NOT FOR QUOTATION

The Theory of Consumer Behavior:

Production and the Allocation of Time

by

Robert R. Wilson

December 28, 1969

File Manuscript No. 153

U.S. NATIONAL MARINE FISHERIES SERVICE
ECONOMIC RESEARCH DIVISION

THE THEORY OF CONSUMER BEHAVIOR: PRODUCTION AND THE ALLOCATION OF TIME*

Robert R. Wilson**

I. Introduction

In neoclassical consumer theory the typical consumer maximizes his well-being, usually in terms of a utility function, subject to a budgetary constraint [25, 26, 41]. In symbols,

$$\text{Maximize } U = U(x_1,\ldots,x_n) \text{ subject to}$$

$$p_1x_1 + p_2x_2 + \ldots + p_nx_n = Y.$$

The $x_1,\ldots,x_n$ are usually regarded as positive flows of commodities, and the prices $p_1,\ldots,p_n$ and income Y are non-negative and given. Utility in this case is not regarded as a function of those commodities not consumed.

If certain mathematical conditions on the utility and constraint functions hold, then the results usually presented are that the demand functions implied by the necessary conditions for utility maximization are single valued, differentiable and homogeneous of order zero in all prices and income in a neighborhood of the maximum point.

In addition, the change in each good with respect to a compensated change in its own price (substitution effect), is negative for all (compensated) price

*Presented at the 1969 Winter Meeting of the Econometric Society, New York, N.Y., December 28, 1969. The author wishes to express his appreciation for the valuable comments and suggestions of Professors John S. Chipman, Clifford G. Hildreth, and M. K. Richter of the University of Minnesota and Professors H. O. Hartley, Thomas R. Saving, and Russell G. Thompson of Texas A&M University on earlier drafts of this paper. The errors are strictly contributed by the author, however.

**Industry Economist, Economic Research Division, National Marine Fisheries Service, Washington, D.C.

changes in a neighborhood of the price income point under consideration. If utility is not restricted to be constant (the price change is uncompensated), the algebraic sign of the change in the good is not deducible. The algebraic sign of the change in each good with respect to an uncompensated change in its own price also depends on the sign and magnitude of the (quantity weighted) change in the good with respect to a change in income (income effect). The income effect logically may be influential enough to cause the change of a good with respect to its own price to be positive (Giffen effect) or zero. These results are often generalized to the case in which $x_i \geq 0$, $i=1,\ldots n$, and $\sum_{i=1}^{n} p_i x_i \geq Y$. The above results constitute the implications of neoclassical consumer theory in terms of properties of the demand functions for the goods involved.

It has been pointed out by several economists [5,6,7,10,30,32,34,37] that the neoclassical theory of consumer behavior has several shortcomings. One of the more elegantly outspoken of these is Kelvin Lancaster [34, p. 132] who declares of value theory that "it has been shorn of all irrelevant postulates so that it now stands as an example of how to extract the minimum of results from the minimum of assumptions." Neoclassical theory yielded no implications concerning intrinsic properties (substitutability, complementarity, neutrality, etc.) of goods and the reaction of consumers to new commodities and to quality variations [34, p. 133]. The neoclassical theory did not treat the allocation of time and production in consumption. Such problems may usually be viewed as involving multiple constraints on the utility maxization analysis.(1) Expositions of neoclassical consumer theory have not usually treated multiple constraint problems with the exception of point rationing. Tobin [53] summarizes the consumer behavior implications of point rationing as deduced by Graaf [20], Samuelson [41], and Scitovski [45]. It will become apparent that in one of its simpler specifications the time allocation problem may be interpreted as one case of point rationing.

Extensions of the neoclassical theory to overcome certain of its inadequacies have usually involved a nation of production; of the transformation of flows of goods into the objects of well-being on the part of consumers [5, 32, 34, 37]. The flows of goods as such are usually inert in these theories and yield no utility. It is their transformation into objects, higher order commodities, or what-have-you of consumption, via production, which conditions the transformed commodities to yield utility (2). Such ideas are in evidence in the writings o many other economists [1, 6, 7, 8, 9, 10, 24, 28, 35, 44, 47, 48, 49, 52, 55].

This paper attempts to build upon the work of Morishima [37], Becker [5], and Lancaster [34] and of earlier economists [20, 26, 24, 41]. Several models of consumer behavior will be synthesized which incorporate certain aspects of the time allocation problem with a generalized mathematical structure for conversion of goods and time into the consumption experiences. Demand functions for goods and time of different types will be obtained and their properties deduced. Some implications of these models for the study of demand for recreation will be indicated.

II. THEORIES OF CHOICE

It seems incredible that economists have endured for forty years with the neoclassical results while making only adjustments in the utility function[3]. They have concentrated their efforts on a minute subset of a class of consumer behavior models. It is difficult to conceive of a consumer behavior model in simpler form than the neoclassical model which specifies that a consumer maximizes a utility as a function of flows of goods subject to a linear budget constraint. [4]

The model of consumer behavior was complicated only slightly by the specification of additional linear budget constraints in a theory of point rationing. If the rationing devices are viewed as additional currencies the neoclassical results for demand functions are deduced for each of the currencies [20, 41, 45]. The result that demand functions are homogeneous of degree zero holds for each of the currencies both individually and collectively. Samuelson [41, p. 168] deduces that increasing the number of additional constraints decreases in magnitude the (negative) response of the demand for a good to compensated changes in its own price.

It seems appropriate to consider some different behavioral postulates. It is relevant to envisage flows of goods and the consumer's time being combined and transformed by a consumer into higher commodities which we shall call consumption activities. A consumption activity is defined to be an action on the part of the consumer. The production of consumption activities may involve the use of goods, but always involves the use of time. An example of a consumption activity is giving a dinner party. The dinner party incorporates various commodities including various foods, various beverages, their preparation, their consumption, visiting with friends or family, etc. into one consumption activity. Other examples of consumption activities include travel, work, a night's sleep, shopping and recreation of all types. It seems plausible that such consumption activites would be quantifiable, directly measurable and purchasable for a fee. Measurability is expressed by direct measurment if the activity is well defined (one dinner party) or in units of time, or of the good consumed or produced.

The utility function may be postulated to depend on consumption activities directly. However, it seems more meaningful to treat consumption activities as intermediate products. The consumption experience can be broadened to include the additional transformation of consumption activities into characteristics of consumption (5). The utility function is specified to have the characteristics as arguments. While consumption activities are related to the physical (and fiscal) act of consuming goods, characteristics are qualities of the internal and external environment of the consumer induced by the consumption activity. They are the sets of attributes of each consumption activity that yield gains and losses to the consumer. A good or activity may be used in the production of many characteristics. Conversely, a characteristic may be produced from many goods or activities. Goods and activities are exchangeable in the marketplace. Characteristics are exchangeable only through the exchange of goods or activities. Lancaster [34, p. 134] assumes that "... the characteristics possessed by a good or a combination of goods are the same for all consumers and, given units of measurement, are in the same quanties...". Following Lancaster this assumption will be made except where indicated (6). With this assumption, subjectivity is confined to the choice problem between groupings and levels of characteristics. The constraints on behavior are all objective. Thus, given the measurability of characteristics, consumer theory may be endowed with an objective consumption technology which can be viewed in terms of efficiency [7].

Time may enter into consumer behavior in a number of ways. The case where time is irrelevant is perhaps the simplest. The simplest nondegenerate case appears to be one in which the time constraint is linear and addative, and production requires time in fixed proportions. Time could be postulated to enter into the production of activities only, of characteristic only or of both activities and characteristics. The time constraints could allow for simultaneous performance of activities rather than one at a time in the linear specification. Time may be differentiated as to night from day, weekday from weekend, etc. Time is a nonpurchasable endowment to the consumer.

It should be noted that the utility function may be specified such that work activities neither directly nor indirectly through characteristics can affect utility. This was an assumption used by Becker [5] to obtain a dependency among the constraints. It seems apparent that work that generates neither satisfaction nor disdain is uncommon. Employed consumers need not necessarily be frustrated.

From the foregoing discussion it seems apparent that there are many possible assumptions related to the production and time allocation problems that could be specified. Some of the more interesting ones include:

A. Utility assumptions
   (1) Work activities do not affect utility.
   (2) Work activities affect utility directly.
   (3) Work activities affect characteristics, affect utility.
   (4) Consumption activities affect utility directly.
   (5) Consumption activities affect characteristics, affect utility.
   (6) Goods affect utility directly.
   (7) Goods affect activities, affect utility.
   (8) Goods affect characteristics, affect utility.
   (9) Goods affect activities, affect characteristics, affect utility.
   (10) Time does not affect utility.
   (11) Time affects utility directly.
   (12) Time affects activities, affects utility.
   (13) Time affects characteristics, affects utility.

(14) Time affects activities, affects characteristics, affects utility.
(15) Time affects activities, affects characteristics and time affects characteristics, affects utility.
(16) Utility function has a special form.

B. Production Assumptions
(1) No Production
(2) Production of activities.
(3) Production of characteristics.
(4) Production of activities and characteristics.
(5) Production without goods.
(6) Production without time.
(7) Production with time in fixed proportions.
(8) Production with time in variable proportions.
(9) Production with time in both fixed and variable proportions.
(10) Production of activities one at a time.
(11) Production of activities simultaneously.
(12) Production functions are linear.
(13) Production functions have special forms.

C. Payments Assumptions
(1) Payment for goods.
(2) Payment for activities.
(3) Payment for goods and activities.
(4) Multiple currencies (rationing).

Many more assumptions could be stated. Furthermore, randomly combining assumptions from the utility group with members of the production and payments groups would not necessarily lead to sensible models. For example, assumptions A-10 and B-8 are not mutually consistent.

III. Models

The first model to be considered is a consumer behavior model with time allocation that is an adaptation of the theory of point rationing.

Model I: The choice problem of a typical consumer is defined as follows:

$$\text{Maximize} \quad U(x_1,\ldots,x_n)$$

$$\text{Subject to} \quad \sum_{i=1}^{n} p_i x_i = W \qquad 1)$$

$$\sum_{i=1}^{n} t_i x_i = T$$

where $x_1,\ldots,x_n$ are positive quantities of consumption activities, the p_i are prices or fees paid by the consumer to participate or wages paid to the consumer for participating in activity i (8), W is a residual wealth parameter (9), t_i is a parameter representing the units of time required to produce one unit of x_i, T is the length of planning period (10). Note that each $t_i > 0$ because x_i is an activity. The model employs assumptions A-2, A-4, A-12, B-2, B-5, B-7. B-12, and C-2 of the last section.

It is assumed that the utility function possesses continuous first and second order partial derivatives. Furthermore, there is one free activity ($p_i = 0$) and one that is not free ($p_j \neq 0$) among the n activities. Hence, not every 2 x 2 determinant vanishes of the matrix dg / dx with elements

$$\frac{\partial g_i}{\partial x_j}, \ (i = 1, 2;\ j = 1, 2,\ldots,n),$$

$$\text{where} \quad g_1 = \Sigma p_i x_i - W \qquad 2)$$

$$g_2 = \Sigma t_i x_i - T.$$

For the constrained utility maximization the following Lagrangean expression is developed:

$$L = U(x_1, x_2, \ldots, x_n) + \lambda(\Sigma p_i x_i - W) + \eta(\Sigma t_i x_i - T).$$

For U to have a maximum with the constraints satisfied, it is (first order) necessary that

$$\begin{aligned} & U_i + \lambda p_i + \eta t_i = 0, \ i=1,\ldots,n \\ & \sum_i p_i x_i - W = 0 \\ & \sum_i t_i x_i - T = 0 \end{aligned} \qquad 3)$$

The first order Lagrange necessary conditions constitute n+2 equations in 3n+3 unknowns $(x_1,\ldots x_n, p_1,\ldots,p_n, t_1,\ldots,t_n, \ , \ ,W,T)$. It is possible to solve the system of equations for $x_1,\ldots,x_n$, λ and η in terms of the $p_1,\ldots,p_n$, $t_1,\ldots,t_n$, W and T by slightly strengthening the assumptions. A unique solution exists in a neighborhood of the prices and times if it is assumed that $\Theta \neq 0$, where

$$\Theta = \det \left[\begin{array}{c|cc} [U_{ij}] & [p_i] & [t_i] \\ \hline [p_i]' & & \\ [t_i]' & 0 & \end{array}\right] = \det \Phi$$

Thus, the formulation yeilds the following (local) demand functions:

$$\begin{aligned} & X_i = f^i\,(p_1,\ldots,p_n, W, t_1,\ldots,t_n, T), \ i=1,\ldots,n \\ & \lambda = f^{n+1}(p_1,\ldots,p_n, W, t_1,\ldots,t_n, T) \\ & \eta = f^{n+2}(p_1,\ldots,p_n, W, t_1,\ldots,t_n, T). \end{aligned} \qquad 4)$$

Those functions f^i of the first n for which p_i is a negative parameter may be interpreted as work supply functions, say $f^1,\ldots,f^r$. Then $f^{r+1},\ldots,f^n$ will represent the demand functions of the consumer for consumption activities. By a result on the homogeneity of functions [54], the linearity of the time and money constraints implies that the equations (4) are homogeneous of degree zero in the parameters of each constraint. The above results are mathematically identical to those derived for the case of point rationing [53, p. 526-7].

Homogeneity of the demand functions in the parameters $t_1, t_2,\ldots,t_n$, T may induce some difficulty in interpretation. It seems grossly hypothetical to consider the per unit time requirements and the number of hours in a day increasing proportionately with activity demand (or supply) remaining constant. However, it is not difficult to visualize demand (supply) remaining at a constant level when proportionate changes in the time requirements are offset by the same change in the length of planning period. Whatever the interpretation of the time parameters, the result is the same.

Using the results about demand (supply) functions and Euler's theorem it can be shown that the price and wealth elasticities all sum to zero and the time elasticities all sum to zero for each activity:

$$\sum_{i=1}^{n} \frac{\partial f^i}{\partial p_j}\frac{p_j}{x_i} + \frac{\partial f^i}{\partial W}\frac{W}{x_i} = 0, \ i=1,\ldots,n$$

$$\sum_{j=1}^{n} \frac{\partial f^i}{\partial t_i}\frac{tj}{x_i} + \frac{\partial f^i}{\partial T}\frac{T}{x_i} = 0, \ i=1,\ldots,n.$$

To investigate the displacement of equilibrium the necessary conditions (equations 3) are differentiated totally as follows:

$$U_{i1}dx_1 + U_{i2}dx_2 + \dots + U_{in}dx_n + p_i d\lambda + t_i d\eta$$

$$= (-\lambda)dp_i + (-\eta)dt_i$$

$$i=1,\dots,n \tag{5}$$

$$p_1dx_1 + p_2dx_2 + \dots + p_ndx_n = dW - x_1dp_1 - x_2dp_2 - \dots - x_ndp_n$$

$$t_1dx_1 + t_2dx_2 + \dots + t_ndx_n = dT - x_1dt_1 - x_2dt_2 - \dots - x_ndt_n.$$

Equations 5) may be solved ($\Theta \neq 0$) to yield the following rates of change of demands (supplies) with respect to parameters:

$$\frac{\partial x_i}{\partial p_j} = \frac{(-\lambda)\Theta_{j,i} - x_j\Theta_{n+1,i}}{\Theta}$$

$$\frac{\partial x_i}{\partial W} = \frac{\Theta_{n+1,i}}{\Theta}$$

$$\frac{\partial x_i}{\partial t_j} = \frac{(-\eta)\Theta_{j,i} - x_j\Theta_{n+2,i}}{\Theta}$$

$$\frac{\partial x_i}{\partial T} = \frac{\Theta_{n+2,i}}{\Theta}$$

for i, $j=1,\dots,n$.

It is desirable to obtain restrictions on the algebraic sign of the partial derivatives. For the utility function to have a maximum subject to the constraint (eq. 1), it is sufficient that the rank condition and equation 3), and in addition that the second order condition hold:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} U_{ij} h_i h_j < 0$$

for all vectors $h \neq 0$ such that

$$\sum_{i=1}^{n} p_i h_i = 0$$

$$\sum_{i=1}^{n} t_i h_i = 0$$

according to Hancock [22]. Debreu [11] shows that the second order condition equation 7) holds if and only if

$$(-1)^r \Theta_r > 0, \text{ for } r=3,\ldots,n,$$

where

$$\Theta_r = \det \left[\begin{array}{c|cc} U_r & P_r & T_r \\ \hline P_r' & & \\ & 0 & \\ T_r' & & \end{array}\right] \qquad U_r = \begin{bmatrix} U_{11}\cdots U_{1r} \\ \\ U_{r1}\cdots U_{rr} \end{bmatrix} \tag{8}$$

$$P_r' = [p_1,\ldots,p_r], \text{ and } T_r' = [t_1,\ldots,t_r].$$

Using equation 8), we have

$$\Theta_{i,i}/\,\Theta < 0, \; i=1,2,\ldots,n+2. \tag{9}$$

$\Theta_{i,j,}/\Theta$ of undetermined sign for $i \neq j$.

Applying equations 9) to equations 6) we deduce that:

$$H_{ii} = \frac{\partial X_i}{\partial P_i} + X_i \frac{\partial X_i}{\partial W} = (-\lambda)\Theta_{i,i}/\Theta < 0$$

$$K_{ii} = \frac{\partial X_i}{\partial t_i} + X_i \frac{\partial X_i}{\partial T} = (-\eta)\Theta_{i,i}/\Theta < 0$$

$$H_{ij} = \frac{\partial X_i}{\partial t_j} + X_j \frac{\partial X_i}{\partial W} = (-\lambda)\Theta_{j,i}/\Theta \lesseqgtr 0, \; i \neq j \qquad 10)$$

$$K_{ij} = \frac{\partial X_i}{\partial t_j} + X_j \frac{\partial X_i}{\partial T} = (-\eta)\Theta_{j,i}/\Theta \lesseqgtr 0, \; i \neq j$$

$$i, j=1,\ldots,n.$$

The expressions 10) are identical to the substitution terms first exhibited by Slutzky [46] for neoclassical theory except that there are more of them and by Graaf [20] and Samuelson [41] for point rationing. It can be shown that by minimizing $\Sigma p_i x_i - W$ subject to $\Sigma t_i x_i - T = 0$ and $U(X_1,\ldots,X_n) - \bar{U} = 0$ that $H_{ij} = \frac{\partial X_j}{\partial p_i}$. That is $H_{ij} = \frac{\partial X_j}{\partial P_i}$ with the restriction that the utility level be constant and the time constraint be satisfied. Equivalently $H_{ij} = (\frac{\partial X_i}{\partial P_i})$ compensated. A similar interpretation holds for K_{ij}. The changes in an activity with respect to a compensated change in its own time and money requirements are both individually negative. However, since wages were defined to be the negative of prices, the supply response to compensated changes in wages is positive. The response to uncompensated changes is not deducible. This implies that the logical possibility exists for the Giffen effect in both time and money. The mathematical assumption of continuous differentiability implies that $H_{ij} = H_{ji}$ and $K_{ij} = K_{ji}$.

Another familiar result from the theory of point rationing [20] is that

$$\frac{\left(\frac{\partial x_i}{\partial p_j}\right) \text{comp.}}{\left(\frac{\partial x_i}{\partial t_j}\right) \text{comp.}} = \frac{\lambda}{\eta} > 0$$

That is, activities that are substitutes with respect to compensated price changes are also substitutes with respect to compensated time requirement changes. Similarly, activities that are complements with respect to one type of compensated price change are complements with respect to another. This need not be true for uncompensated changes in parameters. It can be shown, of course, that λ is the marginal utility of residual wealth and η is that of the time endowment.

Another result is the influence of an increase in the number of constraints on the response of activity demand (supply) to change in its own compensated price. Samuelson [41, p. 168] deduces from the generalized Le-Chatelier-Braun principle that the response of demand (supply) to compensated changes in price or time requirements diminishes in magnitude as the number of constraints increases. That is, compensated demand response becomes less and less negative and compensated supply response less and less positive as the number of constraints increases.

It should now be clear that to some extent, time may behave as a price in simplified models of consumer behavior. In fact, if all goods or activities were free ($p_i = 0$, $i=1,\ldots,n$), under the behavioral assumptions made in Model 1 a consumer choice mechanism could operate in a manner similar to the neoclassical theory. Time would allocate choices according to the neoclassical results if all prices and wages were zero. Such a model would provide a theory of choice for a "Robinson Crusoe" economy without exchange in the usual sense.

Tobin [53] points out the essential differences between money and ration currencies. It is of interest to note that time in Model 1 is analogous to a ration currency and unlike money in that the size of the time endowment is independent of the work supplied by the consumer and that saving (not expending) time is impossible. However, time is like money and unlike ration currencies in that every activity has a time requirement.

It should be apparent also that Model 1 is a generalization of the labor-leisure analysis of Henderson and Quandt [23, p. 234].

Example of Model 1: Suppose for illustration, that the typical consumer engages in three activities; working x_1, resting x_2, and dining x_3. His utility function is specified as:

$$U = U(x_1, x_2, x_3)$$

His constraints are

$$p_1x_1 + p_2x_2 + p_3x_3 = W = 0$$

$$t_1x_1 + t_2x_2 + t_3x_3 = T.$$

Suppose that $p_i \neq 0$ for some i. The first order necessary conditions are that

$$U_i + \lambda p_i + \eta t_i = 0$$

$$i=1,2,3$$

$$\Sigma p_i x_i = 0$$

$$\Sigma t_i x_i = T.$$

Those first order conditions may be solved to yield the supply function,

$$X_1 = f^1 (p_1, p_2, p_3, t_1, t_2, t_3, T)$$

the demand functions,

$$X_2 = f^2\,(p_1,\ p_2,\ p_3,\ t_1,\ t_2,\ t_3,\ T)$$

$$X_3 = f^3\,(p_1,\ p_2,\ p_3,\ t_1,\ t_2,\ t_3,\ T)$$

and the marginal utilities of forced borrowing (saving) and of time,

$$\lambda = f^4\,(p_1,\ p_2,\ p_3,\ t_1,\ t_2,\ t_3,\ T)$$

$$\eta = f^5\,(p_1,\ p_2,\ p_3,\ t_1,\ t_2,\ t_3,\ T)$$

These solution functions $f^1,\ldots,f^5$ are locally unique, differentiable, homogenous of degree zero in p_1, p_2, and p_3 and in t_1, t_2, t_3, and T. The theory provides that $(\partial x_i/\partial p_i)$ compensated is negative for i=1,2,3. With increases in forced borrowings or gifts, we assume that $\partial x_1/\partial W < 0$, $\partial x_2/\partial W > 0$, and $\partial x_3/\partial W > 0$. If $x_1(\partial x_1/\partial W)$ is large enough and negative, it is apparent that a backward bending work activity supply will result even though at the point $(p_1, p_2, p_3, 0)$ we have $W = 0$. For x_2 and x_3 we have $\partial x_2/\partial p_2 < 0$ and $\partial x_3/\partial p_3 < 0$.

Model 2: Model 2 is a formulation particularly rich in results. It allows for both production and purchase of activities, and both fixed and variable proportions in time and basically generalizes the model of Becker [5]. Assumptions specified are A-2, A-4, A-7, A-12, B-2, B-9, B-10, and C-4. The specification is as follows:

$$\text{Maximize} \quad U(x_1,\ldots,x_n) \tag{1}$$

$$\text{Subject to} \quad f(z_1,\ldots,z_n,\ y_1,\ldots,y_m,\ T_1,\ldots,T_n) = 0 \tag{2}$$

$$\sum_1^n w_i q_i + \sum_1^m y_i p_i = W \tag{3}$$

$$\sum_1^n t_i w_i + \sum_1^n T_i = T \tag{4}$$

$$x_i = w_i + f_i,\ i=1,\ldots,n \tag{5}$$

where $x_i,\ldots,x_n$ are n positive activities; $w_1,\ldots,w_n$ and $z_1,\ldots,z_n$ are the amounts of $x_1,\ldots,x_n$ purchased and produced, respectively; $y_1,\ldots,y_m$ are m goods; $T_1,\ldots,T_n$ are n variable nonnegative time inputs; f is an implicit production function; $t_1,\ldots,t_n$ are fixed per unit time requirements for $w_1,\ldots,w_n$; and $p_1,\ldots,p_m$, $q_1,\ldots,q_n$, W, and T have the usual definitions.

By substituting expression 5) into 1) the constrained local extremum problem is:

$$\text{Maximize } U(w_1 + z_1, w_2 + z_2,\ldots, w_n + z_n) \tag{6}$$

subject to expressions 2), 3) and 4). It is apparent that variable time inputs will behave as a good, rather than as a parameter in this problem. Rights to participate in a produced activity are also regarded as a good.

By differentiating the constraints 2), 3) and 4) it can be seen to be sufficient for the Lagrangean rank condition to hold that at least two activities exist, at least one activity be producible from goods and time, and one of the activities have $q_i \neq 0$ while another has $q_i = 0$. Alternatively, it is sufficient that at least one activity be producible and the price of one good be nonzero. With this assumption and continuous differentiability of the first and second order of the utility and constraint functions, the Lagrangean expression may be formed with the arguments as vectors:

$$L = U(W + Z) + \lambda f(Z,Y,T^{*}) + \gamma\left(\sum_{1}^{n} w_i q_i + \sum_{1}^{m} y_i p_i - W\right)$$

$$+ \delta\left(\sum_{1}^{n} t_i w_i + \sum_{1}^{n} T_i - T\right)$$

The first order necessary conditions that expression 6) obtain a maximum subject to constraints 2), 3) and 4) are

$$\frac{\partial L}{\partial w_i} = L_i^w = U_i^x + \gamma q_i + \delta t_i = 0$$

$$i=1,\ldots,n$$

$$\frac{\partial L}{\partial z_i} = L_i^z = U_i^x + \lambda f_i^z = 0$$

$$i=1,\ldots,n$$

$$\frac{\partial L}{\partial y_i} = \lambda f_i^y + \gamma p_i = 0$$

$$i=1,\ldots,n \qquad 7)$$

$$\frac{\partial L}{\partial T_i} = \lambda f_i^T + \delta = 0$$

$$\frac{\partial L}{\partial \lambda} = f(z,y,T^*) = 0$$

$$\frac{\partial L}{\partial \gamma} = \sum_1^n w_i q_i + \sum_1^m y_i p_i - W = 0$$

$$\frac{\partial L}{\partial \delta} = \sum_1^n t_i w_i + \sum_1^n T_i - T = 0$$

Under the appropriate conditions $(\Theta \neq 0)$ [(13)], the first order necessary conditions may be solved for each of the variables $w_1,\ldots,w_n$, $f_1,\ldots,f_n$, $y_1,\ldots,y_m$, $T_1,\ldots,T_n$, λ, γ, and δ as functions of the parameters $q_1,\ldots,q_n$, $p_1,\ldots,p_m$, W, $t_1,\ldots,t_n$, and T in a neighborhood of the optimum point:

$$V_i = h^i(q_1,\ldots,q_n,\ p_1,\ldots,p_m,\ W,\ t_1,\ldots,t_n,\ T),\ i=1,\ 3n+m+3, \qquad 8)$$

where

$$V_i = \begin{cases} w_i, & i=1,\ldots,n \\ z_i, & i=n+1,\ldots,2n \\ y_i, & i=2n+1,\ldots,2n+m \\ T_i, & i=2n+m+1,\ldots,3n+m \\ \lambda, & i=3n+m+1 \\ \gamma, & i=3n+m+2 \\ \delta, & i=3n+m+3 \end{cases} \qquad 9)$$

Each function h^i is single valued and continuous in all of the parameters and homogenous of degree zero in the parameters $q_1,\ldots,q_n$, $p_1,\ldots,p_m$, and W, because the money constraint equation 3) is homogeneous of degree one in the parameters. The time constraint equation 4) is not homogeneous of any degree in $t_1,\ldots,t_n$, and T. Thus the sufficiency condition [54] for homogeneity of degree zero of the solution functions h^i in terms of the time parameters fails. At this point it is uncertain whether the h^i are homogeneous of degree zero in the time parameters.

To investigate the homogeneity of h^i in the time parameters, it is of interest to examine the admissibility of solutions to the constrained extremum problem. A solution vector $\bar{V} = (\bar{w}_1,\ldots,\bar{w}_n,\ \bar{z}_n,\ldots,\bar{z}_n,\ \bar{y}_1,\ldots,\bar{y}_m,\ \bar{T}_1,\ldots,\bar{T}_n,\ \bar{\lambda},$ $\bar{\gamma}$, and $\bar{\delta}$) must remain admissible after a proportionate change in the parameters $t_1,\ldots,t_n$, and T for the corresponding solution functions h^i, for $i=1,\ldots,\ 3n+m+3$ to be homogeneous of degree zero in the parameters. Multiplying the parameters by a constant $\varepsilon \neq 1$ in equation 4 yields

$$\Sigma\varepsilon t_i\bar{w}_i + \Sigma\bar{T}_i - \varepsilon T = (1 - \varepsilon)\Sigma\bar{T}_i \neq 0.$$

Thus, the time constraint is not satisfied by the solution $\bar{V}$ after a proportionate change in all the time parameters. Thus, $\bar{V}$ is not admissible after the change and the demand functions h^i cannot all be homogeneous of degree zero in time parameters. The longstanding homogeneity property has failed for time allocation. It is also possible to modify the model sufficiently to allow nonhomogeneous demand functions in the money parameters (14). Elsewhere [54], I have shown that nonhomogeneity can result under very general conditions. The neoclassical result is a very special case.

By differentiating the first order conditions 7) totally, the following equations are obtained:

$$\sum_{j=1}^{m} U_{ij}dw_j + q_i d\gamma + t_i d\delta = (-\gamma)dq_i + (-\delta)dt_i, \text{ for } i = 1, \ldots, n$$

$$\sum_{j=1}^{n} (U_{ij} + \lambda f^z_{ij})dz_j + f^z_i d\lambda = 0, \text{ for } i = 1,\ldots,n$$

$$\lambda\sum_{j=1}^{n} f^y_{ij}dy_j + f^y_i d\lambda + p_i d\lambda = (-\lambda)\ dp_i, \text{ for } i = 1,\ldots,m \qquad 10)$$

$$\lambda\sum_{j=1}^{n} f^T_{ij}dT_j + f^T_i d\lambda + d\delta = 0, \text{ for } i = 1,\ldots,n$$

$$\sum_{i=1}^{n} f^z_i dz_i + \sum_{i=1}^{m} f^y_i dy_i + \sum_{i=1}^{n} f^T_i dT_i = 0$$

$$\sum_{i=1}^{n} q_i dw_i + \sum_{i=1}^{m} p_i dy_i = dW - \sum_{i=1}^{n} w_i dq_i - \sum_{i=1}^{m} y_i dp_i$$

$$\sum_{i=1}^{n} q_i dw_i + \sum_{i=1}^{m} dT_i = dT - \sum_{i=1}^{n} dw_i dt_i .$$

Under the assumption that $\Theta \neq 0$, the rates of change of the variables v_i (eq. 9)) with respect to changes in the parameters may be obtained:

$$\frac{\partial v_i}{\partial q_j} = (-\gamma)\frac{\Theta_{j,i}}{\Theta} - w_j \frac{\Theta_{3n+m+2,i}}{\Theta}, \qquad 11)$$

$$\frac{\partial v_i}{\partial p_j} = (-\gamma)\frac{\Theta_{2n+j,i}}{\Theta} - y_j \frac{\Theta_{3n+m+2,i}}{\Theta}, \qquad 12)$$

$$\frac{\partial v_i}{\partial t_j} = (-\delta)\frac{\Theta_{j,i}}{\Theta} - w_j \frac{\Theta_{3n+m+2,i}}{\Theta}, \qquad 13)$$

$$\frac{\partial v_i}{\partial w} = \frac{\Theta_{3n+m+2,i}}{\Theta}, \qquad 14)$$

$$\frac{\partial v_i}{\partial T} = \frac{\Theta_{3n+m+3,i}}{\Theta}, \qquad 15)$$

for $i = 1,\ldots, 3n+m+3$.

From the second order conditions, it can be deduced that

$$\frac{\Theta_{i,j}}{\Theta} \begin{cases} < 0 \text{ for } i = j \\ \text{of undetermined sign for } i \neq j. \end{cases} \qquad 16)$$

as for Model 1. Thus, it can be deduced that the substitution terms

$(\frac{\partial w_i}{\partial q_i})_{comp.} < 0$, $(\frac{\partial y_j}{\partial p_j})_{comp.} < 0$, and $(\frac{\partial w_i}{\partial t_i})_{comp.} < 0$, for $i = 1,\ldots,n$; $j=1,\ldots,m$. Furthermore, w_i and z_i are defined to be perfect substitutes in production and consumption. Thus, $\frac{\partial z_i}{\partial w_i} = -1$ by eq. 5) and it follows immediately that

$$\frac{\partial z_i}{\partial q_j} = - \frac{\partial w_i}{\partial q_j} \tag{17)}$$

$$\frac{\partial z_i}{\partial t_j} = - \frac{\partial w_i}{\partial t_j} \tag{18)}$$

$$\frac{\partial z_i}{\partial p_k} = - \frac{\partial w_i}{\partial p_k} \tag{19)}$$

$$\frac{\partial z_i}{\partial W} = - \frac{\partial w_i}{\partial W} \tag{20)}$$

$$\frac{\partial z_i}{\partial T} = - \frac{\partial w_i}{\partial T} \tag{21)}$$

for $i,j = 1,\ldots,n$; $k = 1,\ldots,m$.

These results are certainly empirically testable properties of the theoretically derived demand functions. By substituting 20) into 17) and 18) and 21) into 19) and expanding, the following results are obtained

$$(\frac{\partial z_i}{\partial q_i})_{comp} = - \frac{\partial w_i}{\partial q_i} + w_i \left(- \frac{\partial w_i}{\partial W}\right) = - \left(\frac{\partial w_i}{\partial q_i}\right)_{comp} > 0 \tag{22)}$$

$$(\frac{\partial z_i}{\partial q_j})_{comp} = - \frac{\partial w_i}{\partial q_j} + w_j \left(- \frac{\partial w_i}{\partial W}\right) = - \left(\frac{\partial w_i}{\partial q_j}\right)_{comp} \tag{23)}$$

$$\left(\frac{\partial z_i}{\partial t_i}\right)_{comp} = -\frac{\partial w_i}{\partial t_i} + w_i\left(-\frac{\partial w_i}{\partial W}\right) = -\left(\frac{\partial w_i}{\partial t_i}\right)_{comp} > 0 \qquad 24)$$

$$\left(\frac{\partial z_i}{\partial t_j}\right)_{comp} = -\frac{\partial w_i}{\partial t_j} + w_j\left(-\frac{\partial w_i}{\partial W}\right) = -\left(\frac{\partial w_i}{\partial t_j}\right)_{comp} \qquad 25)$$

$$\left(\frac{\partial z_i}{\partial p_k}\right)_{comp} = -\frac{\partial w_i}{\partial p_k} + y_k\left(-\frac{\partial w_i}{\partial W}\right) = -\left(\frac{\partial w_i}{\partial p_k}\right)_{comp} \qquad 26)$$

for $i,j = 1,\ldots,n;\ k=1,\ldots,m.$

The rate of change of each produced activity with respect to each price and fixed time requirement is of equal magnitude and of opposite sign to that of each purchased activity for compensated changes and uncompensated changes in time and money prices. Furthermore, the demand function for activity x_i, the sum of the amount purchased w_i and the amount produced z_i is

$$x_i = h^i(q_1,\ldots,q_n,\ p_1,\ldots,p_m,\ W,\ t_1,\ldots,t_n,\ T) \qquad 27)$$

$$+\ h^{n+i}(q_1,\ldots,q_n,\ p_1,\ldots,p_m,\ W,\ t_1,\ldots,t_n,\ T) \qquad 28)$$

for $i = 1,\ldots,n$. It is apparent that

$$\frac{\partial x_i}{\partial s_k} = \frac{\partial w_i}{\partial s_h} + \frac{\partial z_i}{\partial s_k} = \frac{\partial w_i}{\partial s_k} - \frac{\partial w_i}{\partial s_k} = 0, \qquad 28)$$

where s_k is any one of the 2n+m+2 parameters in the system. That is, the demand (supply) function for x_i is very flat in a neighborhood of the equilibrium point. This result certainly may imply some rather profound stability properties, if it stands the test of time and applies to a large proportion of the activities available.

That equation 28) appears to be a dubious result is indeed an understatement. In fact, it tends to shed doubt on the entire specification of Model 2. However, a re-examination of the specification and the determinant yeilded no apparent inconsistences or singularities. If result 28) is spurious, it will be embarrassing;

if not, such a result should not be hidden for fear of embarrassment. This would be neither the first nor the last mistake to be made by an economist.

To dispel fears of error, activities could be divided into two classes; those purchased and those produced. If an activity could be both purchased and produced, its purchase and its production could be defined as separate activities. This would be appropriate if production of its own nature affected the utility function. That model would no longer obviously exhibit expression 17) through 28). But those results (17) - 28)) would be deducible for a particular activity as in Model 2 upon the assumption that at least one activity is both producible and purchasible and that production and purchase of the activity are perfect substitutes. Two activities can be perfect substitutes only if they effect the utility in exactly the same way. Imperfect substitution will not yield the frightening results of 28). Such a specification will be given in Model 3.

There are yet results to be deduced from Model 2. If it is assumed that in production increases in inputs increase outputs, $\frac{\partial z_i}{\partial y_k} > 0$ and $\frac{\partial z_i}{\partial T_i} > 0$, then

$$\frac{\partial w_i}{\partial y_k} = - \frac{\partial z_i}{\partial y_i} < 0$$

and
$$\frac{\partial w_i}{\partial T_i} = - \frac{\partial z_i}{\partial T_i} < 0$$

for i = 1,...,n and k = 1,...,m.

It follows under certain conditions on f that

$$\left(\frac{\partial y_k}{\partial q_i}\right)_{comp} < 0$$

$$\left(\frac{\partial T_i}{\partial q_i}\right)_{comp} > 0$$

$$\left(\frac{\partial y_k}{\partial t_i}\right)_{comp} > 0$$

$$\left(\frac{\partial T_i}{\partial t_i}\right)_{comp} > 0$$

$$\left(\frac{\partial w_i}{\partial p_k}\right)_{comp} > 0$$

$$\left(\frac{\partial z_i}{\partial p_k}\right)_{comp} < 0, \text{ for } i = 1,\ldots,n \text{ and } k = 1,\ldots,m.$$

Knowledge of the production functions should yield information on the relationships among goods $y_1,\ldots,y_n$, among time inputs $T_1,\ldots,T_n$, between goods and time inputs, and their influence on activities $z_1,\ldots,z_n$, both for compensated and uncompensated changes in the parameters. Thus the demand functions are much more completely described in terms of their rates of change than either in the neoclassical theory or Model 1. The relationships among activities, however, still are undetermined. The reader should keep in mind throughout the discussion that rates of change are evaluated at the equilibrium points.

The possibility of Giffen effects remains as with Model 1. It is necessary only to recall that the signs of a portion of the compensated rates of change were determined. Uncompensated rates of change are still indeterminate. Also, the symmetry condition is the same as with Model 1 and for the same reasons: $H_{ij} = H_{ji}$ and $K_{ij} = K_{ji}$.

The interpretation of the Lagrangeane multipliers λ, γ, and δ are, respectively, the marginal utility of production $\frac{\partial U}{\partial f}$, the marginal utility of residual wealth $\frac{\partial U}{\partial W}$ and the marginal utility of the length of planning period $\frac{\partial U}{\partial T}$.

One result that may be neatly disposed of in Model 2 is the introduction of a new good. New goods are inputs in production only and do not alter the utility function. Only the objective production relationships vary and the impact of the new good comes through them alone. The utility function must vary, however, when the introduction of a new good induces a new activity.

Results that carry over basically unchanged from Model 1 include the invariance of the intrinsic relationships between activities, goods, or time inputs when going from money prices to time prices. Also included is that the magnitude of the substitution terms is nonincreasing as the number of constraints is increased. The definition that work is a consumption activity is obvious from the specification.

Model 3: This model is a fine turning of Model 2 to alleviate the author's lack of enthusiasm for perfect substitution and subsequent locally flat demand (supply) functions. The specification is as follows:

$$\text{Maximize} \quad U = U(x_1, \ldots, x_n) \tag{1}$$

$$\text{subject to} \quad f(x_{r+1}, \ldots, x_n, y_1, \ldots, y_m, T_{r+1}, \ldots, T_n) = 0 \tag{2}$$

$$\sum_{i=1}^{r} q_i x_i + \sum_{i=1}^{m} p_i y_i = W \tag{3}$$

$$\sum_{i=1}^{r} t_i x_i + \sum_{i=r+1}^{n} T_i = T, \tag{4}$$

where $x_1, \ldots, x_n$ are work and consumption activities, the first r of which are purchased and the remaining n-r are produced and the remaining variables have the same definitions as in Model 2.

Demand (supply) functions may be deduced under the appropriate conditions which express $x_1,\ldots,x_n$, $y_1,\ldots,y_m$ and $T_{r+1},\ldots,T_n$, and as locally differentiable functions of $q_1,\ldots,q_r$, $p_1,\ldots,p_m$, W, $t_1,\ldots,t_r$ and T. They are homogeneous of degree zero in the money parameters $q_1,\ldots,q_r$, $p_1,\ldots,p_m$ and W. However, all of the demand (supply) functions cannot be homogeneous of degree zero in the time parameters $t_1,\ldots,t_r$ and T because of the form of equation 4). If the money constraint equation 3) were respecified to have the same form as 4) the homogeneity result for money (one of the fundamental theorems of economics) would also fail. An example when it does fail is when the consumer plays poker as an activity. Money then may be viewed as a variable input in the production of poker.

The implications for the compensated and uncompensated changes in demand functions are the only areas in which Model 3 yields results that deviate from those of Model 2. Model 3 does not exhibit properties 17) - 28) of Model 2. However, for some pairs of activities, perfect substitution may be appropriate, in which case equations 17) - 28) will indeed hold. There is some modification in equations 10) - 15) and in the determinant Θ for Model 3 as compared to Model 2, but the only substantive changes are with respect to perfect substitution.

It is apparent that perfect substitution is a limiting case of ordinary substitution. That is, $\left(\frac{\partial x_i}{\partial q_j}\right)_{comp} > 0$ if and only if $\frac{\partial x_i}{\partial x_j} < 0$ because $\left(\frac{\partial x_j}{\partial q_j}\right)_{comp} < 0$. Thus, in the case of less than perfect substitution the signs of expressions 17) - 26) are determined, but the magnitudes are not equal. In the limit, as $\frac{\partial x_i}{\partial x_j}$ approaches -1, it is apparent that equations 17) - 28) hold as limits. Thus, as a limiting expression, 28) implies that demand (supply) functions for the sum of related activities become increasingly locally flat as the degree of substitution between the activities approaches perfection. The equilibrium of a consumer that fits the formulation of Model 2 is extremely stable with respect to changes in any one of the parameters. It should be possible to generalize this result for an economy of individuals that obey the assumptions of Model 2. Empirical evidence of perfect substitution would thus be empirical test for stability of equilibrium.

Results of a somewhat similar nature may be deduced for complementary activities and the limiting case of perfect complements. First, it should be noted that in a perfect complementarity relationship, $\frac{\partial x_i}{\partial x_j} = 1$ and that with imperfect complementarity, $\frac{\partial x_i}{\partial x_j} > 0$. Thus, $(\partial x_i/\partial q_j)$ comp < 0 if and only if $\partial x_i/\partial x_j > 0$. Expressions analogous to 17) - 26) of Model 2 can be inferred for complementarity from the foregoing results.

Example of Model 2 and Model 3: Suppose that the typical consumer has available to him three activities, working x_1, resting x_2 and dining x_3. He may obtain dining in either of two ways; by the purchase of the right to dine w_3, or by production of the right to dine by growing food z_3. Growing food requires variable amounts of fertilizer y and time τ. The consumer's choice problem is characterized as follows:

$$\text{Maximize} \quad U = U(x_1, x_2, x_3)$$

$$\text{Subject to} \quad x_3 = w_3 + z_3$$

$$z_3 = a\tau^2 + by^2 - c\tau y$$

$$x_1p_1 + x_2p_2 + w_3p_3 + yp_y = 0$$

$$x_1t_1 + x_2t_2 + w_3t_3 + \tau = 0$$

where it is assumed that $py \neq 0$.

The Lagrangean function

$$L = U(x_1, x_2, w_3 + z_3) + \lambda(a\tau^2 + by^2 - c\tau y - z_3)$$
$$+ \gamma(p_1x_1 + p_2x_2 + p_3w_3 + p_y y)$$
$$+ \delta(t_1x_1 + t_2x_2 + t_3w_3 + \tau - T)$$

yields the following first order necessary conditions for a relative constrained maximum of U:

$$\frac{\partial L}{\partial x_1} = U_1 + \gamma p_1 + \delta t_1 = 0$$

$$\frac{\partial L}{\partial x_2} = U_2 + \gamma p_2 + \delta t_2 = 0$$

$$\frac{\partial L}{\partial w_3} = U_3 + \gamma p_3 + \delta t_3 = 0$$

$$\frac{\partial L}{\partial z_3} = U_3 - \lambda \qquad = 0$$

$$\frac{\partial L}{\partial y} = \lambda(2by - c\tau) + \gamma py = 0$$

$$\frac{\partial L}{\partial \tau} = \lambda(2a\tau - cy) + \delta \qquad = 0$$

$$a\tau^2 + by^2 - c\tau y - z_3 \qquad = 0$$

$$p_1x_1 + p_2x_2 + p_3w_3 + p_y y = 0$$

$$t_1x_1 + t_2x_2 + t_3w_3 + \tau - T = 0$$

Under the condition that $\Theta \neq 0$ [15] the first order conditions 1) may be solved for x_1, x_2, w_3, z_3, y, τ, λ, γ, and δ in terms of the parameters a, b, c, p_1, p_2, p_3, p_y, t_1, t_2, t_3, and T. The solutions may be expressed as

$$v_i = h^i\,(a, b, c, p_1, p_2, p_3, p_y, t_1, t_2, t_3, \text{ and } T), \text{ for } i = 1, \ldots, 9.$$

The example points out a general principle that may be easily overlooked in the mathematics. The solution functions depend on all of the parameters in the extremum problem. This includes those in each constraint and also in the objective function [54].

The demand (supply) functions h^i are each differentiable, unique, and homogeneous of degree zero in the prices p_1, p_2, p_3, and p_y. They are not homogeneous in a, b and c, nor in t_1, t_2, t_3 and T. Again, the demand function for x_3 is $h^3 + h^4$ because of perfect substitution.

With $\Theta \neq 0$ the first order conditions may be differentiated to obtain

$$\frac{\partial v_j}{\partial p_i} = (-\gamma)\frac{\Theta_{i,j}}{\Theta} - x_i\frac{\Theta_{8,j}}{\Theta} \qquad i = 1, 2, \tag{2)}$$

$$\frac{\partial v_j}{\partial p_3} = (-\gamma)\frac{\Theta_{3,j}}{\Theta} - w_3\frac{\Theta_{8,j}}{\Theta}, \tag{3)}$$

$$\frac{\partial v_j}{\partial py} = (-\gamma)\frac{\Theta_{5,j}}{\Theta} - y\frac{\Theta_{8,j}}{\Theta}, \tag{4)}$$

$$\frac{\partial v_j}{\partial t_i} = (-\delta)\frac{\Theta_{i,j}}{\Theta} - x_i\frac{\Theta_{9,j}}{\Theta}, \quad i = 1, 2, \tag{5)}$$

$$\frac{\partial v_j}{\partial t_3} = (-\delta)\frac{\Theta_{3,i}}{\Theta} - w_3\frac{\Theta_{9,j}}{\Theta}, \tag{6)}$$

$$\frac{\partial v_j}{\partial W} = \frac{\Theta_{8,j}}{\Theta}, \qquad \frac{\Theta v_j}{\partial T} = \frac{\Theta_{9,j}}{\Theta}, \tag{7)}$$

$$\frac{\partial v_j}{\partial a} = (-\lambda)\,\frac{2\tau\Theta 6,j}{\Theta} - \frac{2\tau\Theta 7,j}{\Theta}, \tag{8)}$$

$$\frac{\partial v_j}{\partial b} = (-\lambda)\,\frac{2y\Theta 5,j}{\Theta} - \frac{2y\Theta 7,j}{\Theta}, \tag{9)}$$

$$\frac{\partial v_j}{\partial c} = (-\lambda)\left(\frac{-\tau\Theta 5,j}{} - y\,\frac{\Theta 6,j}{\Theta}\right) + \frac{2\tau y\Theta 7,j}{\Theta}, \tag{10)}$$

for j=1,...,9.

Expressions 2) - 7) are rates of change of the demand (supply) functions of the usual form derived for Models 1, 2, and 3. Expressions 8), 9), and 10) are rates of change with respect to the technical parameters. Expressions 17) - 28) of Model 2 hold for w_3 and z_3 so that the demand for x_3, $h^3 + h^4$ is locally flat. The sign and magnitudes of certain of the compensated and uncompensated rates of change can be deduced as with Models 2 and 3. The perfect substitution between purchase of dining and the production of dining, of course, is implied by the implicit assumption that production of food does not affect utility. This is undoubtedly quite heroic.

Model 4: Up to this point we have omitted one important behavioral aspect in order to be able to attribute meaningful conclusions to the appropriate premises. It is apparent from the previous analyses that a knowledge of the intrinsic substitutability or complementarity of goods or activities from the production relationships implied much information about the relative signs of the compensated rates of change of the demand functions. Knowledge of the intrinsic relationships arising from the utility function was not deducible, however.

In order to make deducible more of the relationships between activities, a concept used effectively by Lancaster [34] is employed. The specification that the ordinal utility function depends on the physical and psychological attributes of consumption, or "characteristics" as we have called them will aid in further determining the intrinsic relationships for combinations of goods, activities, and time.

The consumer choice problem is specified as follows:

$$\text{Maximize} \quad U = U(v_1,\ldots,v_s) \tag{1}$$

$$\text{Subject to} \quad f(v_1,\ldots,v_s,\ x_1,\ldots,x_n) = 0, \tag{2}$$

$$g(x_{r+1},\ldots,x_n,\ y_1,\ldots,y_m,\ T_{r+1},\ldots,T_n) = 0, \tag{3}$$

$$\sum_{i=1}^{r} x_i q_i + \sum_{i=1}^{m} y_i p_i = W, \tag{4}$$

$$\sum_{i=1}^{r} x_i t_i + \sum_{i=r+1}^{n} T_i = T, \tag{5}$$

where $v_1,\ldots,v_s$ are characteristics of consumption; $x_1,\ldots,x_n$ are work and consumption activities; f is an implicit production function transforming activities into characteristics; activities $x_{r+1},\ldots,x_n$ are producible through the implicit production process g with goods $y_1,\ldots,y_m$ and variable times $T_{r+1},\ldots,T_n$ as inputs; and activities $x_1,\ldots,x_r$ are purchasible with prices $q_1,\ldots,q_r$ and fixed time inputs $t_1,\ldots,t_r$.

Model 3 employs assumptions A-3, A-5, A-9, A-14, B-4, B-9, B-10, and C-3. It does not explicitly assume any perfect substitution in production. However, since Model 4 is a generalization of Models 2 and 3, results on perfect substitution can be obtained as a special case.

By forming the Lagrangean expression in the usual manner and with the appropriate companion assumptions, the first order necessary conditions for a maximum of 1) subject to 2) - 5) are as follows:

$$\frac{\partial L}{\partial v_i} = U_i^v + \lambda f_i^v = 0, \qquad i=1,\ldots,s, \tag{6}$$

$$\frac{\partial L}{\partial x_i} = \lambda f_i^x + \delta q_i + \eta t_i = 0, \qquad i=1,\ldots,r, \tag{7}$$

$$\frac{\partial L}{\partial x_i} = \lambda f_i^x + \gamma g_i^x = 0, \qquad i=r+1,\ldots,n \qquad 8)$$

$$\frac{\partial L}{\partial y_i} = \gamma g_i^y + \delta p_i = 0, \qquad i=1,\ldots,m, \qquad 9)$$

$$\frac{\partial L}{\partial T_i} = \gamma g_i^T + \eta = 0, \qquad i=r+1,\ldots,n \qquad 10)$$

$$\frac{\partial L}{\partial \lambda} = f(v_1,\ldots,v_s, x_1,\ldots,x_n) = 0 \qquad 11)$$

$$\frac{\partial L}{\partial \gamma} = g(x_{r+1},\ldots,x_n, y_1,\ldots,y_m, T_{r+1},\ldots,T_n) = 0 \qquad 12)$$

$$\frac{\partial L}{\partial \delta} = \sum_{i=1}^{r} q_i x_i + \sum_{i=1}^{n} p_i y_i - W = 0 \qquad 13)$$

$$\frac{\partial L}{\partial \eta} = \sum_{i=1}^{r} t_i x_i + \sum_{i=r+1}^{n} T_i - T = 0. \qquad 14)$$

Under the condition that $\Theta \neq 0$ [16], equations 6) - 14) may be solved in the usual way for the demand and supply functions $z_i = h^i(q_1,\ldots,q_r, p_1,\ldots,p_m, W, t_1,\ldots,t_r, T)$ where z_i is any one of the $s+r+2(n-r) + m+4$ variables in the problem. The h_i are unique, differentiable, and homogeneous of degree zero in the money parameters $q_1,\ldots,q_r, p_1,\ldots,p_m$, and W. They are not homogeneous of degree zero in $t_1,\ldots,t_r$, and T. They also depend on whatever parameters there are in the utility function U or in either of the production functions, f or g, but zero degree homogeneity does not, in general, hold. Again, the money constraint 4) and the activity production function 3) may be respecified so that zero degree homogeneity of the h^i does not hold in the money parameters. Thus, (demand) functions for each characteristic, each work (supply) or consumption (demand) activity, each good, each variable time input (time good), and each Lagrangean multiplier are obtained.

Wading through the mathematics to investigate the displacement of equilibrium again seems annoyingly redundant. However, certain results should be noted. First, there are no direct time and money price effects for the characteristics, producible acitvities, or time inputs. The direction of compensated price effects for these quantities must be deduced from the compensated price effects of goods and activities that have time and money prices and the marginal rates of substitution of quantities one for the other. Under the appropriate mathematical conditions and the second order sufficiency conditions for a constrained extremum, the direct compensated rates of change are:

$$\left(\frac{\partial x_i}{\partial q_i}\right)_{\text{comp.}} = (-\delta)\frac{\Theta_{s+i,s+i}}{\Theta} < 0, \text{ for } i = 1,\ldots,r.$$

$$\left(\frac{\partial x_i}{\partial t_i}\right)_{\text{comp.}} = (-\eta)\frac{\Theta_{s+i,s+i}}{\Theta} < 0, \text{ for } i = 1,\ldots,r$$

$$\left(\frac{\partial y_i}{\partial p_i}\right)_{\text{comp.}} = (-\delta)\frac{\Theta_{s+n+i,s+n+i}}{\Theta} < 0, \text{ for } i = 1,\ldots,m.$$

To the extent that the signs of $\frac{\partial f_i}{\partial x_i}$ and $\frac{\partial z_j}{\partial y_i}$ are obtainable from the production relationships at the equilibrium point, the signs of

$$\left(\frac{\partial z_j}{\partial q_i}\right)_{\text{comp.}}, \qquad \left(\frac{\partial z_j}{\partial t_r}\right)_{\text{comp.}}, \qquad \text{and} \qquad \left(\frac{\partial z_j}{\partial p_i}\right)_{\text{comp.}}$$

for $j = 1,\ldots, s+r+2(n-r) + m + 4$, $i = 1,\ldots,r$, and $k = 1,\ldots,m$, are known. The intrinsic relationships between the characteristics v_j depend directly on the utility function, so that the production relationships provide incomplete information about those of the characteristics. All of the other results derived for Models 2 and 3 may be obtained as special cases of Model 14.

In addition, the introduction of a new good or activity affects only the production relationships, provided that no new characteristics are introduced into the space of characteristics. The creation of a completely new characteristic is regarded as a relatively rare event. That is, under the assumptions that new characteristics are relatively rarely introduced and that preferences are relatively constant, the choice mechanism undergoes relatively unimportant changes with the introduction of new goods.

Another aspect of interest is product differentiation and advertising. If more information helps the consumer to better distinguish between inputs in his characteristics generating process, consumer technology is increased and more efficient choice is made. Since this analysis extends and makes the analysis of Lancaster more complete, the reader is referred to Lancaster's discussion on the same topics [34, p. 149 ff.].

Using the framework in Model 4, it seems possible to shed light on the notion of the quality of a good or activity. Basically, the quality of a good refers to its potential for yielding characteristics. Quality differences generally refer to different levels of potential for generating characteristics. If a good or an activity has variations in quality, those quantities of exactly the same quality constitute a good, while those of a different quality constitute a different good. The goods that differ only by quality variations are expected to be very nearly perfect substitutes. I discuss these issues in greater detail elsewhere [56].

A note on the measurability of characteristics seems necessary at this point. It seems unlikely that the set of characteristics of consumption will be isolated without the aid of psychologists. Indeed, will they then be isolated?

Is it too much to expect them to be common to all consumers and, furthermore, measurable? It seems possible that all of the empirically observable consumer behavior may be embodied in activities, goods, time inputs, money and time parameters, and the production relationships and demand functions associated with these goods and parameters. At least, this much provides us with considerably more to work with than we had with neoclassical theory.

IV. SOME APPLICATIONS

The interest and much of the behavioral intuition going into this analysis have arisen from efforts at the specification of empirical econometric models of outdoor recreation demand and supply structures. The literature is extensive[17], but some summarization of practice seems necessary. Economists and others studying recreation often obtain some cross sectional data on time spent participating in a particular recreation activity per time period, number of trips to a particular recreation site, fees paid, time and distance required for travel to the recreation site, facilities used, total quantity of facilities and possibly their capacity in some surrounding geographical region, socioeconomic variables, etc. Usually, a linear relationship is estimated the dependent variable of which is either the number of trips to the site per period or the amount of time spent participating in a particular recreation activity. The independent variables used may be any combinations of the above variables. The resulting equation is usually labeled a "demand function". The economic theory referred to is generally of the two dimensional diagram variety which is inadequate even to illustrate Model 1. The resulting confusion is a mish-mash of references to travel distance or cost as a proxy for price, fixed and variable recreation costs as replacements for prices, "supply creating its own demand", etc.

Employing the theory of Model 4, it is apparent that a structure specified for recreation demand should view participation in a particular recreation activity, visits to particular sites, recreational travel, etc. as consumption activities. The recreation facilities, transportation facilities, special equipment, nonrecreational goods, etc. are goods inputs and the variable participation times, travel times, visit times, etc. are time inputs into the recreation production process. The money and time parameters are respectively fees for participation, entrance fees, prices of goods and facilities employed in travel, use fees for facilities, rentals on equipment, wages, prices of other related goods and activities, and the wealth parameter for money and the fixed time inputs for purchased activities and the length of planning period for time. If the production process is known or has been estimated, the intrinsic substitution, complementarity and independence relationships will be available to be used as hypotheses. The theory implies the specification and estimation of demand relationships for each of the recreation and travel activities, the services of facilities, equipment and goods (along with the joint supply relationships), and the variable time requirements as functions of all of the parameters.

The services of a freeway or superhighway as a transportation good may be viewed in the context of Model 4. If transportation from point A to point B is viewed as a class of substitute activities and all routes from A to B yield the same characteristics, then the route chosen should be the one with the smallest fixed time requirement. Consumers then substitute the services of the superhighway or freeway for other routes because by so doing they are able to reduce their expenditures of time in transportation.

VI. CONCLUDING REMARKS

This paper has developed the essential components of a complete theory of choice that incorporates the time allocation problem and its economic relevance. The line of theory that culminates in Model 4, of course, generalizes the neoclassical theory and those theories of Becker [5] and Lancaster [32, 34] and increases the empirical relevance of each. From the list assumptions provided, it is apparent that many alternative models of behavior can be investigated and their implications for demand analysis deduced.

The mathematical technique utilized is about as simple as is available having been used in economic theory at least for most of the present century. Indeed, the technique is usually neither regarded as modern nor contemporary. However, it is well understood throughout the economics profession and lends readily to generalization and abstraction.

It is apparent that the time or money substitution and resource effects as components of the **rates** of change of quantities with respect to changes in time or money prices is indeed as Hicks pointed out [25, p. 309] the "Fundamental Equation of Value Theory". The fundamental equation is basic to Models 1 through 4 as an outgrowth of the mathematical technique.

Other results obtained include the illustration of a dual role for time as a price and as a good; the nonhomogeneity of demand functions in general in time parameters and also in money parameters; some results on perfect substitution as a limiting case of substitute goods; that intrinsic properties of goods and activities may be completely specified through knowledge of the production processes in terms of the algebraic signs of compensated rates of change; that intrinsic relations between characteristics depend on the utility function in addition to the production processes; some results on the introduction of new goods and activities, advertising, and product differentiation that agree closely with these of Lancaster [34]; and a definition of the quality of a good or activity.

It is apparent that Model 4 very likely is not the final form of the "New General Theory of Value" but is indeed a step in that direction. There probably is enough of interest contained in the class of models visualized here to keep economists (and psychologists) doodling for a while.

FOOTNOTES

(1) Becker [5] built a dependency into his constraints that allowed several constraints to be combined into one "resource" contraint. Thus his model of time allocation is not a multiple constraint problem.

(2) These objects of consumption are called objectives, higher commodities, characteristics or attributes by Hicks [24] and Morishima [37], Becker [5], Lancaster [34], and May [35] respectively.

(3) I do not wish to belittle any of the efforts in the areas of separable utility, additive utility, measurable utility, integrability of demand functions, revealed preference, dynamic consumer choice, or any of the mathematical abstractions [3, 4, 12, 13, 14, 15, 16, 17, 18, 19, 24, 27, 29, 31 33, 35, 36, 37, 38, 39, 40, 42, 50, 51, 56]. In fact, the work on separable and additive utility functions has much in common with the approach in this paper [3, 18, 19, 27, 37, 39, 50, 51]. However, the point remains that with the exceptions of the theory of rationing and some abstract notions of budget sets, the single linear budget constraint on behavior has remained as a sacred entity in economics up until Becker [5] and Lancaster [32, 34].

(4) A linear additive utility function may be regarded as simpler.

(5) This analysis subdivides Lancaster's production relationship. Goods and time are transformed into intermediate quantities called consumption activities by one production process and then consumption activities and are transformed into characteristics of consumption. The model reduces to Lancaster's model if the transformation of goods into consumption activities is one-to-one and onto, and time is omitted. Subdividing the production relationship in this way incorporates a mechanism in the theory to handle choices between various levels of putting the goods and time together into consumption activities by the consumer or of direct purchase of the consumption activity. Its relevance to time allocation problems is obvious.

(6) It has been pointed out by Brems [7] and Chipman [10] that measurability and commonality of characteristics among consumers are heroic assumptions at best and that commonality should be empirically tested. Continuity of the utility function requires that the consumer act as if he can measure infinitesimal differences in characteristics [56]. Engineers typically require only finite error tolerances on the finest of precision measuring instruments. It seems strange to assume that consumer judgments are calibrated to infinitesimal precision, but not so strange to assume that consumers act as if they are. If consumers act as if their measurement of characteristics is perfect, but in reality it is not, then behavior will not be repeatable. I have referred to this as the behavior of the "spastic consumer" [56]. This lack of measurability may be a principal reason that estimates of demand relationships from individual consumer or household data tend to fit poorly.

(7) For example, see the discussions in Brems [7], Chipman [10], Lancaster [32], and Scitovski [44].

(8) The n activities contain all activites participated in including all income earning and income expending activities. The p_i for work activities are negative but for all other activities, nonnegative. In one sense, the p_i may be the fee for the right to perform an activity.

(9) W does not represent income as in the neoclassical model. All income and expenditures are generated within the activities. Usually W=0. It is a residual that must remain after the earnings are balanced with expenditures. If $W < 0$ it may represent a previously prescribed long term level of saving. If $W = 0$ it represents that all income is spent, except that current saving may be viewed as an activity. If $W > 0$, it may represent a debt parameter that must be achieved exactly. These may be called parameters of forced saving and borrowing.

(10) The parameter T may be viewed in more than one way. If T is the amount of time in a specified time period, say one day, it is impossible in practice to vary T parametrically and interpret its meaning. That is, if a day were longer or shorter, consumers would adjust in a certain manner. However, it seems much more sensible to ask for results in a planning period context. If the planning period were one day, what would happen if it were one-half day or two days? The assumption is also being made that there is only one kind of time.

(11) Of course, other constraints are likely relevant in the short run. The work week might be restricted to 40 hours. There might be restrictions on credit. The amount of sleep required per day averages approximately 8 hours. These restrictions will vary from consumer to consumer and will be deleted from this analysis.

(12) Also we obtain

$$\frac{\partial(-\lambda)}{\partial P_i} = -\frac{(-\lambda)\Theta_{i,n+1} - X_i\Theta_{n+1,n+1}}{\Theta}$$

$$\frac{\partial(-\lambda)}{\partial t_i} = -\frac{(-\eta)\Theta_{i,n+1} - X_i\Theta_{n+2,n+1}}{\Theta}$$

$$\frac{\partial(-\lambda)}{\partial T} = -\frac{\Theta_{n+2,n+1}}{\Theta}$$

$$\frac{\partial(-\eta)}{\partial P_i} = -\frac{(-\lambda)\Theta_{i,n+2} - X_i\Theta_{n+1,n+2}}{\Theta}$$

$$\frac{\partial(-\eta)}{\partial t_i} = -\frac{(-\eta)\Theta_{i,n+2} - X_j\Theta_{n+2,n+2}}{\Theta}$$

$$\frac{\partial(-\eta)}{\partial T} = -\frac{\Theta_{n+2,n+2}}{\Theta}$$

for i, j=1, ...,n.

(13)

$$
\Theta = \det \left[\begin{array}{cccc|ccc}
\underset{n\times n}{[U_{ij}]} & \underset{n\times n}{[U_{ij}]} & \underset{n\times m}{[\,0\,]} & \underset{n\times n}{[\,0\,]} & \underset{n\times 1}{[\,0\,]} & \underset{n\times 1}{[\,q_i\,]} & \underset{n\times 1}{[\,t_i\,]} \\
\underset{n\times n}{[U_{ij}]} & \underset{n\times n}{[U_{ij}+\lambda f_{ij}^{zz}]} & \underset{n\times n}{[\lambda f_{ij}^{zy}]} & \underset{n\times n}{[\lambda f_{ij}^{zT}]} & \underset{n\times 1}{[\,f_i^{z}\,]} & \underset{n\times 1}{[\,0\,]} & \underset{n\times 1}{[\,0\,]} \\
\underset{m\times n}{[\,0\,]} & \underset{m\times n}{[\lambda f_{ij}^{yz}]} & \underset{m\times m}{[\lambda f_{ij}^{yy}]} & \underset{m\times n}{[\lambda f_{ij}^{yT}]} & \underset{m\times 1}{[\,f_i^{y}\,]} & \underset{m\times 1}{[\,P_i\,]} & \underset{m\times 1}{[\,0\,]} \\
\underset{n\times n}{[\,0\,]} & \underset{n\times n}{[\lambda f_{ij}^{Tz}]} & \underset{n\times m}{[\lambda f_{ij}^{Ty}]} & \underset{n\times n}{[\lambda f_{ij}^{TT}]} & \underset{n\times 1}{[\,f_i^{T}\,]} & \underset{n\times 1}{[\,0\,]} & \underset{n\times 1}{[\,1\,]} \\
\hline
\underset{1\times n}{[\,0\,]} & \underset{1\times n}{[\,f_i^{z}\,]'} & \underset{1\times m}{[\,f_i^{y}\,]} & \underset{1\times n}{[\,f_i^{T}\,]} & & & \\
\underset{1\times n}{[\,q_i\,]'} & \underset{1\times n}{[\,0\,]} & \underset{1\times m}{[\,P_i\,]'} & \underset{1\times n}{[\,0\,]} & & \underset{3\times 3}{[\,0\,]} & \\
\underset{1\times n}{[\,t_i\,]'} & \underset{1\times n}{[\,0\,]} & \underset{1\times m}{[\,0\,]} & \underset{1\times n}{[\,1\,]'} & & &
\end{array}\right]
$$

(14) If the consumer participates in an activity that uses money as an input, e.g. gambling, speculation, or even investment in stocks, bonds, etc., the money constraint becomes

$$\Sigma w_i q_i + \Sigma y_i p_i + \Sigma w_i = W$$

and the production function is $f(Z, Y, T^*, W^*) = 0$. By the above argument, the solution (demand and supply) functions h^i are no longer homogenous of degree zero in $q_1, \ldots, q_n, p_1, \ldots, p_m$ and W. Proportionate changes in prices (inflations and deflations) now effect quantities consumed!

(15)

$$
\Theta = \det \begin{bmatrix}
U_{11} & U_{12} & U_{13} & U_{13} & 0 & 0 & 0 & p_1 & t_1 \\
U_{21} & U_{22} & U_{23} & U_{23} & 0 & 0 & 0 & p_2 & t_2 \\
U_{31} & U_{32} & U_{33} & U_{33} & 0 & 0 & 0 & p_3 & b_3 \\
U_{31} & U_{32} & U_{33} & U_{33} & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda 2b & -\lambda c & (2b_y - c\tau) & p_y & 0 \\
0 & 0 & 0 & 0 & -cy & \lambda 2a & (2a\tau - cy) & 0 & 1 \\
0 & 0 & 0 & -1 & (2b_y - c\tau) & (2a\tau - cy) & 0 & 0 & 0 \\
P_1 & P_2 & P_3 & 0 & P_y & 0 & 0 & 0 & 0 \\
t_1 & t_2 & t_3 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
$$

(16)

$$\Theta = \det \left[\begin{array}{ccccc|cccc}
\underset{\text{sxs}}{[U^{vv}_{ij} + \lambda g^{vv}_{ij}]} & \underset{\text{sxr}}{[\lambda f^{vx}_{ij}]} & \underset{\text{sxn-r}}{[\lambda f^{vx}_{ij}*]} & \underset{\text{sxm}}{[0]} & \underset{\text{sxn-r}}{[0]} & \underset{\text{sx1}}{[f^{r}_{i}]} & \underset{\text{sx1}}{[0]} & \underset{\text{sx1}}{[0]} & \underset{\text{sx1}}{[0]} \\
\underset{\text{rxs}}{[\lambda f^{vx}_{ij}]} & \underset{\text{rxr}}{[\lambda f^{xx}_{ij}]} & \underset{\text{rxn-r}}{[\lambda f^{xx}_{ij}*]} & \underset{\text{rxn-r}}{[0]} & \underset{\text{rxn-r}}{[0]} & \underset{\text{rx1}}{[f^{x}_{i}]} & \underset{\text{rx1}}{[0]} & \underset{\text{rx1}}{[q_{i}]} & \underset{\text{rx1}}{[t_{i}]} \\
\underset{\text{n-rxs}}{[\lambda f^{vx}_{ij}*]} & \underset{\text{n-rxr}}{[\lambda f^{xx}_{ij}*]} & \underset{\text{n-rxn-r}}{[\lambda f^{xx}_{ij}** + \gamma g^{xx}_{ij}]} & \underset{\text{n-rxm}}{[\gamma g^{xy}_{ij}]} & \underset{\text{n-rxn-r}}{[\gamma g^{xT}_{ij}]} & \underset{\text{n-rx1}}{[f^{x}_{i}*]} & \underset{\text{n-rx1}}{[g^{x}_{i}]} & \underset{\text{n-rx1}}{[0]} & \underset{\text{n-rx1}}{[0]} \\
\underset{\text{mxs}}{[0]} & \underset{\text{mxr}}{[0]} & \underset{\text{mxn-r}}{[\gamma g^{xy}_{ij}]'} & \underset{\text{mxm}}{[\gamma g^{yy}_{ij}]} & \underset{\text{mxn-r}}{[\gamma g^{vT}_{ij}]} & \underset{\text{mx1}}{[0]} & \underset{\text{mx1}}{[g^{y}_{i}]} & \underset{\text{mx1}}{[p_{i}]} & \underset{\text{mx1}}{[0]} \\
\underset{\text{n-rxs}}{[0]} & \underset{\text{n-rxr}}{[0]} & \underset{\text{n-rxn-r}}{[\gamma g^{xT}_{ij}]'} & \underset{\text{n-rxm}}{[\gamma g^{yT}_{ij}]} & \underset{\text{n-rxn-r}}{[\gamma g^{TT}_{ij}]} & \underset{\text{n-rx1}}{[0]} & \underset{\text{n-rx1}}{[g^{T}_{i}]} & \underset{\text{n-rx1}}{[0]} & \underset{\text{n-rx1}}{[1]} \\
\hline
\underset{\text{1xs}}{[f^{v}_{i}]'} & \underset{\text{1xr}}{[f^{x}_{i}]'} & \underset{\text{1xn-r}}{[f^{x}_{i}*]'} & \underset{\text{1xm}}{[0]} & \underset{\text{1xn-r}}{[0]} & & & & \\
\underset{\text{1xs}}{[0]} & \underset{\text{1xr}}{[0]} & \underset{\text{1xn-r}}{[g^{x}_{i}]'} & \underset{\text{1xm}}{[g^{y}_{i}]'} & \underset{\text{1xn-r}}{[g^{T}_{i}]'} & \multicolumn{4}{c}{\underset{\text{4x4}}{[0]}} \\
\underset{\text{1xs}}{[0]} & \underset{\text{1xr}}{[q_{i}]'} & \underset{\text{1xn-r}}{[0]} & \underset{\text{1xm}}{[p_{i}]'} & \underset{\text{1xn-r}}{[0]} & & & & \\
\underset{\text{1xs}}{[0]} & \underset{\text{1xr}}{[t_{i}]'} & \underset{\text{1xn-r}}{[0]} & \underset{\text{1xm}}{[0]} & \underset{\text{1xn-r}}{[1]'} & & & &
\end{array} \right]$$

(17) The author will provide a bibliography of the recreation economics literature upon request.

REFERENCES

(1) Adelman, Irma, and Griliches, Zvi, "On An Index of Quality Change", American Statistical Association Journal, 1961.

(2) Allen, R. G. D., Mathematical Economics, 2nd Ed., MacMillan, London, 1963.

(3) Arrow, Kenneth J., "Additive Logarithmic Demand Functions and the Slutsky Relations", Review of Economic Studies, 1961.

(4) Baumol, William J., "The Neumann-Morgenstern Utility Index -- An Ordinalist View", Journal of Political Economy, 1951.

(5) Becker, Gary S., "A Theory of the Allocation of Time", The Economic Journal, 1965.

(6) Bilkey, Warren J., The Basic Relationships in Consumer Expenditure Behavior, Cambridge, Massachusetts, 1951.

(7) Brems, Hans, "Allocation and Distribution Theories: Technological Innovation and Progress, Discussion", American Economic Review, 1966.

(8) Brems, Hans, Product Equilibrium under Monopolistic Competition, Harvard University Press, Cambridge, 1963.

(9) Chamberlin, Edward, The Theory of Monopolistic Competition, Harvard University Press, Cambridge, 1963.

(10) Chipman, John S., "Allocation and Distribution Theories: Technological Innovation and Progress, Discussion", American Economic Review, 1966.

(11) Debreu, Gerard, "Definite and Semidefinite Quadratic Forms", Econometrica, 1952.

(12) Debreu, Gerard, Theory of Value, an Axiomatic Analysis of Economic Equilibrium, Wiley, New York, 1959.

(13) Diamond, Peter A., "The Evaluation of Infinite Utility Streams", Econometrica, 1950.

(14) Farrell, M. J., "Some Aggregation Problems in Demand", The Review of Economic Studies, 1953-54.

(15) Friedman, Milton, and Savage, L. J., "The Expected-Utility Hypothesis and the Measurability of Utility", The Journal of Political Economy, 1952.

(16) Gale, David, "The Law of Supply and Demand", Matematica Scandanavia, 1955.

(17) Georgescu-Roegen, Nicholas, "Choice Expectations and Measurability", Quarterly Journal of Economics, 1954.

(18) Gorman, W. M., "Additive Logarithmic Preferences a Further Note", Review of Economic Studies, 1963.

(19) Gorman, W. M., "Separable Utility and Aggregation", Econometrica, 1959.

(20) Graaf, J. de V., "Towards an Austerity Theory of Value", South African Journal of Economics, 1948.

(21) Hadley, G., Nonlinear and Dynamic Programming, Addison-Wesley, Reading, Massachusetts, 1964.

(22) Hancock, Harris, Theory of Maxima and Minima, Dover Pub. Inc., New York, 1960.

(23) Henderson, J. M., and Quandt, R. E., Microeconomic Theory, A Mathematical Approach, McGraw-Hill, New York, 1958.

(24) Hicks, J. R., A Revision of Demand Theory, Oxford University Press, 1956.

(25) Hicks, J. R., Value and Capital, 2nd Ed., Clarendon Press, Oxford, 1946.

(26) Hicks, J. R., and Allen, R. G. D., "A Reconsideration of the Theory of Value", Economica, 1934.

(27) Houthakker, H. S., "Additive Preferences", Econometrica, 1960.

(28) Houthakker, H. S., "Compensated Changes in Quantities and Qualities Consumed", Review of Economic Studies, 1951-52.

(29) Houthakker, H. S., "Revealed Preference and the Utility Function", Econometrica, 1950.

(30) Johnson, Harry G., "Demand Theory Further Revised or Goods are Goods", Economica, 1958.

(31) Koopmans, T. C., Diamond, Peter A., and Williamson, Richard E., "Stationary Utility and Time Perspective", Econometrica, 1964.

(32) Lancaster, Kelvin, "Allocation and Distribution Theory", Technological Innovation and Progress - Change and Innovation in the Technology of Consumption, American Economic Review, 1966.

(33) Lancaster, Kelvin J., "An Axiomatic Theory of Consumer Time Preference", International Economic Review, 1963.

(34) Lancaster, Kelvin J., "A New Approach to Consumer Theory", Journal of Political Economy, 1966.

(35) May, Kenneth, "Intransitivity, Utility, and the Aggregation of Preference Patterns", Econometrica, 1954.

(36) McKenzie, Lionel, "Demand Theory Without A Utility Index", Review of Economic Studies, 1956-57.

(37) Morishima, Michio, "The Problem of Intrinsic Complementarity and Separability of Goods", Metroeconomica, 1959.

(38) Richter, Marcel K., "Revealed Preference Theory", Econometrica, 1966.

(39) Samuelson, Paul A., "Constancy of the Marginal Utility of Income", Studies in Mathematical Economics and Econometrics, The University of Chicago Press, 1942.

(40) Samuelson, Paul A., "Consumption Theory in Terms of Revealed Preference", The Collected Scientific Papers of Paul A. Samuelson, 1966.

(41) Samuelson, Paul A., Foundations of Economic Analysis, Harvard University Press, Cambridge, 1963.

(42) Samuelson, Paul A., "The Problem of Integrability in Utility Theory", Economica, 1950.

(43) Samuelson, Paul A., "Prices of Factors and Goods in General Equilibrium", The Review of Economic Studies, 1953-54.

(44) Scitovsky, Tibor, "Allocation and Distribution Theories: Technological Innovation and Progress, Discussion", American Economic Review, 1966.

(45) Scitovsky, Tibor, "The Political Economy of Consumers Rationing", The Review of Economics and Statistics, 1942.

(46) Slutsky, Eugen E., "On the Theory of the Budget of the Consumer", Reprinted in A.E.A. Readings in Price Theory, Vol. VI, Stigler and Boulding, Editors, Richard D. Irwin, Inc., 1952.

(47) Smith, Victor, "Linear Programming Models for the Determination of Palatable Human Diets", Journal of Farm Economics, 1959.

(48) Stigler, George J., "The Cost of Subsistence", Journal of Farm Economics, 1945.

(49) Stone, Richard, "Quantity and Price Indexes in National Accounts", Organization for European Economic Co-operation, 1956.

(50) Strotz, Robert H., "The Empirical Implications of a Utility Tree", Econometrica, 1957.

(51) Strotz, Robert H., "The Utility Tree -- A Correction and Further Appraisal", Econometrica, 1959.

(52) Theil, H., "Qualities, Prices and Budget Enquiries", Review of Economic Studies, 1953.

(53) Tobin, J., "A Survey of the Theory of Rationing", Econometrica, 1952.

(54) Wilson, Robert R., "A Note on Maximization Subject to Homogeneous Constraints", Tech. Article No. 7925, Texas Agri. Exp. Sta., (Mimeo, March 1969).

(55) Wilson, Robert R., "The Theory of Consumer Behavior: A Schematic Extension to the Demand for Outdoor Recreation", Tech. Art. 7922, Texas Agri. Exp. Sta., (Mimeo, February 1969).

(56) Wilson, Robert R., "The Theory of Consumer Behavior in Commodity Grading: Theoretical Results and Implications for Public Policy", Tech. Article No. TA-8064, Texas Agri. Exp. Sta., (Mimeo, 1969).