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Risk-Taking, Global Diversification, and Growth

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Department of Economics

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Abstract

This paper develops a stochastic continuous-time model in which international risk sharing can yield substantial welfare gains through its positive effect on expected consumption growth. The mechanism linking global diversification to growth is an attendant world portfolio shift from safe but low-yield capital into riskier high-yield capital. The presence of these two types of capital is meant to capture the idea that growth depends on the availability of an ever-increasing array of specialized, hence inherently risky, production inputs. Calibration exercises based on international consumption and stock market data imply that most countries reap large steady-state welfare gains from global financial integration

RISK-TAKING, GLOBAL DIVERSIFICATION, AND GROWTH

By MAURICE OBSTFELD*

This paper develops a stochastic continuous-time model in which international risk sharing can yield substantial welfare gains through its positive effect on expected consumption growth. The mechanism linking global diversification to growth is an attendant world portfolio shift from safe but low-yield capital into riskier high-yield capital. The presence of these two types of capital is meant to capture the idea that growth depends on the availability of an ever-increasing array of specialized, hence inherently risky, production inputs. Calibration exercises based on international consumption and stock-market data imply that most countries reap large steady-state welfare gains from global financial integration.
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Standard models of international asset trade lack mechanisms linking an economy's long-run output growth rate to its financial openness.[1] Within such models the gains from asset trade, at least between industrial economies, typically are estimated to be modest under common specifications of individuals' preferences. The contribution of this paper is a simple model of global portfolio diversification in which a link between growth and financial openness emerges very naturally. Within that model, an economy that opens its asset markets to trade may experience an increase in expected consumption growth and a substantial rise in national welfare.

Recent analyses of economic growth due to Paul M. Romer (1986, 1990), Lucas (1988), and others explore mechanisms through which growth rates are endogenously determined by technological parameters, intertemporal preferences, market structures, and government policies. Extensions of these mechanisms to multi-economy frameworks, notably those contained in the treatise by Gene M. Grossman and Elhanan Helpman (1991), show that international trade in goods may accelerate or slow growth by shifting resources among alternative productive uses. The model set out below pursues this line of analysis, showing that a pure expansion of opportunities for trade across states of nature may itself promote resource reallocations favorable to long-term economic growth.

The paper's model supposes that each country can invest in two linear projects, one safe and one risky. This setup is a stylized rendition of the idea, developed by Romer (1990) and by Grossman and Helpman (1991), that ongoing growth depends on investments in supplying specialized, hence inherently risky, production inputs. Because risky technologies in my model have higher expected returns than safe ones, international asset trade, which allows each country to hold a globally diversified portfolio of risky investments, encourages all countries simultaneously to shift from low-return, safe investments toward high-return, risky investments. Provided risky returns are imperfectly correlated across countries, and provided some risk-free assets are initially held, a small rise in diversification opportunities always raises expected growth as well as national welfare.[2]

The basic theme of this paper recalls Arrow's (1971, p. 137) observation that "the mere trading of risks, taken as given, is only part of the story and in many respects the less interesting part. The possibility of shifting risks, of insurance in the broadest sense, permits individuals to engage in risky activities that they would not otherwise undertake."

Several recent papers have explored ideas related to those illustrated below.

Jeremy Greenwood and Boyan Jovanovic (1990) develop a model in which financial intermediaries encourage high-yield investments and growth by performing dual roles, pooling idiosyncratic investment risks and eliminating *ex ante* downside uncertainty about rates of return. The analysis below shows, however, that even the former role of financial diversification can be an important spur to growth. The much simpler framework I choose allows closed-form solutions for an unrestrictive class of isoelastic preferences. As a result, quantitative welfare comparisons become simple and links between preferences and growth are clarified.[3]

Valerie R. Bencivenga and Bruce D. Smith (1991) assume that financial intermediaries, by providing liquidity, encourage savings to flow into relatively productive uses. The random element in their model is not investment productivity, as below, but a preference shock that creates a demand for liquid assets. Because the payoffs on liquid assets, by definition, are relatively nonspecialized across dates and state of nature, the role of intermediaries in promoting more productive illiquid investments is another example of the mechanism emphasized in this paper.

Finally, Michael B. Devereux and Gregor W. Smith (1991) examine an explicitly multi-economy model of diversification and growth and illustrate how the risk reduction implied by diversification may promote or retard growth, with the outcome depending on assumptions about intertemporal consumption substitutability and the nature of uncertainty. Their analysis does not, however, allow for aggregate shifts in the global portfolio of risky assets. A special case of this paper's model, one in which countries initially hold no riskless assets and asset returns are symmetrically distributed, yields some of the main conclusions reached by Devereux and

Smith.[4]

The paper is organized as follows. Section I describes a closed economy in which technological uncertainty follows a continuous-time diffusion process. Section II studies the closed economy's competitive equilibrium and shows how a reduction in uncertainty can spur economic growth. The section also explains the relationship among growth, consumers' risk aversion, and consumers' attitudes toward intertemporal substitution. The impact of global financial integration in a multi-economy world is studied in section III. This section contains the paper's central results concerning international diversification, growth, and real interest rates. Section IV presents a pair of simple two-country examples to illustrate how some structural assumptions can lead to large welfare gains from financial integration, while others result in smaller gains of the type often found in contexts where long-run growth rates are exogenously determined.

Examples calibrated to international consumption and stock-market data are explored in section V. Even when the reallocation of international capital stocks must occur gradually, the estimated gains from moving to a regime of perfect global financial markets can be large. Limitations of section V's calibration exercise suggest, however, that the numerical welfare gains it implies should be viewed only as tentative indicators of the potential strength of endogenous growth effects. Section VI summarizes what has been learned.

I. Individual Choice in a Closed Economy with Uncertainty

The closed economy is populated by identical infinitely-lived individuals who face the choice between consuming or investing a single good. The economic decision interval has length

h. At time t a representative household maximizes the intertemporal objective $U(t)$ defined by the recursion

$$(1)\quad f[(1 - R)U(t)] = [(1 - R)/(1 - (1/\varepsilon))]C(t)^{1-(1/\varepsilon)}h + e^{-\delta h}f[(1 - R)\mathrm{E}_t U(t+h)],$$

where the function $f(x)$ is given by

$$(2)\quad f(x) = x^{[1-(1/\varepsilon)]/(1-R)}/\{[1 - (1/\varepsilon)]/(1 - R)\}.$$

In (1), E_t is a mathematical expectation conditional on time-t information, $C(t)$ is time-t consumption, and $\delta > 0$ is the subjective rate of time preference. The parameter $R > 0$ in (1) and (2) measures the household's relative risk aversion and the parameter $\varepsilon > 0$ measures its intertemporal substitution elasticity. When $R = 1/\varepsilon$, so that $f(x) = x$, we have the standard state- and time-separable expected-utility setup, which does not allow independent variation in risk aversion and consumption substitutability over time.

The more general preference setup assumed in (1) is proposed by Larry G. Epstein and Stanley E. Zin (1989) and by Philippe Weil (1989, 1990). There are two main reasons for considering such preferences. First, dynamic welfare comparisons that confound risk aversion and intertemporal substitutability can be misleading.[5] Second, one would like to answer the positive question of how preference parameters influence growth. The effects of intertemporal substitutability on growth have been analyzed extensively (for example, by Romer 1990, Grossman and Helpman 1991, and Rebelo 1991). The effects of attitudes toward risk have not.[6]

Individuals save by accumulating capital and by making risk-free loans that pay real interest at the instantaneous rate $i(t)$. One unit of consumption can be transformed into one unit of capital, or vice versa, at zero cost. Capital comes in *two* varieties, however, riskless capital offering a sure instantaneous yield of r (a constant) and risky capital offering a random instantaneous yield with constant expected value $\alpha > r$. So individuals face a portfolio decision—how to allocate their wealth among the two types of capital and loans—as well as a saving decision. The fact that there is no nondiversifiable income (such as labor income) means that asset markets in this closed economy are complete.

The analysis is simplified by observing that when $i(t) > r$, individuals wish to hold no safe capital and cannot go short in that asset. The opposite configuration $i(t) < r$ is inconsistent with equilibrium because it implies a sure arbitrage profit from borrowing for investment in safe capital. Finally, if $i(t) = r$, the division of an individual's safe assets between safe capital and loans is indeterminate.

Given this behavior of the interest rate, the individual's portfolio problem reduces to a choice over two assets only, risky capital and a composite safe asset offering the sure instantaneous real return $i(t)$. To simplify the derivations I assume that the real interest rate is constant at level i. As the next section shows, the economy's equilibrium is indeed characterized by a constant real interest rate.

Let $V^B(t)$ denote the cumulative time-t value of a unit of output invested in safe assets at time 0 and $V^K(t)$ the cumulative time-t value of a unit of output invested in risky capital at time 0. Clearly $V^B(0) = V^K(0) = 1$. With payouts reinvested and continuously compounded, $V^B(t)$ obeys the ordinary differential equation

(3) $dV^B(t) = iV^B(t)dt.$

The stochastic law of motion for $V^K(t)$ is described by the geometric diffusion process

(4) $dV^K(t)/V^K(t) = \alpha dt + \sigma dz(t).$

In (4), $dz(t)$ is a standard Wiener process, such that $z(t) = z(0) + \int_0^t dz(s)$, and σ^2 is the instantaneous variance of $dz(t)$.[7]

Per capita wealth $W(t)$ is the sum of per capita holdings of the composite safe asset, $B(t)$, and per capita holdings of risky capital, $K(t)$:

(5) $W(t) = B(t) + K(t).$

Equations (3), (4), and (5) imply that

(6) $dW(t) = iB(t)dt + \alpha K(t)dt + \sigma K(t)dz(t) - C(t)dt.$

Let $\omega(t)$ denote the fraction of wealth invested in risky capital. An alternative way to write (6) is as

(7) $dW(t) = \{\omega(t)\alpha + [1-\omega(t)]i\}W(t)dt + \omega(t)\sigma W(t)dz(t) - C(t)dt.$

Epstein and Zin (1989) and Weil (1989, 1990) assume that time is discrete in their expositions of nonexpected-utility preferences. But continuous-time extensions by Svensson (1989) and Duffie and Epstein (1992) provide formulations that are readily applied to the problem of maximizing the continuous-time limit of $U(t)$ in (1) subject to (7) and an initial wealth endowment $W(t) = W_t$.

Let $J(W_t)$ denote the maximum feasible level of lifetime utility when wealth at time t equals W_t. Itô's Lemma shows that in continuous time the stochastic Bellman equation resulting from maximizing $U(t)$ in (1) is

$$(8)\quad 0 = \max_{\omega, C} \{[(1-R)/(1-(1/\varepsilon))]C^{1-(1/\varepsilon)} - \delta f[(1-R)\,J(W)] \\ + (1-R)f'[(1-R)\,J(W)][J'(W)(\omega\alpha W + (1-\omega)iW - C) + \tfrac{1}{2} J''(W)\omega^2\sigma^2 W^2]\}.$$

(Recall the definition of $f(x)$, equation (2). Time subscripts henceforth are suppressed when they are unnecessary.) From (8) follow the first-order conditions with respect to ω and C,

$$(9)\quad J'(W)(\alpha - i) + J''(W)\omega\sigma^2 W = 0,$$

$$(10)\quad C^{-1/\varepsilon} - f'[(1-R)\,J(W)]\,J'(W) = 0.$$

Equation (1)'s form suggests a guess that maximized lifetime utility U is given by $J(W) = (aW)^{1-R}/(1-R)$ for some constant $a > 0$. Given this functional form for $J(W)$, (9) and (10) simplify. Equation (9) now implies that demand for the risky asset is

$$(11)\ \omega = (\alpha - i)/R\sigma^2,$$

a constant fraction of wealth. Equation (10) becomes $C = a^{1-\varepsilon}W$, so that the consumption-wealth ratio also is a constant, denoted by μ. Substitution into (8) shows that

$$(12)\ \ \mu = C/W = \varepsilon\{\delta - [1 - (1/\varepsilon)][i + (\alpha - i)^2/2R\sigma^2]\}$$

and confirms that the value function is

$$(13)\ \ J(W) = [\mu^{1/(1-\varepsilon)}W]^{1-R}/(1 - R).$$

Consumption behavior depends on attitudes toward intertemporal substitution as well as toward risk, whereas portfolio choice, given the i.i.d. uncertainty assumed, depends only on risk aversion. When $R = 1/\varepsilon$, (11) therefore is unchanged while (12) reduces to the formula derived by Merton (1971) in the expected-utility case, $\mu = (1/R)\{\delta - (1 - R)[i + (\alpha - i)^2/2R\sigma^2]\}$.

II. Closed-Economy Equilibrium

Equilibrium growth in this closed economy can now be described. Because the two capital goods can be interchanged in a one-to-one ratio, instantaneous asset-supply changes always accommodate the equilibrium asset demand given by (11). There are two types of equilibrium, one in which both types of capital are held and one in which only risky capital is.

The first type of equilibrium occurs when $(\alpha - r)/R\sigma^2 \leq 1$. In this case the interest rate

i is equal to r and the share of the economy's wealth held in the form of risky capital is, by (11), $\omega = (\alpha - i)/R\sigma^2 \leq 1$.

An alternative possibility, however, is that $(\alpha - r)/R\sigma^2 > 1$. Given this inequality, an interest rate of $i = r$ is impossible: it would imply that the closed economy, in the aggregate, wishes to go short in risk-free assets. The second type of equilibrium occurs in this case of an incipient excess supply of risk-free assets at an interest rate equal to r. In this equilibrium, the interest rate i rises above r until the excess supply of risk-free assets is eliminated, that is, until $\omega = (\alpha - i)/R\sigma^2 = 1$. The implied equilibrium interest rate is $i = \alpha - R\sigma^2 > r$. (This confirms the constancy of i that was assumed in the last section.)

The equilibrium interest rate helps to determine an equilibrium rate of economic growth. Equations (7) and (12) imply the wealth-accumulation equation

$$(14)\quad dW = [\omega\alpha + (1-\omega)i - \mu]Wdt + \omega\sigma Wdz.$$

By (12) and (14), per capita consumption follows the stochastic process

$$(15)\quad dC = [\omega\alpha + (1-\omega)i - \mu]Cdt + \omega\sigma Cdz.$$

Define g as the instantaneous expected growth rate of consumption:

$$g \equiv \frac{1}{C(t)}\frac{\mathrm{E}_t\, dC(t)}{dt}.$$

Equation (15) shows that g is endogenously determined as the average expected return on wealth, $\omega\alpha + (1-\omega)i$, less the consumption-wealth ratio, μ.[8] Combination of this result with (11) and (12) leads to a closed-form expression for the expected consumption-growth rate,

$$(16)\quad g = \varepsilon(i - \delta) + (1 + \varepsilon)(\alpha - i)^2/2R\sigma^2.$$

In an equilibrium in which no riskless capital is held, the growth rate g can be expressed as

$$(17)\quad g = \varepsilon(\alpha - \delta) + (1 - \varepsilon)R\sigma^2/2,$$

which follows upon substitution of $\alpha - R\sigma^2$ for i in (16).

To gain some preliminary insight into the determinants of growth, consider the effects of a fall in σ. If the economy holds some risk-free capital, so that i may be held constant at r in (16) for small reductions in σ, the growth rate rises unambiguously. Equation (12) discloses that the effect of the fall in σ on the consumption-wealth ratio is ambiguous. The dominant effect on mean consumption growth, however, is that of the induced portfolio shift from risk-free to risky capital [equation (11)], which increases the average return to saving sufficiently to swamp any increase in the propensity to consume out of wealth. (The dominance of the portfolio-shift effect results from the specific isoelastic class of preferences assumed in (1).)

When all of the economy's capital is already in risky form, however, there can be no equilibrium portfolio shift for a closed economy. In this case equation (17) applies; it shows that a fall in σ raises growth when $\varepsilon > 1$ but lowers it when $\varepsilon < 1$. This is the result found by

Devereux and Smith (1991). With the economy's production side held fixed, a fall in σ raises growth if and only if it lowers the consumption-wealth ratio. But a fall in σ now affects consumption by pushing up the real interest rate, $i = \alpha - R\sigma^2$. Since ε is the elasticity of intertemporal substitution, a rise in the real interest rate lowers C/W (and raises growth) when $\varepsilon > 1$, but raises C/W (lowering growth) when $\varepsilon < 1$.

The preference setup used in this paper allows an evaluation of the separate impacts of intertemporal substitution and risk aversion on growth. Consider intertemporal substitution first. In deterministic growth models, the rate of growth is determined by

$$g = \frac{1}{C(t)} \frac{dC(t)}{dt} = \varepsilon(i - \delta).$$

Thus, a rise in the intertemporal substitution elasticity ε raises growth provided i, the private rate of return to investment, exceeds δ, the rate of time preference.

In the present model with uncertainty, however, equation (16) can be written as

$$(18) \quad g - \tfrac{1}{2}R\omega^2\sigma^2 = \varepsilon[\omega\alpha + (1-\omega)i - \tfrac{1}{2}R\omega^2\sigma^2 - \delta].$$

The left-hand side of (18) is the *risk-adjusted* expected growth rate: the negative risk adjustment, $-\tfrac{1}{2}R\omega^2\sigma^2$, is proportional to the degree of risk aversion and to the instantaneous variance of growth. The right-hand side is the difference between the risk-adjusted expected rate of return to investment and the time-preference rate. We therefore have a result analogous to the certainty case. Since the portfolio weight ω is independent of intertemporal substitutability, a rise in the elasticity ε raises the growth rate whenever the risk-adjusted expected return on the optimal portfolio exceeds δ.

Equations (16) and (17) also reveal how the degree of risk aversion influences expected growth. The effect of lower R parallels that of lower σ. If some riskless capital is held, lower risk aversion is associated with higher expected growth. But if only risky capital is held, the effect of R on g is proportional to $1 - \varepsilon$.

In either case expected economic growth is decreasing in the impatience parameter δ and increasing in α. The effect of a rise in return on safe capital, r , is ambiguous when the economy is nonspecialized, because a rise in r diverts investment away from more productive risky capital.[9] Of course, a small rise in r has no effect when $i > r$.

When the economy holds both types of capital, the technological parameters α and σ influence the individual's lifetime utility only through their effect on the growth rate, g. This property of the model turns out to be useful in evaluating the growth effects of international asset-market integration. To prove it, I use (12) and (13) to calculate $J(W)$, the maximized value of the intertemporal objective U in (1):

$$(19)\quad J(W) = W^{1-R}\{[2\varepsilon\delta + (1-\varepsilon)(g + i)]/(1 + \varepsilon)\}^{(1-R)/(1-\varepsilon)}/(1 - R).$$

Notice that because i is constant at r when some risk-free capital is held, the technology parameters α and σ influence lifetime utility only through their effects on g in that case.[10] Clearly an increase in g due to a rise in α or a fall in σ raises lifetime utility.

For an economy specialized in risky capital, $J(W) = W^{1-R}(\alpha - g)^{(1-R)/(1-\varepsilon)}/(1 - R)$.

Given α and the preference parameters, changes in g can come about only through changes in σ when no riskless capital is held (see (17)). As we have seen, a fall in σ may stimulate or depress growth in this case, despite its unambiguously positive welfare effect.

III. Growth Effects of International Economic Integration

All of the results above can be extended to a multi-country world economy. This extension yields predictions about the effect of economic openness on growth.

Let there be N countries, indexed by $j = 1, 2, \ldots, N$. Each country has a representative resident with preferences of the form specified in (1). Preferences may be country specific, however. Country j's representative individual has a relative risk aversion coefficient R_j, an intertemporal substitution elasticity ε_j, and a rate of time preference δ_j.

The rate of return on safe capital, r, is common to all countries (a condition relaxed in section V). The cumulative value of a unit investment in country j's risky capital follows the geometric diffusion

(20) $dV_j^K(t)/V_j^K(t) = \alpha_j dt + \sigma_j dz_j(t), \quad j = 1, 2, \ldots, N.$

Country-specific technology shocks in (20) display the instantaneous correlation structure

(21) $dz_j dz_k = \rho_{jk} dt.$

The symmetric $N \times N$ covariance matrix $\Omega \equiv [\sigma_j \sigma_k \rho_{jk}]$ is assumed to be invertible.

Our goal is to characterize a global equilibrium with free asset trade. The first step is to describe individuals' decision rules when they can invest in the N risky technologies described by (20) and (21) as well as in safe assets.

Let **1** denote the $N \times 1$ column vector with all entries equal to 1, let α denote the $N \times 1$ column vector whose kth entry is α_k, and let ω_j denote the $N \times 1$ column vector whose kth entry is the demand for country k's risky capital by a resident of country j. A generalization of the last section's argument (as in Svensson 1989) shows that an individual from country j has the following vector of portfolio weights for the N risky assets

$$(22) \quad \omega_j = \Omega^{-1}(\alpha - i^*\mathbf{1})/R_j,$$

where i^* is the world real interest rate that all countries face.

The task of describing individual decision rules is simplified by the availability of a mutual-fund theorem identical to the one proved by Merton (1971) in a similar setting. Asset demands of the form (22) imply that every individual will wish to hold the *same* mutual fund of risky assets. The ratio of risk-free wealth to wealth invested in the mutual fund is, however, an increasing function of investor risk aversion. What is convenient about the mutual-fund theorem is its implication that (11) and (12) remain valid, with α replaced by the weighted expected return on the risky mutual fund and σ^2 replaced by the variance of that weighted return.

Equation (22), as noted above, implies that the proportions in which individuals wish to hold the risky assets are independent of nationality. The $N \times 1$ vector of portfolio weights for the resulting mutual fund is

(23) $\theta \equiv \Omega^{-1}(\alpha - i^*1)/1'\Omega^{-1}(\alpha - i^*1)$,

where a "prime" (′) denotes a matrix transpose. Since the portfolio weights in (23) are constants, the analysis can proceed as if there is a single risky asset in the world with mean return $\alpha^* = \theta'\alpha$ and with return variance $\sigma^{*2} = \theta'\Omega\theta$.[11]

To envision equilibrium, imagine that N autarkic economies are opened up to free multilateral trade. Since all types of capital may be freely transformed into each other, there can be no changes in the relative prices of assets, which are fixed at 1. Instead, available *quantities* adjust to balance demands, given the world real interest rate, i^*, and the technological parameters in α and Ω. For example, there may be an initial global excess demand for country 61's risky capital, in which case risky capital resident in country 61, K_{61}, expands under foreign ownership, while other countries' capital stocks shrink.

It will generally turn out that world investors desire to go short in some countries' risky capital stocks. Since this is not possible in the aggregate, these capitals will be swapped into other forms and the associated activities will simply shut down. In equilibrium, the remaining $M \leq N$ risky capital stocks make up a market portfolio whose proportions are specified by the mutual-fund theorem.

Notice that individual countries can now go short in risk-free capital, that is, can invest a share of wealth greater than 1 in the global mutual fund of risky assets. They do this by net issues of risk-free bonds to foreigners. It may happen as in the closed economy analysis above, however, that there is an *ex ante* global excess demand for the mutual fund. In this case, the world real interest rate, i^*, rises above r until the global excess demand for risky capital disappears.

More formally, assume that $M \leq N$ risky capital stocks remain in operation after trade is opened and that they are available in the positive quantities $K_1, K_2, \ldots, K_M$. To conserve on notation, let α now denote the $M \times 1$ subvector of mean returns and Ω the associated $M \times M$ covariance matrix of returns. Define the $M \times 1$ vector of mutual-fund weights θ by equation (23), $\theta \equiv \Omega^{-1}(\alpha - i^*1)/1'\Omega^{-1}(\alpha - i^*1)$, and denote the fund return's weighted mean and variance by α^* and σ^{*2}, respectively. Then an equilibrium must satisfy the conditions

$$K_j / \sum_{j=1}^{M} K_j = \theta_j > 0 \qquad \text{(for all } j = 1, 2, \ldots, M),$$

$$\sum_{j=1}^{M} K_j = \sum_{j=1}^{N} (\alpha^* - i^*) W_j / R_j \sigma^{*2},$$

where θ_j is the jth component of θ and W_j is country j's wealth.

In an integrated world equilibrium, national consumption levels can grow at different rates on average despite the single risk-free interest rate i^* prevailing in all countries. Country j's mean growth rate is

$$(24) \quad g_j^* = \varepsilon_j(i^* - \delta_j) + (1 + \varepsilon_j)(\alpha^* - i^*)^2/2R_j\sigma^{*2}.$$

Given the world interest rate, country j grows more quickly the greater its tolerance for risk and the lower its degree of impatience. Subject to the condition discussed in the last section, an increase in willingness to substitute intertemporally also is associated with higher growth. Provided any risk-free capital is held in the world, $i^* = r$; but if not, a decrease in all countries' risk aversion implies a higher world interest rate and an ambiguous effect on growth.

Consider next the impact of economic integration on growth. The most straightforward case is that in which all countries hold riskless capital before integration and some continue to hold it afterward. In this case countries share a common risk-free interest rate, r, both before *and after* integration. Equation (19) shows that the expected growth rate must rise in all countries. Why? Economic integration does not change any country's wealth because different types of capital are costlessly interchangeable. But in the present distortion-free setting, trade must raise welfare; and equation (19) shows that at an unchanged interest rate, welfare rises if and only if growth rises. The intuition behind this result follows from the discussion in section II. International portfolio diversification encourages a global shift from (relatively) low-return, low-risk investments into high-return, riskier investments.

A similar argument, again based on equation (19), shows that any country whose risk-free interest rate falls upon integration with the rest of the world must experience increased expected growth. Growth can fall only in a country whose real interest rate rises. For such a country, however, the risk-reducing benefits of diversification necessarily outweigh the adverse welfare effect of lower expected growth. (Once again, the specific class of preferences assumed here is responsible for the strong predictions about growth described above.)

IV. Two Simple Examples

This section works out two numerical examples to show how the growth effects of international diversification can imply a large welfare payoff from financial integration. A number of applied studies, for example Lucas (1987), Cole and Obstfeld (1991), and van Wincoop (1991), take consumption growth to be exogenous in their evaluations of the costs of income variability. By comparing the welfare effects in the examples to the numbers a researcher would find if consumption growth were assumed to be exogenous, I can quantify the difference that endogenous consumption growth makes.

Example 1. Imagine a symmetric two-country world ($N = 2$) in which $r = 0.02$, $\alpha_1 = \alpha_2 = 0.05$, $\sigma_1 = \sigma_2 = 0.1$, and returns to capital are uncorrelated, $\rho_{12} = 0$. Preferences are the same in the countries, with $\varepsilon = 0.5$, $R = 4$, and $\delta = 0.02$. Under financial autarky, residents of each country hold a fraction of wealth $\omega = (\alpha - r)/R\sigma^2 = 0.75$ in the domestic risky asset. Equations (11) and (16) imply a mean consumption growth rate of $g = \frac{1}{2}(1 + \varepsilon)(\alpha - r)\omega - \varepsilon(\delta - r) = 1.6875$ percent. In both countries the risk-free real rate of interest, i, is equal to r, i.e., $i = 0.02$.

Now let the two countries trade. The optimal global mutual fund is divided equally between the two risky capitals. This portfolio's mean rate of return is $\alpha^* = 0.05$ with instantaneous return variance $\sigma^{*2} = (0.1)^2/2 = 0.005$. Each country's total demand for risky assets will now be $\omega^* = (\alpha^* - i^*)/R\sigma^{*2}$; it is simple to check that at a world real interest rate of $i^* = 0.03$, $\omega^* = 1$. Thus, financial integration leads to a rise in the real interest rate, from 0.02 to 0.03, and an equilibrium in which risk-free assets are no longer held. The increase in the world real interest rate reflects lower precautionary saving due to a reduction in the variability of wealth.[12]

From (24) we can calculate the expected consumption growth rate g^* in the integrated equilibrium. Equilibrium growth averages 2 percent per period, as compared with the rate of 1.6875 percent per period characterizing the pre-trade situation.

The present value of the welfare gain from economic integration can be calculated as a *compensating variation*: by what percentage λ must wealth be increased under financial autarky so that people enjoy the same level of utility as under financial integration? Using (19) and (24), one finds that $\lambda = 0.371$, or 37.1 percent of initial wealth.[13] This is a very large welfare gain. It is derived from two sources: the opportunity to trade consumption risks *given* the stochastic process governing consumption growth, and the endogenous effect of this risk sharing on the consumption-growth process itself.

Notice that this example assumes an instantaneous reallocation of capital from risk-free to risky uses. Such speedy adjustment would not be observed in practice. Instead, the shift in relative capital stocks would be spread out over time; the post-trade portfolio proportions just described would be reached eventually, but not in the short run. The welfare gain just calculated thus is more realistically viewed as the steady-state increase in wealth to diversification; it provides no more than an upper bound on the short-run income effect.

Example 2. Let's look at an example in which (1) the induced growth effects of financial integration are essentially zero, and (2) the variance of consumption is closer to the type of number characteristic of the richer industrialized economies. In this case, the welfare effects of financial integration will turn out to be much smaller than above. Let all parameters be as in the previous example, with the exception that now $\sigma_1 = \sigma_2 = 0.02$. Given this change, both countries hold only risky capital in the pre-trade equilibrium, and their real interest rates will coincide at $i = 0.0484$. In each country, therefore, equation (16) or (17) gives $g = 0.0154$ as the expected growth rate of consumption.

Under financial integration people hold a risky asset, the equal-shares mutual fund, with return variance half that of either country's capital and with a mean rate of return of 5 percent. In the pooled equilibrium the real interest rate is $i^* = 0.0492$, slightly above its level under autarky, and the growth rate of consumption declines very slightly, to $g^* = 0.0152$ percent. The compensating variation measure of the welfare gain from financial integration is now $\lambda = 0.0116$, or 1.16 percent of initial wealth.

The only difference between examples 1 and 2 is that the variance of the risky-capital shock is 25 times larger in the first case than in the second. This leads to a welfare gain from financial integration that is about 32 times larger in the first case. Without knowing about endogenous growth, we might have guessed naively that the welfare gains would be 25 times as great in the first example, not 32 times as great. The resulting underestimate of the gains from financial integration is economically substantial.

A more rigorous way to assess the contribution of endogenous growth is to ask *what conclusion a researcher would reach in the examples above if he took the observed growth rate of consumption to be exogenous.* The assumption that reducing economic variability does not greatly affect the growth rate of consumption has been typical in recent applied studies of the cost of consumption variability.

Equation (15) and Itô's Lemma imply[14] that under financial autarky, a researcher using annual data would observe the per capita consumption process

$$\log C(t) - \log C(t-1) = 0.0141 + v(t), \quad \sigma_v^2 = 0.00563,$$

given the assumptions of example 1, and the process

$$\log C(t) - \log C(t-1) = 0.0152 + v(t), \quad \sigma_v^2 = 0.00040,$$

given those of example 2. Taking the consumption growth rates as exogenous, the researcher might suppose that international diversification would halve each of the two variances above, leaving expected growth—which equals the regression constant plus $\frac{1}{2}\sigma_v^2$—unchanged. It is easy to compute the implied compensating-variation measures of welfare gain, which are reported in table 1 (left-hand column) beside the true gains calculated earlier (right-hand column).[15]

In example 2 the growth effects of international financial diversification are minimal. Thus, assuming exogenous consumption growth makes little difference to the answer. But when larger growth effects are present, analyses that fail to account for them can be misleading. Under the parameters of example 1, the true gain from financial integration is 73 percent higher than the number one finds ignoring the endogeneity of growth.

V. Examples Based on Global Consumption and Stock-Market Data

This section is devoted to two final examples of the gains from international financial integration. These examples are based, respectively, on actual consumption-growth data (as reported by Robert Summers and Alan Heston (1991) in the Penn World Table (Mark 5)), and on international data on stock-market returns. The welfare effects reported below should not be taken as a literal prediction about reality; they simply indicate that, when matched to some realistic parameters, the preceding model could imply very large gains from asset trade.

The first example considers an eight-region world, consisting of North America, South America, Central America, East Asia, Noneast Asia, Northern Europe, Southern Europe, and Africa. Within each region, real per capita consumption is a population-weighted average of national per capita consumptions. I use data spanning the period 1960-87. Only countries with data available over this entire period, and with data quality of at least C− according to Summers and Heston, are included.[16]

Equation (15) implies that the logarithm of per capita consumption follows the random walk with drift,

$$\log C(t) - \log C(t-1) = g - \tfrac{1}{2}\sigma_v^2 + v(t),$$

where $v(t) = \omega\sigma[z(t) - z(t-1)]$ and $\sigma_v = \omega\sigma$.[17] Table 2 reports the information one extracts by fitting this equation to the data: estimates of g and σ_v for the eight regions, as well as an estimate of the correlation matrix of regional consumption shocks. Because production shocks are the only source of consumption uncertainty in the model, the matrix in table 2 is also the correlation matrix of regional productivity shocks, $z(t) - z(t-1)$. With perfect risk pooling, all entries in this correlation matrix would be 1.

The moments in table 2 provide a basis for calibrating the model empirically. Any such attempt runs immediately, however, into two well-known problems: the equity-premium puzzle of Mehra and Prescott (1985) and the risk-free rate puzzle of Weil (1989).

To appreciate the equity-premium puzzle, let i again be a country's risk-free rate. By (10), the equity premium can be expressed in terms of the consumption variance σ_v^2 as

$$(29) \quad \alpha - i = R\sigma_v^2/\omega.$$

Table 2 shows that in most countries, the variability of consumption growth is too small to generate equity premia on the plausible order of 5 percent per year without some combination extremely high risk aversion and a very low portfolio share for risky assets. In an attempt to meet the data half way, I will assume that $R = 18$ and that the equity premium is 4 percent per year in all regions. Under these assumptions, (29) yields the estimates of ω reported in table 3. (The ω values in table 3 describe an initial allocation in which limited trade may occur, but in which economic integration is incomplete.) With the exceptions of South America and Africa, where the variability of annual consumption growth is exceptionally high (standard deviations of 4.57 and 3.59 percent, respectively), these portfolio shares for risky assets seem implausibly low. I nonetheless use them to infer estimates of $\sigma = \sigma_v/\omega$, the standard deviation of the underlying annual production shock. These, too, are reported in table 3.[18]

Table 3 highlights a counter-intuitive empirical implication of the model. Equation (10) implies that for given values of the equity premium and R, there is an *inverse* relation between the observed variability of consumption growth, σ_v, and the variability of the underlying technology shock, σ: $\sigma = (\alpha - i)/R\sigma_v$. Thus, table 3 suggests that in those regions where the variability of consumption growth is lowest, the variability of technology shocks is greatest. In Northern Europe, for example, the standard deviation of the annual consumption growth rate is only 1.3 percent (table 2), yet that of the return to risky capital is reckoned at 17 percent. Conversely, the corresponding standard deviation for risky capital held by South Americans is estimated to be only 4.9 percent. The result could be overturned if the equity premium had a sufficiently strong positive cross-sectional correlation with consumption variability; but the empirical basis for such an assumption has not been established. Risky nontradable income, which is present in reality, would also break the tight link between consumption variability and the riskiness of capital investments.

Consider next the implications of the risk-free rate puzzle. Equation (21) can be rewritten in the general form

$$(30)\quad g = \tfrac{1}{2}(1 + \varepsilon)R\sigma_v^2 - \varepsilon(\delta - i).$$

Given the low values for σ_v^2 suggested by table 2, however, the mean growth rates g in the table cannot be matched unless some combination of the following is true: R is very large, ε is very large, δ is negative, or i is high. Maintaining the assumption that $R = 18$ and setting $\delta = 0.02$ and $\varepsilon = 1.1$, I compute region-specific risk-free interest rates that generate, through (30), the mean consumption growth rates reported in table 2.

Table 4 reports these rates. Even though an unrealistically high intertemporal substitution elasticity ($\varepsilon = 1.1$) was assumed, the interest rates in the table are still on the high side for some of the regions, in line with the risk-free rate puzzle. Notice that the risk-free rate is calculated to be relatively low in countries where consumption variability is relatively high. This pattern results mainly from the low risk tolerance assumed earlier, and reflects the precautionary motive for saving.[19] Mean national rates of return to risky capital are calculated as $\alpha = 0.04 + i$. For convenience, I report these rates in the second row of table 4.

The numbers reported in tables 2 through 4 allow computation of the covariance matrix of risky capital returns, and hence of the global equilibrium that would obtain after financial integration (recall section III). Table 5 reports the equilibrium portfolio shares in the optimal global mutual fund of risky assets, along with the mean and standard deviation of the fund's annual return (α^* and σ^*), the share of the fund in global wealth (ω^*), the prevailing world interest rate (i^*), and the common new world growth rate (g^*). Notice that Northern European capital disappears entirely from the world portfolio, essentially because it is highly correlated with East Asian capital (the correlation coefficient is 0.753 according to table 2) but has a slightly lower expected return (table 4). Equilibrium holdings of risk-free capital are located exclusively in East Asia.

A note of interpretation is in order at this point. The 1960-87 data are already based on some international risk sharing. For example, the high correlation between East Asian and Northern European log-consumption innovations probably reflects some cross-holding of capital. The non-appearance of Northern Europe in the optimal global portfolio therefore does not really mean that no Northern European capital is held in equilibrium. Prior to *full* market pooling, East Asians already hold a portfolio that includes some Northern European capital; after pooling, it is this portfolio, rather than the one Northern Europeans hold, for which demand is positive. Nothing in the calculations requires literal autarky in the pre-integration equilibrium.

Although the expected return on the global portfolio is significantly below that on East Asian capital, for example, global pooling does lead to a substantial reduction in risk (refer back to the second row of table 3). In addition, the consumption growth rate rises everywhere. At 4.37 percent per year, equilibrium growth is substantially above even East Asia's initial high of 3.64 percent (table 2). This sharp increase comes partly from a drop in the consumption-to-wealth ratio, but primarily from the shift of world wealth into riskier high-yield capital.

The gains from asset trade, reported in table 6, are very large, ranging from 478.4 percent of wealth for Noneast Asia to "only" 22.6 percent for East Asia. The uneven regional distribution of trade gains is easy to understand. Areas where returns initially are low gain disproportionately from access to more productive investment technologies. (These gains are especially large because of the assumed absence of diminishing returns to investment.) Naturally, the gains in table 6 also reflect the advantages of worldwide risk sharing.

For a given country, what share of the gain in table 6 is due purely to the adoption of a new production technology, as opposed to the channels my theoretical model stresses? A simple measure of the gain from pure international technology transfer is the welfare effect, in a *deterministic* setting, of changing the average rate of return on domestic investment from $\omega\alpha + (1 - \omega)i$ to $\omega\alpha^* + (1 - \omega)i^*$. This experiment holds fixed the allocation of inputs to risky and riskless activities, but moves the rates of return on those activities to expected world equilibrium levels.

Table 7 reports the resulting measures of welfare gain.[20] These gains are large in most cases, but they are all far below the *total* gains shown in table 6. The present example therefore implies large gains from diversification even after subtracting the gains from technology transfer.

The model unrealistically assumes that capital can relocate immediately; but allowing gradual adjustment could reduce the gains in table 6 dramatically. A crude way to capture gradual adjustment is to suppose that after financial integration takes place, the current annual welfare gain converges toward the long-run annual gain implied by table 6 at an instantaneous rate of γ percent. This convergence assumption means that the actual capitalized welfare gain, λ', is related to the measure λ in table 6 by $\lambda' = \int_0^\infty i\lambda(1 - e^{-\gamma t})e^{-it}dt = \gamma\lambda/(i + \gamma)$. As a numerical example, suppose that the world real interest rate is 4.54 percent per year and that the annual rate of convergence, as suggested by the work of Barro, N. Gregory Mankiw, and Xavier Sala-i-Martin (1992), is 2.2 percent per year. Then the welfare gains from financial integration would be just under a third of those in table 6 (and higher for lower interest rates). Such gains remain large.

A major shortcoming of this first example is that it must assume preference-parameter values that may seriously overstate both risk aversion and willingness to substitute consumption over time. Welfare gains even a twentieth as large as those in table 6 would be significant, however, particularly for countries in the developing world.

A second numerical example of gains from financial integration is based on data on stock-market rates of return. As before, it is difficult to reconcile these data with aggregate consumption data within the class of models explored here. My procedure also assumes that stock-market returns are an accurate measure of the returns to risky investments.

In table 8 I have used data on total annual stock-market returns, 1976-92, to estimate α_j, σ_j, and Ω for a world of three countries: Germany, Japan, and the United States. The periodical *Morgan Stanley Capital International Perspective* publishes U.S. dollar indexes of stock-market value, including reinvested dividends. I deflated the dollar indexes for Germany, Japan, and the U.S. by the U.S. consumer price index (CPI) and used these data to estimate expected annual returns and variances, along with the covariance matrix of annual returns.[21] Also shown in table 8 are the pre-integration consumption growth rates and portfolio proportions that (16) and (11) imply. The values shown for g and ω were derived on the assumptions that in all three countries, $R = 6$, $\varepsilon = 0.5$, and $\delta = r = i = 0.02$.

The implied average per capita growth rates in the table's upper panel are underestimates of the true growth rates, especially in the cases of Germany and Japan. The implied consumption-growth standard deviations, $\omega\sigma$, exceed the actual ones, σ_v in the bottom panel, by very wide margins in all three cases.[22] These results suggest that stock-market returns may be poor proxies for the aggregate returns to high-risk but productive investments.

In table 9 I report the effects of full financial integration. If the data sample used here were typical, people would want to concentrate their stock portfolios (69 percent) in low-return but low-risk United States assets. The intermediate mean German risky return, coupled with the relatively high correlation of German with both U.S. and Japanese returns (table 8), leads to an incipient negative demand for German assets. In equilibrium, therefore, no risky German assets are held. Average consumption growth rates increase in all regions.

The gains from trade remain substantial in this example; they are around 28 percent of wealth for Japan and the U.S., around 70 percent for Germany, which experiences the largest growth increase. Observe that the gains in table 9 are expected to be smaller than those in table 6 because I have now assumed smaller values of both R and ε. (Earlier I assumed $R = 18$ and $\varepsilon = 1.1$.) Realistic adjustment costs would suggest scaling down the gains in table 9, as before.

VI. Conclusion

This paper has demonstrated that international risk sharing can yield substantial welfare gains through its positive effect on expected consumption growth. The mechanism linking global diversification to growth was the attendant world portfolio shift from safe, but low-yield, capital into riskier, high-yield capital.

The model makes this theoretical point cleanly, but its empirical applicability is limited by several factors. One set of factors is related to the equity-premium and risk-free rate puzzles familiar from United States data. Another, not entirely separate, issue is the probable importance of nontradable income risk. The model assumes a single consumption good and ignores the roles of goods that do not enter international trade and of variation in real exchange rates. Finally, the absence of capital-adjustment costs and related capital-gains effects are drawbacks, except, perhaps, for analyzing comparative steady states. Further empirical and theoretical work is needed before accurate welfare evaluations can be made using models based on the one presented here. Even welfare gains much smaller than those found in section V above would, however, be important.

References

Arrow, Kenneth J., "The Economic Implications of Learning by Doing," *Review of Economic Studies*, June 1962, *29*, 155-73.

Arrow, Kenneth J., *Essays in the Theory of Risk-Bearing*, Chicago: Markham, 1971.

Atje, Raymond and Jovanovic, Boyan, "Stock Markets and Development," *European Economic Review*, April 1993, *47*, 632-40.

Backus, David K., Kehoe, Patrick J. and Kydland, Finn E., "International Real Business Cycles," *Journal of Political Economy*, August 1992, *100*, 745-75.

Barro, Robert J., "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, October 1990, *98*, S103-25.

Barro, Robert J., Mankiw, N. Gregory, and Sala-i-Martin, Xavier, "Capital Mobility in Neoclassical Models of Growth," Working Paper No. 4206, National Bureau of Economic Research, November 1992.

Bertola, Giuseppe, "Flexibility, Investment, and Growth," Working Paper No. 3864, National Bureau of Economic Research, October 1991.

Bencivenga, Valerie R. and Smith, Bruce D., "Financial Intermediation and Endogenous Growth," *Review of Economic Studies*, April 1991, *58*, 195-209.

Campbell, John Y., "Intertemporal Asset Pricing without Consumption Data," *American Economic Review*, 1993, 83, forthcoming.

Cole, Harold L. and Obstfeld, Maurice, "Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?" *Journal of Monetary Economics*, August 1991, *28*, 3-24.

Cooley, Thomas F. and Smith, Bruce D., "Financial Markets, Specialization, and Learning by Doing," Working Paper No. 276, Rochester Center for Economic Research, May 1991.

Corsetti, Giancarlo, "A Model of Taxation, Saving and Risk-Taking with and without Risk Disposal," mimeo, Università di Roma, 1991.

Cox, John C., Ingersoll, Jonathan E., Jr. and Ross, Stephen A., "An Intertemporal General Equilibrium Model of Asset Prices," *Econometrica*, March 1985, *53*, 363-84.

Dellas, Harris, "Stabilization Policy and Long Term Growth: Are They Related?" mimeo, University of Maryland, February 1991.

Devereux, Michael B. and Smith, Gregor W., "International Risk Sharing and Economic Growth," mimeo, Queen's University, July 1991.

Duffie, Darrell and Epstein, Larry G., "Stochastic Differential Utility," *Econometrica*, March 1992, *60*, 353-94.

Eaton, Jonathan, "Fiscal Policy, Inflation and the Accumulation of Risky Capital," *Review of Economic Studies*, July 1981, *48*, 435-45.

Epstein, Larry G. and Zin, Stanley E., "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, *57*, July 1989, 937-69.

Gertler, Mark and Grinols, Earl, "Monetary Randomness and Investment," *Journal of Monetary Economics*, 1982, *10*, 239-58.

Greenwood, Jeremy and Jovanovic, Boyan, "Financial Development, Growth, and the Distribution of Income," *Journal of Political Economy*, October 1990, *98*, 1076-1107.

Grinols, Earl L. and Turnovsky, Stephen J., "Exchange Rate Determination and Asset Prices in a Stochastic Small Open Economy, mimeo, University of Illinois and University of Washington, 1992.

Grossman, Gene M. and Helpman, Elhanan, *Innovation and Growth in the Global Economy*, Cambridge, MA: MIT Press, 1991.

Jones, Larry E. and Manuelli, Rodolfo, "A Convex Model of Equilibrium Growth: Theory and Policy Implications," *Journal of Political Economy*, October 1990, *98*, 1008-38.

Kandel, Shmuel and Stambaugh, Robert F., "Asset Returns and Intertemporal Preferences," *Journal of Monetary Economics*, February 1991, *27*, 39-71.

King, Robert G., Plosser, Charles I. and Rebelo, Sergio T., "Production, Growth and Business Cycles: II. New Directions," *Journal of Monetary Economics*, March/May 1988), *21*, 309-41.

Levine, Ross, "Stock Markets, Growth, and Tax Policy," *Journal of Finance*, September 1991, *46*, 1445-65.

Lucas, Robert E., Jr, "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics*, November 1982, *10*, 335-59.

_____, *Models of Business Cycles*, Oxford: Basil Blackwell, 1987.

_____, "On the Mechanics of Economic Development," *Journal of Monetary Economics*, July 1988, *22*, 3-42.

Mehra, Rajnish and Prescott, Edward C., "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, March 1985, *15*, 145-61.

Mendoza, Enrique G., "Capital Controls and the Gains from Trade in a Business Cycle Model of a Small Open Economy," *International Monetary Fund Staff Papers*, September 1991, *38*, 480-505.

Merton, Robert C., "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory*, December 1971, *3*, 373-413.

Obstfeld, Maurice, "How Integrated Are World Capital Markets? Some New Tests," in Guillermo Calvo et al., eds., *Debt, Stabilization and Development: Essays in Memory of Carlos Díaz-Alejandro*, Oxford: Basil Blackwell, 1989, pp. 134-55.

_____, "Evaluating Risky Consumption Paths: The Role of Intertemporal Substitutability," Technical Working Paper No. 120, National Bureau of Economic Research, May 1992.

Pagano, Marco, "Financial Markets and Growth: An Overview," *European Economic Review*, April 1993, *37*, 613-22.

Rebelo, Sergio, "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy*, June 1991, *99*, 500-21.

Romer, Paul M., "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, October 1986, *94*, 1002-37.

_____, "Endogenous Technological Change," *Journal of Political Economy*, October 1990, *98*, S71-102.

Saint-Paul, Gilles, "Technological Choice, Financial Markets and Economic Development," *European Economic Review*, 1992, *36*, 763-81.

Stulz, René M., "An Equilibrium Model of Exchange Rate Determination and Asset Pricing with Nontraded Goods and Imperfect Information," *Journal of Political Economy*, October 1987, *95*, 1024-40.

Summers, Robert, and Heston, Alan, "The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988," *Quarterly Journal of Economics*, May 1991, *106*, 327-68.

Svensson, Lars E. O., "Trade in Risky Assets," *American Economic Review*, June 1988, *78*, 375-94.

_____, "Portfolio Choice with Non-Expected Utility in Continuous Time," *Economics Letters*, 1989, *30*, 313-17.

Uzawa, Hirofumi, "Optimum Technical Change in an Aggregative Model of Economic Growth," *International Economic Review*, January 1965, *6*, 18-31.

van Wincoop, Eric, "Welfare Gains from International Risk Sharing," mimeo, Boston University, October 1991.

Weil, Philippe, "The Equity Premium Puzzle and the Risk-Free Rate Puzzle," *Journal of Monetary Economics*, November 1989, *24*, 401-21.

_____, "Nonexpected Utility in Macroeconomics," *Quarterly Journal of Economics*, February 1990, *105*, 29-42.

Economic Report of the President, Washington, D.C.: U.S. Government Printing Office, 1992.

Morgan Stanley Capital International Perspective, Geneva: Morgan Stanley, various issues..

Table 1 Endogenous Growth and Welfare Comparisons

	Welfare gain assuming exogenous growth (percentage of wealth)	*Welfare gain assuming endogenous growth* (percentage of wealth)
Example 1	21.5	37.1
Example 2	1.1	1.2

Table 2 Global Regions and Their Consumption Processes, 1960-87

Mean and standard deviation of annual per capita consumption growth rate (percent)

	NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
g	2.35	3.11	1.68	3.64	0.91	2.87	3.13	1.31
σ_v	1.76	4.57	2.96	2.12	3.02	1.31	3.03	3.59

Correlation coefficients of regional per capita consumption growth rates

	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
NAm	-0.248	-0.113	0.393	0.117	0.366	0.118	-0.415
SAm		0.147	0.134	-0.467	0.440	0.391	0.139
CAm			0.365	-0.136	0.289	0.115	0.525
EAsia				-0.048	0.753	0.369	0.074
NAsia					-0.299	-0.166	-0.299
NEur						0.474	-0.035
SEur							0.321

Regional groupings

North America (NAm): Canada, United States. South America (SAm): Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Paraguay, Peru, Uruguay, Venezuela. Central America (CAm): Costa Rica, Dominican Republic, El Salvador, Guatemala, Honduras, Jamaica, Mexico, Trinidad. East Asia (EAsia): Hong Kong, Japan, South Korea, Malaysia, Philippines, Thailand, Australia, New Zealand. Noneast Asia (NAsia): India, Israel, Pakistan, Sri Lanka, Syria. Northern Europe (NEur): Austria, Belgium, Denmark, Finland, France, West Germany, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Sweden, Switzerland, United Kingdom. Southern Europe (SEur): Cyprus, Greece, Malta, Portugal, Spain, Turkey, Yugoslavia. Africa (Afr): Cameroon, Côte d'Ivoire, Kenya, Morocco, Senegal, South Africa, Tanzania, Tunisia, Zimbabwe.

Table 3 Initial Portfolio Share of Risky Assets (ω), Expressed as a Fraction, and Standard Deviation of the Annual Return to Risky Investment (σ), in Percent

	NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
ω	0.14	0.94	0.40	0.20	0.41	0.08	0.41	0.58
σ	12.60	4.86	7.50	10.47	7.35	16.98	7.34	6.19

Table 4 Riskless and Risky Rates of Return, in Percent per Year

	NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
i	3.60	1.25	2.02	4.54	1.26	4.32	3.28	0.98
α	7.60	5.25	6.02	8.54	5.26	8.32	7.28	4.98

Table 5 Characterizing Equilibrium under Global Financial Integration: First Example

*Equilibrium shares in the risky mutual fund**

NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
0.105	0.225	0.098	0.101	0.205	0.000	0.207	0.058

*Shares sum to 0.999 because of rounding.

Other characteristics of the financially integrated equilibrium

Expected annual return on the risky mutual fund (α^*), percent	6.31
Standard deviation of mutual-fund annual return (σ^*), percent	3.41
Share of mutual fund in total wealth (ω^*), fraction	0.85
World annual real rate of interest (i^*), percent	4.54
Expected annual growth rate of consumption (g^*), percent	4.37

Table 6 Gains from International Financial Integration, as a Percentage of Wealth

NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
124.5	237.6	299.1	22.6	478.4	61.1	98.8	463.4

Table 7 Gains from Switching Deterministic Technologies, as a Percentage of Wealth

NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
43.3	106.7	154.4	−23.4	274.2	2.9	22.3	262.6

Table 8 Stock-Market Returns and Implied Growth Rates for Germany, Japan, and the United States, 1976-92

Mean (α) and standard deviation (σ) of annual risky return and implied expected annual per capita consumption growth rate (g), in percent, and implied portfolio share of risky assets (ω), expressed as a fraction

	Germany	Japan	United States
α	9.10	12.79	6.72
σ	28.35	28.07	12.41
g	0.78	1.85	1.81
ω	0.15	0.23	0.51
$\omega\sigma$	4.25	6.46	6.33

Correlation coefficients of national stock-market returns

	Germany	Japan
United States	0.554	0.284
Germany		0.420

Actual annual mean growth rate (g) and standard deviation (σ_v) of consumption per capita, 1976-88

	Germany	Japan	United States
g	2.09	3.06	2.49
σ_v	1.80	1.40	1.82

Table 9 Characterizing Equilibrium under Global Financial Integration: Second Example

Equilibrium shares in the risky mutual fund

Germany	Japan	United States
0.00	0.31	0.69

Other characteristics of the financially integrated equilibrium

Expected annual return on the risky mutual fund (α^*), percent	8.60
Standard deviation of mutual-fund annual return (σ^*), percent	13.84
Share of mutual fund in total wealth (ω^*), fraction	0.57
World annual real rate of interest (i^*), percent	2.00
Expected annual growth rate of consumption (g^*), percent	2.84

Gains from international financial integration (λ), as a percentage of wealth

Germany	Japan	United States
70.3	27.2	28.3

Note: The values reported above are based on the assumptions that $R = 6$, $\varepsilon = 0.5$, and $i = \delta = 0.02$.

Footnotes

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[1]Alternative theoretical models are proposed by Robert E. Lucas, Jr. (1982), René M. Stulz (1987), and Lars E. O. Svensson (1988). Recent attempts to quantify the welfare gains from international asset trade include Harold L. Cole and Maurice Obstfeld (1991), Enrique G. Mendoza (1991), David K. Backus, Patrick J. Kehoe, and Finn E. Kydland (1992), and Eric van Wincoop (1991).

[2]With its linear technologies, this paper's model is a special case of the continuous-time stochastic model of John C. Cox, Jonathan E. Ingersoll, Jr., and Stephen A. Ross (1985). Their focus, however, is on asset pricing rather than on growth, and their assumptions on preferences are more restrictive than those entertained below. Similar stochastic models have been used to study effects, including growth effects, of fiscal or monetary policies; see Jonathan Eaton

(1981), Mark Gertler and Earl Grinols (1982), Giancarlo Corsetti (1991), and Grinols and Stephen J. Turnovsky (1992). Explicit production externalities of the type first posited by Kenneth J. Arrow (1962), and featured in much of the literature on endogenous growth, are not modeled explicitly below. Instead, endogenous growth springs from a constant private marginal product of investment, as in work of Hirofumi Uzawa (1965), Robert G. King, Charles I. Plosser, and Sergio T. Rebelo (1988), Robert J. Barro (1990), Larry E. Jones and Rodolfo Manuelli (1990), and Rebelo (1991). In endogenous-growth models based on Arrow-type externalities, the social marginal product of investment is effectively constant. Obviously, nothing below depends on the existence of literally risk-free assets; all that is needed is that relatively safe assets have low expected returns.

[3]The Greenwood-Jovanovic assumption of a sunk cost of entering the financial intermediation network leads, however, to much richer dynamics than those that emerge from my model.

[4]The foregoing capsule review lists only a few papers that are especially relevant to the approach taken below to model the effects of uncertainty and financial markets on growth. A number of other related studies have appeared. See, for example, Giuseppe Bertola (1991), Thomas F. Cooley and Bruce D. Smith (1991), Harris Dellas (1991), Ross Levine (1991), and especially Gilles Saint-Paul (1992), who presents a formal model of the link between technological specialization and markets for risk. Marco Pagano (1993) surveys this literature. Raymond Atje and Jovanovic (1993) present evidence of a positive cross-sectional association between national output growth rates and proxies that measure levels of domestic financial intermediation.

[5]For further discussion, see Obstfeld (1992).

[6]The paper will focus on the economy's behavior in the limit as h becomes infinitesimally small. When $R = 1/\varepsilon$, (1) implies that as $h \to 0$, $U(t)$ approaches the familiar expected-utility form

$$U(t) = \mathrm{E}_t\{(1 - R)^{-1} \int_t^\infty C(s)^{1-R} e^{-\delta(s-t)} ds\}.$$

[7]Equation (4) implies that $V^K(t)$ is lognormally distributed: by Itô's Lemma, $V^K(t) = V^K(0)\exp\{(\alpha - \frac{1}{2}\sigma^2)t + \sigma[z(t) - z(0)]\}$. Since $\mathrm{var}[z(t) - z(0)] = t$, the expected growth rate of $V^K(t)$ is α, that is, $\mathrm{E}_0 V^K(t)/V^K(0) = e^{\alpha t}$. The assumption of i.i.d. uncertainty is analytically convenient, but it compromises the model's empirical fit. Log U.S. consumption, for example, does not follow an exact random walk, as the model will imply. I.i.d. uncertainty is in part responsible for the extreme equity-premium and risk-free rate puzzles noted below (section V). Potentially, a linearized model such as the one proposed by John Y. Campbell (1993) could be used to approximate the effects of serially correlated shocks.

[8]Itô's Lemma, applied to equation (15), reveals the time-t consumption level to be $C(t) = C(0)\exp\{(g - \frac{1}{2}\omega^2\sigma^2)t + \omega\sigma[z(t) - z(0)]\}$. Note that for any $t > 0$, $\mathrm{E}_0\, C(t)/C(0) = e^{gt}$.

[9]For plausible parameter values, however, $dg/dr > 0$.

[10]Equation (19) follows from the observation that $\mu = [2\varepsilon\delta + (1-\varepsilon)(g + i)]/(1 + \varepsilon)$. The condition $\mu > 0$ is required for the existence of an individual optimum.

[11]For example, let the scalar quantity ω_j^* denote $\mathbf{1}'\omega_j$, the share of country j's wealth invested in risky assets. Then by (22),

$$\omega_j^* = \mathbf{1}'\Omega^{-1}(\alpha - i^*\mathbf{1})/R_j$$

$$= [1'\Omega^{-1}(\alpha - i^*1)/R_j][(\alpha - i^*1)'\Omega^{-1}(\alpha - i^*1)/(\alpha - i^*1)'\Omega^{-1}\Omega\Omega^{-1}(\alpha - i^*1)]$$

$$= (\alpha - i^*1)'[\Omega^{-1}(\alpha - i^*1)/1'\Omega^{-1}(\alpha - i^*1)]/R_j\theta'\Omega\theta$$

$$= (\theta'\alpha - i^*)/R_j\theta'\Omega\theta = (\alpha^* - i^*)/R_j\sigma^{*2}.$$

[12]The instantaneous variability of wealth falls from $(0.75)^2(0.1)^2 = 0.005625$ to $(0.1)^2/2 = 0.005$.

[13]For country j, the welfare gain λ_j is given by

$$\lambda_j = (\mu_j^*/\mu_j)^{1/(1-\varepsilon_j)} - 1$$

$$= \{[2\varepsilon_j\delta_j + (1 - \varepsilon_j)(g^*_j + i^*)]/[2\varepsilon_j\delta_j + (1 - \varepsilon_j)(g_j + i_j)]\}^{1/(1-\varepsilon_j)} - 1.$$

[14]See footnote 8.

[15]The formulas for computing the welfare gains can be found in Obstfeld (1992) (see equation (11), p. 12).

[16]National consumption per capita is measured at 1985 international prices as PWT variable 3 times PWT variable 6 (see Summers and Heston 1991, p. 362, for exact definitions). Consumption of nondurables and services only would be a superior consumption measure for the purpose at hand, but data are unavailable for most countries.

[17]Recall that investments in a country's risky capital have cumulative payoffs that follow (4), and that ω denotes the share of risky capital in the optimal portfolio.

[18]Many would regard a value of $R = .18$ as being unrealistically high. Kandel and Stambaugh (1991) marshall arguments to the contrary.

[19]Consumption variability is highest in less-developed regions of the Western Hemisphere and Asia. The low real interest-rate levels that these regions therefore display are consistent with the "financial repression" hypothesis of the economic development literature.

[20]Optimal consumption in a deterministic model with rate of return $\rho = \omega\alpha + (1 - \omega)i$ is given by $C/W = \mu_d = [\varepsilon\delta - (\varepsilon - 1)\rho]$. The coefficient μ_d^*, in which ρ is replaced by $\rho^* = \omega\alpha^* + (1 - \omega)i^*$, governs optimal consumption after the technology transfer described in the text. The measure of welfare gain reported in table 7 is $\lambda = (\mu_d^*/\mu_d)^{1/(1-\varepsilon)} - 1$. A more exact measure of the technology transfer effect than the one used to construct table 7 would be based on defining $\rho^* = \omega\alpha^* + (1 - \omega)i^* + (\omega^* - \omega)[(\alpha^* - i^*) - (\alpha - i)]$. Making this change only reduces the numbers in table 7, given the assumptions of the present example.

[21]By using the U.S. CPI to deflate dollar returns, I am implicitly assuming that German and Japanese investors evaluate the real returns on dollar assets as Americans do. A more detailed treatment would allow for deviations from relative purchasing-power parity; this change would recognize that investors in different countries may perceive different real returns on the same asset. In my 1989 paper I present evidence that international differences in consumption growth are systematically related to purchasing-power parity deviations. The annual CPI data I use here come from the first column of table B-56 in *Economic Report of the President*.

[22]Empirical properties of per capita consumption growth rates were derived from the 1976-88 data in Summers and Heston (1991). (See footnote 16 above for details.)

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