AN APPLICATION OF AN INVESTMENT MODEL TO CHANNEL CATFISH FARMING

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ABSTRACT

An Application of an Investment Model
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A dynamic operations and investment simulation model was applied to catfish farming. The purpose was to identify some important aspects that should influence investment decisions in channel catfish farming enterprises. These results exhibited a number of economic relations of which the following were most important: (1) When initial average profits were $0.20 per pound and the initial price of land, buildings, and equipment was close to $800 per acre, the initial investment policy of the firm was one of continuous purchase of new capacity. (2) Higher initial average profits resulted in larger maximum capacities up to a limiting size, beyond which further increases in profits resulted in increases in net worth, but not in capacity. (3) The investment policy of the firm was found to be very sensitive to initial prices of capacity higher than $800 per unit, and no new capacity was added if prices of capacity reached $1,500 per acre. (4) Profit accumulation and, thus, investment decisions were found to be sensitive to changes in the interest rate paid for financing new capacity.

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In recent years, fish farming has developed as an economically viable industry in the United States primarily because of two factors. First, fish farming in southern areas of the United States represents an important alternative use for land. Second, fish are efficient converters of concentrate feeds into high-protein human food.

Many farmers have earned high per acre net returns from fish farming. Some farmers have financed the complete investment involved in shifting from rice to fish farming and have paid off the indebtedness within seven years.

Objectives of the study

This study has three basic objectives. The first was to adapt a theoretical investment model developed by Thompson and

George[1] and interpreted and made operational by Mange[2] to channel catfish farming. The second objective was to estimate the underlying functions regarded as known in the Thompson-George model and to calculate empirical solutions for the decisions that face channel catfish farmers in Arkansas, the present center of catfish farming activities. The third objective was to deduce the economic relations implied by the results and express them in a form which can be useful to decision makers interested in fish farming.

Sources of data

Extensive biological experimentation has been performed at the U. S. Bureau of Sport Fisheries and Wildlife Fish Farming Experimental Station at Stuttgart, Arkansas. The growth data analyzed in this study are based on the results obtained from these experiments by Hastings.[3]

1/ Thompson, Russell G., and Melvin D. George, "Optimal Operations and Investments of the Firm." Management Science, in press.

2/ Mange, Frank A., "An Optimal Investment Model of the Firm: An Interpretation and Application to Fish Farming." Ph.D. dissertation, University of Missouri, Columbia, Missouri, 1967.

3/ These results have been prepared for publication by W. Hastings, U. S. Bureau of Sport Fisheries and Wildlife, Fish Farming Experimental Station, Stuttgart, Arkansas.

Useful supplemental information was obtained from fish hatcheries and selected channel catfish farmers in the southeastern United States. Information from the farmers was particularly helpful in delimiting investment planning which is the most important problem faced by fish farmers. Desirable operating procedures, such as stocking and feeding rates, are fairly well-known, and price and cost data are available; however, decision makers actually associated with fish farming and servicing would like to know more about planning investments in fish farming. This study focused solely upon channel catfish farming for human food, where the fish are regularly fed the best types[4/] and amounts of prepared feeds. We assumed that the farmer stocks fingerlings large enough at the start of the growing season to attain an average size of $1\frac{1}{4}$ pounds by the end of the season. In addition, we made the following specific assumptions:

(1) The growing season is 210 days

(2) Fish are stocked at the rate of 1,500 per acre

(3) Fish are fed daily at the rate of 3 percent of body weight

(4) Fingerlings, at the time of stocking, average 18 weeks in age, 7 inches in length, and 0.1 pounds in weight

(5) The ponds lie fallow for 154 days each year.

4/ By definition, the best ration was one giving the cheapest rate of gain per pound of live weight fish produced.

The Thompson-George model is a recent development, so it is (as most pioneering works commonly are) more restrictive than the applied economist and the decision maker might wish.[5/] This is especially true with respect to the subjective expression of the management limitation on new capacity purchases and the lack of a time lag in the investment process.

5/ Since the time of this study, the Thompson-George model has been transformed into a dynamic linear programing model and extended to allow for a savings account as well as a borrowing account. The transformation is especially important for fish-farming applications since ponds are typically stocked and harvested once a year. The extension of the model makes it possible to have three rates of interest: (1) The rate paid for borrowed money, (2) the rate received for savings, and (3) the rate of temporal time preference. The first and second rates would be expected to differ by the amount reflecting the risk involved in fish farming, whereas the second and third rates would differ by the amounts reflecting the appreciation (or depreciation) of the value of money. The model has also been extended to allow for a debt-equity constraint, so it is possible to evaluate the effects of capital rationing on the growth of the firm.

Adaptation of the Model

To use the Thompson-George model, it was necessary to estimate and/or specify all the functions regarded as known in that model: (1) $g(t)$ is the output of a unit of capacity in a unit of time, (2) $\pi(t)$ is the profit per unit of output, (3) $c(t)$ is the price of a unit of capacity, (4) $M(t)$ is the management limitation on the purchase of new capacity, (5) $i(t)$ is the money rate of interest at which the firm can borrow or lend, (6) $\alpha(t)$ is the rate of attrition of capacity, (7) $\delta(t)$ is the discount rate reflecting the money rate of interest less (plus) the depreciation (appreciation) of the value of money, and (8) $\phi(t)$ = exponent $\delta(t-t_1)$ is the discount function by which revenue and costs are reduced to present values.

For this study, a unit of output is considered as 1 live-weight pound of fish and a unit of capacity is 1 acre of pond. Further, the stocking rate is specified in terms of the number of fish per acre and the unit of time used was specified as 1 week.

The management limitation, $M(t)$, was specified in acres of capacity per week and represents the maximum amount of new capacity that can be brought into production each week.

Output function, g(t)

Channel catfish stop assimilating food and gaining weight (for all practical purposes) when the water temperature falls below 60° F. The water temperature in the area being considered is below 60° F. from about late October until late March, so the growing season for channel catfish is about 210 days. Thus, the function g(t) in the Thompson-George model reflects the growth in that 210-day period, and is zero otherwise. The function g(t) used in this study was specified to be of the form illustrated in figure 1, where g(t) has the same value at every multiple of 52n+t (where n is a natural number and t is a week).

In the Stuttgart experiments by Hastings, channel catfish were stocked at 1,500, 2,500, and 5,000 per acre and fed daily at a rate of 3 percent of body weight. When those results were related to time, the following exponential function gave the best fit:

$$W(t) = \text{exponent } (\alpha + \beta_1/t + \beta_2/t^2),$$

where W(t) represents per acre weight in pounds, and t is time in weeks.

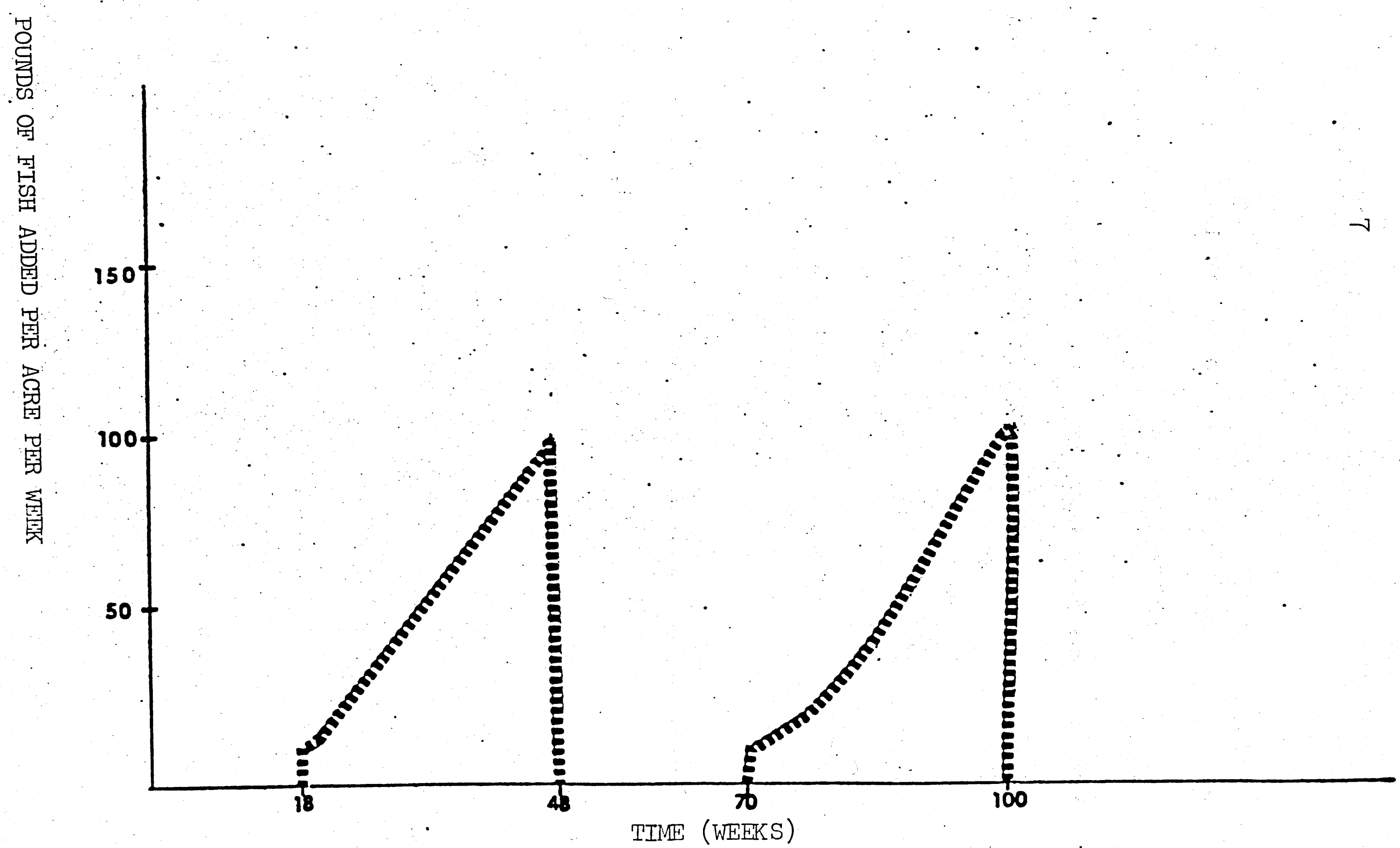

Figure 1. The output function, g(t)

This function was fit statistically by regression analysis. The following estimates were obtained:

Case 1. Stocking rate of 1,500 fish per acre:

$$\hat{W}_1(t) = .0022 \text{ exponent } (16.45 - 1{,}087/t + 44{,}136/t^2)$$

$$R^2 = .990 \qquad N = 42$$

Case 2. Stocking rate of 2,500 fish per acre:

$$\hat{W}_2(t) = .0022 \text{ exponent } (16.63 - 978/t + 40{,}803/t^2)$$

$$R^2 = .992 \qquad N = 7$$

Case 3. Stocking rate of 5,000 fish per acre:

$$\hat{W}_3(t) = .0022 \text{ exponent } (18.68 - 1{,}791/t + 102{,}657/t^2)$$

$$R^2 = .992 \qquad N = 8$$

In each case, the statistical fit was significant and the coefficients had the expected signs.

With these estimated functions, it was possible to obtain the growth segment of the function $g(t)$ in the Thompson-George model by differentiating the selected $\hat{W}_i(t)$, $i = 1, 2, 3$. Unfortunately, because of the form of $W(t)$, we regarded this form of $g(t)$ as too complex for an initial application of the Thompson-George model. Instead, we used the following functional form which fits the data reasonably well (figure 2) and is easier

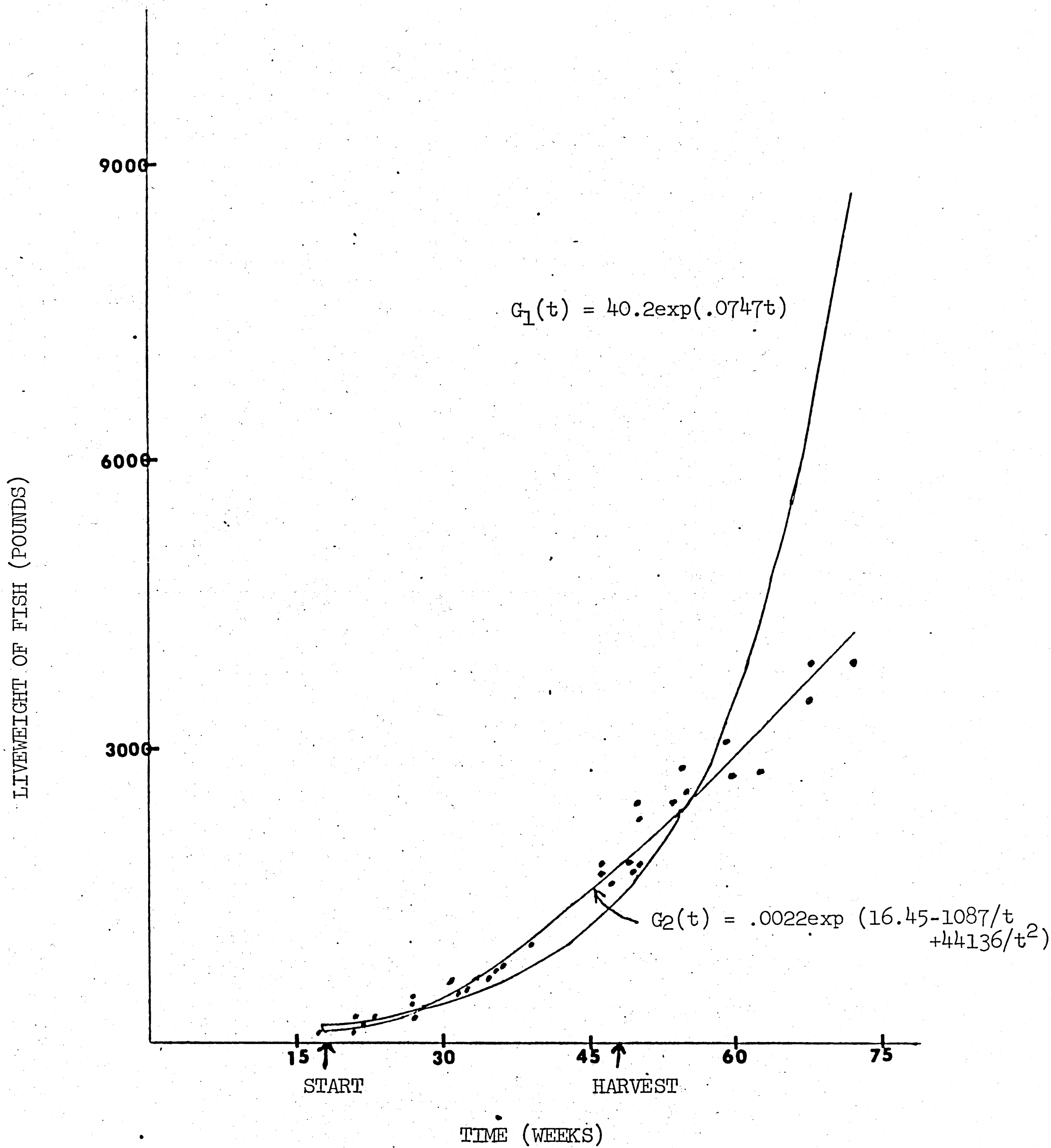

Figure 2. Live weight of fish produced per acre where 1,500 fish were stocked per acre.

to compute:

$\hat{G}(t) = \alpha$ exponent βt, where $G(t)$ represents pounds of fish per acre, and t is time in weeks.

This form was fit statistically by regression analysis for a stocking rate of 1,500 fish per acre:

$$\hat{G}(t) = 40.2 \text{ exponent } .0747t$$

$$R^2 = .915 \qquad N = 42$$

This estimate represents a reasonably good statistical fit, explaining 91.5 percent of the variation in live weight of fish produced per acre.

In the analysis which follows, $g(t)$ was specified to be the derivative of $\hat{G}(t)$, $\hat{g}(t) = 3$ exponent $.0747t$, for the 210-day growing season and zero for the 154-day fallow period.

Profit function, $\pi(t)$

To estimate profits per pound of live-weight fish produced, we estimated both production costs and prices received.

Production costs.--To estimate production costs, we obtained five sets of data: one set from the Stuttgart Experimental Station and four from commercial operations. The former represented the production costs involved in the experiments cited above, whereas the latter represented the costs actually incurred by fish farming operations.

In each case, production costs were classified into seven cost categories: water pumping, labor, feed, chemicals and fertilizers, fingerlings, harvesting, and miscellaneous. For each category, we estimated costs per pound and calculated the weighted average costs (table 1).

Fish prices.--It was much more difficult to estimate prices received than costs incurred because of the lack of a central market or marketing agency. To date, it has been common practice for a farmer to pass the word that he is going to harvest fish in specific ponds on certain days. The local people show up at these times and places and buy as many fish as they desire.

To estimate prices received, we obtained available historical price information from published sources and private conversations. The live-weight price of $0.50 per pound was most often quoted, but some sources reported prices at $0.35.[6/] Because of limited price information, we could not use statistical techniques to estimate a price function. Therefore, for the

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6/ U. S. Department of the Interior, Fish and Wildlife Service, Bureau of Sport Fisheries and Wildlife, "Catfish Feeding and Growth." Fish Farming Experimental Station, Stuttgart, Arkansas, 1966, mimeographed report.

Hulsey, A. H., "Trends in Commercial Fish Farming Practices in Arkansas." Paper presented at the Eighteenth Annual Conference of the Southeastern Association of Game and Fish Commissioners, Clearwater, Florida, October 1964.

Table 1. Costs of production per pound of live-weight fish produced

Type of expenditure	Production cost per pound
	Dollars
Feed	.108
Water pumping	.010
Labor	.032
Chemicals and fertilizer	.010
Fingerlings	.046
Harvesting	.012
Miscellaneous	.032
Total cost per pound	.250

analysis which follows, the price function was specified to be of the following form:

$P(t) = E_1$ exponent $\beta t + F \cos St$, where E_1 is the initial price, β is the rate of increase (or decrease) in the initial price, F is the seasonal variation in price, and S is the length of the seasonal price cycle.

This price function was positioned to give the lowest prices in early October and the highest in early April.

Because production costs were $0.25 per pound, the following profit fuction was obtained by subtracting 25 from E_1:

$\pi(t) = (E_1 - 25)$ exponent $\beta(t) + F \cos (St)$.

Capacity price, c(t)

The cost of a unit of capacity (1 acre of pond) was based on data from Mullins.7/ His report estimated all of the costs shown in table 2, except land costs.

Land is not a homogenous commodity, so land prices vary widely in eastern Arkansas. The minimum price quoted was $300 per acre; the maximum was $500 per acre. For this study, the price of land was specified as $400 per acre.

7/ Mullins, Troy, "Producing Food Fish Requires High Capital," Arkansas Farm Research, 16:1, 1967.

Table 2. Estimates of the component costs for a unit of fish farming capacity (1 acre of pond)

Item	Per acre costs
	Dollars
Land	400
Reservoir construction, pipes, gates, and other reservoir equipment	225
Wells, pumps, and pump motors	101
Storage buildings and tanks	21
Boats and boat motors	8
Sieves	7
Agitators, containers, miscellaneous	7
Total costs per acre	799

The price of a capacity unit, c(t), was specified to be of the form Y exponent (Zt), making it possible to allow for rising as well as constant capacity prices over the decision-making period.

Limitation on the purchase of new capacity, M(t)

In the area of firm growth, there are factors limiting the scale of a new firm. Hicks[8/] states that these factors are: (1) The increasing difficulty of management and control as the firm grows, and (2) the amount of risk that the decision maker is willing to accept.

The function M(t) can be viewed in at least two different ways. First, it can be regarded as a management constraint that varies with the decision maker's age and experience; that is, the ability of the manager to coordinate larger operations over time. For example, that managers are promoted progressively in most organizations to higher positions of responsibility indicates the common belief that the ability to manage a larger unit accrues with experience. Second, the function M(t) can be regarded as a supply constraint. In any given period of time, the availability of new capacity is limited. That is, the amount of new capacity

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8/ Hicks, John, Value and Capital, 2nd. ed. Clarendon Press, Oxford, 1946.

that can be built, delivered, and brought into production in a specific period, such as a week, is limited.

In this study, M(t) was specified arbitrarily in the first sense so that it would reflect an initial period of learning and increasing management ability, a following period of constant management ability, and a final twilight period of declining management ability (figure 3). This function was specified mathematically as follows:

$$M(t) = \begin{cases} \nu_o \text{ exponent } \delta_1 t & t_1 \leq t \leq \tau_1 \\ \gamma & \tau_1 < t < \tau_2 \\ \nu_1 \text{ exponent } \delta_1 t & \tau_2 \leq t \leq t_2 \end{cases}$$

where ν_o is the initial amount of new capacity purchased, γ is the maximum level of new capacity purchases, τ_1 is the time when M(t) reaches its maximum, and τ_2 is the time when M(t) begins to decline.

It should be noted that the function M(t) is determined by four of the six parameters: ν_o , γ, δ_1, ν_1, τ_1, τ_2. The particular four used is arbitrary. In this study, ν_o , γ, τ_1, and τ_2 were the parameters specified.

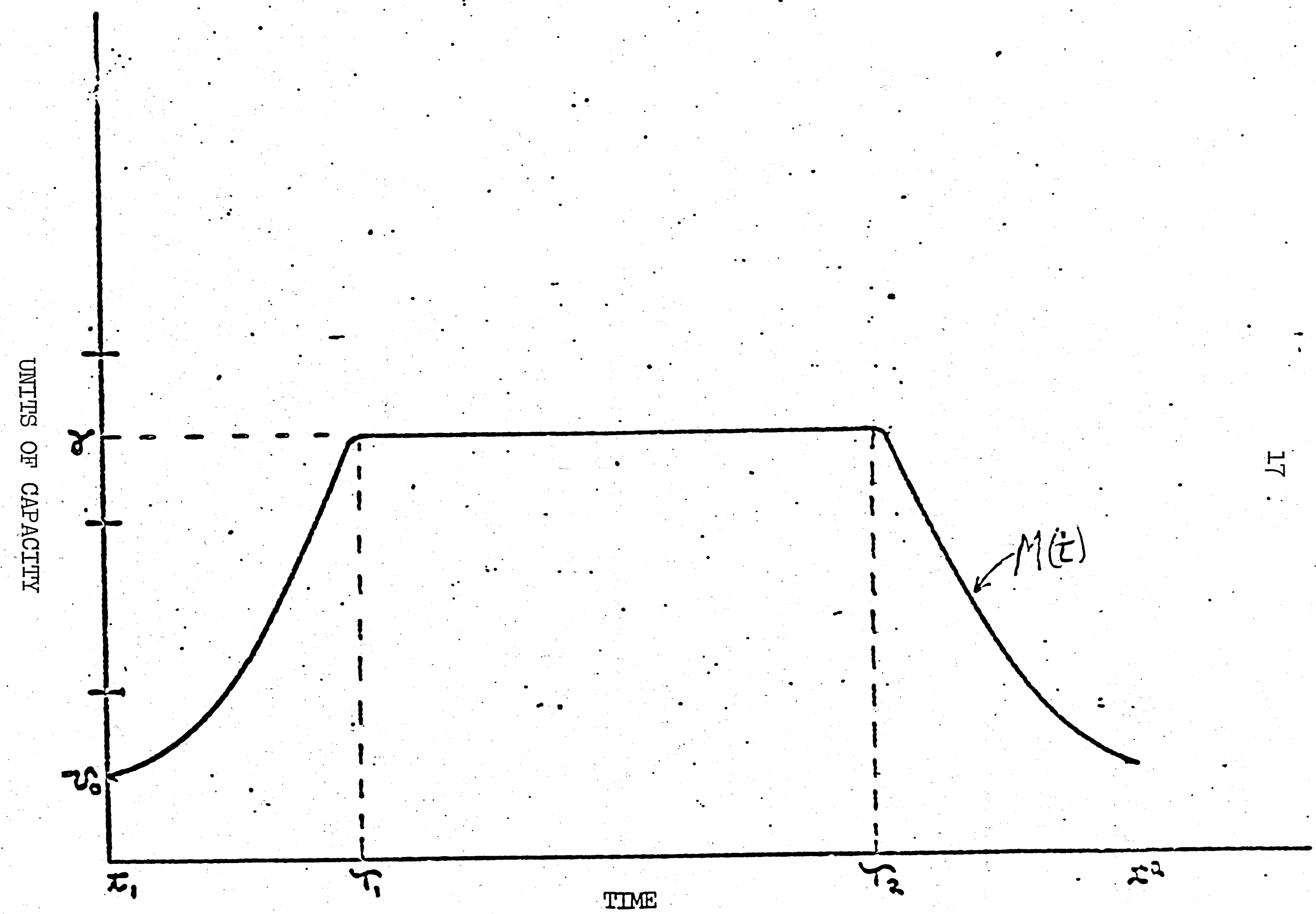

Figure 3. The upper bound on the rate of purchase of new capacity

Interest rate, $i(t)$

In this study, the interest rate, $i(t)$ was held constant over each decision-making interval. The interest rate was varied parametrically around 7 percent for different decision-making intervals in order to determine the economic effect of different interest rates.[9/]

Because the firm can both borrow money and save money, the rate $i(t)$ paid for borrowed money is the same as that received for savings. We realize that savers receive less interest than borrowers pay, but this is a limitation of the Thompson-George model.

Attrition rate, $\alpha(t)$

Fish farmers and service representatives indicated that a unit of capacity would have a usable life of about 10 years. Thus $\alpha(t)$ was varied around 10 percent per year. That rate is likely a conservative specification, especially because one-half of the value of a capacity unit is represented by the land, which might well have an infinite life.

Initial capacity of farm, x_1

A basic assumption of the Thompson-George model is that

9/ Seven percent was the typical rate charged by the Lonoke, Arkansas Production Credit Association at the time of this study.

the initial amount of capacity must be positive. That is, at the beginning of the period, the decision maker has some property rights at his command.

In this study, we allowed initial capacity, x_1, to vary around 120 acres. This reflects the fact that a good well in eastern Arkansas will generally supply enough water for 120 acres. In addition, one man can operate a 120-acre farm reasonably well.[10/]

Initial debt, D_1

The amount of indebtedness was not constrained by the model, so we also arbitrarily specified the initial amount of indebtedness. In making this specification, we reasoned that the initial indebtedness would not be larger than costs of initial capacity plus costs of production in the first season. At a price of $800 per acre, initial costs of a 120-acre fish farm would be $96,000 (including land and equipment, trucks, and pumps), whereas production costs would vary with the level of stocking. If the ponds were stocked to produce 1,200 pounds of live-weight fish per acre, production costs would be $36,000 (using a production

10/ This statement reflects only normal requirements. More labor would be needed seasonally, in particular at stocking and harvesting times.

cost figure of 25 cents per pound), whereas if the ponds were stocked to produce 2,400 pounds of live-weight fish per acre, production costs would be $72,000. Hence, at a maximum, initial indebtedness would be $168,000 for a 120-acre farm. Moreover, the farmer would normally be expected to have some equity at the start of the decision-making period. It seemed reasonable, therefore, to specify the initial indebtedness norm as $100,000.

Discount rate, $\delta(t)$

Because the "proper" discount rate has been debated by economists for years, we will not reopen that debate. Instead, we state that the discount rate, which was a constant over the decision-making interval, was specified to reflect the interest rate plus (or minus) the appreciation (or depreciation) of the value of money. We specified that it had a norm of 4 percent and was varied around this figure to reflect conditions in the economy and to determine how this parameter affects the decision-making problem.

Terminal endpoint of the problem, t_2

In this analysis we specified the terminal endpoint, t_2, as 1,040 weeks (20 years) in most of the problems studied;

however, to evaluate the effect of a longer as well as a shorter period, we specified t_2 as 2,080 weeks (40 years) in one case and as 520 weeks (10 years) in another.

Selected solutions of the Thompson-George model

Discussion of the cases studied

Using a computer program developed by Mange[11] specifically for the Thompson-George model, we calculated complete solutions for 53 problems. To evaluate the economic effects of each parameter in the model, we varied the parameters in each problem. This was done by specifying one problem as the standard problem and by varying only one parameter in the model at a time. For example, when the attrition rate was varied, all of the remaining parameters in the problem were held fixed at the values specified in the standard problem.

The standard problem was:

$i = .07$, $\delta = .04$, $\alpha = .10$, $E = .20$, $F = .05$, $R = 3$,

$t_2 = 1{,}040$, $Y = 800$, $Z = .05$, $\nu_o = .50$, $\gamma = 2.5$,

$\beta = .03$, $\theta = .0747$, $x_1 = 120$, $D_1 = 100{,}000$, $\tau_1 = 260$,

and $\tau_2 = 780$.[12]

11/ Frank Mange, *op. cit.*

12/ We believe that values of the parameters i, δ, E, F, R, Y, Z, X_1, D_1, β, and θ, as specified in the standard problem, reflect present-day fish farming operations.

Form of presentation

By use of the computer, we examined the firm's decision-making position by week, over the fixed decision-making interval. This examination provided investment policy as well as physical and monetary balance sheets. Moreover, the analysis indicated the marginal present values in terms of net worth of additional units of capacity and debt as well as the current imputed prices of additional capacity and improved management.

We performed a considerable amount of computing, and the figures presented here only summarize the results. Some of the specific results not presented will be discussed in terms of what appeared to be general relations.

In tables 3 and 4, summaries of the results are presented for all of the 53 cases. In table 3, the results reflect basically what happened during the period, whereas the results in table 4 represent the terminal values of capacity, debt, and net worth. Table 3 is organized to give both explicit and implied information about the firm's economic position. So that table 3 can be interpreted readily, the following remarks are provided:

(1) Columns headed "First" and "Last switch points," give the investment policy of the firm. In every case but two, the initial investment policy from the beginning of the period to

Table 3. Values of the parameters: first and last switch points in the investment decision rule (D.R.), maximum capacity of the firm and time of occurrence, maximum indebtedness and time of occurrence, and time of final debt-free position

Case	Parameter varied	Value of parameter	First and last switch points: First	First and last switch points: Last	Maximum capacity and time of occurrence: Capacity	Maximum capacity and time of occurrence: Time	Maximum indebtedness and time of occurrence: Indebt.	Maximum indebtedness and time of occurrence: Time	Time of final debt-free position: Time
			Week	Week	Acres	Week	Dollars	Week	Week
1	Standard	Standard	913	1,010	963	828	379,466	475	804
2	i	.02	1,014	1,014	963	828	283,985	423	735
3	i	.04	1,016	1,016	963	828	314,291	424	749
4	i	.06	964	1,011	963	828	349,083	424	798
5	i	.08	911	1,009	963	828	405,419	476	842
6	i	.10	706	1,005	933	805	478,596	528	853
7	i	.12	498	745	797	647	561,926	530	803
8	α	.14	601	895	606	650	513,539	581	861
9	α	.12	809	1,007	864	808	438,849	579	891
10	α	.08	965	1,012	1,083	845	332,992	424	701
11	α	.06	1,013	1,013	1,232	871	295,091	422	643
12	δ	.08	654	1,002	902	753	379,466	475	756
13	δ	.06	810	1,007	961	808	379,466	475	804
14	δ	.02	965	1,012	963	828	379,466	475	804
15	E	.25	1,013	1,013	963	828	187,304	321	495
16	E	.15	342	582	629	537	587,978	484	888
17	E	.10	No switch	D.R.Neg.	120	1	100,000	1	Always in debt
18	F	0	965	1,012	963	828	273,099	426	648
19	F	.07	912	1,009	963	828	431,183	577	856
20	F E	.10 .25	810	1,005	961	829	547,144	576	913
21	F E	.07 .25	966	1,013	963	828	214,662	370	545
22	F E	.10 .30	966	1,012	963	828	257,989	369	633
23	F E	.10 .30	1,014	1,014	963	828	129,123	316	436
24	F	.13	1,013	1,013	963	828	160,739	316	483
25	t_2 t_2	520 2,080	446	492	680	491	370,156	476	Always in debt

(continued)

Table 3 (continued). Values of the parameters: first and last switch points in the investment decision rule (D.R.), maximum capacity of the firm and time of occurrence, maximum indebtedness and time of occurrence, and time of final debt-free position

Case	Parameter varied	Value of parameter	First and last switch points		Maximum capacity and time of occurrence		Maximum indebtedness and time of occurrence		Time of final debt-free position
			First	Last	Capacity	Time	Indebt.	Time	Time
			Week	Week	Acres	Week	Dollars	Week	Week
26	τ_2	1,560	1,381	1,526	1,189	1,380	379,466	475	804
27	Y	600	1,013	1,013	963	828	164,043	319	496
28	Y	1,000	602	1,000	859	701	791,701	686	941
29	Y	1,200	185	330	364	329	423,580	329	688
30	Y	1,500	No switch	D.R.Neg.	120	1	100,000	1	285
31	Z	.03	1,013	1,013	963	828	280,478	422	644
32	Z	.01	1,015	1,015	963	828	225,031	370	545
33	Z	0	1,016	1,016	963	828	205,574	320	533
34	ν_o	.10	913	1,010	921	808	411,416	528	852
35	ν_o	.30	913	1,010	946	819	387,669	476	840
36	ν_o	.70	913	1,010	978	838	371,687	475	798
37	γ	3.0	913	1,010	1,143	825	456,237	578	840
38	γ	2.0	913	1,010	783	833	287,268	475	793
39	γ	1.5	913	1,010	601	841	202,269	423	739
40	X_1	40	913	1,010	947	831	503,962	630	897
41	X_1	60	913	1,010	951	830	512,221	579	892
42	X_1	160	913	1,010	971	827	295,634	475	747
43	D_1	120,000	913	1,010	963	828	415,399	529	841
44	D_1	50,000	913	1,010	963	828	284,697	475	748
45	D_1	0	913	1,010	913	828	191,375	424	649
46	β	.05	1,014	1,014	965	828	286,219	372	596
47	β	.01	446	583	675	582	492,321	479	750
48	β	0	290	380	485	379	356,068	379	646
49	β	-.03	80	121	166	120	118,434	120	471
50	τ_1	130	913	1,010	1,005	801	384,888	422	745
51	τ_1	390	913	1,010	921	863	359,449	579	855
52	τ_2	1,040	913	1,010	1,027	912	379,466	475	804
53	τ_2	650	913	1,010	879	713	379,466	475	753

Table 4. Final capacity, final debt, and final discounted net worth of the firm for the cases in table 3.

Case	Parameter varied	Value of parameter	Final capacity	Final debt	Discounted net worth
			Acres	Dollars	Dollars
1	Standard	Standard	580	-1,246,924	1,452,286
2	i	.02	799	-1,301,344	1,544,121
3	i	.04	800	-1,295,382	1,526,922
4	i	.06	789	-1,273,509	1,485,626
5	i	.08	767	-1,209,045	1,408,010
6	i	.10	683	-1,159,454	1,275,784
7	i	.12	433	-1,252,730	1,068,020
8	α	.14	357	- 588,144	655,975
9	α	.12	619	- 776,692	1,022,714
10	α	.08	941	-1,862,197	1,959,106
11	α	.06	1,132	-2,604,099	2,562,238
12	δ	.08	620	-1,487,034	641,633
13	δ	.06	743	-1,315,208	970,888
14	δ	.02	794	-1,220,957	2,155,866
15	E	.25	798	-3,084,376	2,563,069
16	E	.15	248	- 272,354	356,916
17	E	.10	16	96,428	27,829
18	F	0	791	-1,950,290	1,898,190
19	F	.07	773	- 970,631	1,273,987
20	F	.10	748	- 585,853	1,006,246
21	E	.25			
	F	.07	798	-2,795,602	2,384,697
22	E	.25			
	F	.10	796	-2,364,379	2,117,166
23	E	.30			
	F	.10	799	-4,232,539	3,227,952
24	E	.30			
	F	.13	798	-3,798,485	2,960,418
25	t_2	520	643	267,320	444,584

(continued)

Table 4 (continued). Final capacity, final debt, and final discounted net worth of the firm for the cases in table 3

Case	Parameter varied	Value of parameter	Final capacity	Final debt	Discounted net worth
			Acres	Dollars	Dollars
26	t_2	2,080			
	τ_2	1,560	377	-15,931,277	5,160,871
27	Y	600	798	- 2,499,238	2,040,548
28	Y	1,000	550	- 494,010	863,713
29	Y	1,200	93	- 430,405	397,080
30	Y	1,500	16	- 301,403	306,136
31	Z	.03	798	- 2,084,111	1,703,467
32	Z	.01	800	- 2,779,346	1,937,315
33	Z	0	800	- 3,073,398	2,046,963
34	ν_o	.10	679	- 1,115,084	1,272,495
35	ν_o	.30	739	- 1,190,554	1,377,440
36	ν_o	.70	814	- 1,295,745	1,515,359
37	γ	3.0	914	- 1,409,313	1,649,170
38	γ	2.0	645	- 1,082,728	1,252,589
39	γ	1.5	507	- 916,016	1,048,990
40	X_1	40	769	- 856,746	1,167,907
41	X_1	60	772	- 893,463	1,210,219
42	X_1	160	786	- 1,482,564	1,613,664
43	D_1	120,000	780	- 1,165,820	1,413,909
44	D_1	50,000	780	- 1,449,683	1,548,227
45	D_1	0	780	- 1,652,443	1,644,168
46	β	.05	799	- 3,199,206	2,570,377
47	β	.01	280	- 653,456	635,762
48	β	0	136	- 435,028	415,682
49	β	-.03	28	- 122,473	183,171
50	τ_1	130	732	- 1,626,539	1,668,191
51	τ_1	390	789	- 909,646	1,290,962
52	τ_2	1,040	962	- 920,806	1,470,022
53	τ_2	650	613	- 1,468,553	1,413,405

the first switch point[13/] (which represents a change in investment policy), represented continuous purchase of new capacity at the maximum level, M(t). Between the time of the first switch point and the last, there was at least one additional switch point, which is indicated by the same value of t in the first and last switch point columns. The firm never purchased any new capacity between the last switch point and the endpoint of the period.

(2) In the column headed "Maximum capacity and time of occurrence," the largest capacity attained by the firm and the time when it was attained is given. The minimum capacity and the time it was attained is not given, since it can be found in all cases (where the initial investment policy was one of purchase) from the information in the standard problem and table 4. That is, if the final capacity in table 4 is less than the initial capacity in the standard problems, the final capacity is the minimum capacity; however, if it is not, the initial capacity is the minimum one.

(3) The maximum amount of indebtedness and the time when it occurred is given in the columns headed accordingly. The maximum amount of savings is now shown, because when savings

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13/ At this point, the marginal efficiency of capital is equal to the money rate of interest, i.
Keynes, John Maynard, The General Theory of Employment, Interest, and Money. New York: Harcourt-Brace and Company, 1963.

occurred, they were always a maximum at the end of the period.

(4) In the last column of table 3, the time is given when the firm became debt free; that is, the point when the firm became a creditor rather than a debtor. In several instances, however, the firm did oscillate between being a debtor and creditor. When that happened, the figure given represents the last change of this type.

Explanation of the specification for the standard problem

In the standard problem, the decision maker with a 20-year enterprise horizon, starts farming with 120 acres of capacity and $100,000 of indebtedness. This capacity costs him $800 per unit initially and increases in price at the rate of 5 percent per year. The firm's capacity depreciates at an annual rate of 10 percent. Live-weight fish are produced from each unit of capacity during the production season in accordance with the growth·function R exponent θt, where, R = 3 and θ = .0747. This results in initial (average) profits of 20 cents per pound, which vary seasonally from a low of 15 cents to a high of 25 cents. With the passage of time, average profits increase 3 percent per year.

To buy new capacity, the firm may either borrow money or invest earnings. If it does the former, the firm must pay 7 percent interest per year, whereas for the latter, the firm uses money that could earn 7 percent per year.

Initially, the decision maker can purchase one-half a unit of new capacity per week and, as he gains experience, he can increase his purchases to a maximum of 2.5 units per week. He can purchase new capacity at this level between the 260th and 780th week. Thereafter, in the twilight period, the decision maker is limited to smaller purchases each passing week.

Discussion of the results for the standard problem

In the standard problem the firm's initial investment policy was one of continuous purchase for 913 weeks. It was followed by another period of purchase which ended at the 1,010th week. The firm's maximum capacity, 963 units, was attained at the 828th week.

In the first 475 weeks, production profits were generally less than interest charges and new investment costs. Indebtedness generally increased as a result and reached a maximum of $379,466 in the 475th week. Thereafter, production profits generally exceeded interest charges and new capacity costs. These profits were used to retire debt until the 804th week, when the firm became debt free.

Table 4 shows that the discounted value of the firm's net worth was $1,452,286 at the end of the period. This represents an increase in net worth of $1,448,286 in 20 years.

Discussion of the results for the 53 cases

General.--In 51 of the 53 cases studied, the initial investment policy of the firm was one of continuous purchase. The length of the period in which this policy was in effect varied from a minimum of 80 weeks in Case 49, where average profits decreased 3 percent per year, to a maximum of 1,381 weeks in Case 26, where the operating period was 2,080 weeks. Cases 17 and 30 were the exceptions; in Case 17, initial average profits were relatively low--10 cents per pound--and in Case 30, the price of a capacity unit was relatively high--$1,500 per unit.

In 40 of the 51 cases where there was an initial period of investment, there was at least one period of investment following some time between the first and last switch point. This implies that the marginal efficiency of (physical) capital, as defined by Keynes[14/] was equal to the rate of interest, which represented the money rate of interest, more than once during the period. In the remaining 11 cases, the marginal efficiency of (physical) capital was equal just once during the period to the money rate of interest. Investment equilibrium for the firm

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14/ Keynes, John Maynard, The General Theory of Employment Interest and Money. New York: Harcourt-Brace and Company, 1936.

would be expected to exist when the marginal efficiency of (physical) capital equals the interest rate. Therefore, the investment equilibrium of the firm was characterized by instability, as postulated by Keynes.

The investment policy of the firm was found to be very sensitive to the level of average profits. No new investments were made when initial average profits were relatively low. As these profits were increased, longer periods and larger amounts of new investments were observed at first, but as these profits were further increased, definite limits in the investment policy and the firm's maximum capacity emerged. In particular, the firm's maximum capacity reached a limit of 963 units when initial average profits reached 20 cents per pound. Of course, the firm's discounted net worth continued to increase as these profits increased above 20 cents per pound, even though the firm's maximum capacity remained the same. This means that the decision maker's primary role changed from that of a producer to that of a banker for initial average profits of 20 cents per pound and higher. Therefore, higher profits stimulated new investments in the firm up to a point, beyond which further increases in profits only added to net worth.[15/]

15/ It is possible, of course, that this result might be the form of a "local" rather than a "global" maximum.

In general, we observed that low interest rates, low discount rates, and low attrition rates stimulated growth, whereas high interest rates, high discount rates, and high attrition rates stifled growth. In addition, the initial price of capacity had a significant inverse effect on the investment policy of the firm and, in turn, the amounts of capacity operated by the firm.

Bankers will be interested in the fact that the largest amount of indebtedness ($791,701) was incurred when the initial price of capacity was $1,000 per unit. Less indebtedness was incurred for both lower and higher capacity prices. This implied that a price of $1,000 was not high enough to stifle growth, yet high enough to result in the largest amount of indebtedness when growth took place.

Some results obtained for variations in specific parameters

Interest rate, i(t).--It can be seen that the positive investment policy carried out when the interest rate was low, was longer than when it was high. This was true for the initial investment period and every investment period thereafter.

It should be noted that the firm's maximum capacity was 963 units in the 828th week for each rate considered between 2 and 8 percent. Thus, the maximum capacity operated by the firm and the time when it was operated were invariant with respect to this range of variation in the interest rate.

Discount rate.--In Cases 12, 13, and 14, inverse relations were evident between the firm's initial investment policy and the discount rate. When this rate was 8 percent, the initial investment period was 654 weeks and the firm's maximum capacity was 902 units in the 753rd week. This period increased by 156 weeks and capacity increased by 59 units when the discount rate was decreased to 6 percent. Further decreases in the discount rate resulted in a longer investment period but a maximum capacity that was only two units larger.

Attrition rate.--Variation in the attrition rate had a noticeable effect on the investment policy carried out by the firm. For example, when this rate was increased from 6 to 14 percent, the first switch point decreased from 1,013 to 601 weeks. This means that the time of the first change in the investment policy from one of purchase to no purchase decreased by 412 weeks with a change of 8 percentage points in the attrition rate. In addition, the last switch point, which represented the beginning of the final period of no investment, decreased from 1,013 weeks at 6 percent to 895 weeks at 14 percent.

Upper bound on new capacity purchases, M(t).-- Since the investment rule in the Thompson-George model is independent of the parameters determining M(t), the investment policy of the firm was not influenced by the magnitude of upper bound on new capacity purchases. Thus, variation in the parameters underlying M(t) had no direct effect on the investment policy of the firm. These variations only had an indirect effect through the capacity level of the firm (c.f., the first and last switch points in Cases 34 to 39).

Variations in the parameters underlying M(t) did affect the maximum capacity and its time of occurrence, the debt-free position, the final capacity, and the discounted value of net worth. Specifically, the maximum and final capacities increased as ν_0, γ, and τ_2 increased, whereas the maximum indebtedness generally decreased as ν_0 and τ_2 increased, and generally increased as γ increased. The effects of the changes in γ were particularly large. For example, in Cases 37 to 39 and Case 1, when γ increased from 1.5 to 3.0 units by increments of 0.5 units, maximum capacity increased from 601 to 1,143 units and maximum indebtedness increased from $202,269 to $456,237.

The effects of variation in γ upon the final discounted values of net worth were sizable. Net worth was $1,048,990 in Case 39 when γ was 1.5, and $1,649,170 in Case 37 when was 3.0.

Profits.--It is evident from the cases analyzed that the investment policy of the firm was very sensitive to the level of average profits. When average profits were 10 cents per pound at the beginning of the period, the investment decision rule was always negative. In this case, the firm never purchased any new capacity during the period, and its initial capacity was its maximum capacity. There was, however, an initial period of investment followed by another period of investment in Case 16 when initial average profits were 15 cents per pound, or 5 cents higher than in Case 17. No new capacity was purchased after the 582nd week, and the maximum capacity operated was 629 units in the 537th week. When initial average profits increased by another 5 cents per pound in Case 1, the initial period of investment increased to 913 weeks; it was followed by another relative short period of investment. The firm quit purchasing new capacity for the last time in the 1,010th week. Its maximum capacity was 963 units in the 828th week. In Case 15, initial average profits were increased again by 5 cents per pound. The initial period of investment, which was the only period of investment in this case, lasted until the 1,013th week. Surprising as it might seem, however, these profits did not result in a larger maximum capacity. The maximum capacity in this case was the same as it was in Case 1, and it was attained in the same week. Therefore, larger initial average profits up to 20 cents per pound resulted in larger capacities up to a limiting size of 963 units, beyond which further increases in profits had no effect on size.

It can be seen from Cases 18 to 20 that, by varying the seasonal variation in profits, the length of the initial investment period did vary, but its effect on maximum capacity was slight. The firm's maximum capacity varied by only two units when the seasonal variation in profits changed by 6 cents.

In studying the results from Cases 15 to 17 and Case 1, the relation between initial average profits and maximum indebtedness was interesting. The largest amount of indebtedness was incurred when initial average profits were 15 cents per pound, and the smallest amount was incurred when these profits were 10 cents per pound.

When initial average profits were 15 cents per pound, discounted net worth was $356,916; when initial average profits were 20 cents per pound, discounted net worth was $1,452,286; and when initial average profits were 25 cents per pound, discounted net worth was $2,563,069. The change in net worth between the last two cases was particularly important because the firm's maximum capacity was the same in both cases.

Therefore, higher initial profits resulted in larger discounted values of net worth, even though they did not necessarily stimulate larger firms.

Capacity price, c(t).--In cases 27 to 29, the firm's investment policy was very sensitive to prices of capacity higher than $800 per unit. In cases 27, 28, and 29, when the initial price was $600, $1,000, and $1,200, respectively, the first switch points were in the 1,013th, 602nd, and 185th weeks, and the last switch points were in the 1,013th, 1,000th, and 330th weeks. In addition, the firm never invested during the period when the initial price of capacity was $1,500 per unit.

In Cases 31 to 33, the effect of an upward trend in the initial price of capacity was evaluated. The firm's investment policy was not affected by these trends. First and last switch points as well as maximum and final capacities were virtually the same, whether this price was constant over the period or increased as much as 3 percent per year.

Initial amount of capacity, x_1.--Since the investment decision rule in the model is unaffected by the initial amount of capacity, the investment policy of the firm was unaffected. The larger capacities, therefore, acted solely as an endowment effect and resulted in larger maximum and final capacities and larger discounted values of net worth.

Endpoint of the decision making period, t_2.--To evaluate the effect of the length of the horizon, the endpoint of the decision making period was decreased to 520 weeks in Case 25 and increased to 2,080 weeks in Case 26. (The parameters underlying M(t) were changed accordingly). In both cases, the first and last switch points changed in the direction of the change in the length of the interval.

The firm's maximum capacity increased as the length of the period increased, whereas the firm's final capacity decreased as the length of the period increased above 1,040 weeks. In fact, the final capacity was considerably smaller for the 2,080 week period than for the 520-week period; yet the discounted value of net worth was $5,160,871 for the longer period and was only $444,584 for the shorter. Fish farmers who stay in business for longer periods of time would be expected (under conditions approximating those analyzed) to become richer individuals, but they will not necessarily be farming larger capacities.

Major findings

(1) When initial average profits were close to 20 cents

per pound and the initial price of capacity was close to $800 per unit, the initial investment policy of the firm was one of continuous purchase of new capacity.

(2) Higher initial average profits resulted in larger maximum capacities up to a limiting size, beyond which further increases in profits resulted in increases in net worth, but not in capacity.

(3) The investment policy of the firm was found to be very sensitive to initial prices of capacity higher than $800 per unit. The initial investment period decreased continuously from 913 to 185 weeks as the initial price of a capacity unit increased from $800 to $1,200. The firm did not invest in more capacity at prices above $1,500 per unit (per acre).

(4) Inverse relations were found between the initial investment policy of the firm and the rates of interest, discount, and attrition. In general, low rates of interest, discount, and attrition were found to stimulate growth, whereas high rates of interest, discount, and attrition were found to stifle growth.

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