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The Resource Misallocation Costs of Inefficient Regulation of the Pacific Coast Halibut Industry: A Preliminary Discussion

by

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*This is a rough draft of a preliminary final report on a research grant, No. 14-17-0007-993 from the U.S. Department of Commerce, National Marine Fisheries Service, to the University of Minnesota. It is preliminary in the sense that it does not report on some research currently in process that will be included in the final draft. It is rough in that it probably contains factual errors, and fails either to describe accurately or, indeed, to acknowledge the related work of others. Comments and corrections would be greatly appreciated.

I Introduction

It has long been recognized that a fishery to which access is unrestricted will inevitably be exploited inefficiently. This inefficiency may involve either a "wrong" level of output or "wrong" input combinations, or both. As for output level, if demand for a fishery's product is modest (in a sense to be defined more exactly in Section II) relative to its productive capacity, it will be used to produce fish at too high a rate. That is, output from it will expand to a level at which the total cost of producing an additional pound of fish exceed the value of that pound to those who consume it. Regarding input combinations, unrestricted exploitation can (but not necessarily will) lead to situations in which the costs incurred by fishermen exceed those at which the output levels they achieve could have been produced. The evidence (see Section III) strongly suggests that such a state of affairs existed in the Pacific halibut fisheries at least through the early 1940s.

By suitably restricting the behavior of fishermen, regulatory bodies could virtually eliminate both sorts of inefficiency. The controls currently imposed by regulatory bodies undoubtedly reduced these inefficiencies to some degree. Unfortunately, however, current control devices succeed in reducing the inefficiency associated with unrestricted exploitation only by introducing a second set of inefficiencies. These controls--limitations on, for example, boat size, season length, port turn-around time, fishing gear employed, and areas in which fish may be caught--succeed by increasing the costs of catching any given quantity of fish to a level greater than the minimum cost at which that quantity can be caught.[1] In assessing the effectiveness of current

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[1] A qualification is in order: the long run effect of gear restrictions may not on balance be to increase fishing costs. See Section II.

regulatory procedures, these latter inefficiencies together with the costs of regulation must be balanced against the inefficiencies of unrestricted exploitations.

This, then, is the basic task to which this study has been directed: determining the nature of optimal regulatory procedures for the Pacific halibut fisheries and developing quantitative comparisons of the results that could be anticipated from applying them with those of (a) current regulatory procedures and (b) unrestricted exploitation. To this end, Section II discusses the biological process that takes place in a fishery, the way in which the long run bio-economic equilibrium of that fishery can be inferred from this biological process, the nature of an efficient regulatory procedure, and the resource costs resulting from inefficient or no regulation. Section III describes the procedures used to infer the functional forms and specific paramenter values for some of the relationships required to quantify the inefficiencies discussed in Section III and the results of this work. Section IV deals with the future research necessary fully to compare the optimal regulation, current regulation, and no regulation alternatives. This additional work falls into two categories: some would require a very modest extension of the work described here--the algebraic analysis and computer programming requiring at most a few additional weeks effort. Much of this work is either currently in process or could be undertaken before writing the final draft of this study. The second category involves analysis that would require considerably more substantial effort and therefore must await the availability of both more time and more money.

II Long Run Equilibrium of a Fishery and the Losses Involved in Its Inefficient Exploitation

For the purpose at hand, the biological process that transpires in a fishery can be summarized by a simple differential equation:

$$dS/dt = g(S) - h(E, S) \qquad (1)$$

This equation says that the instantaneous growth rate, dS/dt, of a stock of fish, S, equals the rate at which fish would grow in the absence of exploitation, g(S), minus the rate at which they are caught, h(E, S). This latter function indicates that the catch rate depends on both the effort, E, expended by fishermen and the stock of fish.

Two distinct lines of analysis can be identified in the biological literature regarding the nature of the growth function, g(S). In one, the eumetric theory developed initially by Beverton and Holt, S is a vector $(S_1, \ldots S_n)$ where S_i is the weight or biomass of i year old fish. At least in its simplest form, this theory assumes the growth rate of S_i to be a function only of i. That is, an age cohort has a percentage growth rate—an own-rate of interest--that is independent of the size of either the age cohort or of all other age cohorts. In the absence of exploitation, an age cohort's own-rate of interest equals the rate at which surviving fish grow minus the rate at which fish are lost through natural mortality. Except possibly at the youngest stages, the sum of these two growth rates diminishes with age and ultimately becomes negative when the slow rate of growth of surviving older fish is more than offset by their natural mortality.

In a second line of analysis associated with, e.g. Schaeffer, S is regarded as a single number, the biomass of an entire fish population. In this line of analysis, the net growth rate in a function of two offsetting forces. On the one hand, the larger the biomass, the greater is the number of fish breeding and growing. On the other hand, the greater the biomass, the greater is the pressure on the limited capacity of the fishery to support life. When the biomass is small, the former force dominates and the growth rate is an increasing function of S. Beyond some point, however, the capacity of the fishery to support life comes to dominate and the growth rate declines with increases in biomass until some large population size--M in subsequent analysis--the biomass no longer grows, i.e., dS/dt is zero.

Both lines of reasoning seemed to have merit. Contrary to the implications of the Schaeffer approach, young fish do grow more rapidly than old fish. On the other hand, contrary to the implications of the Beverton-Holt approach, the capacity of a fishing bank to sustain life is limited. An initial goal of this study was to synthesize these two approaches. That is, it was hoped that it would prove possible to develop and test a model in which growth depends on both the total biomass and the age distribution of the fish population. Unfortunately, data, time, and intellectual restrictions made it impossible to achieve this goal. It proved possible to work only with variations on the basic Schaeffer model.

For purposes of this section, it is useful to work only with long run equilibrium relationships. That is, it is useful to consider fisheries in which the instantaneous catch rate equals the instantaneous natural growth rate and in which dS/dt is therefore zero. In such an equilibrium, the following equalities hold:

$$g(S) = h(E, S) = C$$

where C represents the equilibrium catch rate. If g(S) is a reasonably well behaved function, the relationship g(S) = C can be inverted to yield S = G(C)--a relationship giving equilibrium stock as a function of equilibrium catch.[2] Using this relationship to replace S in h(E, S) = C and again inverting yields

[2]As will be developed below, the function h is not, in general, single valued. Except where C takes a value equal the maximum sustained yield of a fishery, any given catch rate can be sustained at either of two stock levels. The same consideration applies to each of the additional relationships discussed in the remainder of this paragraph.

$E = H(C)$— the effort level required to yield the equilibrium catch. Finally, the equilibrium cost of catching C pounds of fish is a function, $f(E)$, of effort level. That is, $f(E) = f[H(C)] = F(C)$.

To illustrate these calculations it is useful to employ the simplest formulation of equation (1) that has been analyzed in this existable statistical work described in Section III:

$$dS/dt = G(M - S) - qES \tag{2}$$

In this expression G is a growth rate coefficient, M is the maximum sustainable stock of fish—the biomass which the fishery would approach if left unexploited—and q is a "catchability coefficient"—the fraction of the existing fish stock that a single unit of effort would catch. It should be noted that the catch rate function in equation (2) implies what could be termed "instantaneous constant returns to scale for fishing effort." That is, it implies that the percentage of the existing fish stock that would be caught by an additional unit of effort is independent of the level at which effort is applied. Instantaneous diminishing returns to effort—a state of affairs in which successive equal increments to the effort level would yield successively smaller increments to total catch—might seem a more plausible assumption. The possible existence of diminishing returns to effort is one of the phenomena tested in the statistical analysis described in Section III.

If equation (2) does describe the functioning of a fishery, then in equilibrium:

$$C = G(M - S) = qES \tag{3}$$

Differentiating C with respect to S and setting the result equal to zero reveals that the maximum equilibrium yield—more commonly termed the maximum sustained yield—is achieved when capital $S = M/2$. Subsatuting M/2 for S in $C = G(M - S)S$ yields $GM^2/4$ as the maximum sustainable yield, C_{max}.

Solving = G(M - S)S to determine S as a function of equlibrium catch yields:

$$S = M/2 \pm (M^2 - 4C/G)^{1/2}/2 \quad (4)$$

This relationship can be simplified by expressing equlibrium catch as a fraction, a of maximum sustained yield. Substituting $C = aGM^2/4$ into equation (4) yield

$$S = [1 \pm (1-a)^{1/2}]M/2 \quad (5)$$

Equation (4) and (5) indicate that, except for the maximum sustained yield, any level of output can be achieved at either of two stock levels. One of these is greater and the other less than that which would maximize the sustained yield.

If $C = a\,C_{max} = qES$ the effort required to catch C pounds of fish a year is $E = aC_{max}/qS$. That is, required effort is inversely proportional to equilibrium stock. Thus, the effort required to produce aC_{max} pounds of fish a year would be smaller at the higher of the two equlibrium stock levels given by equations (4) and (5). Specifically, substituting $aC_{max} = aGM^2/4$ and equation (5) into $E = aC_{mas}/qS$ yields:

$$E = aGM/[2q(1 + (1-a)^{1/2})] \quad (6a)$$

and

$$E = aGM/[2q(1 - (1-a)^{1/2})] \quad (6b)$$

as the required effort levels when the biomass is respectively greater than and less than M/2. If effort is costly, it would clearly be desirable to produce any given quantity as fish with the least possible effort. Production of aC_{max} pounds of fish at the effort level given by equation (6b) would be inefficient. Indeed, it can be shown[3] that, if a fishery is in long run equilibrium under the conditions implied by equation (6b), a <u>dec</u>rease in

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[3]By differentiating equation (6b) with respect to A and re-arranging terms.

the equilibrium effort level would lead to an increase in equilibrium yield. That is, under equation (6b) cost conditions, the marginal product of fishing effort is negative.

Suppose that effort can be regarded as a composite input to fishing that is supplied by a competitive industry at a price that is independent of the rate at which fishing activity takes place. Then the units in which effort and q are measured can be specified so that one unit of effort costs $1. With units specified in this fashion, equation (6a) and (6b) give the total costs in long run equilibrium of catching aC_{max} fish per year when, to repeat the equilibrium stock of fish is respectively greater than less than M/2. Division of equations 6 by $aC_{max} = aGM^2/4$ yields the average cost of a pound of fish as a function of output level while differentiation of (6a) respect to aC_{max} yields the associated marginal cost:

$$\text{Efficient average cost} = AC = 2/[qM(1 + (1-a)^{1/2})] \quad (7a)$$

$$\text{Inefficient average cost} = AC' = 2/[qM(1 - (1-a)^{1/2})] \quad (7b)$$

$$\text{Efficient marginal cost} = MC = AC + a/[qM(1 + (1-a)^{1/2})^2(1-a)^{1/2}] \quad (8)$$

These schedules are listed in Table 1 and are plotted in Figure I. The data plotted in Figure I form the basis for Figures II-VI albeit with changed values on the horizontal and vertical axes.

Suppose the fish demand schedule is $ABCD_1$ as shown in Figure II. Efficiency would then dictate producing OX pounds of fish a year—70% of maximum sustainable yield. At this output level the marginal cost of the effort required to produce a pound of fish equals the price a marginal consumer would be willing to pay for it. At an output of $0.7C_{max}$, the marginal and average costs of fish are respectively $1.82/qM and $1.19/qM. The authority responsible for the fishery could induce fishermen to supply the effort level

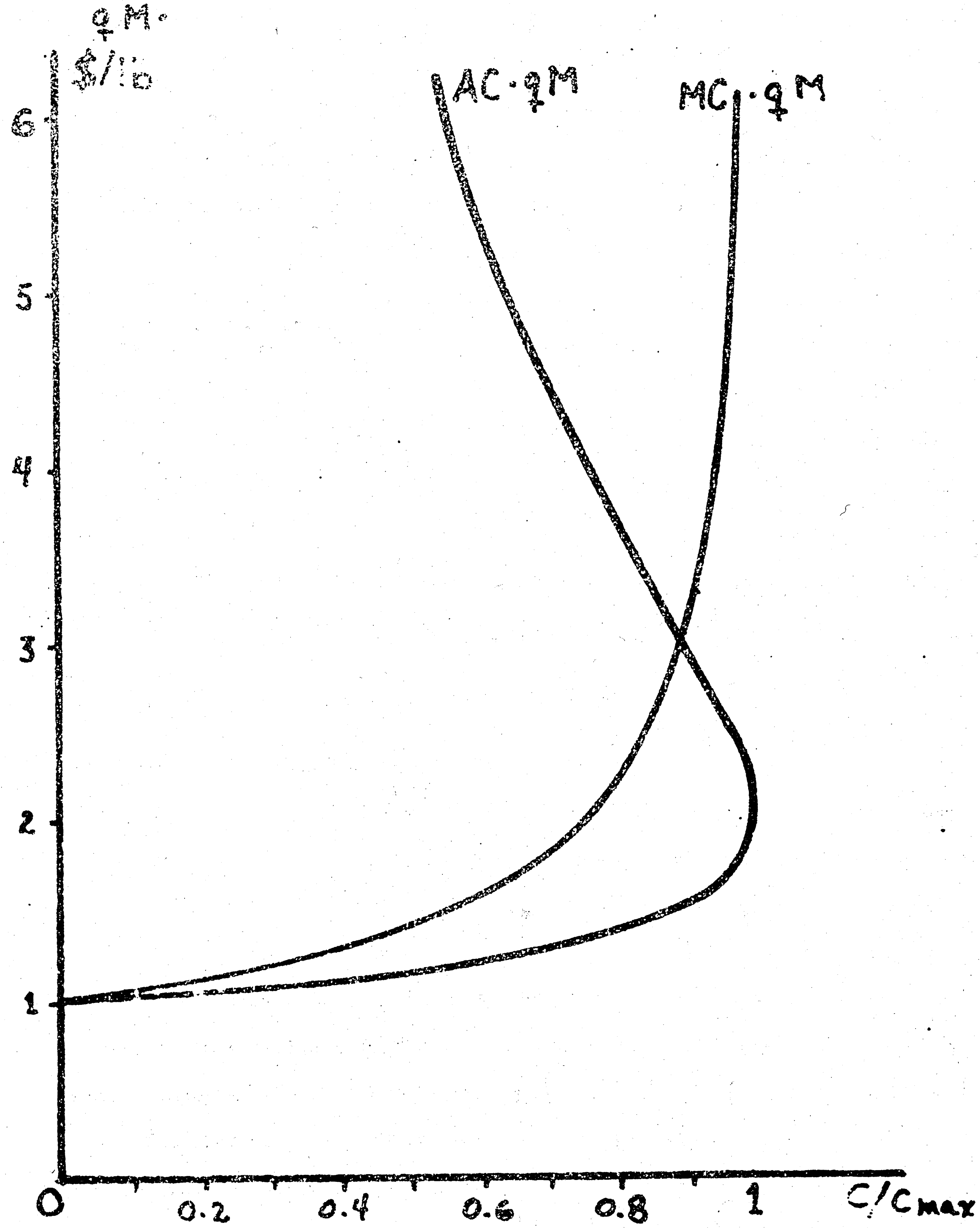

FIGURE 1

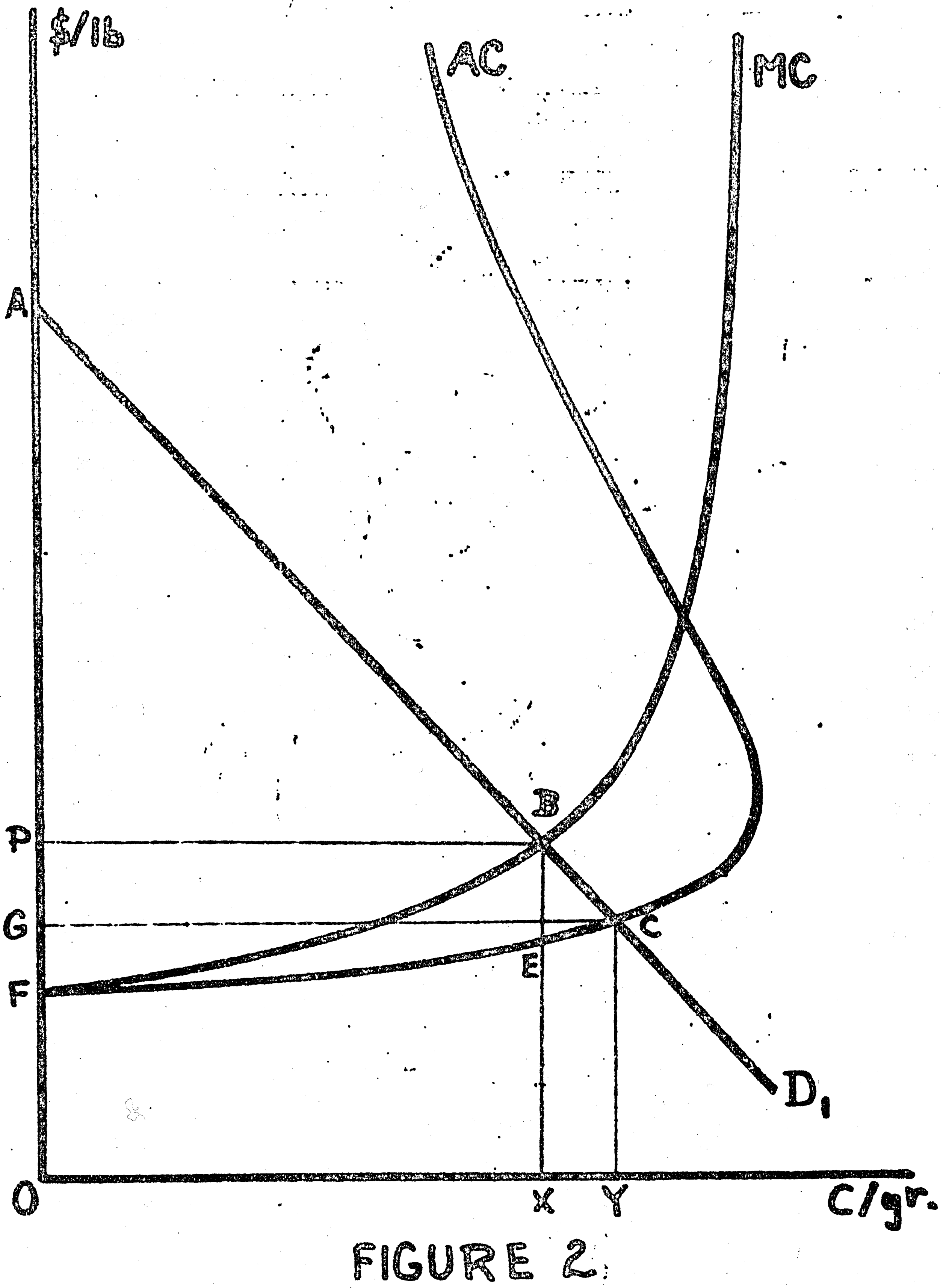

FIGURE 2

Table 1: qM Times Average and Marginal Costs of Fish at Alternative Ratios of Actual Catch (c) to Maximum Sustained Yields (C_{max})

C/C_{max}	qM Times		
	Efficient AC	Inefficient AC	Efficient MC
0	$1.00	∞	$1.00
0.1	1.03	$40.00	1.06
0.2	1.06	18.20	1.12
0.3	1.09	12.12	1.20
0.4	1.13	8.70	1.29
0.5	1.17	6.83	1.41
0.6	1.23	5.40	1.59
0.7	1.29	4.42	1.82
0.8	1.38	3.64	2.23
0.9	1.53	2.90	3.17
0.95	1.64	2.52	4.48
1.0	2.00	2.00	∞

necessary to produce this output level in a variety of ways. However, the most straightforward technique would be to impose a tax of BE = $0.53/qM per pound of fish caught.

Efficient utilization of the fishery under these circumstances would generate a net benefit to society with a dollar value equal to the area ARF. This area can be broken into two parts: first is a consumer's surplus, ABP which equals the sum over all OX pounds of fish of the difference between the price some consumer would be willing to pay for each pound and the price, OP he actually does pay for that pound. Second is what, for a more normal production process[4] would be termed a rent or producer's surplus PBF. It equals the difference between the total revenue, OPBX, received from the sale of 0.7 C_{max} pounds of fish and the cost, OFBX(=OMEX) of the fishing effort required for annual production of 0.7 C_{max} pounds of fish. As in any other production process, this rent/producer's surplus would reflect the value of the marginal product of a fixed input--in this case, the limited capacity of the fishery to grow fish. If annual output is restricted to OX pounds through the imposition of a tax, this rent would accrue in the form of tax collections entirely to the regulatory authority and hence the society in whose interests it presumably operates.

A brief aside is in order regarding the way in which the conclusions reached in the preceding paragraphs would be affected if, contrary to the assumptions underlying Figure I fish growth rates depend on their age distribution as well as their total weight. As Turvey and perhaps others have noted, if the own-rate of interest of a fish decreases with increases in its age, a tax based simply on pounds of fish caught would not, in general, result in

[4] That is, one for which a single producer supplies both fixed and variable inputs.

efficient exploitation of a fishery. Rather, efficiency would require additional incentives to avoid taking small, high growth rate fish. Turvey suggests the appropriate incentives to be restrictions on fishing technology such as limitations on hook or mesh size. Such restrictions undoubtedly would do the trick. However, a tax schedule under which the rate per pound for a fish is inversely related to its size would, at least in principle, be perhaps even more effective.

Reverting again to Figure II, suppose that no restrictions are imposed on the exploitation of the fishery. Output would then expand to OY = $0.8C_{max}$ a year. At this level, the cost incurred in catching a pound of fish, \$1.38/qM, equals the market price for fish. At this output level, the consumer's surplus generated by the fishery increases from ABP to ACG. However, this increase in consumer benefit is more than offset by the fact that equilibrium at OY pounds per year yields zero rent to the fishery.

To put the matter differently, expansion of output by $0.1\ C_{max}$ from OX to OY yields fish on which consumers place an aggregate value of XBCY. However, the additional cost incurred in producing these fish equals XBDY. Thus, the cost of output expansion exceeds the resulting benefits--the decrease in fishery rent exceeds the increase in consumer surplus--by an amount equal to area BCD in Figure II. To suggest the orders of magnitude involved, as demand curve $ABCD_1$ is drawn, BCD has an area of approximately $0.043C_{max}$/qM. The market value of fish (which equals the costs incurred by fishermen) OGCY at an output of OY per year is $1.38C_{max}$/qM. Hence, the loss involved in increasing from output from OX to OY amount to 0.043/1.38 or 3.1% of total outlays on fish in the inefficient equilibrium.

Suppose, now, that the demand for fish is as shown in Figure III, $GHJD_2$. Efficiency would then dictate an output level of OZ = $0.92C_{max}$. As with the

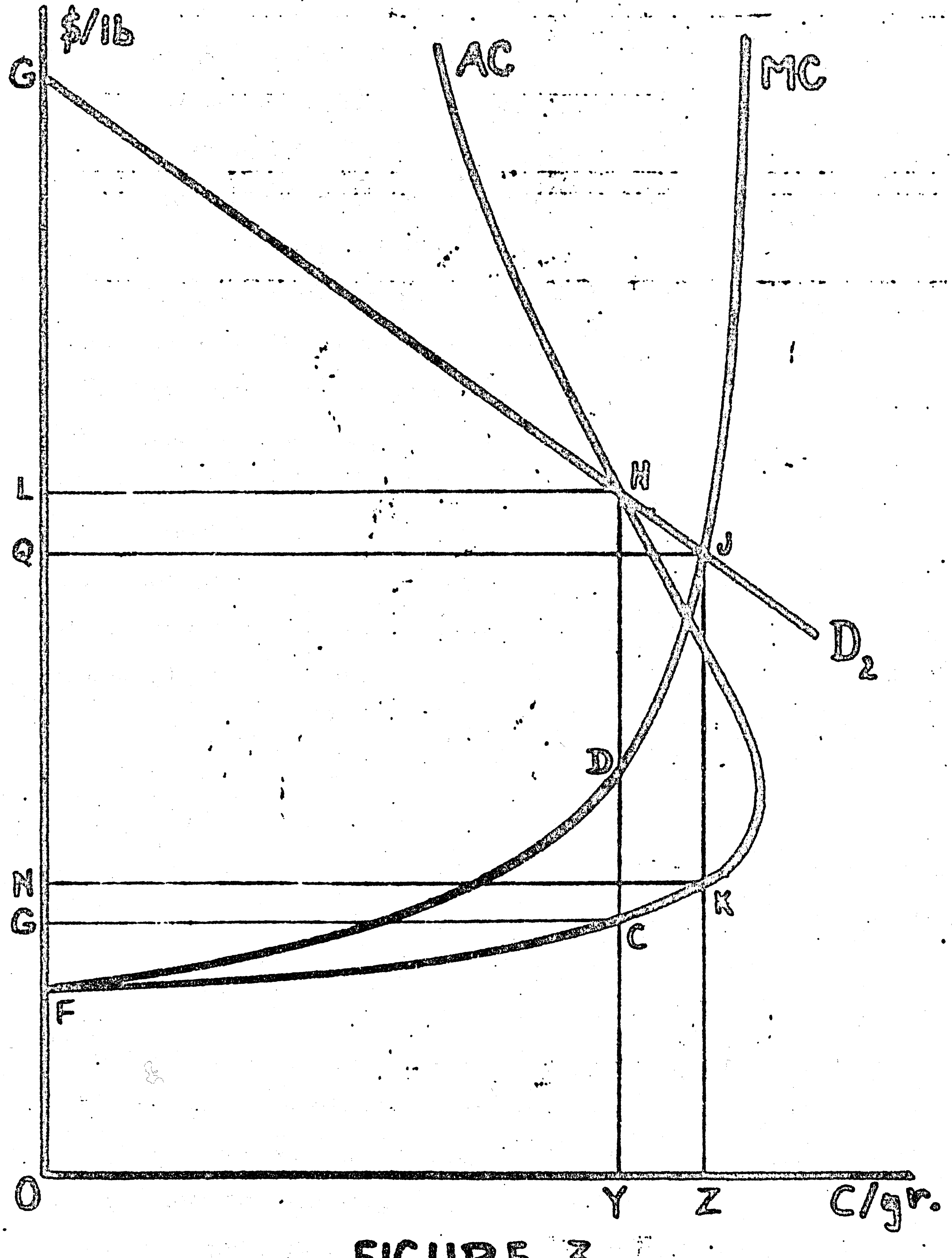

FIGURE 3

efficient equilibrium associated with demand schedule $ABCD_1$ in Figure II, OZ is the output level at which demand intersects marginal costs. At this output level, the marginal and average costs per pound of fish are respectively \$3.47/qM and \$1.59/qM. A tax of \$1.88/qM per pound would therefore be required to induce fishermen to supply the efficient effort level. In this equilibrium, the net benefit of the fishery to society would be the area FJG in Figure III. As before, this benefit can be broken into two parts. First is a consumer's surplus of GJQ; second is a rent or producer's surplus of FJQ(=QJKN).

In the absence of restrictions on entry, the fishery would reach equilibrium at H in Figure III. As when demand schedule $ABCD_1$ in Figure II was assumed to be in effect, equilibrium at H involved equality of the cost incurred in producing a pound of fish, \$3.64/qM, with the market price of fish. Under these circumstances, the only benefit derived from exploitation of the fishery would be GHL, the consumer's surplus generated when OY pounds of fish are consumed at a price of YH = \$3.64/qM per pound.

The difference between the benefits resulting from equilibrium at H and those possible with efficient exploitation of the fishery is the area LHJF. This area can be broken into two parts: first is a loss in potential consumer surplus, LHJQ, resulting from consumption of OY pounds of fish at \$3.64/qM per pound rather than OZ pounds at \$3.47/qM. Second is the loss in rent on the fishery, QJF(=QJKN).

The loss associated with equilibrium at H can be interpreted in an alternative fashion. The total cost of producing OY pounds of fish can be interpreted as OY times the average cost associated with that output level or alternatively as the area between O and Y under the marginal cost of producing schedule. This being the case, area OGCY in Figure III equals area OFDY. Hence, area LHJF—the loss associated with production at H rather than at J—

is equal to area LHJDCG. This latter area can be broken into two parts: first is LHCG, the additional cost of the fishing effort required to produce OY pounds of fish inefficiently rather than efficiently. The second part is the area HJD. It equals the additional benefit that could be derived from increase in fish output from OY to OZ if these two output levels were produced at minimum cost. To put the matter in different terms, LHCG is the loss associated with using an inefficient combination of inputs (i.e., an inefficient combination of fish stock and effort) while HJD is the loss resulting from producing an inefficient level of output.

This latter interpretation of the loss associated with producing at H rather than at J in Figure III can be used to interpret the loss resulting from a third possible type of equilibrium depicted in Figure IV. In this Figure, an efficient allocation of resources will require producing OX pounds of fish at a marginal cost of BX. Total benefits from exploiting the fishery would then equal the area ABF. With unrestricted entry, the fishery would be in equilibrium at C. The loss involved in operating at this equilibrium rather than at B can be interpreted as the sum of areas HCDG and BCJ. The latter area is the loss resulting from producing an output level (OY) different from that (OX) at which price equals marginal cost. Area HCDG is the loss resulting from producing this (inefficient) equilibrium output at a cost (CY) greater than that (DY) at which it could be produced.

To summarize briefly, unrestricted access to a fishery can lead to inefficient resource allocation of two sorts.[5] First, use of restricted access will almost certainly result in an inefficient output rate, i.e., an output rate different from that at which, given minimum cost production, price equals

[5] In addition to the losses that may result from catching fish at too young an age if the percentage growth rate of a fish declines with its age.

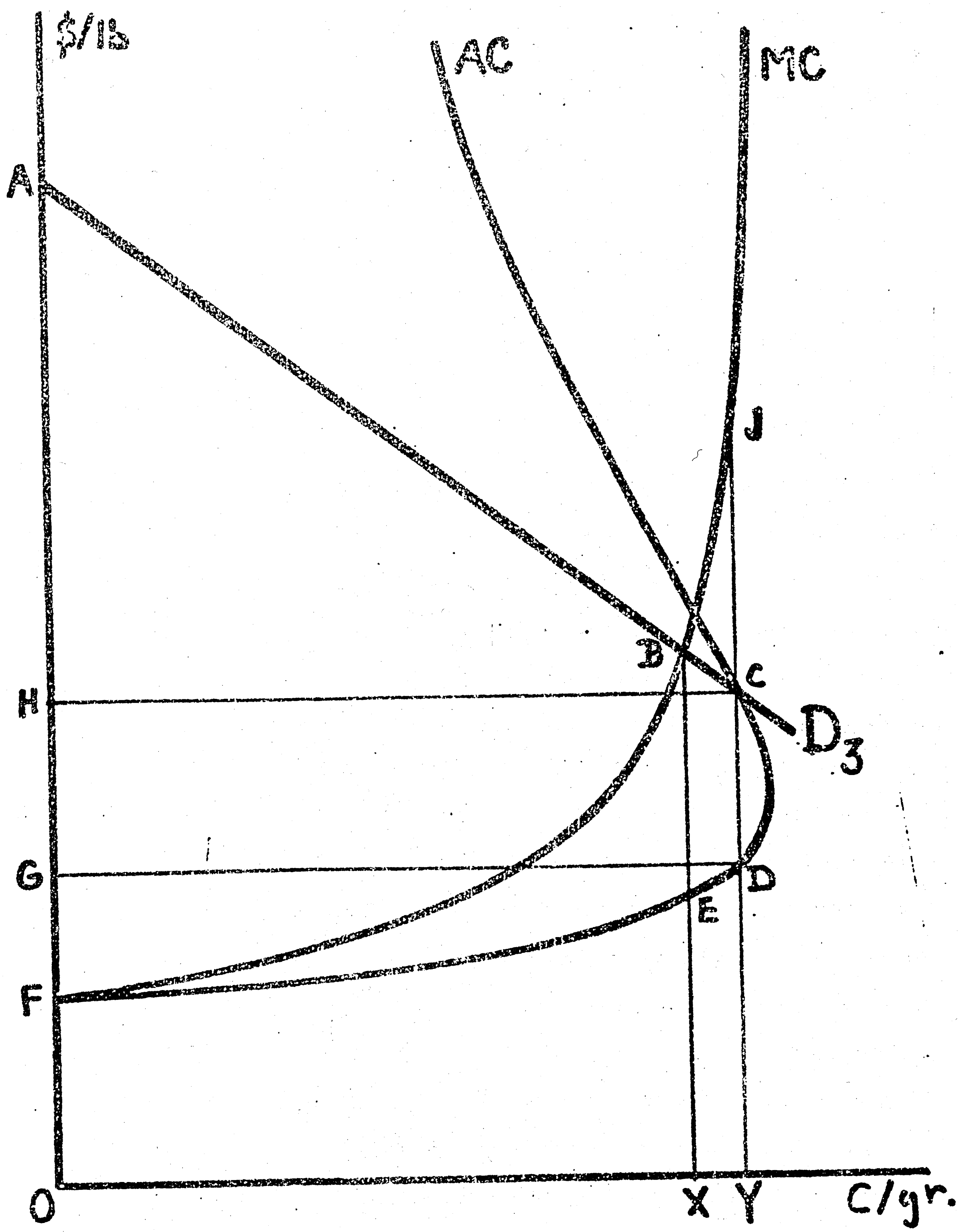

FIGURE 4

marginal cost. Second, given interrelationships between demand and average cost schedules of the sort depicted in Figures III and IV, unrestricted access can result in wasteful production of whatever level of output market forces settle upon. That is, unrestricted entry can result in the state of affairs in which it would be possible to produce whatever level of output is settled upon at (perhaps substantially) lower costs than those which are actually incurred.

Any of a variety of controls could be adopted to eliminate these inefficiencies. At least conceptually, the simplest control device would be to levy a tax equal to the difference between the marginal and the average cost of fish at that output rate at which demand and marginal cost schedules intersect. If the growth rate of a fish population depends only on its biomass, the appropriate tax per pound would be independent of the size of fish caught. If the population's growth rate depends on its age distribution as well as its biomass, the appropriate tax per pounds would vary inversely with size of fish.

Regulatory bodies do, of course, control access to many fisheries. However, there is no fishery (at least no fishery of which I am aware) in which controls take the form of taxes on output. Rather, under the typical control system, output is regulated by allowing free access to any fishing vessel that abides by restrictions on such things as vessel and crew size, gear employed, length of season, area fished, and port turn-around time. Except for those gear restrictions which serve efficiently to limit catch to slow growing age groups, all of these control devices are effective in restricting output only to the extent that they raise the long run average cost of catching any given quantity of fish. Imposition of season length limitations, for example, result in either vessels, gear, and crews spending more idle time in port than they otherwise would or in the less specialized vessels and duplicate sets of gear necessary to exploit different fish populations during the open seasons for them.

The geometry of Figures II-IV can be used to suggest the effects of cost increasing restrictions on the benefits derived from fishery exploitation. Suppose, first, that a fishery is an initial equilibrium with unrestricted entry of the sort depicted in Figure II. Specifically, annual catch is OY pounds in Figure V while OZ is the catch at which marginal cost and demand schedules intersect. In this equilibrium, the loss due to over-exploitation of the bank of the area ACE. Suppose that a cost increasing restriction is imposed on fishermen that leads to a new equilibrium output of OX and equilibrium costs per pound of XG. At this new equilibrium, resources valued at XBCY are released for production elsewhere in the economy. Since the fish produced by these resources are valued at only XGEY, the output reduction yields a saving of BCEG. It does so, however, only at the expense of making the total cost of producing OX pounds of fish per year HGFJ greater than the minimum cost, OJFX, at which this quantity of fish could be produced. In Figure V, BCEG is clearly smaller than JGFJ. Imposition of the restriction therefore would result in a net loss. In specific situations, cost and demand elasticities could be such that the counterpart of BCEG would exceed that of HGFJ. However, the gain possible would likely be small. Furthermore, Figure V takes no account of the costs incurred by the regulatory body in imposing the restrictions involved in reducing output from OY to OX. These costs could well eliminate whatever gain Figure V would suggest to result in a specific situation.

In brief, <u>if</u> the equilibrium associated with unrestricted entry to a fishery is the sort depicted in Figures II and V—an equilibrium in which the only loss involved is that associated with producing an output in excess of that which would equate demand and marginal cost--and <u>if</u> the only way in

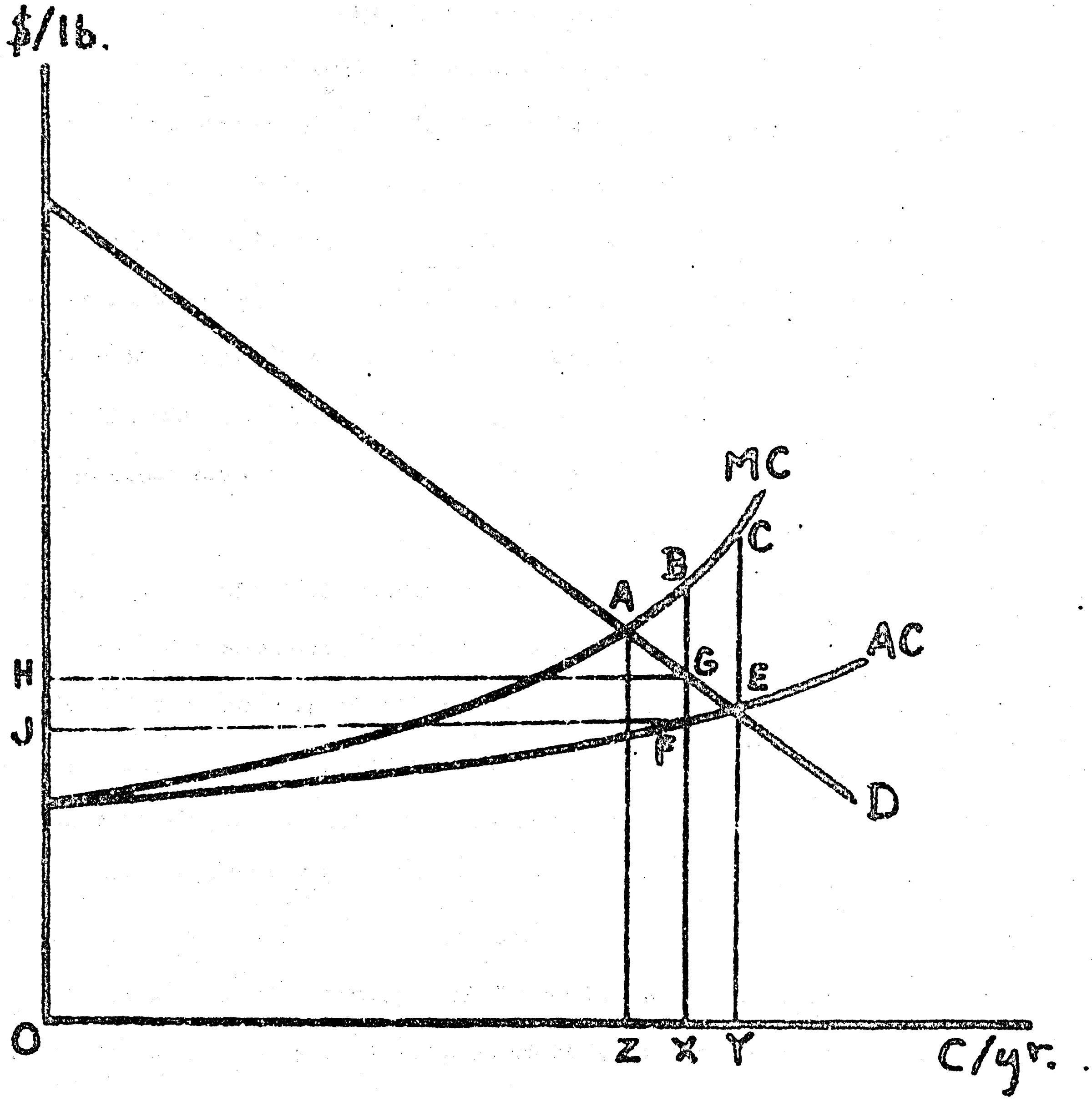

FIGURE 5

which output can be reduced is to impose cost increasing restrictions on fishermen, <u>then</u> it seems unlikely that the gains to be derived from restrictions would be worth the costs of imposing them.

It seems safe to assert, however, that it is equilibria of the sort depicted in Figures III and IV that typically if not variably evoke fishery regulation, not equilibria like those in Figures II and V. That is, it seems safe to assert that regulation is typically evoked when, in its absence, the equilibrium effort level would be one in which the marginal product of effort is negative. In such circumstances, although cost increasing restrictions are inevitably less beneficial than other controls would be, they almost certainly yield greater benefits to fishery exploitation than would eventuate with unrestricted exploitation.

Figure VI depicts what seems to be the typical situation. In the absence of controls, equilibrium in a fishery would involve producing OX pound of fish a year at a cost of BX per pound. Exploitation at this level would yield a net benefit of ABH, the consumer's surplus generated by OX pounds of fish. Imposition of a tax per pound of CE would lead to efficient exploitation of the fishery and generate a net benefit of ACF. Being unable to levy such a tax, the regulatory body in control of the fishery imposes restrictions designed to maximize the sustained yield. If successful in this aim, output would increase to OZ at a cost of DZ per pound. This cost per pound is DG greater than the minimum cost, CZ, at which the maximum sustained yield could be produced. Nevertheless, equilibrium at D does generate net benefits to exploitation of the fishery equal to ABJ—an amount equal to HDBJ greater than the benefits resulting from unrestricted access.

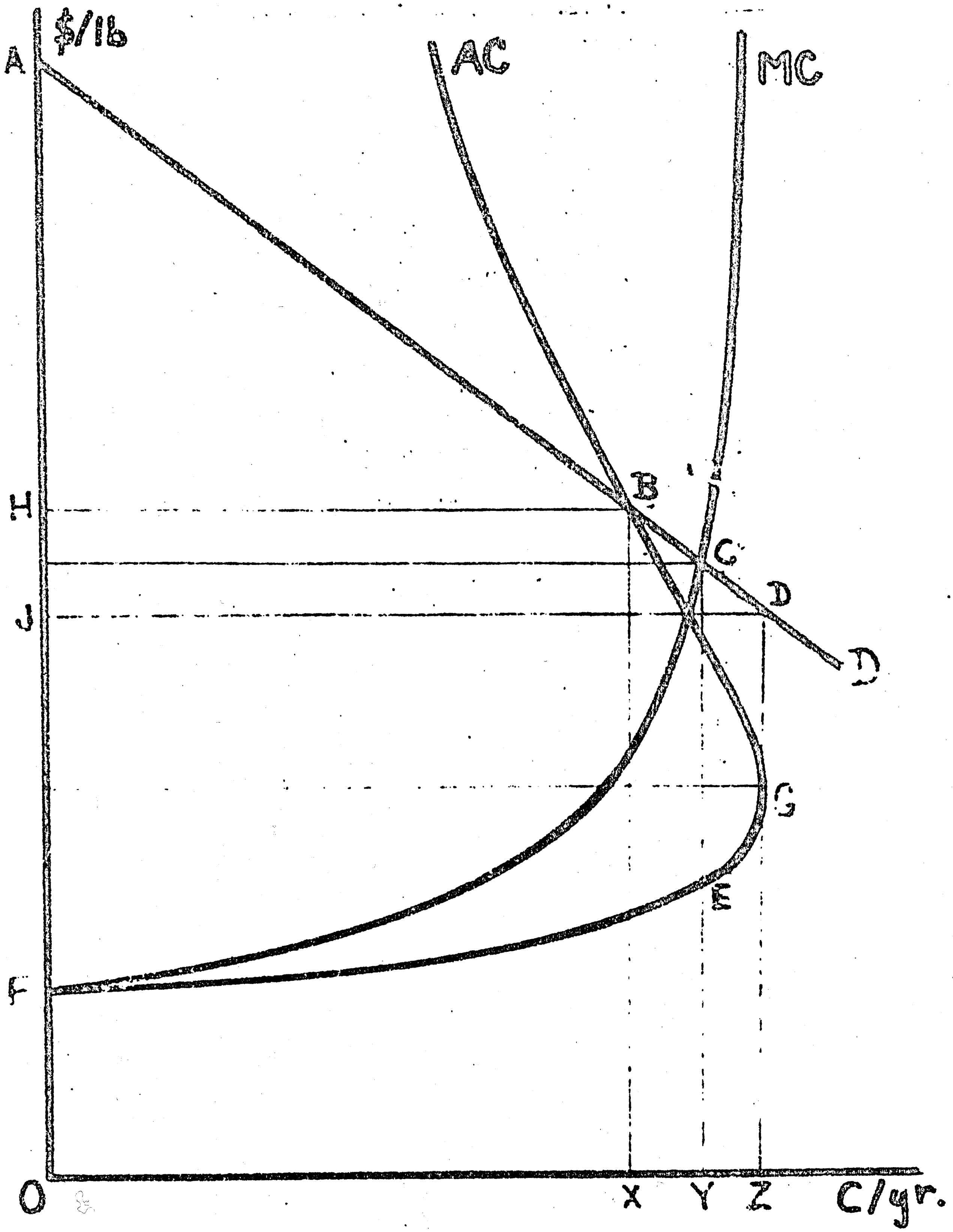

FIGURE 6

III Statistical Estimation of Growth and Catch Relationship in the Pacific Halibut Fishery

The assumption that a fishery is a long-run equilibrium is a convenient device for drawing long-run cost curves and describing the losses that result from failing to exploit it efficiently. The assumption that observations are drawn from systems in long-run equilibrium is common--perhaps almost universal--in statistical analyses of fishery behavior. Although common, such an assumption seems highly suspect in attempting to estimate the relationship discussed in Section II from data on the operations of real world fisheries.

This section first develops a general procedure which does not require assuming long-run equilibrium for estimating the functional forms and specific parameter values of the differential equation

$$dS/dt = g(S) - h(E, S) \qquad (1)$$

which formed the basis of the cost functions discussed in Section II.[6] It then describes the results of applying this procedure to data from the Pacific halibut fisheries. Since the data required--annual series on catch, effort, and length of season--are modest, the procedure may prove useful in analyzing other fisheries.

To provide an overview of the procedure, suppose, for the moment, that exploitation of a fishery is allowed only for an instant of time on January 1st of each year. Immediately prior to the beginning of the season in year i,

[6]The procedure used in this study is, I find, quite similar to that employed by Tomlinson and Pella to analyse the Pacific tuna industry. There are two basic differences between the Tomlinson-Pella approach and that used in this analysis: first, I incorporate length of fishing season into the analysis. Second, they use an estimate of fish stock at the beginning of the time series analyzed to estimate catch in all subsequent time periods whereas I employ a recurrsive relationship which generates an estimate of catch in time period i + 1 from data on catch in time i and effort in period i and i + 1.

the stock of fish in each square mile of the fishery is S_i where S_i is, unfortunately, unknown. The fraction of the stock in any given square mile caught in year i is a function, $F(E_i)$ of the effort, E_i, expended on the given square mile by the fishing fleet during year i. That is,

$$C_i = F(E_i)S_i \tag{9}$$

where C_i is catch per square mile in year i. The stock of fish in the square mile under examination at the end of the instantaneous fishing season in year i is $S_i - C_i$. Between the end of this season and the beginning of the season in year i + 1, the stock of fish grows. Specifically,

$$S_{i+1} = G(S_i - C_i) \tag{10}$$

In this system, C_i and E_i—or rather C_i and E_i times the number of square miles in the fishery—are observed. However, neither S_i nor the specific functional forms of F and G are known. Nevertheless, with luck, equations (9) and (10) can be employed to infer which of several alternative specifications of the functional forms and specific parameter values of F and G are most nearly compatible with the available catch and effort data.

Assume a specific functional form and parameter values for F. Then an estimate, S_i*, of the pre-season stock in year i can be obtained from equation (9):

$$S_i^* = C_i/F(E_i) \tag{11}$$

Assume also a functional form for G. Substituting S_i^* for S_i in equation (2) yields an estimate, S_{i+1}^{**}, of the pre-season stock in year i + 1:

$$S_{i+1}^{**} = G(S_i^* - C_i) \tag{12}$$

Inserting E_{i+1} and the estimate of S_{i+1}^{**} obtained from equation (12) into equation (9) yields a prediction of catch in year i + 1, C_{i+1}^*, which can be compared with the actual catch in that year, C_{i+1}. The objective of the game, then, is to select those functional forms and specific parameter values for

F and G which minimize,

$$\sum(C_i{*} - C_i)^2 \quad (13)$$

To elaborate on this procedure, the instantaneous rate, dS/dt, at which the weight, S, of the fish population in a square mile of a fishery grows depends, among other things, on attributes of its environment, its current weight,[7] and the rate at which its members are caught. The rate per square mile at which fish are caught depends on the rate per square mile at which effort, E, is expended by fishing boats and on the stock of fish. Thus, to repeat:

$$dS/dt = g(S) - h(E, S) \quad (1)$$

The available data provide information only on total fishing effort and total catch for the entire seasons (i.e., $\int E\, dt$ and $\int h\, dt$), not on instantaneous effort or catch rates. In what follows, the value of E in each fishing season will therefore be treated as a constant equal to the average effort rate:

$$E = \int E\, dt/t^*$$

where t^* is the length of the season. Also, in each of the specific examples to be dealt with, the instantaneous effort rate per square mile will be assumed to determine the proportion of the current stock in a square mile that is caught. That is, h(E, S) will be assumed to equal f(E) S. Making this substitution in equation (1), rearranging terms, and integrating yields:

$$\int [g(S) - f(E)S]^{-1}\, dS = t + k \quad (14)$$

where k is a constant of integration.

If g(S) is a "nice" function, the integral in equation (14) can be evaluated and the equation can be rearranged to yield, say,

$$S = S(f(E),\ t+k) \quad (15)$$

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[7] And, quite likely, its age distribution as well. As has already been noted, data restrictions make it impossible to take the effects of age distribution into account.

In turn, equation (15) can be substituted in

$$C = \int_0^{t^*} f(E)\ S\ dt = f(E) \int_0^{t^*} S\ dt \qquad (16)$$

where t* is the length of the fishing season and C is the total catch from a square mile of fishery. If S = S(f(E), t+k) is also a nice function, the integral in equation (16) can be evaluated and used to determine the constant of integration in quation (15). On inserting this value of k and setting t equal to 0, the estimated level of the stock of fish at the beginning of the fishing season results. This estimate is the equivalent of that given by equation (11).

Furthermore, setting t equal to t* in equation (11) yields that estimated level of the fishing stock at the end of the fishing season. This end of season stock level can be used to determine the constant of integration in the solution to equation (14) when f(E) is set equal to 0. The resulting expression can then be used to extrapolate the stock to the beginning of the following season. This extrapolation is the equivalent of that given by equation (12). Finally, this estimate of fishing stock at the beginning of the following season serves to determine the constant of integration, k, when the level of effort corresponding to the following season is inserted in equation (16). The result is an estimate of catch in the following year based on effort in that year and catch and effort in the preceding year.

Two opposing forces appear to affect the instantaneous growth rates of unexploited fish populations. On the one hand, the greater the population, the more fish there are to breed and to grow. On the other hand, the larger the population, the greater is competition for food and possibly other amenities of the environment. At low populations, the first of these forces dominates. The instantaneous growth rate increases albeit at a decreasing rate with increases in the fish stock. Beyond some critical stock level, however, the second force comes to dominate. The instantaneous growth rate declines with further increases in the stock. Several relationships exhibiting this property have been used in the literature on this subject. The one to which most attention has so far been devoted in this study is:

$$dS/dt = G(M - S)S - B \qquad (17)$$

of B equals zero in equation (17), M is the maximum sustainable fish stock. Having B equal to zero implies that the population of a fishery will grow as long as it contains at least two fish of opposite sexes. Under such circumstances, it would be impossible to fish the population to extinction. With B equal to zero, however, there is some minimum stock below which the fish population will not reproduce itself. With B greater than zero, unrestricted exploitation of a fishery could result in complete destruction of its population:

$$dC/dt = qES \qquad (18a)$$

The constant of proportionality in this expression, q, is sometimes referred to as the "catchability coefficient."

It seems clear that instantaneous constant returns to scale could not hold for <u>all</u> effort levels. If it did, an effort rate of 1/q would result in instantaneous obliteration of a fish stock. Instantaneous constant returns to effort would clearly <u>not</u> hold if, at some instant of time, a representative fish is torn between biting a hook from one boat and biting a hook for another. That fisheries ever get so crowded that this condition prevails seems implausible. Still, the possibility of instantaneous diminishing returns seems worth taking into account. The simplest way of doing so would seem to be:

$$dC/dt = qE^{b}S \qquad (18b)$$

where $0 < b \leq$.

Setting $B = 0$ in equation (17) and denoting the right hand side of equation (18b) by $f(E)S = fS$, the solution to equation (14) can be written:

$$S(t) = H/[G + K\exp(-Ht)] \quad (19)$$

where S(t) is fish stock at time t, H = GM - f, K is a transformation of the constant of integration, and exp(-Ht) denotes e^{-Ht} where e is the base of the natural logarithm, 2.71828. . . . Inserting equation (19) into equation (16) and using the solution to eliminate K from equation (19) yields

$$S^* = S(t^*) = H[1 - \exp(-GC/f)]/[G(1 - \exp(-Ht^*))] \quad (20)$$

where S* denotes stock at the end of the fishing season during year i and C is total catch during that year. Letting t = t* = f = 0 in equation (19) and solving for the constant of integration yields

$$K^* = G(M/S^* - 1) \quad (21)$$

Substituting equation (21) for K in equation (19) and letting t** denote the interval between the end of the fishing season in year i and the beginning of the season in year i + 1 yields

$$S^{**} = S(t^{**}) = M/[1 + K^* \exp(-GMt^{**})] \quad (22)$$

as the fish stock at the beginning of year i + 1. Again setting t equal to zero in equation (19) and solving for the constant of integration yields

$$K^{**} = (H - GS^{**})/S^{**} \quad (23)$$

where H reflects the effort level during year i + 1. The solution to equation (16) can be written:

$$C = (f/G) \ln[(G\exp(Ht^*)+K)/(G+K)] \quad (24)$$

where ln[ ] denotes the natural logarithm of [ ]. On inserting in this expression, equation (23) for K, the effort level in year i + 1 in f and H, and the length of season in year i + 1 for t* a prediction of catch in that year results.

With f(E) set equal to qE, the system defined in the preceding paragraph has three parameters: a growth rate coefficient (G), the maximum possible stock (M), and the catchability coefficient (q). In using standard non-linear least squares regression programs, initial approximations must be provided for the parameter values being estimated. Spelling out the procedure used in the case at hand may prove of value to someone interested in applying the procedure employed in this study to estimate relationships for other fisheries.

Maximizing sustained yields seems to be a common goal of fishery regulatory bodies. If a fishery is in a long run - maximum sustained yield equilibrium, the following equalities hold (see Section II):

$$C = qEM/2 = GM^2/4 \tag{25}$$

Inspection of the data (see Table 2 below) suggests that the annual catch and effort series were reasonably stable between 1952 and 1968, the last year for which data were available. Inserting average annual values during this period for C and E in equations (25) provides two equations. To find the necessary third equation, Tomlinson and Pella indicate that tuna, a fast growing fish, has an own-rate of interest somewhat in excess of 100% a year. Halibut apparently are a slow growing fish. An annual growth rate of 33% therefore seemed as reasonable an assumption as other. If the total annual growth of the fish stock in a long-run maximum sustained yield equilibrium is $GM^2/4$, its percentage growth is 100GM/2. The third equation used to find initial approximations, then, was 33 = 100GM/2.

Table 2 contains the basic data employed in the analysis. These are catch, effort, length of season, and interval between seasons for each of the three fishery areas defined by the International Pacific Halibut Commission (IPHC) in which fishing occurred during each of the years 1929-68. Area boundaries have changed slightly from time to time. Basically, however, the Area 2 data cover the Pacific coast from Willapa Bay, Washington to Cape Spencer, Alaska. Area 3A extends from Cape Spencer to the Shumagin Islands. Area 3B reaches from these islands to 175° east longitude.

Table 2: Effort, Catch, Length of Season, and Interval Between Season Data

	Area 2				Area 3A				Area 3B			
	Effort	Catch	T*	T**	Effort	Catch	T*	T**	Effort	Catch	T*	T**
1929	620.3	24.565	273	105	404.7	29.058	273	105	28.2	2.076	273	105
1930	618.1	21.367	260	108	396.5	24.700	260	106	30.1	2.475	260	106
1931	535.3	21.627	242	108	271.7	18.967	244	108	33.8	2.594	244	107
1932	446.0	21.988	249	101	254.3	20.622	257	93	11.6	0.976	257	93
1933	437.5	22.530	206	167	278.6	23.097	268	125	5.3	0.410	268	125
1934	410.9	22.638	172	224	269.8	23.097	241	155	2.2	0.198	241	155
1935	365.7	22.817	159	191	239.5	22.898	270	80	1.4	0.143	270	80
1936	458.8	24.911	148	217	248.4	23.700	233	132	1.4	0.147	233	132
1937	430.9	20.024	135	246	208.5	23.182	218	163	2.2	0.317	218	163
1938	363.0	24.975	120	245	210.0	24.238	212	153	2.2	0.321	212	153
1939	452.1	27.354	120	246	199.3	23.080	211	155	1.8	0.278	211	155
1940	439.7	27.615	104	261	220.0	25.345	179	186	3.6	0.569	179	186
1941	425.6	26.007	91	289	217.8	26.206	167	213	2.6	0.519	167	213
1942	378.2	24.321	75	290	196.6	20.105	163	202	0.2	0.047	163	202
1943	345.8	25.311	66	333	204.8	26.495	146	254	7.7	1.615	146	254
1944	312.7	26.517	51	295	171.8	24.566	195	152	13.7	2.227	195	152
1945	302.8	24.378	46	319	201.9	25.947	147	218	21.0	3.199	147	218
1946	351.2	29.678	42	323	221.8	26.974	111	254	23.4	3.556	111	254
1947	333.6	28.652	35	327	224.2	25.608	109	257	11.8	1.757	109	257
1948	312.2	28.409	32	333	216.9	24.381	72	293	21.9	3.043	72	293
1949	293.0	26.942	34	333	249.8	26.201	73	292	19.5	2.246	73	292
1950	282.3	27.046	32	333	262.0	27.959	66	299	17.5	2.211	66	299
1951	318.8	30.640	28	351	227.4	24.399	56	323	9.5	1.016	56	303
1952	266.5	30.893	26	342	224.5	29.863	60	308	12.6	1.293	17	351
1953	240.6	33.007	24	340	187.7	25.589	52	312	8.5	1.308	25	259
1954	244.2	36.699	29	332	240.9	32.834	68	293	6.2	0.917	94	266
1955	219.9	28.744	31	342	228.3	27.900	93	280	11.5	1.771	116	256
1956	263.1	35.412	45	300	231.6	30.614	105	240	4.4	0.613	126	217
1957	283.6	39.626	54	314	225.0	28.931	144	223	10.1	1.352	168	166
1958	275.5	30.558	66	295	207.9	29.731	119	242	17.1	2.391	198	166
1959	277.3	30.804	75	290	187.9	30.257	94	273	41.2	6.261	198	167
1960	280.5	31.809	98	275	198.7	29.958	85	288	38.0	4.238	198	190
1961	270.9	28.849	120	243	223.8	33.901	108	258	20.4	2.544	159	199
1962	309.9	28.663	122	242	264.2	34.608	96	270	35.7	4.214	164	200
1963	288.3	26.151	205	152	270.5	32.973	92	255	35.6	3.958	179	173
1964	214.6	19.610	137	227	280.3	23.134	110	254	46.8	4.752	192	178
1965	252.8	24.349	137	235	315.2	33.697	117	255	44.5	3.891	171	199
1966	245.4	23.435	108	256	305.2	34.426	98	266	31.8	3.086	141	244
1967	207.7	20.019	159	201	275.8	30.948	159	201	18.3	2.157	190	149
1968	168.6	16.637	164		241.3	27.215	164		27.7	3.667	194	

Effort: 1000 Skate-Soaks; Catch: 1,000,000 pounds; T*: Season Length (days); T**: Interval between seasons

During each of the years 1951-60, one or more of these fisheries operated with split seasons. Thus, in 1951, Area 2 was open between May 1-28 and again between July 26 and August 4. No attempt was made to perform the algebra necessary to take this fact into account in the statistical analysis. Rather, for the years with split seasons, the interval between seasons was determined on the assumption that fishing began on the opening date of the first part of the season and extended for a continuous period equal in length to the sum of the lengths of the two periods. Thus, for Area 2 in 1951, the season was treated as the 38 day interval beginning May 1 and ending June 7.

Except for the effort series, the data seem self-explanatory. The basic unit of effort appears to be a "standardized skate-soak." A skate-soak is a skate placed in the water and withdrawn after a period of time on the order of two days in length. A skate, in turn, is a long (1,800 or more feet) line onto which shorter lines with bated hooks are attached at varying intervals. The design of hooks discriminates against small fish and apparently has not changed significantly during the period studied. For most of this period, the standard interval between side lines was 13'. However, 9' was common during the years immediately following 1929 and intervals as great as 24' have come increasingly to be used since about 1960. Recent data strongly suggest that catch per hook increases with the space between side lines. Unfortunately, the data do not take this fact into account. In converting an actual skate into a standard skate, the implicit assumption made is that the interval between lines in excess of 13' has a zero marginal product. Thus, a standard skate is effectively a standard number of bated hooks.

To repeat, in the work undertaken so far, it has been assumed that the catch data in Table 2 are generated by a differential equation that, to change notations slightly, can be written:

$$ds/dt = g(m - s)s - qe^b s \qquad (26)$$

In this equation, s and m are actual and maximum attainable stocks respectively. They have dimensions pounds/square mile. The growth rate coefficient, g, has dimensions 1/day. The catchability coefficient, q, has dimensions of (say) 1/hook-day while effort, e, has dimensions hook-days/square mile-day. The exponent, b, is dimensionless and should have a value in the range $0 < b = 1$.

Catch and effort data are, of course, reported for an entire fishery, not per square mile of fishery area. Multiplying equation (26) through by A (for area) yields:

$$Ads/dt = dS/dt = (g/A)(M - S)S - (q/A^b)E^b S \qquad (27)$$

Where capital letters denote values for an entire fishery rather than for an individual square mile. This relationship says that the number by which (M - S)S is multiplied can be interpreted as the estimated growth coefficient for a square mile of a fishery divided by the size of the fishery. Similarly, the number by which $E^b S$ is multiplied can be interpreted as the catchability coefficient for a fishery divided by the area of the fishery to the power b.

Three separate specifications of equation (27) have so far been investigated in the statistical analysis. In the first, b was set equal to 1 and values of g/A, M, and q/A were estimated. Denoting the value of q/A obtained from the first specification by Q*, the second involved estimating g/A, M, and b in

$$dS/dt = (g/A)(M - S)S - (Q^*E)^b S \qquad (28)$$

The final specification involved estimating g/A, M, k/A^b, and b without further restrictions. The results of these calculations are shown in Table 3. Perhaps the first thing worth noting about this Table is that the basic Schaffer model does appear to fit the data reasonably well. Depending on area, the models underlying Table 3 account for 60-90% of the variance in the catch data

Table 3: Parameter Estimates for Basin Halibut Fishery

Differential Equation: $dS/dt = (g/A)(M - S)S -)q/A^b)E^bS$

Area	$G/A \cdot 10^{-4}$	M	$q/A^b \cdot 10^{-2}$	b	Adjusted R^2
2	0.1094 (-0.0506 - .2693)*	176.1 (47.14 - 305.1)*	0.1008 (0.0187 - 0.1828)*	1.000#	0.642
3A	0.1702 (0.0291 - .3113)*	136.5 (80.54 - 192.4)*	0.1742 (0.0948 - 0.2537)*	1.000#	0.762
3B	0.9122 (0.0083 - 1.816)*	20.45 (11.01 - 29.88)*	0.9200 (0.4220 - 1.418)*	1.000#	0.898
2	0.0988 (-0.0068 - 2043)*	185.4 (88.79 - 282.1)*	0.1008#	1.011 (0.9173 - 1.105)*	0.625
3A	0.1992 (0.0728 - .3256)*	126.2 (85.65 - 166.8)*	0.1742#	0.9823 (0.9242 - 1.041)*	0.743
3B	0.6343 (0.1893 - 1.079)*	24.36 (16.75 - 31.97)*	0.9200#	1.039 (0.9745 - 1.104)*	0.874
2	0.1287 (-0.0596 - .3171)*	161.7 (27.43 - 295.9)*	0.1062 (0.0226 - .1898)*	1.021 (0.8189 - 1.222)*	0.625
3A	0.1567 (0.0210 - .2923)*	142.3 (81.54 - 203.1)*	0.1774 (0.0931 - .2617)*	0.931 (0.8232 - 1.040)*	0.748
3B	0.9900 (0.0634 - 1.917)*	19.37 (11.00 - 27.74)*	1.079 (0.4558 - 1.702)*	1.077 (0.9861 - 1.168)*	0.875

* Approximate 95% confidence interval

Value assigned, not estimated

analyzed. Why the Model's performance differs so substantially from area to area is something of a mystery. Fishery 3B appears (see below) to be considerably smaller than fishery 3A which, in turn, appears smaller than fishery 2. Differences in correlation coefficients might therefore reflect differences among fisheries in the variability of conditions within an individual fishery. The ratio of the standard deviation of annual catch to average catch is 0.752 in Area 3 but only 0.163 and 0.145 in Areas 2 and 3A respectively. That there is considerably more relative variability to be explained in Area 3B may partly account for the high correlation coefficient obtained for it. But these are ad hoc hypotheses. To repeat, why the model's performance differs so substantially from Area to Area must remain a mystery at least for the time being.

The next thing worth noting about the Table 3 results is the unhappy fact that individual parameter estimates are generally surprisingly highly correlated. To take the most extreme example, when b is set equal to 1, the correlation between the estimated values of g/A and M for Area 2 is -0.9977. That between g/A and q/A is 0.9744 while that between M and q/A is -0.9735. Such high correlations have the same sort of implication for a non-linear estimation problem as do high correlations between independent variables in a linear regression problem. That is, if the parameter estimates shown in the first line of Table 3 were respectively increased, decreased, and increased by 10%, the result equation system would predict catch in Area 2 almost as well as do the numbers actually shown in this line. With high correlations among parameter values, little reliance can be placed on the specific value estimated for any particular parameter--thus the very large 95% confidence intervals for parameter value estimates shown in parentheses in Table 3.

Allowing the exponent of hook days/square mile-day to differ from one adds nothing to the explanatory power of equation (28). For any given area, the correlation coefficient between actual and predicted catch adjusted for degrees of freedom actually declines when the additional parameter, b, is introduced into the system. Furthermore, for two of the three areas, the estimated value of b is greater than 1. Taken at face value, such parameter estimates imply that an increase in the instantaneous effort rate is associated with an increase in the rate per hook day at which fish are caught--a clearly implausible finding. Thus, while it would seem reasonable to assume that, as a general proposition, effort is subject to a law of diminishing instantaneous returns, it also seems reasonable to assert that, in the halibut fisheries, the range of effort levels that has been experienced is one in which the effects of operation of this law are not discernible. This being the case, subsequent discussion will be restricted to the results of the first specification of the system analyzed, that in which b was set equal to 1.

To repeat, because of high correlations, the estimated values of individual system parameters are unreliable. Still, it seems worth ignoring this fact for a moment to explore some of the implications of the estimated parameter values for similarities and differences among the three fisheries studied. Among the parameters involved in equation (28), it seems plausible to suppose that the catchability coefficient, q, is likely to vary least among fisheries. Suppose, therefore, that q has the same value in each fishery and that b equals 1 in all fisheries. Then the estimated value of q/A for fishery i can be written q/A_i and $(q/A_1)/(q/A_2) = A_2/A_1$--the ratio of the area of fishery 2 to that of fishery 1. Multiplying the value of M for fishery 1 by this ratio would then give the maximum sustainable fish stock in fishery 1 if that fishery had the same area as does fishery 2. That is,

$$M_i(A_2/A_i) = m_iA_i(A_2/A_i) = m_iA_2 \qquad (29)$$

Differences in this expression among fisheries would reflect differences only in m_i--maximum sustainable density per square mile. Similar conclusions would apply to the ratio of the estimated value of g_i/A_i to $(q/A_i)/(q/A_2) = A_2/A_i$. That is, if the catchability coefficient is independent a fishery, the result of the division would equal g_i/I_1--a value which differs from fishery to fishery only to the extent that g_i differs. The results of these calculations are:

Area	m_iA_1	mA_1/M_1	g_i/A_1	$(g_i/A_1)/(g_1/A_1)$
2	176.1	1.000	0.1090	1.000
3A	235.9	1.340	0.09895	0.904
3B	186.6	1.060	0.0999	0.917

If[8] these numbers can be taken at face value, it would appear that the maximum sustainable densities in Areas 3A and 3B are respectively 34 and 6% greater than those in Area 2. On the other hand, growth rates in areas 3A and 3B are respectively 9.5 and 8.6% less than those in Area 2.

As was suggested in Section 2, with the formulation of the basic fishery differential equation currently under examination, the maximum sustainable yield from a fishery is obtainable when the stock of fish equals M/2, i.e., half the level toward which the fishery would approach in the absence of exploitation. Since the daily rate at which fish would grow is $(g/A)(M - S)S$, annual growth with a stock of $M/2 = 365(g/A)M^2/4$—the maximum sustainable yield, C_{max}. Dividing $100C_{max}$ by M/2 yields the percentage rate of growth of a fish stock--its own rate of interest. Since total catch equals $(q/A)ES$, catch per unit

[8] Once more, given the high correlations among parameter values, this is a very big "if."

of effort under maximum sustained yield conditions would be (q/A)M/2. The results of these calculations are:[9]

Area	M/2 (million lb.)	C_{max} (million lb/yr)	Own Rate of interest	C_{max}/E (pounds)
2	88.05	30.96	35.2%	88.75
3A	68.25	28.94	42.4%	118.89
3B	10.23	3.48	34.0%	94.07

These numbers suggest that the costs of exploiting fishery 3A are considerably lower than those for the remaining two fisheries. Because of the apparently higher density of its population, the effort required to catch a pound of fish under maximum sustained yield conditions is a bit less than 75% of that associated with Area 2.

It is of interest to compare these figures for maximum sustained yield conditions with the historical data from the individual fisheries exhibited in Tables 4A, 4B, and 4C. A few words of explanation are in order about the data in these Tables. In the process of predicting catch in year i + 1 from data on catch in year i and efforts in years i and i + 1, it was necessary to obtain, among other things, an estimate of the stock of fish left in an individual fishery at the end of the season in year i and the level to which that remaining stock had grown by the beginning of the season in year i + 1. Estimated stock at the beginning of the season in year i + 1 together with the effort level in that season yield a prediction of catch in year i + 1. These numbers are respectively reproduced in the SOH, S and F columns of Tables 4. The column lablelled C gives actual catch during year i + 1 while the value

[9] It should perhaps be noted that, since each of these calculations depends on two of the parameter estimates, they are probably more reliable than those discussed in the preceding paragraph.

Table 4A: Estimated Stock and Catch in Area 2 from $dS/dt = (g/A)(M-S)S - (q/A)ES$

Year (1929+I)	Stock (Million lb.) End of Last Season	Stock (Million lb.) Beginning of This Season	Catch (Million lb.) Predicted	Catch (Million lb.) Actual	F-C
I= 1	SOH= 35.269	S= 41.322	F= 22.983	C= 21.387	R= 1.596
I= 2	SOH= 30.756	S= 36.400	F= 18.106	C= 21.627	R=-3.521
I= 3	SOH= 36.660	S= 43.066	F= 18.568	C= 21.988	R=-3.420
I= 4	SOH= 46.496	S= 53.453	F= 21.797	C= 22.530	R=-0.733
I= 5	SOH= 47.212	S= 60.640	F= 22.870	C= 22.638	R= 0.232
I= 6	SOH= 49.831	S= 66.561	F= 22.541	C= 22.817	R=-0.276
I= 7	SOH= 56.886	S= 71.866	F= 28.935	C= 24.911	R= 4.024
I= 8	SOH= 47.156	S= 62.893	F= 24.055	C= 26.024	R=-1.969
I= 9	SOH= 52.503	S= 71.427	F= 23.434	C= 24.975	R=-1.541
I=10	SOH= 61.003	S= 80.900	F= 31.539	C= 27.354	R= 4.185
I=11	SOH= 51.495	S= 70.263	F= 26.708	C= 27.615	R=-0.907
I=12	SOH= 53.153	S= 73.410	F= 26.951	C= 26.007	R= 0.944
I=13	SOH= 51.695	S= 74.024	F= 24.468	C= 24.321	R= 0.147
I=14	SOH= 55.102	S= 78.065	F= 23.808	C= 25.311	R=-1.503
I=15	SOH= 63.216	S= 90.773	F= 25.148	C= 26.517	R=-1.369
I=16	SOH= 73.608	S= 98.455	F= 26.430	C= 24.378	R= 2.052
I=17	SOH= 70.130	S= 96.909	F= 29.446	C= 29.678	R=-0.232
I=18	SOH= 71.538	S= 98.690	F= 28.674	C= 28.652	R= 0.022
I=19	SOH= 73.243	S= 100.753	F= 27.581	C= 28.409	R=-0.828
I=20	SOH= 78.111	S= 106.063	F= 27.982	C= 26.942	R= 1.040
I=21	SOH= 77.978	S= 105.770	F= 26.537	C= 27.046	R=-0.509
I=22	SOH= 83.469	S= 111.164	F= 30.881	C= 30.640	R= 0.241
I=23	SOH= 81.983	S= 111.197	F= 26.454	C= 30.893	R=-4.439
I=24	SOH= 101.251	S= 127.388	F= 27.629	C= 33.007	R=-5.378
I=25	SOH= 121.030	S= 142.449	F= 31.285	C= 36.699	R=-5.414
I=26	SOH= 132.233	S= 149.888	F= 29.970	C= 28.744	R= 1.226
I=27	SOH= 116.898	S= 139.547	F= 32.864	C= 35.412	R=-2.548
I=28	SOH= 118.068	S= 138.047	F= 34.796	C= 30.626	R= 4.170
I=29	SOH= 94.683	S= 119.838	F= 29.712	C= 30.558	R=-0.846
I=30	SOH= 98.040	S= 121.371	F= 30.337	C= 30.804	R=-0.467
I=31	SOH= 98.437	S= 121.353	F= 30.865	C= 31.809	R=-0.944
I=32	SOH= 101.104	S= 122.582	F= 30.437	C= 28.849	R= 1.588
I=33	SOH= 96.620	S= 116.238	F= 32.562	C= 28.663	R= 3.899
I=34	SOH= 83.084	S= 103.450	F= 28.344	C= 26.151	R= 2.193
I=35	SOH= 85.834	S= 98.677	F= 20.353	C= 19.610	R= 0.743
I=36	SOH= 86.819	S= 105.831	F= 25.134	C= 24.349	R= 0.785
I=37	SOH= 89.453	S= 108.983	F= 24.891	C= 23.435	R= 1.456
I=38	SOH= 87.900	S= 109.199	F= 21.887	C= 20.019	R= 1.868
I=39	SOH= 92.451	S= 109.093	F= 18.111	C= 16.637	R= 1.474

Table 4B: Estimated Stock and Catch in Area 1A from
$dS/dt = (g/A)(M-S)S - (q/A)ES$

Year (1929+I)	Stock (million lb.) End of last Season	Stock (million lb.) Beginning of This Season	Catch (Million lb.) Predicted	Catch (Million lb.) Actual	F-C
I= 1	SOH= 36.184	S= 43.026	F= 26.151	C= 24.700	R= 1.451
I= 2	SOH= 31.649	S= 38.021	F= 17.458	C= 18.967	R=-1.509
I= 3	SOH= 38.656	S= 45.964	F= 19.922	C= 20.622	R=-0.700
I= 4	SOH= 45.429	S= 52.192	F= 24.185	C= 23.097	R= 1.088
I= 5	SOH= 45.766	S= 54.972	F= 24.274	C= 23.097	R= 1.177
I= 6	SOH= 46.520	S= 58.095	F= 23.618	C= 22.898	R= 0.720
I= 7	SOH= 53.762	S= 59.919	F= 24.439	C= 23.700	R= 0.739
I= 8	SOH= 51.933	S= 62.087	F= 21.665	C= 23.182	R=-1.517
I= 9	SOH= 60.977	S= 73.851	F= 25.355	C= 24.238	R= 1.117
I=10	SOH= 62.722	S= 74.811	F= 24.533	C= 23.080	R= 1.453
I=11	SOH= 63.446	S= 75.688	F= 26.493	C= 25.345	R= 1.148
I=12	SOH= 60.858	S= 75.537	F= 26.068	C= 26.206	R=-0.138
I=13	SOH= 62.961	S= 79.715	F= 25.077	C= 26.105	R=-1.028
I=14	SOH= 69.932	S= 85.549	F= 27.458	C= 26.495	R= 0.963
I=15	SOH= 67.199	S= 86.839	F= 24.459	C= 24.566	R=-0.107
I=16	SOH= 77.493	S= 88.921	F= 28.113	C= 25.947	R= 2.166
I=17	SOH= 67.000	S= 83.982	F= 28.391	C= 26.974	R= 1.417
I=18	SOH= 61.272	S= 81.211	F= 27.732	C= 25.608	R= 2.124
I=19	SOH= 57.576	S= 77.786	F= 25.430	C= 24.381	R= 1.049
I=20	SOH= 55.721	S= 78.708	F= 28.862	C= 26.201	R= 2.661
I=21	SOH= 50.635	S= 73.353	F= 27.906	C= 27.959	R=-0.053
I=22	SOH= 50.664	S= 73.935	F= 24.956	C= 24.399	R= 0.557
I=23	SOH= 52.199	S= 77.428	F= 25.881	C= 29.863	R=-3.982
I=24	SOH= 64.581	S= 88.362	F= 25.232	C= 25.589	R=-0.357
I=25	SOH= 68.063	S= 91.774	F= 32.401	C= 32.834	R=-0.433
I=26	SOH= 65.422	S= 88.046	F= 30.143	C= 27.900	R= 2.243
I=27	SOH= 60.531	S= 82.469	F= 28.837	C= 30.614	R=-1.777
I=28	SOH= 65.430	S= 84.144	F= 29.218	C= 28.931	R= 0.287
I=29	SOH= 65.579	S= 83.008	F= 26.716	C= 29.731	R=-3.015
I=30	SOH= 72.413	S= 90.719	F= 26.343	C= 30.257	R=-3.914
I=31	SOH= 81.239	S= 100.294	F= 30.267	C= 29.958	R= 0.309
I=32	SOH= 75.429	S= 96.471	F= 32.462	C= 33.901	R=-1.439
I=33	SOH= 74.779	S= 93.912	F= 36.044	C= 34.608	R= 1.436
I=34	SOH= 62.605	S= 83.709	F= 32.915	C= 32.973	R=-0.058
I=35	SOH= 58.091	S= 78.922	F= 32.301	C= 33.134	R=-0.833
I=36	SOH= 56.577	S= 76.544	F= 34.465	C= 33.697	R= 0.768
I=37	SOH= 50.111	S= 69.869	F= 30.527	C= 34.426	R=-3.899
I=38	SOH= 52.502	S= 73.280	F= 30.543	C= 30.948	R=-0.405
I=39	SOH= 55.872	S= 71.656	F= 26.974	C= 27.215	R=-0.241

Table 4C. Estimated Stock and Catch in Area 3B
from dS/dt = (g/A)(M-S)S - (q/A)ES

	Stock (million lb.)		Catch (million lb.)		
Year (1929+I)	End of Last Season	Beginning of This Season	Predicted	Actual	P-C
I= 1	SOH= 8.199	S= 9.176	F= 2.530	C= 2.475	R= 0.055
I= 2	SOH= 8.921	S= 9.924	F= 2.978	C= 2.594	R= 0.384
I= 3	SOH= 8.175	S= 9.171	F= 1.054	C= 0.976	R= 0.078
I= 4	SOH= 9.860	S= 10.745	F= 0.572	C= 0.410	R= 0.162
I= 5	SOH= 9.443	S= 10.633	F= 0.236	C= 0.198	R= 0.038
I= 6	SOH= 10.826	S= 12.276	F= 0.172	C= 0.143	R= 0.029
I= 7	SOH= 12.291	S= 13.011	F= 0.179	C= 0.147	R= 0.032
I= 8	SOH= 12.421	S= 13.584	F= 0.290	C= 0.317	R=-0.027
I= 9	SOH= 16.225	S= 17.153	F= 0.354	C= 0.321	R= 0.033
I=10	SOH= 16.380	S= 17.231	F= 0.291	C= 0.278	R= 0.013
I=11	SOH= 17.219	S= 17.929	F= 0.596	C= 0.569	R= 0.027
I=12	SOH= 17.346	S= 18.153	F= 0.436	C= 0.519	R=-0.083
I=13	SOH= 21.260	S= 20.986	F= 0.038	C= 0.047	R=-0.009
I=14	SOH= 24.625	S= 23.141	F= 1.562	C= 1.615	R=-0.053
I=15	SOH= 21.709	S= 21.215	F= 2.514	C= 2.227	R= 0.287
I=16	SOH= 17.035	S= 17.762	F= 3.193	C= 3.199	R=-0.006
I=17	SOH= 15.458	S= 16.830	F= 3.333	C= 3.556	R=-0.223
I=18	SOH= 15.148	S= 16.790	F= 1.762	C= 1.757	R= 0.005
I=19	SOH= 15.669	S= 17.199	F= 3.181	C= 3.043	R= 0.138
I=20	SOH= 13.901	S= 16.066	F= 2.683	C= 2.246	R= 0.437
I=21	SOH= 11.754	S= 14.308	F= 2.170	C= 2.211	R=-0.041
I=22	SOH= 12.933	S= 15.343	F= 1.302	C= 1.016	R= 0.286
I=23	SOH= 11.383	S= 14.077	F= 1.549	C= 1.293	R= 0.256
I=24	SOH= 10.599	S= 13.789	F= 1.045	C= 1.308	R=-0.263
I=25	SOH= 16.154	S= 17.567	F= 0.987	C= 0.917	R= 0.070
I=26	SOH= 15.923	S= 17.431	F= 1.781	C= 1.771	R= 0.010
I=27	SOH= 16.204	S= 17.590	F= 0.710	C= 0.613	R= 0.097
I=28	SOH= 15.302	S= 16.700	F= 1.527	C= 1.352	R= 0.175
I=29	SOH= 14.532	S= 15.745	F= 2.397	C= 2.391	R= 0.006
I=30	SOH= 14.749	S= 15.931	F= 5.284	C= 6.261	R=-0.977
I=31	SOH= 14.213	S= 15.475	F= 4.813	C= 4.238	R= 0.575
I=32	SOH= 10.995	S= 12.754	F= 2.310	C= 2.544	R=-0.234
I=33	SOH= 12.988	S= 14.644	F= 4.305	C= 4.214	R= 0.091
I=34	SOH= 11.543	S= 13.353	F= 3.987	C= 3.958	R= 0.029
I=35	SOH= 10.936	S= 12.546	F= 4.726	C= 4.752	R=-0.026
I=36	SOH= 9.677	S= 11.368	F= 4.110	C= 3.891	R= 0.219
I=37	SOH= 8.440	S= 10.318	F= 2.795	C= 3.086	R=-0.291
I=38	SOH= 9.719	S= 12.026	F= 2.004	C= 2.157	R=-0.153
I=39	SOH= 12.591	S= 13.885	F= 3.326	C= 3.667	R=-0.341

in the R column equals the difference between predicted and actual catch. Thus, the numbers in the first row of Table 4A indicate for Area 2 an estimated end of season stock in 1929 of 35.269 million pounds. By the beginning of the season in 1930, the stock had grown to 41.322 million pounds. This stock together with the effort level in 1930 from Table 2A yielded a predicted catch of 22.983 million pounds in 1930. The difference between this prediction and the actual 1930 catch of 21.387 million pounds is 1.596 million pounds.

Tables 4A and 4B clearly suggest that Areas 2 and 3A were both heavily over-exploited at the beginning of the period studied. Thus, in 1930, the average of the beginning and end of season estimated stocks was $(41.322 + 30.756)/2 = 36.089$ million pounds. The estimated maximum sustained yield stock for the fishery is the substantially greater number, 88.05 million pounds. The analysis of Section II indicates that, with $dS/dt = (g/A)(M - S)S$ in the absence of exploitation, any given long run equilibrium catch other than that required for C_{max} could be supported with either of two equilibrium stock levels, one above and the other below that required for maximum sustained yield. Specifically, equilibrium stock can be written:

$$S(C) = M/2 \pm D(C) \tag{30}$$

where S(C) and D(C) are respectively the equilibrium stock and the deviation from the maximum sustained yield stock, M/2, associated with an equilibrium catch of C. Inserting 36.09 and 88.05 million pounds respectively for S(C) and M/2 in equation (30) yields a deviation of minus 51.96 million pounds. A sustained yield equal to the 1930 catch could therefore also be obtained with a stock level of approximately $88.05 + 51.96 = 140.01$ million pounds of fish. Since the effort required to achieve any given sustained yield is proportional to the equilibrium stock with which it is associated, these calculations suggest that the effort expended on fishery 2 in 1930 was

140.01/36.09 = 3.88 times that at which a sustained yield equal to 1930 catch could have been obtained. Similar conclusions apply to Area 3A. However, while Area 3B also appears to have been over-fished during the early 1930s, the difference between average and maximum sustained yield stock was modest.

Regulation of the halibut fisheries does appear to have succeeded in reducing and ultimately eliminating their over-exploitation. If maximizing the sustained yield is, in fact, the objective of the IPHC, it seems to have come very close to meeting that objective at least since about 1950 in Areas 2 and 3A and since about 1959 in Area 3B. The Commission took a substantial amount of time to achieve this objective, however. In the years following 1930, the average of the estimated beginning and end of year stocks in Area 2 increased steadily. By 1944, the estimated beginning of season stock increased to a level slightly in excess of that required for maximum sustained yield. And, from about 1948 on, the average of the estimated beginning and end of year stocks almost invariably exceeded the level required for maximum sustained yield. The corresponding adjustments took place more rapidly in Area 3A and more rapidly still in Area 3B. As for 3A, estimated beginning of season stock first came to exceed estimated maximum sustained yield stock in 1938 and estimated average stock during the season exceeded this level during most of the years since 1942. In Area 3B, the corresponding years are 1933 and 1934.

Inspection of the catch data in Tables 4 suggest that some additional tinkering with the estimating relationships may be in order. In Area 2, negative differences between predicted and actual catch prevail in the earlier years studied while positive differences predominate in the later years. The reverse pattern characterizes Area 3A. Furthermore, in all but one of the years since 1957, catch in this area has exceeded its estimated maximum sustainable yield, typically by 10% or more. The basic biological model used does not preclude the possibility of catch occasionally exceeding maximum sustained yield. That this phenomenon could occur consistently for a decade does seem bothersome, however.

IV Future Work

A. The Immediate Future

Two extensions to the work discussed in Section III can be undertaken with a modest amount of additonal effort. The first involves limited further exploration into the nature of the differental equation from which catch predictions are generated. Implicit in the system analysed in Sections II and III is the assumption that, if two fish of opposite sexes remain in a fishery, they will find each other and reproduce. Under such circumstances, it would be virtually impossible to exploit a fishery to extinction. An alternative possibility is that a fish population will be unable to sustain itself if size falls below some minimum levels substantially greater than two fish. Determining expressions to predict catch as well as beginning and end of season stocks while allowing for the possibility that there is a minimum stock below which a fishery will die out requires a fairly considerable amount of algebraic bull work. This work is currently in progress.

The second extension of the analysis described in the preceding sections that can be undertaken with a limited amount of additional work involves the determining the number of skates-soaks that were, in effect, wasted between 1930 and 1968. Two sorts of waste can be taken into account: those resulting from the sub-optimal stocks that characterised the halibut fishery during the early years of the period and those resulting from the short seasons that have been imposed to restrict catch. To elaborate, suppose that the 1930 fish stock in Area 2 was the higher of the two levels for which the catch in that year would have been in long run equilibrium—an average of 140 million pounds rather than the actual 36 million pounds. To obtain the 1930 catch from the higher stock would have required substantially less effort than that which was actually expended. Further, starting with an initial 140 million stock would have led to substantially higher stock from which to take the following year's catch. It would be a fairly easy matter to determine the approximate effort that would have been required to achieve the actual catch in each year studied had the Area 2 stock been 140 million or any other number of pounds that might be of interest. The difference between the effort levels determined in this fashion and those actually expended can be regarded as a measure of waste effort. In addition to these wastes, had effort been applied at lower instantaneous rates but over longer seasons, actual catches could have been obtained with a smaller number of skate soaks. By following a procedure similar to that suggested above, it would be a fairly easy job to estimate the reductions in total effort that longer seasons would have made possible.

B. The Longer Run

The relationship $dS/dt = (q/A)(M - S)S$ is symmetrical about the line $S = M/2$. There is no biological reason to suppose that such symmetry prevails in real world fisheries. Working with the one asymmetrical relationship I have encountered in literature—that suggested by Tomlinson and Pella—presents some unpleasant algebraic problems. To predict catch according to the procedures sketched out in Section III requires that a differential equation giving dS/dt as a function of other variables and parameters be integrated to obtain a relationship of the general form

$$S = S(E, t + k) \qquad (31)$$

To determine the constant of integration, k, in this relationship requires solving a second differential equation which can be written

$$dC/dt = h(E)S(E, t + k) \tag{32}$$

while the growth relationship suggested by Tomlinson and Pella can be solved to determine the equivalent of equation (31), it does not appear possible to solve the equivalent of equation (32) to which this stock equation gives rise. Numerical methods do, of course, exist for obtaining approximate values for expressions that cannot be integrated exactly. Since the initial purpose of solving equation (32) is to determine a constant of integration rather than the (known) value of a definite integral, applying numerical integration techniques is likely to prove quite messy. Additional work is in order to find either solutions to this algebraic mess or a more tractable asymmetric growth relationship.

Fully to determine the welfare losses resulting from non-optimal or non-existent fishery regulation would, requires in addition to analysis of the sort described in Section III, information on or assumptions about fish demand relationships and the conditions under which effort is supplied to fisheries—in particular, short and long run supply schedules, the nature of the dynamic adjustment process involved in moving from one long run equilibrium to another, and the additional costs incurred by applying a given total amount of effort over a short rather than a long season. By making judicious assumptions about the nature of the functional relationships involved and experimenting to determining the effects of changing different parameter values, it would be possible to proceed with this sort of analysis without being dependent on additional data. Alternatively and preferably, it may be possible to adapt to the task data from Crutchfield and Zellner's pioneering study of the halibut fisheries together with information from the study of halibut and other fisheries that Crutchfield and others are currently conducting at the University of Washington. Both full determination of welfare losses and the development of optimal regulatory rules would require, in addition to this information, adaptation of recent work by Plourde and others on the dynamic properties of fishery optimization--a task requiring mathematical skills which I should acquire by do not presently possess.

Finally, there remains a task which I had initially hoped this study would accomplish: marriage of the eumetric and logistic theories of fishery biodynamics. I expect this marriage can be performed. There is some doubt about whether the data exists which would make it possible actually to apply the resulting theory. Still, an attempt to develop the theory would seem to me to be well worthwhile.