z_n$ produced; $y_1,\ldots,y_m$ are goods; $q_1,\ldots,q_n$ and $p_i,\ldots,p_m$ are exogenous prices of purchased activities and input goods respectively; $s_1,\ldots,s_r$ are exogenous production parameters; $t_1,\ldots,t_n$ are exogenous time coefficients for purchased activities; $v_1,\ldots,v_n$ are variable non-negative endogenous time inputs for the produced activities and I and τ are as defined previously. The constraint $F = 0$ is an implicit strictly concave production function, and the utility function U is strictly quasi-concave. All functions possess continuous first and second order partial derivatives by assumption.

By substituting constraint 3) into the objective function 1), a new objective function may be written as

$$U = U(z_1 + w_1,\ z_2 + w_2,\ldots,z_n + w_n) \qquad 6)$$

The consumer allocation problem is then characterized by maximizing objective 6) subject to the constraints 2), 4), and 5).

By differentiating constraints 2), 4) and 5) partially with respect to the variables, it may be seen that the Lagrange rank condition holds if at least one activity is producible from goods and time and that at least one of the goods used in producing the activity is not free. It seems most reasonable to make that assumption.

The following Lagrange expression may be specified:

$$L = U(w_1 + z_1, \ldots, w_n + z_n) + \lambda F(z_1, \ldots, z_n, v_1, \ldots, v_n, y_1, \ldots, y_m, s_1, \ldots s_r)$$

$$\gamma(I - \sum_1^n w_i q_i - \sum_1 y_i p_i) + \delta(\tau - \sum_1^n w_i t_i - \sum_1^n v_i)$$

The first-order Lagrange conditions are as follows:

$$\frac{\partial L}{\partial w_i} = U_i^x - \gamma q_i - \delta t_i = 0$$

$$i = 1, \ldots, n$$

$$\frac{\partial L}{\partial z_i} = U_i^x + \lambda F_i^z = 0$$

$$i = 1, \ldots, n$$

$$\frac{\partial L}{\partial v_i} = \lambda F_i^v - \delta = 0$$

$$i = 1, \ldots, n$$

$$\frac{\partial L}{\partial y_i} = \lambda F_i^y - \gamma p_i = 0 \qquad 7)$$

$$i = 1, \ldots, m$$

$$\frac{\partial L}{\partial \lambda} = F(z_1, \ldots, z_n, v_1, \ldots, v_n, y_1, \ldots, y_m, s_1, \ldots, s_r) = 0$$

$$\frac{\partial L}{\partial \gamma} = I - \sum_1^n w_i q_i - \sum_1^m y_i p_i = 0$$

$$\frac{\partial L}{\partial \delta} = \tau - \sum_1^n w_i t_i - \sum_1^n v_i = 0$$

Under appropriate conditions[7] the first-order conditions 7) may be solved for each of the variables $z_1,\ldots z_n, w_1,\ldots w_n, y_1,\ldots y_m, v_1,\ldots, v_n$, λ, γ, and δ, as locally differentiable functions of the parameters s_1, $\ldots, s_r, q_1,\ldots, q_n, p_1,\ldots p_m, I, t_i,\ldots,$ tn and τ.[8] The functions are all homogeneous of degree zero in the money parameters $q_1,\ldots, q_n, p_1$, $\ldots, p_m$, and I. However, all of them cannot be homogeneous of degree zero in the time parameters because of the form of equation 5). Equation 5) will not necessarily preserve its equality with a proportional change in $t_1,\ldots,t_n$, and τ (14).[9]

Certain points should be noted about the functions obtained from the first order conditions. The produced activities do not possess market prices, but their demand functions are well defined[10] and depend on the other prices and parameters. Furthermore, the prices in the system are all attached to either inputs or purchased activities. Time, as a variable factor input in the production of an activity, behaves as a good in that a demand function for its use in each activity is deduced. However, the different time demands do not have associated market prices.

The production parameters $s_1,\ldots,s_r$ have an interesting interpretation in the case of outdoor recreation. Amongst these parameters are included such items as the minimum distance that must be traveled if a particular recreation site is to be visited (an activity). The actual travel distance is an activity jointly demanded with the site visit. Also included as parameters would be the minimum required values of travel time, total travel expenditure, total time expenditure, total expenditure, outfitting expenditure, etc. Such production parameters are obviously exogenous, but the actual levels chosen in the allocation process for these items are either endogenous activities or activity total costs, as the case may be. Neither the production parameters nor the values of related activities nor their total costs in money or time would appear to be surrogates for prices for produced activities on theoretical grounds.

By partially differentiating the first order conditions 7) with respect to the parameters, one at a time, it is possible to solve the resulting sets of second-order partial derivatives of the Lagrange function for the rates of change of each of the variables with respect to each of the parameters.[11] The following rates of change are obtained:

$$\frac{\partial d_i}{\partial s_k} = \frac{\lambda}{\Theta}\left[\sum_{h_1=1}^{n} F^{z\ s}_{h_1 k}\Theta_{j,i} + \sum_{h_2=1}^{m} F^{y\ s}_{h_2 k}\Theta_{j,i} + \sum_{h_3=1}^{n} F^{v\ s}_{h_3 k}\Theta_{j,i}\right] \qquad 8)$$

$$+ \frac{F^{s}_{k}\Theta_{3n+m+1,i}}{\Theta}, \quad k=1,\ldots,r$$

$$\frac{\partial d_i}{\partial q_k} = \gamma\frac{\Theta_{k,i}}{\Theta} + \frac{w_k\Theta_{3n+m+2,i}}{\Theta} \qquad 9)$$

$$k=1,\ldots,n$$

$$\frac{\partial d_i}{\partial p_k} = \gamma\frac{\Theta_{k,i}}{\Theta} + y_k\frac{\Theta_{3n+m+2,i}}{\Theta} \qquad 10)$$

$$k=1,\ldots,m$$

$$\frac{\partial d_i}{\partial t_k} = \delta \frac{\Theta_{k,i}}{\Theta} + w_k \frac{\Theta_{3n+m+3,i}}{\Theta} \tag{11}$$

$$k=1,\ldots,n$$

$$\frac{\partial d_i}{\partial I} = \frac{\Theta_{3n+m+2,i}}{\Theta} \tag{12}$$

$$\frac{\partial d_i}{\partial \tau} = \frac{\Theta_{3n+m+3,i}}{\Theta} \tag{13}$$

for $i = 1,2,\ldots,3n+m+3$

$h_1 = 1,\ldots,n$

$h_2 = 1,\ldots,m$

$h_3 = 1,\ldots,n$

$j = n+h_1,\ 2n+h_2,\ 2n+m+h_3;$

where

$$d_i = \begin{cases} w_i, & \text{for } i = 1,\ldots,n \\ z_j, & \text{for } i = n+1,\ldots,n+j,\ldots,2n \\ y_j, & \text{for } i = 2n+1,\ldots,2n+j,\ldots,2n+m \\ v_j, & \text{for } i = 2n+m+1,\ldots,2n+m+j,\ldots,3n+m \\ \lambda, & \text{for } i = 3n+m+1 \\ \gamma, & \text{for } i = 3n+m+2 \\ \delta, & \text{for } i = 3n+m+3 \end{cases}$$

and $\Theta_{j,i}$ is the cofactor of the j^{th} row and the i^{th} column of the bordered Hessian matrix Φ.[12/] From the second-order conditions[13/] it can be deduced that

$$\frac{\Theta_{j,i}}{\Theta} \begin{cases} < 0 \text{ for } i=j \\ \text{of undetermined sign for } i \neq j. \end{cases}$$

By substituting 12) and 13) into 9), 10) and 11), it is obtained that

$$\frac{\partial d_i}{\partial q_k} = \gamma \frac{\Theta_{k,i}}{\Theta} + w_k \frac{\partial d_i}{\partial I} \qquad 14)$$

$$k=1,\ldots,n$$

$$\frac{\partial d_i}{\partial p_k} = \gamma \frac{\Theta_{k,i}}{\Theta} + y_k \frac{\partial d_i}{\partial I} \qquad 15)$$

$$k=1,\ldots,m$$

$$\frac{\partial d_i}{\partial t_k} = \delta \frac{\Theta_{k,i}}{\Theta} + w_k \frac{\partial d_i}{\partial \tau} \qquad 16)$$

$$k=1,\ldots,n.$$

$$i=3n+m+3.$$

By application of the second-order conditions [13/] to 14), 15) and 16) and rearranging,

$$H_{i,i} = \gamma \frac{\Theta_{i,i}}{\Theta} = \frac{\partial w_i}{\partial q_i} - w_i \frac{\partial w_i}{\partial I} < 0 \qquad 17)$$

$$i=1,\ldots,n$$

$$J_{i,i} = \gamma \frac{\Theta_{j,i}}{\Theta} = \frac{\partial y_i}{\partial p_y} - y_j \frac{\partial y_j}{\partial I} < 0 \qquad 18)$$

$$i=2n+j$$

$$j=1,\ldots,m$$

$$K_{i,i} = \delta \frac{\Theta_{i,i}}{\Theta} = \frac{\partial w_i}{\partial t_i} - w_i \frac{\partial w_i}{\partial \tau} < 0 \qquad 19)$$

$$i=1,\ldots,n$$

$$H_{k,i} = \gamma\frac{\Theta_{k,i}}{\Theta} = \frac{\partial d_i}{\partial q_k} - w_k\frac{\partial d_i}{\partial I} \lesseqgtr 0 \qquad 20)$$

$i \neq k$

$i=1,\ldots,3n+m+3$

$k=1,\ldots,n$

$$J_{j,i} = \gamma\frac{\Theta_{j,i}}{\Theta} = \frac{\partial d_i}{\partial p_j} - y_j\frac{\partial d_i}{\partial I} \lesseqgtr 0 \qquad 21)$$

$i \neq 2n+j$

$j=1,\ldots,m$

$i=1,\ldots,3n+m+3$

$$K_{k,i} = \delta\frac{\Theta_{k,i}}{\Theta} = \frac{\partial d_i}{\partial t_k} - w_k\frac{\partial d_i}{\partial \tau} \lesseqgtr 0 \qquad 22)$$

$i \neq k$

$i=1,\ldots,3n+n+3$

$k=1,\ldots,n$

It can be shown that by minimizing $\Sigma q_i w_i + \Sigma p_i y_i - I$ subject to $U(x_1,\ldots,x_n) - \bar{U}$ and constraints 2), 3) and 5) that $H_{j,i} = \frac{\partial d_i}{\partial q_i}$ and $J_{j,i} = \frac{\partial d_i}{\partial p_j}$. Stated differently this says that $H_{j,i} = \frac{\partial d_i}{\partial q_j}$ and $J_{j,i} = \frac{\partial d_i}{\partial p_j}$ with utility held at a constant level and the other constraints satisfied. A similar interpretation may be derived for $K_{h,i}$. The notation $H_{j,i} = (\frac{\partial d_i}{\partial q_j})_c$, $J_{j,i} = (\frac{\partial d_i}{\partial p_j})_c$ and $K_{k,i} = (\frac{\partial d_i}{\partial t_k})_c$ for the compensated rates of change will be adopted.

The compensated rates of change of variables with respect to their own money and time parameters 17), 18), 19) provide a set of hypotheses to be tested in empirical demand investigations. Uncompensated rates of change of variables with respect to their own prices (cf.14), 15), and 16)) may be positive, zero or negative depending on the magnitude and direction of residual wealth effects. $w_k \frac{\partial d_i}{\partial I}$ or $y_k \frac{\partial d_i}{\partial I}$ and time effects $w_k \frac{\partial d_i}{\partial \tau}$.

Hypotheses about compensated or uncompensated responses to other parameter changes may not be deduced. The direction of movement with respect to other time and money parameters cannot be deduced without additional specification of the utility and/or production functions. Directions of response with respect to production parameters 8), may be deduced via more restrictive specification of the production function F.

In addition, it is not generally possible to deduce the compensated or uncompensated rates of change in the total activities x_i with respect to changes in any of the parameters. That is

$$\frac{\partial x_i}{\partial \rho_j} = \frac{\partial w_i}{\partial \rho_j} + \frac{\partial z_i}{\partial \rho_j} \lesseqgtr 0$$

for $i=1,\ldots,n$

$j=1,\ldots,2n+m+2$,

where ρ is any of the parameters in the problem.

In particular,

$$\frac{\partial x_i}{\partial q_i} = \gamma \frac{\Theta_{i,i}}{\Theta} + w_i \frac{\partial w_i}{\partial I} + \gamma \frac{\Theta_{k,i}}{\Theta} + w_i \frac{\partial z_k}{\partial I} \qquad 23)$$

for $i=1,\ldots,n$ and $k=n+i$. Rewriting 23) as

$$\gamma\left[\frac{\Theta_{i,i}}{\Theta} + \frac{\Theta_{k,i}}{\Theta}\right] = \frac{\partial x_i}{\partial q_i} - w_i\left[\frac{\partial w_i}{\partial I} + \frac{\partial z_h}{\partial I}\right] \qquad 24)$$

it is not possible to deduce the algebraic sign of $\left(\frac{\partial x_i}{\partial q_i}\right)_c$ because the sign of $\frac{\Theta_{k,i}}{\Theta}$ in 24) is unknown. By a similar argument, the directions of $\frac{\partial x_i}{\partial t_i}$ and $\left(\frac{\partial x_i}{\partial t_i}\right)_c$ are unknown.

Knowledge of the production function F should allow derivation of certain of the rates of change in produced activities z_i, goods y_i, and variable time v_i with respect to the own price of goods p_i. For example, if $\frac{\partial z_i}{\partial y_j} > 0$ then $\frac{\partial z_i}{\partial p_j} = \frac{\partial z_i}{\partial y_j} \frac{\partial y_j}{\partial p_j}$ and if $\frac{\partial y_j}{\partial p_j} < 0$ then $\frac{\partial z_i}{\partial p_j} < 0$.

Thus, hypotheses about the system of demand functions are more completely developed than in models discussed previously. In case a new recreation facility does not provide the capability for new activities the facility effects only the constraints in the problem in known ways and does not disturb the utility relationship. Changes in demand parameters for goods and time inputs can be deduced from the changes in the production function. Directions of changes in activities are not deducible. All other theorems attainable from the fixed proportions model are also deducible for variable proportions.

It should be mentioned that with produced activities such as recreation, the activity quantities may be measured in amounts of time spent. In such circumstances, the fixed time parameters will be equal to 1 and the variable time for such an activity will be identical to the quantity of activity. For activities measured in time units, demand functions for associated time inputs will be redundant.

B. Psychological Goods

Intrinsic complementarity, substitution, and independence among activities may be deduced by employing a refinement of the variable proportions model. Lancaster (10) first developed the psychological goods model in a linear activity analysis context that was generalized by Wilson (18). The psychological goods, called characteristics of consumption, are arguments of a strictly quasi-concave utility function and produced from activities. The choice problem is specified as follows:

Maximize $U = U(u_1,\ldots,u_s)$ 25)

Subject to $G(u_1,\ldots,u_s,x_1,\ldots x_n) = 0$ 26)

and 2), 3), 4) and 5), where $u_1,\ldots,u_s$ are characteristics of consumption; $x_1,\ldots,x_n$ are work and consumption activities; and G is an implicit strictly concave production function transforming activities into characteristics.

The solutions to this choice problem are demand functions for each of the characteristics of consumption, work and consumption activities, goods and variable time inputs that each depend on the money, time and production parameters. The usual Slutsky equations and substitution effects are obtainable.

In case both F and G are known, the complementarity, substitutability and independence relations are obtainable for all activities, all goods, and all variable time inputs. The effects of the introduction of new activities and new goods (facilities) are also deducible.

C. The Becker Model

Becker (1) has developed a theory which is similar to the model with variable time proportions. In one specification he was able to impute the money price for time used in consumption as the wage rate. This was accomplished through several key assumptions, including homogeneity of time within types, fixed amounts of time at work and a fixed wage rate for each time type, substitutability among alternative time uses, linear activity production functions, and independence of utility from work. It would seem plausible to conjecture that such a measure might be an upper bound for the price of time in some circumstances and a lower bound in others.

Becker points out the heroism in his own assumptions and generalizes his model, but retains all except the fixed work time and fixed wage rate assumption. For that he substitutes a "full" or potential income concept obtained by maximizing earnings, subject to expenditure, time, and production function constraints and independent of utility. Money costs of diverting time and goods from pursuit of income to pursuit of utility are then calculable as foregone income, provided that full income is measurable and actual income is a known function of consumption activities. The foregone income overlooked by the contemporary theory contains both cost of goods and cost of time components. Becker's model implies marginal costs for goods and time that could be used if measurable in the computation of opportunity costs for time. The marginal costs, however, are not parameters in general and vary with commodity bundles.

D. Example

Suppose that the typical consumer has available to him three activities, working x_1, dining x_2, and recreation x_3. He may obtain recreation in either of two ways; by the purchase of a fixed recreation package w_3, or by production of recreation, utilizing variable amounts of a recreation facility y and time v. The production parameter d might represent distance to the facility. The consumer's choice problem is characterized as follows:

$$\text{Maximize} \qquad U = U(x_1,x_2,x_3)$$

$$\text{Subject to} \qquad x_3 = w_3 + z_3$$

$$z_3 = av^2 + by^2 - cvy - d$$

$$x_1p_1 + x_2p_2 + w_3p_3 + yp_y = 0$$

$$x_1t_1 + x_2t_2 + w_3t_3 + v = \tau$$

where it is assumed that $p_y = 0$.

The Lagrangean function

$$L = U(x_1,x_2,w_3 + z_3) + \lambda(av^2 + by^2 - cvy - d - z_3)$$

$$+ \lambda(p_1x_1 + p_2x_2 + p_3w_3 + p_yy)$$

$$+ \delta(t_1x_1 + t_2x_2 + t_32_3 + v - \tau)$$

yields first order conditions for a relative constrained maximum of U which, under cettain conditions, may be solved for x_1,x_2, w_3, z_3, y, v, λ, γ, δ, in terms of the parameters a, b, c, d, p_1, p_2, p_3, p_y, t_1, t_2, t_3, and τ. The solutions may be expressed as:

$$v_i = h^i(a,b,c,d,p_1,p_2,p_3,p_y,t_1,t_2,t_3,\tau), \text{ for } i = 1,\ldots,9.$$

The demand (supply) functions h^i are each differentiable, unique and homogeneous of degree zero in the prices p_1, p_2, p_3, and p_y provided that U is strictly quasi-concave. They are not homogeneous in a, b, c, and d, nor t_1, t_2, t_3, and τ.

The demand function for x_3 is $h^3 + h^4$. Its rate of change cannot be deduced for compensated changes in p_3 or p_y.

The sign and magnitude of certain of the compensated and uncompensated rates of change in demand can be deduced.

IV. DISCUSSION AND CONCLUSIONS

Consumer behavior theories have been summarized and their most relevant implications pointed out. The variable proportions time allocation model lucidly describes the manner in which activities, goods and variable time inputs are related to prices and other known money, time, and production parameters. It has intuitive appeal as a decision framework representing consumers of outdoor recreation.

There should be little doubt concerning the meaning of a demand function for a produced activity. Such demand functions are well defined whether or not the activities or goods each have money prices that can be nonzero. The demand functions have as arguments all parameters in the problem.

If an activity is both purchased and produced the price of the activity as purchased does not hold an equivalent relationship to the activity as produced and to the total of purchased and produced. This is evidenced by the indeterminateness in the response of the produced activity and consequently, total activity to a change in the purchase price. Thus, purchase price may be no surrogate for a money price for a produced activity. Similar statements may be made about time parameters.

Recreational facilities are themselves physical inputs for which a derived demand function is obtainable. In the event that the facilities are public goods they are often accorded zero prices by fiat. The application of contemporary theory to recreational problems has led to a lack of appreciation for the distinct roles of facility inputs and activity outputs. Indeed, none of the models provides insight into possible surrogates for prices for the use of non-priced recreational facilities or activities.[14/]

It has been suggested for many years (9), and again recently (2) that a proper surrogate for the price of a recreational facility (input) or facility visit (activity) paid by a visitor might be the distance from the residence of the visitor to the recreational site. Confusion exists, of course, as to whether this distance should be accorded as a price to the visit or to the facility. The variable proportions time allocation model puts this problem in focus. The distance from the residence to the recreational site is a parameter in the production of activities from a facility. As such, it is a parameter in the consumer's demand function both for the facility and for activities associated with it.

There is no evidence that distance is properly a surrogate for price except that as distance diminishes, one would expect both the amounts of activities and facility use to increase via time substitution. The distance parameter may be viewed as a lower bound for recreational travel, an activity demanded jointly with activities at each recreational site. Travel cost is the total cost of the recreational travel activity.

Distance, recreational travel, and travel cost are <u>not</u> prices. The use of production parameters, activity quantities, or total costs of activities as surrogates for prices would appear to lack economic justification. The cost per unit of recreational travel would, however, be one element in the vector of prices associated with recreational activity.

Samuelson (14) has pointed out that consumer's surplus as a tool for the measurement of welfare is both superfluous to the analysis and expressible in at least a half a dozen mutually inconsistent forms in contemporary theory. Burt and Brewer (2), on the other hand, accept these shortcomings and point to the usefulness of such a measure. It appears that such positions are justified for commodities for which contemporary theory appears adequate. Such commodities are purchased rather than produced, have prices with a nonzero range, and have minimal time allocation effects. At present there has not been developed a companion consumer's surplus theory for the variable-proportions time-allocation demand theory. Therefore, any relationship of the quantities computed by Burt and Brewer (2) to utility changes is unknown and, furthermore, may be coincidental.

The point cannot be overemphasized. The computation of recreational benefits as consumer's surplus by using distance or total travel cost as a price may have been intuitively appealing to Hotelling (9), Clawson (3) and Burt and Brewer (2) but its meaning is at best nebulous and at worst, nonsense. Such measures were suggested before a sufficiently reflective-demand theory was developed, and now appear spurious. With an appropriate demand theory at hand it is now apparent that there is no companion theory of consumer's surplus for produced activities. At such time as economic theory provides a consumer's surplus framework for produced activities, the benefits question may be settled.

Opportunity costs for time used in recreation have been observed to be a significant factor in demand determination by Milam and Pasour (12) and fractions of the wage rate have been used as values for time in recreation by Pearce (13). Becker's simplified model (1) suggested the wage-rate as a value for time in consumption. His generalized version suggests another value, the marginal foregone earnings from using time in consumption, that depends on many things in addition to the wage rate.

The wage rate as a measure of time value may be too high when gainful alternative opportunities yield less than the wage and too low when alternative uses for time yield more than the wage. For example, if a consumer substitutes an afternoon of golf for an afternoon of work at a loss of that afternoon's pay then the golf was more valuable than the work in terms of money (12). However, if the consumer did not lose the pay and enjoyed the golf as well, the value of time is not revealed. If the consumer had the afternoon off and had no gainful opportunity for its use, the value of golf could not be assessed, but would certainly be less than the wage.

At best, it can be said that there is no rigorous method that has ready application to the evaluation of time in consumption. Becker's marginal foregone earnings approach may be the best alternative if it can be measured. The time valuation problem is of much importance because much of the value attributable to recreation likely is incorporated in the time spent.

FOOTNOTES

1/ In case non-negativity is assumed, the existence of solutions may always be mathematically assured by using the Kuhn-Tucker Theorems.

2/ Slightly weaker mathematical conditions than Samuelson's are strict quasi-concavity of the utility function, and continuous first and second order partial derivatives of the utility and constraint functions.

3/ Others, notably Becker (1), might assume several kinds of time. This assumption would only replicate the time constraint for each kind: for example; daytime, night-time, weekday, weekend, holiday, etc.

4/ The utility function is defined for a specified planning horizon τ. Changing τ implies changing U so that questions as to changes in the optimum implied by changing τ may not be well posed.

5/ This is the famous Le Chetalier-Braun Principle.

6/ The assumption that activities are consumed one at a time is retained.

7/ The bordered Hessian determinant θ of the first-order conditions must not vanish. The second order conditions usually applied are as follows (Hancock (6, p. 115)):

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \phi_{ij} h_i h_j < 0$$

for all vectors $h \neq 0$ such that

$$\sum_{i=1}^{n} g_i h_i + \sum_{i=2n+1}^{2n+m} p_i h_i = 0$$

$$\sum_{i=1}^{n} t_i h_i + \sum_{i=2n+m+1}^{N} h_i = 0$$

$$\sum_{i=n+1}^{N} F_i^d h_i = 0$$

Where $N = 3n + m$ and ϕ_{ij} is the element in the i^{th} row and j^{th} column of the matrix Φ and

$$\Phi = \left[\begin{array}{cccc|ccc}
\underset{n\times n}{[U_{ij}]} & \underset{n\times n}{[U_{ij}]} & \underset{n\times m}{[0]} & \underset{n\times n}{[0]} & \underset{n\times 1}{[0]} & \underset{n\times 1}{[q_i]} & \underset{n\times 1}{[t_i]} \\
\underset{n\times n}{[U_{ij}]} & \underset{n\times n}{[U_{ij}+\lambda F^{zz}_{ij}]} & \underset{n\times m}{[\lambda F^{zy}_{ij}]} & \underset{n\times n}{[\lambda F^{zv}_{ij}]} & \underset{n\times 1}{[F^{z}_{i}]} & \underset{n\times 1}{[0]} & \underset{n\times 1}{[0]} \\
\underset{m\times n}{[0]} & \underset{m\times n}{[\lambda F^{yz}_{ij}]} & \underset{m\times m}{[\lambda F^{yy}_{ij}]} & \underset{m\times n}{[\lambda F^{yv}_{ij}]} & \underset{m\times 1}{[F^{y}_{i}]} & \underset{m\times 1}{[P_i]} & \underset{m\times 1}{[0]} \\
\underset{n\times n}{[0]} & \underset{n\times n}{[\lambda F^{vz}_{ij}]} & \underset{n\times m}{[\lambda F^{vy}_{ij}]} & \underset{n\times n}{[\lambda F^{vv}_{ij}]} & \underset{n\times 1}{[F^{v}_{i}]} & \underset{n\times 1}{[0]} & \underset{n\times 1}{[1]} \\
\hline
\underset{1\times n}{[0]} & \underset{1\times n}{[F^{z}_{i}]'} & \underset{1\times m}{[F^{y}_{i}]'} & \underset{1\times n}{[F^{v}_{i}]'} & & & \\
\underset{1\times n}{[q_i]'} & \underset{1\times n}{[0]} & \underset{1\times m}{[P_i]'} & \underset{1\times n}{[0]} & & \underset{3\times 3}{[0]} & \\
\underset{1\times n}{[t_i]'} & \underset{1\times n}{[0]} & \underset{1\times m}{[0]} & \underset{1\times n}{[1]'} & & &
\end{array}\right]$$

The second order conditions of Hancock are equivalent to the following (Samuelson (14) p. 378, and Debrew (4) p. 298):

$$0 < (-1)^r \left|\begin{array}{c|c} \underset{r\times r}{[\phi_{ij}]} & \underset{r\times 3}{[\phi_{ik}]} \\ \hline \underset{r\times 3}{[\phi_{kj}]} & \underset{3\times 3}{[0]} \end{array}\right| , \text{ for } \left\{ \begin{array}{l} i, j = 1, \ldots, r \\ k = 1, \ldots, 3 \\ 4 \leq r \leq N. \end{array}\right.$$

By these conditions, obviously $\Theta \neq 0$.

8/ Solutions will not exist in general using the Lagrange method for nonpositive variables. Here it is assumed that all variables are positive. Solutions for cases in which some of the variables have zero values may be obtained using the Kuhn-Tucker method.

9/ Statements about homogeneity in $S_1, \ldots, S_r$ depend on the form of F.

10/ Again subject to the requirement that all variables take only positive values.

11/ By differentiating partially the first-order Lagrange conditions with respect to parameters, one at a time, 2n+m+r+2 systems of linear equations are obtained:

$$\Phi[\frac{\partial d_i}{\partial \rho_k}] = \Psi_k, \ k=1,2,\ldots,2n+m+r+2,$$

where

Φ is the bordered Hessian matrix obtained in Footnote 7/ above, d_i is the i^{th} quantity variable, $i=1,\ldots,3n+m+3$, ρ_k is the k^{th} parameter in the system and Ψ_k is the k^{th} constant vector, associated with differentiating the first-order conditions with respect to ρ_k, $k=1,\ldots,2n+m+r+2$. Each system is solved 3n+m+3 times using Cramer's rule to obtain $\frac{\partial d_i}{\partial \rho_k}$, for $i=1,\ldots,3n+m+3$ and $k=1,2,\ldots,2n+m+r+2$.

12/ Φ is defined in Footnote 7 above.

13/ Footnote 7 above.

14/ Indeed, the demand functions are well defined without some prices. The question of proxies for prices arises only with respect to the computations of benefits via consumer's surplus. Since at this point there is little reason to suspect that the conventional consumer's surplus approach is applicable, it may be that the question of proxies for prices is irrelevant.

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