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The Demand for Food in the UK, 1974-1984.

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Introduction.

In an earlier paper (Burton, 1988) the direct and indirect Translog models were applied to aggregate data on the demand for wet fish. The direct and indirect utility/cost functions can both be viewed as approximations to the true underlying functions, and thus either model may be expected to give a reasonable approximation of consumer preferences. However, when applied empirically, the direct model gives rise to an inverse demand system (with prices being determined by exogenous quantities), while the indirect model gives rise to a direct demand system, with quantities being determined by prices. Thus the application of either system implies competing hypothesis about the market structure in which the goods are purchased. In the case of wet fish, the implications of the two alternatives for consumer preferences was considerable, with the assumption of quantity exogenaity (which is not the usual assumption in demand studies, but which may be the more reasonable for a harvested, perishable good like wet fish) resulting in very large own and cross price elasticities compared with the conventional system. In the current paper, the same analysis is applied to all food consumption. Although the argument for considering the supply of all food types as exogenous is less strong than for wet fish alone, it would still seem a relevant option if compared with the alternative of a perfectly elastic supply.

The first two sections of this paper, covering theory, are taken directly from the previous report. The data set used and the results from the alternative models are then given, followed by a comparison of the elasticities generated.

Theory.

The cornerstone of neo-classical demand theory is the concept of the utility function. It is assumed that the observed consumption decisions of consumers can be rationalized in terms of the outcome of maximising the utility function subject to some budget constraint. For any given budget and set of commodity prices, explicit knowledge of the utility function allows the analyst to identify key variables, such as income and price elasticities. Thus the aim of much empirical demand analysis is to explicitly (or implicitly) identify the un-observable form and parameters of the utility function. These will not be a function of the market structure through which goods are obtained ie. in a direct utility function, the level of utility attained depends solely on the quantities of the commodities consumed. Thus, nowhere in the literature is it suggested that the level of utility achieved by a consumer as a result of consuming a bundle of goods will be different if that bundle has been imposed by a rationing system, or if an identical bundle is chosen through a free market system. This is an extreme case, but it will be equally true that, in a market system, the optimization process of the individual consumer will be unchanged whether or not the market supply of the good is perfectly elastic or perfectly inelastic. Each consumer will behave as a price taker, and, for a particular price vector, derive their demand for commodities. At the market level there is a difference: if supply is perfectly elastic at a particular price, then market demand is simply given by the summation of the individual demands evaluated at that price. If the supply is perfectly inelastic, then the sum of the individual demands will be constrained to equal that exogenous supply, and it is the price vector that will change in order to ensure this. Thus, the market structure will have no implications for the form of the optimality conditions of the consumer, but will have implications for any empirical analysis that utilises market level data, as the variables that will be considered as exogenous will be different.

The majority of empirical demand analysis that uses complete systems has been conducted on the basis that prices are exogenous, implying a perfectly elastic supply curve. Given the econometric complexity of most systems, it is perhaps not surprising that researchers have avoided the difficulties in determining both price and quantity simultaneously, but it is by no means clear why the Gordian knot has so resolutely been cut in this way, in particular when the commodities under consideration are foodstuffs. In this case, given the nature of the production process, a more reasonable assumption would seem to be that supply is exogenous, and thus prices are determined within the market. There have been some studies that have followed this approach, for example, Houthakker (1960), Christensen and Manser (1977). In this note, consumer preferences with respect to wet fish are to be investigated, and here in particular it would seem appropriate to specify that supply is exogenous, especially when using quarterly data. The results from a conventional, quantity dependant, demand system will also be presented.

The models used are the direct and indirect translogs, as outlined in Christensen et al (1975). In the direct translog, a direct utility function is specified of the form

1) $$-\ln(U) = \alpha_0 + \sum_i \alpha_i.\ln(X_i) + \tfrac{1}{2}.\sum_i\sum_j \beta_{ij}.\ln(X_i).\ln(X_j) \quad i,j=1....n$$

where X_i is the quantity of commodity i consumed. Maximization of utility subject to the budget constraint $\Sigma P_i X_i = M$ yield first order conditions of the form

2) $$\alpha_i + \sum_j \beta_{ij}.\ln(X_j) - [\ \sum_j \alpha_j + \sum_j\sum_i \beta_{ij}.\ln(X_j)\ ].P_i.X_i/M \quad = 0$$

As we have argued above, these conditions are independent of the market structure that is assumed, and in theory could be used to generate direct or indirect demand functions. In fact, the specification of 2) lends itself to indirect demand functions of the form

3) $$W_i = \frac{\alpha_i + \sum_j \beta_{ij}.\ln(X_j)}{-1 + \sum_j \beta_{mj}.\ln(X_j)}$$

where W_i is the share of expenditure spent on good i, and $\beta_{mj} = \Sigma_i \beta_{ij}$. It is also necessary to impose some normalization rule on the parameters, as the utility function is homogeneous of degree one, and the first order condition homogeneous of degree zero, in the parameters. The normalization used is that $\Sigma_j \alpha_j = -1$. Although the parameters are not invariant to the rule used all elasticities and test statistics are (for a proof of this, see Christensen and Manser, pp 50-51).

The direct demand functions are derived from the indirect translog utility function

4) $$\ln(V) = \alpha_0 + \sum_i \alpha_i.\ln(p_i) + \tfrac{1}{2}.\sum_i\sum_j \beta_{ij}.\ln(p_i).\ln(p_j) \quad i,j=1....n$$

where $p_i = P_i/M$. Using Roys' identity the budget shares can be derived.

5) $$W_i = \frac{\alpha_i + \sum_j \beta_{ij}.\ln(p_j)}{-1 + \sum_j \beta_{mj}.\ln(p_j)}$$

Equations 3 and 5 bear a close resemblance to each other, but it is important to remember that the translog function is not self-dual and therefore they do not represent the same preferences. This would be the case only if $\beta_{ij}=0$ for all i,j, as the utility and cost functions then collapse to the double log form, which is self-dual. From an econometric viewpoint, these are competing models, containing alternative assumptions about the source of exogeneity in the market, a point that was emphasised in the original article, but which seems to have been lost on others (e.g. Bewley 1986, McLaren 1982).

Data

The demand systems have been estimated using quarterly data on the consumption of foodstuffs over the period 1974:1 to 1984:4. The data is drawn from the national food expenditure survey annual reports, aggregated into six categories:

Dairy Products	: Milk, Cream, Cheese and Eggs
Basics	: Fats, Sugar, Cereals, Beverages and Processed Fish
Meat	
Fish	: All Fish except Processed
Vegetables	
Fruit	

Processed fish is mainly fish fingers and fish cakes, products that would not appear to be close substitutes for the other fish products. Its removal from the 'Fish' category is important, as it makes up a fairly high proportion of total 'Fish' expenditure (some 50%), and in the absence of a convenience food category, its inclusion with Basics would seem most appropriate, although here its importance is much reduced (less than 10% of expenditure). The shares of total expenditure accounted for by each group are reported in Table 1 below.

Table 1 Food Expenditure Shares: 1974:1 to 1984:4

	Max	Mean	Min
Dairy	0.184	0.173	0.150
Basics	0.315	0.295	0.271
Meat	0.342	0.325	0.301
Fish	0.230	0.020	0.018
Vegetables	0.165	0.125	0.092
Fruit	0.078	0.063	0.054

Estimation

Before estimation, the exogenous data series in each model were normalised. Thus, in equation 3, the quantities (X_j) were normalised to have a value of 1 in 1984:4, and in equation 5 the prices and income were normalised to have a value of 1 in 1984:4. Again, these transformations will change the parameter values, but not the estimated elasticities or test statistics. The reason for indexing the series in this manner is that it greatly eases the calculation of the elasticities.

Given that adding up holds within the data, and will hold within the estimation, the residual variance covariance matrix will be singular if all 6 share equations within a system are estimated together. The standard procedure is to exclude one equation, and then recover the non-estimated parameters using the adding up constraint. The alternative used here is to estimate each system twice, excluding a different equation each time. The common parameters and the log likelihood value should be invariant between the two, thus providing a check on the computer coding.

In addition to the standard specification reported above, some additional modifications to the model were used. Given that the data is quarterly, seasonal dummies were included to allow for any seasonal change in consumers perception of food. Thus it would may be expected that during the hotter summer months, the utility attained by consumption of meats would fall as compared with lighter meals, and (ceteris paribus) demand would decline in those periods. The dummies were introduced on only the α_i parameters in equations 3 and 5 above, as this is the most parsimonious method of allowing for some seasonal effect. The restriction that the sum of the α_i equals unity was maintained over all 4 quarters of the year. The second modification to the model investigated was that of explicit additivity between goods. This was achieved by imposing the restrictions of the form $\beta_{ij}=0$ for $i \neq j$. The relevant log likelihood test statistics are reported in Table 2 below, for both the Direct and Indirect models. The use of the seasonal dummies can not be rejected, and in the direct model, additivity is accepted.

Table 2 LL Test Statistics

Indirect Model

	No Dummies		Dummies
Additivity	966.7	87 (15)	1016.0
	77 (15)		81 (15)
No Additivity	1012.0	83 (15)	1064.0

Direct Model

	No Dummies		Dummies
Additivity	927.6	66 (15)	964.8
	41 (15)		21 (15)
No Additivity	950.6	44 (15)	977.8

Small sample adjusted test statistics reported between LL values, with number of restrictions in parentheses.

The parameter values for the preferred models are reported in Table 3 below. Note that symmetry has been imposed, with the restrictions that $\beta_{ij}=\beta_{ji}$ $i>j$ = 1....n. Although these may not be valid restrictions for some utility functions that generate demand equations in the form of 3) and 5) (see Simmons and Weiserbs 1979), thereby invalidating the claim that the translogs are second order approximations to any underlying function, the symmetry restrictions were accepted in the preferred indirect model (test statistic = 19.5 for 15 restrictions). In the preferred direct model the symmetry restrictions cannot be tested independently while maintaining additivity.

Table 3 Estimation Results for the Preferred Translog Models

Parameter	Direct		Indirect	
A1	-0.1795	(0.004)	-0.1742	(0.014)
D11	-0.0090	(0.005)	-0.0052	(0.015)
D12	-0.0066	(0.004)	-0.0038	(0.014)
D13	-0.0012	(0.005)	0.0001	(0.013)
B11	0.1099	(0.036)	-0.0714	(0.072)
B12			0.0502	(0.107)
B13			0.0475	(0.139)
B14			0.0045	(0.019)
B15			0.0077	(0.057)
B16			-0.0177	(0.032)
A2	-0.3084	(0.004)	-0.3126	(0.007)
D21	0.0115	(0.003)	0.0063	(0.005)
D22	0.0099	(0.004)	0.0071	(0.007)
D23	0.0083	(0.005)	0.0063	(0.007)
B22	0.1431	(0.105)	-0.1401	(0.192)
B23			0.1184	(0.304)
B24			-0.0009	(0.019)
B25			0.0111	(0.106)
B26			-0.0225	(0.083)
A3	-0.3223	(0.004)	-0.3163	(0.009)
D31	-0.0055	(0.004)	-0.0069	(0.008)
D32	0.0013	(0.004)	-0.0042	(0.009)
D33	-0.0016	(0.004)	-0.0017	(0.009)
B33	-0.0925	(0.057)	0.0900	(0.318)
B34			0.0043	(0.027)
B35			0.0533	(0.139)
B36			0.0252	(0.076)

Table 3. cont.

A4	-0.0199	(0.001)	-0.0206	(0.003)
D41	-0.0008	(0.001)	0.0001	(0.004)
D42	-0.0004	(0.001)	-0.0003	(0.005)
D43	0.0001	(0.001)	0.0011	(0.004)
B44	-0.0142	(0.003)	-0.0055	(0.010)
B45			0.0029	(0.013)
B46			0.0057	(0.011)
A5	-0.1084	(0.003)	-0.1117	(0.005)
D51	0.0022	(0.005)	0.0016	(0.003)
D52	-0.0007	(0.008)	0.0017	(0.004)
D53	-0.0004	(0.008)	0.0012	(0.009)
B55	0.2475	(0.041)	-0.0863	(0.043)
B56			-0.0058	(0.050)
A6	-0.0614	(0.002)	-0.0646	(0.009)
D61	0.0016	(0.002)	0.0042	(0.012)
D62	-0.0034	(0.002)	-0.0004	(0.014)
D63	-0.0056	(0.003)	-0.0070	(0.011)
B66	-0.0285	(0.011)	-0.0346	(0.031)

(standard errors in parenthesis)

Commodity Code: 1=Dairy, 2=Basics, 3=Meat, 4=Fish, 5=Vegetables, 6=Fruit

There appears to be very little difference in the fit of each model, as revealed from a visual inspection of the simulation results (Figures 1 and 2), with both models reproducing the changes in expenditure shares over the period. The System R^2_L[(1)] provides some measure of the goodness of fit, in a similar fashion to the R^2 for a single equation. It is derived by comparing the system log likelihood value with that generated by a base model.

[(1)] The statistic is defined as:-

$$R^2_L = 1 - \frac{1}{1+2^*(LL^* - LL^0).\frac{(T-k)}{T}.\frac{1}{T(n-1)}}$$

LL^* = log likelihood of unrestricted model.
LL^0 = log likelihood of base model.
T = number of observations.
k = number of parameters in each equation.
n = number of equations in the system.

Figure 1 Simulation Results for the Direct Model.

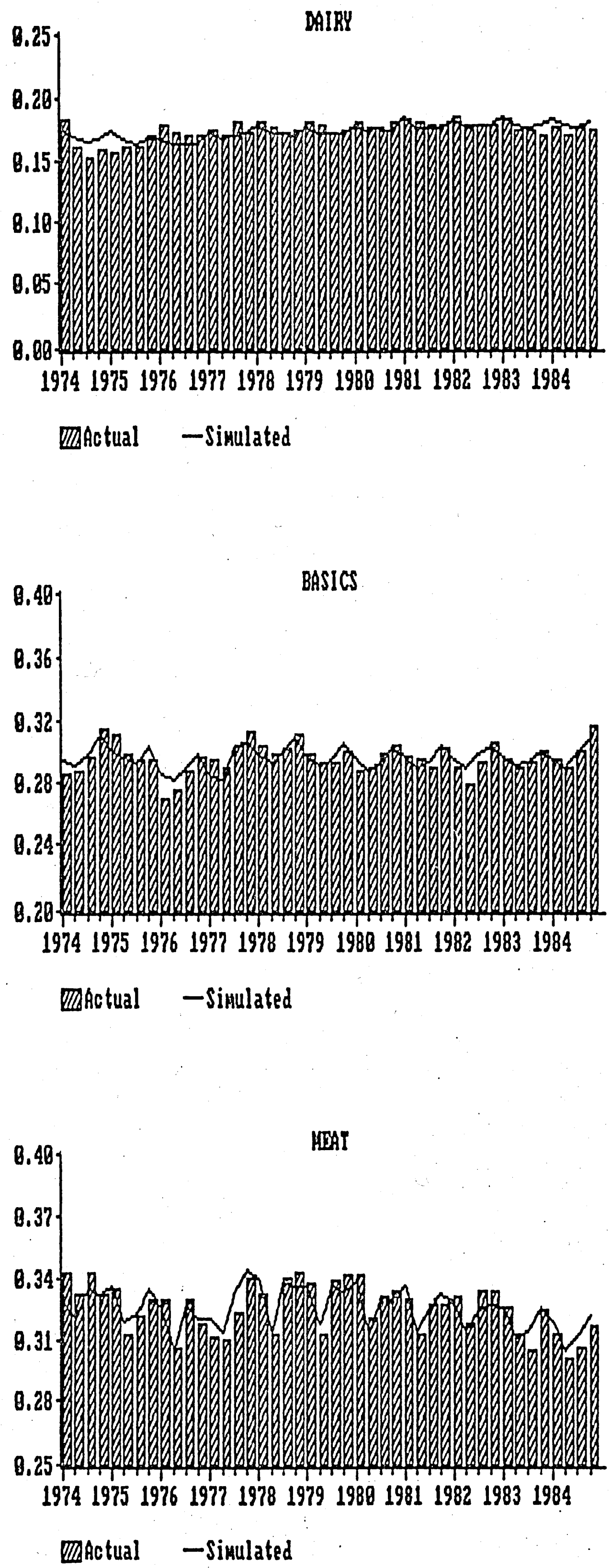
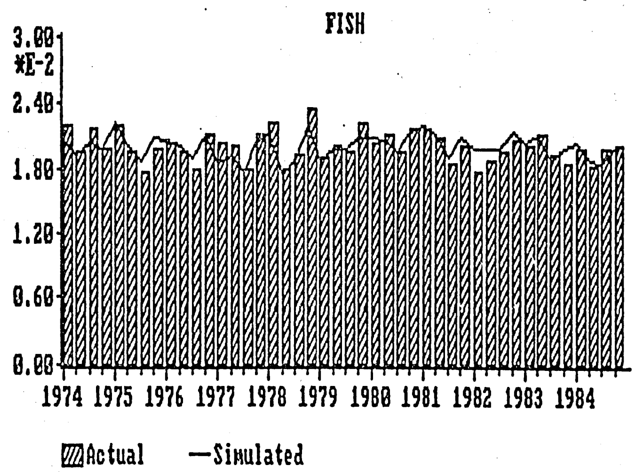
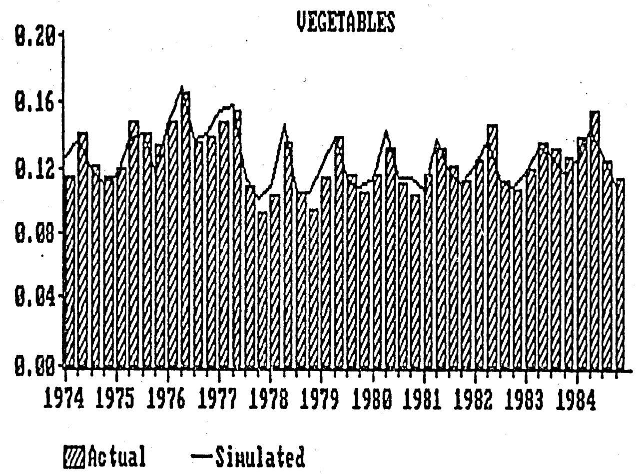


Figure 1 cont.





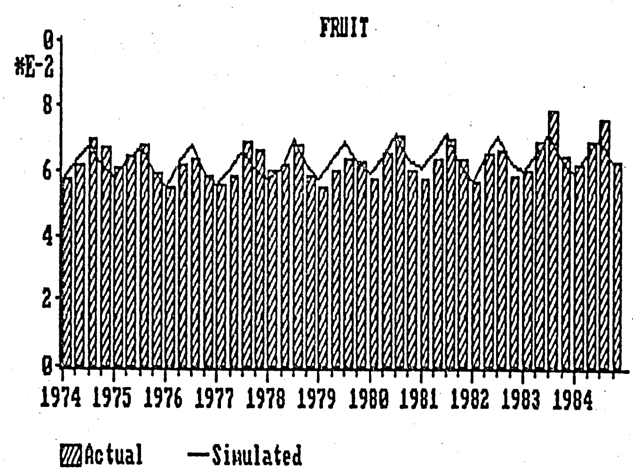


Figure 2 Simulation results for the Indirect Model

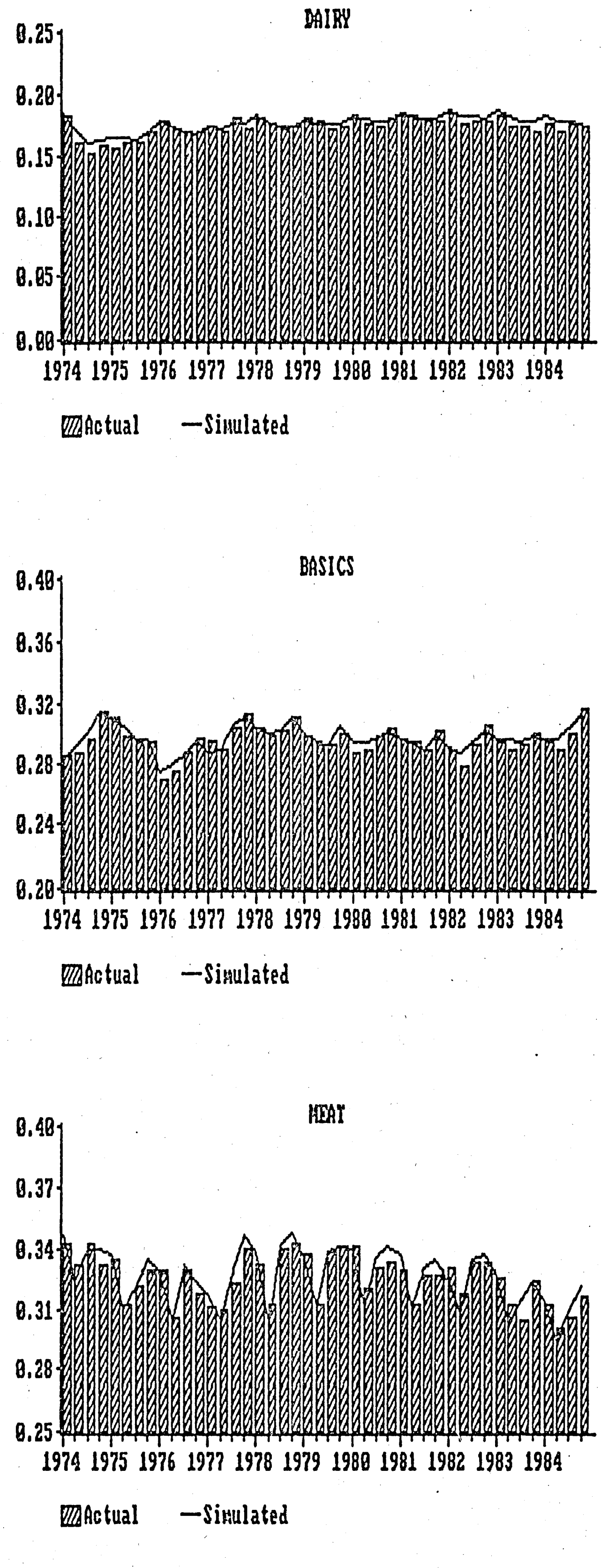
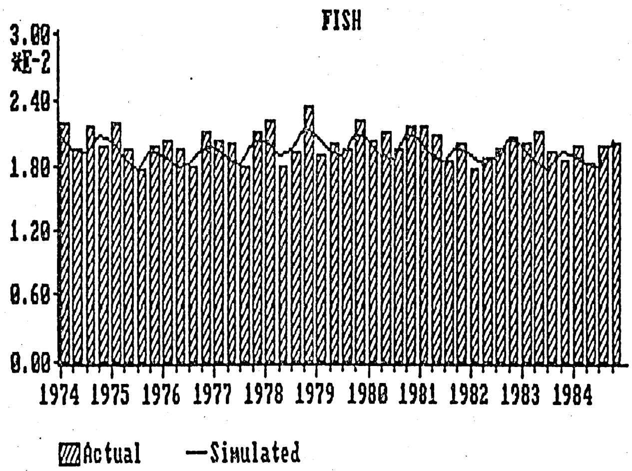
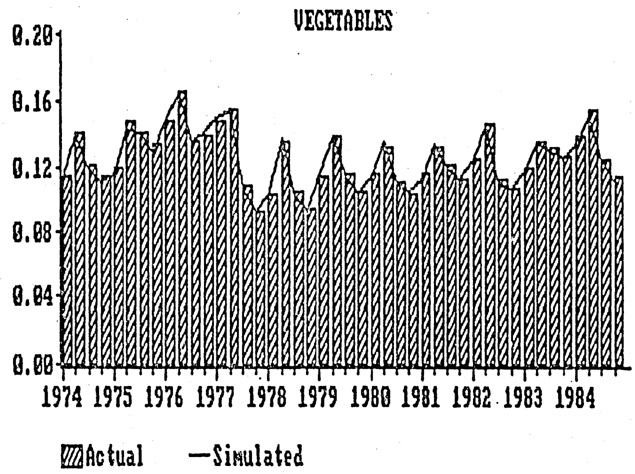
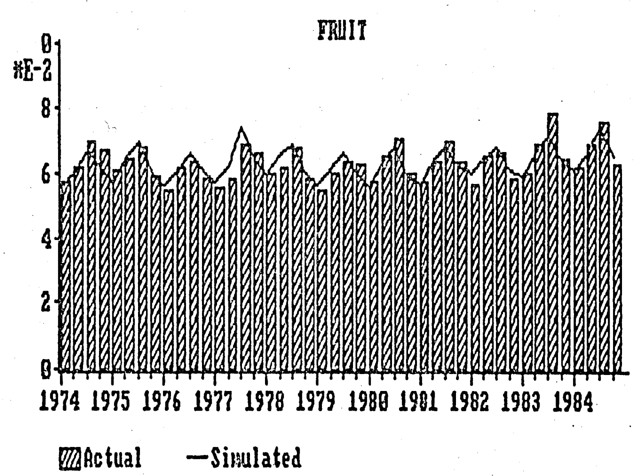


Figure 2 cont.







It is usual to exclude all economic terms from the base model (see Bewley p189), so in this case the translog functions were estimated with the restriction that $\beta_{ij}=0$ for all i,j. The values so generated were 0.388 for the Direct system and 0.404 for the indirect system. Thus, the models explained approximately 40% of the variation after the seasonal pattern of shares is accounted for.

The income and price elasticities of the two models are reported in tables 4 and 5 below (the method of calculation is given in Appendix 3). Both the total price elasticity and the Allen price elasticity of substitution (which compensates for any income effect) are reported. There are some significant differences in the representation of consumer preferences made by the two models. Both have Dairy products, Basics and Vegetables as 'necessities', with income elasticities less than one, and meat and fish as luxuries, although the direct system reverses the ordering of the latter two, with fish having a significantly higher income elasticity in the direct system. The major difference is in Fruit, which changes from a luxury good in the direct model, to an inferior good in the indirect system. Given that additivity is imposed on the direct system, all goods are automatically substitutes in that system. There is a relatively high level of substitution between meat and fish. In the indirect system there is some complementarity, between Dairy products and Basics, between Meat and Fruit, and between Fish and both Vegetables and Fruit. There are no extreme values in the own price elasticities, with all goods in the indirect model showing an inelastic response. In the direct model, the most significant changes occur in Fish and Fruit, the elasticities of which increase substantially, sufficient to give them elastic responses to their own price. The problem of proportionality between income and own price elasticities that is possible within additive demand systems is clearly present, with a correlation coefficient between the two of 0.988. However, Additivity, which generates this result, was accepted empirically, implying that, although this relationship is not required by general demand theory, in this case it is genuinely present in the data rather than being imposed. This is confirmed by inspection of the elasticities from the direct model when additivity is not imposed (Appendix 2). The most important impact of additivity is on the income elasticity of fruit, which changes from -0.082 to 1.915 as aresultof imposing additivity.

Table 4. Elasticities for Direct Model: with dummies and additivity imposed.

With Respect To:-

	Income	Price (total)					
		1	2	3	4	5	6
1	0.609	-0.660	-0.061	0.077	0.030	-0.048	0.033
2	0.710	-0.047	-0.754	0.090	0.035	-0.056	0.038
3	1.403	-0.092	-0.140	-1.223	0.070	-0.111	0.075
4	3.552	-0.233	-0.355	0.452	-3.297	-0.281	0.190
5	0.326	-0.021	-0.033	0.041	0.016	-0.330	0.017
6	1.915	-0.126	-0.191	0.243	0.095	-0.152	-1.760

Price (Allen)

	1	2	3	4	5	6
1	-3.092					
2	0.424	-1.719				
3	0.838	0.978	-2.415			
4	2.121	0.227	4.891	-161.6		
5	0.195	0.227	0.449	1.136	-2.705	
6	1.144	1.334	2.637	6.675	0.612	-26.72

Commodity Code: 1=Dairy, 2=Basics, 3=Meat, 4=Fish, 5=Vegetables, 6=Fruit

Table 5. Elasticities for Indirect Model: With Dummies, Additivity not Imposed.

With Respect To:-

	Income	Price (total) 1	2	3	4	5	6
1	0.799	-0.570	-0.272	0.066	-0.015	-0.062	0.052
2	0.732	-0.140	-0.536	-0.040	0.014	-0.053	0.022
3	1.751	-0.129	-0.358	-0.946	-0.003	-0.186	-0.129
4	1.216	-0.197	0.060	0.127	-0.720	-0.157	-0.329
5	0.527	-0.048	-0.083	-0.139	-0.015	-0.245	0.002
6	0.089	0.296	0.364	-0.052	-0.078	0.073	-0.514

Price (Allen)

	1	2	3	4	5	6
1	-2.471					
2	-0.071	-0.982				
3	1.009	0.606	-1.239			
4	0.086	1.409	1.617	-33.82		
5	0.248	0.262	0.089	-0.186	-1.662	
6	1.609	1.076	-0.254	-3.885	0.564	-8.048

Commodity Code: 1=Dairy, 2=Basics, 3=Meat, 4=Fish, 5=Vegetables, 6=Fruit

It is important to note that these elasticities have been calculated for the 4th quarter of 1984. Given that seasonal dummies have been included in the specifications, the elasticities will change over the year. The elasticities have been calculated for each quarter of 1984, and are reported in Appendix 1. The changes are all fairly small.

Conclusions.

In this paper the demand for food in the UK has been analyzed using the direct and indirect translog models. Although both systems are capable of approximating any underlying set of consumer preferences, their empirical application will represent competing hypothesis about the market structure in which the goods are traded. Using the current data set there appears to be little difference in the explanatory power of the two models, although there are some differences in the representation of preferences. The direct system seems to indicate greater levels of substitution, both to own price and between goods. This conclusion was also arrived at in the original study by Christensen and Manser. Whether this is an implicit feature of the direct translog approach, as opposed to a coincidence over two data sets, can only be resolved by extending the applications further.

Appendix 1.

Elasticities for the indirect model, by season.

Quarter 4.

	Income	Price (total) 1	2	3	4	5	6
1	0.799	-0.570	-0.272	0.066	-0.015	-0.062	0.052
2	0.732	-0.140	-0.536	-0.040	0.014	-0.053	0.022
3	1.751	-0.129	-0.358	-0.946	-0.003	-0.186	-0.129
4	1.216	-0.197	0.060	0.127	-0.720	-0.157	-0.329
5	0.527	-0.048	-0.083	-0.139	-0.015	-0.245	0.002
6	-0.089	0.296	0.364	-0.052	-0.078	0.073	-0.514

Quarter 3.

	Income	Price (total) 1	2	3	4	5	6
1	0.799	-0.570	-0.271	0.067	-0.015	-0.061	0.052
2	0.734	-0.146	-0.519	-0.054	0.014	-0.054	0.025
3	1.786	-0.134	-0.370	-0.955	-0.003	-0.191	-0.132
4	1.277	-0.221	0.065	0.102	-0.690	-0.172	-0.361
5	0.545	-0.041	-0.072	-0.086	-0.012	-0.330	-0.003
6	0.013	0.260	0.318	-0.001	-0.066	0.061	-0.584

Quarter 2.

	Income	Price (total) 1	2	3	4	5	6
1	0.805	-0.576	-0.269	0.062	-0.014	-0.060	0.052
2	0.742	-0.148	-0.515	-0.065	0.014	-0.054	0.027
3	1.788	-0.134	-0.369	-0.962	-0.003	-0.190	-0.130
4	1.250	-0.207	0.060	0.107	-0.703	-0.163	-0.343
5	0.580	-0.030	-0.056	-0.014	-0.008	-0.460	-0.011
6	-0.055	0.286	0.352	-0.047	-0.075	0.070	-0.531

Quarter 1.

	Income	Price (total) 1	2	3	4	5	6
1	0.797	-0.580	-0.265	0.072	-0.014	-0.060	0.050
2	0.736	-0.149	-0.511	-0.062	0.014	-0.054	0.026
3	1.766	-0.131	-0.363	-0.951	-0.003	-0.188	-0.130
4	1.244	0.209	0.064	0.116	-0.708	-0.164	-0.343
5	0.555	-0.037	-0.066	-0.056	-0.010	-0.380	-0.007
6	-0.138	0.313	0.387	-0.078	-0.084	0.079	-0.479

Commodity Code: 1 = Dairy, 2 = Basics, 3 = Meat, 4 = Fish, 5 = Vegetables, 6 = Fruit

Elasticities for the Direct Model, by Season.

Quarter 4.

	Income	Price (total) 1	2	3	4	5	6
1	0.609	-0.660	-0.061	0.077	0.030	-0.048	0.033
2	0.710	-0.047	-0.754	0.090	0.035	-0.056	0.038
3	1.403	-0.092	-0.140	-1.223	0.070	-0.111	0.075
4	3.552	-0.233	-0.355	0.452	-3.297	-0.281	0.190
5	0.326	-0.021	-0.033	0.041	0.016	-0.330	0.017
6	1.915	-0.126	-0.191	0.243	0.095	-0.152	-1.760

Quarter 3.

	Income	Price (total) 1	2	3	4	5	6
1	0.617	-0.663	-0.059	0.077	0.036	-0.050	0.031
2	0.685	-0.045	-0.746	0.086	0.040	-0.056	0.035
3	1.389	-0.092	-0.133	-1.234	0.080	-0.113	0.070
4	4.188	-0.277	-0.400	0.524	-3.714	-0.339	0.211
5	0.337	-0.022	-0.032	0.042	0.019	-0.375	0.017
6	1.824	-0.121	-0.174	0.228	0.106	-0.148	-1.578

Quarter 2.

	Income	Price (total) 1	2	3	4	5	6
1	0.618	-0.666	-0.057	0.075	0.030	-0.060	0.031
2	0.704	-0.044	-0.744	0.086	0.034	-0.068	0.035
3	1.433	-0.090	-0.133	-1.230	0.069	-0.138	0.071
4	3.702	-0.232	-0.344	0.451	-3.483	-0.357	0.183
5	0.421	-0.026	-0.039	0.051	0.020	-0.420	0.021
6	1.883	-0.118	-0.175	0.230	0.091	-0.182	-1.633

Quarter 1.

	Income	Price (total) 1	2	3	4	5	6
1	0.624	-0.671	-0.059	0.077	0.027	-0.057	0.035
2	0.700	-0.045	-0.744	0.086	0.030	-0.064	0.039
3	1.417	-0.092	-0.134	-1.221	0.060	-0.129	0.078
4	3.186	-0.206	-0.301	0.393	-3.019	-0.291	0.177
5	0.392	-0.025	-0.037	0.048	0.017	-0.378	0.022
6	2.067	-0.134	-0.195	0.255	0.088	-0.189	-1.794

Commodity Code: 1 = Dairy, 2 = Basics, 3 = Meat, 4 = Fish, 5 = Vegetables, 6 = Fruit

Appendix 2.

Elasticities for the Direct Model, with Dummies but without Additivity imposed.

With Respect To:-

	Income	Price (total)					
		1	2	3	4	5	6
1	0.841	-0.722	-0.106	0.012	-0.009	-0.031	0.006
2	0.588	-0.012	-0.721	0.084	0.052	-0.050	0.073
3	1.795	-0.157	-0.302	-1.209	0.084	-0.164	-0.053
4	3.392	-0.515	-0.084	0.839	-3.920	-0.303	0.623
5	0.336	0.045	-0.059	0.011	0.008	-0.329	-0.000
6	-0.082	0.171	0.553	0.315	0.260	0.045	-1.319

Price (Allen)

	1	2	3	4	5	6
1	-3.284					
2	0.512	-1.721				
3	0.873	0.847	-2.011			
4	0.403	3.165	5.995	-192.1		
5	0.587	0.151	0.369	0.759	-2.644	
6	0.892	1.698	0.904	12.88	0.321	-20.48

Commodity Code: 1=Dairy, 2=Basics, 3=Meat, 4=Fish, 5=Vegetables, 6=Fruit

Appendix 3.

The income and price elasticities for the indirect translog can be derived directly.

The expenditure elasticities are

$$\text{A1)}\quad n_i = 1 + \frac{d\ln(W_i)}{d\ln(M)} = 1 + \frac{-\sum_j \beta_{ij}/W_i + \sum_j\sum_i \beta_{ij}}{-1 + \sum_j\sum_i \beta_{ij}.\ln(p_j)}$$

The price elasticities are given by

$$\text{A2)}\quad n_{ij} = -\delta_{ij} + \frac{d\ln(W_i)}{d\ln(p_i)} = -\delta_{ij} + \frac{\beta_{ij}/W_i - \sum_j \beta_{ij}}{-1 + \sum_j\sum_i \beta_{ij}.\ln(p_j)}$$

where δ_{ij} is the Kronecker delta.

Derivation of the elasticities for the direct translog has to utilize the bordered Hessian. If the bordered Hessian is defined as the (n+1,n+1) matrix H, and the (n+1,1) vector N is defined as

$$N = H^{-1} . \begin{bmatrix} -M \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

then the income elasticity of the ith good is given by the i+1 element of N.

i.e. $n_i = N[i+1,1]$

Similarly, if the (n+1,n) matrix E is defined as

$$E = H^{-1} . \begin{bmatrix} P_1 & P_2 & P_3 & .. & .. & P_n \\ U_1 & 0 & 0 & .. & .. & 0 \\ 0 & U_2 & 0 & .. & .. & 0 \\ 0 & 0 & U_3 & .. & .. & 0 \\ : & 0 & 0 & .. & .. & 0 \\ : & 0 & 0 & .. & .. & 0 \\ : & .. & .. & .. & .. & 0 \\ 0 & 0 & .. & .. & .. & U_n \end{bmatrix}$$

where U_i is dU/dX_i, then the price elasticities of the ith good with respect to the jth price is given by the (i+1,j) element of E.

i.e. $\quad n_{ij} = E[i+1,j]$

The Allen elasticities of substitution can then be retrieved via the Slutsky equation.

$$\sigma^*_{ij} = n_{ij}/W_j + n_i$$

If the elasticities are evaluated at the point of the normalization of the exogenous variables (i.e. where $X_i=1$) then the derivation is fairly easy. The bordered Hessian is defined as:-

$$H = \begin{bmatrix} 0 & -P_1 & -P_2 & .. & .. & -P_n \\ -P_1 & U_{11} & U_{12} & .. & .. & U_{1n} \\ -P_2 & U_{21} & U_{22} & .. & .. & U_{2n} \\ -P_3 & U_{31} & U_{32} & .. & .. & \vdots \\ \vdots & .. & .. & .. & .. & \vdots \\ \vdots & .. & .. & .. & .. & \vdots \\ \vdots & .. & .. & .. & .. & \vdots \\ -P_n & .. & .. & .. & .. & U_{nn} \end{bmatrix}$$

where $U_{ij} = d^2U/dX_idX_j$.

If $X_i,X_j=1$ then $\quad U_{ij} = \alpha_i.\delta_{ij} - \beta_{ij}$

and $\quad U_i = -\alpha_i$

Thus the elements of the bordered Hessian and the other matrix needed to calculate the elasticities can be derived directly from the estimated parameters. A point to note is that the <u>prices</u> used have to be re-calculated, to allow for the normalization of the quantities in that period.

The price flexabilities for the direct translog are derived in an exactly analogous fashion to the price elasticities in the indirect translog.

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