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A Non Parametric Test of the Expected Utility Hypothesis

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Abstract

A nonparametric test of the expected utility hypothesis is developed. It is shown that the expected utility hypothesis holds if there exists a feasible solution to a system of linear inequalities. Furthermore, when a feasible solution exists boundaries on the coefficient of absolute risk aversion can be calculated explicitly. The test is applied to data on land allocations that are modeled as choices over lottery sets. The result is that the expected utility hypothesis cannot be rejected in most of the cases. This result is in contrast to results obtained in many laboratory experiments.

A Non Parametric Test of the Expected Utility Hypothesis

1. Introduction

The expected utility hypothesis has been used extensively in most models of decisions under uncertainty. The expected utility property is a direct consequence of a number of axioms introduced by von Neumann and Morgenstern four decades ago. The empirical studies of decisions under uncertainty, that are based on the expected utility hypothesis, are valid only when the empirical data are consistent with the expected utility hypothesis. That is, the individuals' preferences have to obey the von Neumann-Morgenstern's axioms in order for the empirical analysis to be valid. In the early fifties economists, like Allais, started studying the validity question: is their set of axioms consistent with reality? Allais constructed an example, a lab test, that shows that the way people rank lotteries is inconsistent with the expected utility hypothesis. During the following years this question was often addressed in the economic and the psychology literature. It was shown that in a large number of cases people make decisions that are inconsistent with the expected utility hypothesis (i.e. Kahneman and Tversky, MacCrimmon and Larsson). As a result of these contradictions economists introduced new theories that relax some of the axioms, mainly the independence axiom. The relaxation of the independence axiom caused some of the inconsistencies to disappear. In general, the employed method was to make the decision function nonlinear in the probabilities (i.e. Kahneman and Tversky (1979), Quiggin (1982), Machina (1982), Fishburn (1983). Recently, Machina (1989) strengthened the nonexpected utility side in the on going expected utility debate by showing that nonexpected utility agents behave, in a contrast to the common belief, in a dynamically consistent manner.

So far, most of the tests concerning the expected utility hypothesis were lab tests. They were based on questionnaires with hypothetical questions that were distributed in a class of students, for example. The objective of this paper is to construct a test of the expected utility hypothesis which is based on a real data set, namely the actual decisions of input allocation made by farmers. By using this test, it will be possible to conclude whether the expected utility hypothesis is the appropriate assumption for analyzing decisions under uncertainty and behavior toward risk for a given data set. The test is a nonparametric one, namely it does not depend on neither the functional form of the preferences nor the functional form of the random variable. Thus, it avoids an inherited problem in parametric tests of testing a joint hypothesis of the functional form and the original hypotheses. Recently, Chalfant and Alston showed that while parametric test indicates taste changes, a nonparametric one indicates non. Thus, the taste changes indicated by the parametric test are a result of the specific functional form used.

The next section is expository in nature, however necessary for a whole picture. It starts by stating the expected utility hypothesis and its axiomatic basis. It, then, reviews some cases where the expected utility fails to predict as in reality. Finally, it discusses the strength of the evidence against the expected utility hypothesis. Section three describes the formalism of the test, for the three dimensional case and for the multidimensional case and, then, derives boundaries on the coefficient of absolute risk aversion. Section four deals with the empirical application, it explains how to model the choice of land allocation as a choice among different lotteries and, then, suggests some tests that check the reliability of the model. Section 6 concludes the chapter with results and conclusions.

2. Preliminaries

2.1. The Expected Utility Hypothesis

Before stating the expected utility hypothesis, note that the individual is facing some choice set, G. A typical elment in G, g_j, is a prospect (lottery) of the form $(x_1, \cdots, x_n; p_1^j, ..., p_n^j)$, where x_i is a prize and p_i^j is its associated probability. As the notation suggests, different prospects differ in the probabilities they assign to the same outcome space. The expected utility hypothesis[1] states that

$$U(x_1, \cdots, x_n; p_1, \cdots, p_n) = \sum_{i=1}^{n} p_i u(x_i),$$

that is the utility from a prospect is just the expected utilities of its prizes.

The existence of a general utility function that represents preferences defined over a prospect set is guaranteed upon the completeness, reflexivity, transitivity, and continuity axioms (Varian 1984). In other words, if the above axioms hold there is a utility function such that $U(g_i) > U(g_j)$ if and only if $g_i \geq g_j$, where $\geq$ is the preference relationship defined over the choice set G. For the expected utility property to hold one needs an additional axiom, namely the independence axiom (Samuelson 1952).[2] It states that

$$g_1 \geq g_2 \text{ if and } \textit{only} \text{ if } (p, 1-p; g_1, g_3) \geq (p, 1-p; g_2\, g_3) \text{ for } \textit{all } g_1, g_2, g_3 \textit{ in } G.$$

The following interpretation of the independence axiom, given by Samuelson (1966), helps one to see its attractiveness. Consider an individual who prefers g_1 to g_2. This

[1] More than two hundred years ago, Bernoulli suggested the expected utility criterion for ranking lotteries as a resolution of St. Petersburg paradox. For extended discussion of this paradox see Samuelson (1977).

[2] The axiomatic basis for the expected utility hypothesis was first given by von Neumann and Morgenstern (1953). Although, the independence axiom is not an explicit part their set of axioms, it has been shown by Malinvaud to be an implicit assumption underlying their axioms.

individual is offered the following two lotteries, $(p, 1-p; g_1, g_3)$ and $(p, 1-p; g_2, g_3)$. This is equivalent to being offered a toss of a coin with probability p of landing heads. If the coin comes tails the individual gets g_3 independently of his/her choice. If it comes heads it is only rational to assume that the original choice (g_1 over g_2) would be made.

2.2. Violation of The Expected Utility Hypothesis

This section presents examples,[3] obtained in laboratory experiments, of empirical evidence against the expected utility hypothesis. The most famous one is the Allais paradox. Consider a choice of one prospect from the following two pairs:[4]

a: (1; 1M) versus b: (0.01, 0.89, 0.1; 0, 1M, 5M)

and

c: (0.9, 0.1; 0, 5M) versus d: (0.89, 0.11;0, 1M)

where M stands for one million dollars. Researchers, such as Allais, Raiffa, and MacCrimmon and Larsson have found that the great majority of the individuals who were asked to rank these prospects preferred a to b and c to d. Clearly, these choices are not consistent with the expected utility maximization since for any expected utility maximizer who prefers a to b the expression {-0.01u(0) + 0.11u(1) - 0.1u(5)} is greater than zero, while the same expression is less than zero for any expected utility maximizer who prefers c to d. To see how these choices violate the independence axiom, define e as (1/11, 10/11; 0, 5M) and rewrite the Allais paradox as:

aa: (0.11, 0.89; 1M, 1M) versus bb: (0.11, 0.89; e, 1M)

and

[3] More cases where the expected utility hypothesis is violated can be found in Machina 1987.

[4] Allais paradox is drawn as a decision tree at the end of the paper.

cc: (0.11, 0.89; e, 0) versus dd: (0.11, 0.89; 1M, 0)

The independence axiom states that preference of aa over bb implies preference of dd over cc, and vice versa. When the choices are as in the Allais paradox, it appears that the certainty equivalent of e is less than 1M when the alternative is 1M, while for the 0 alternative the certainty equivalent of e is greater than 1M. In other words, a better alternative (in a stochastically dominating sense) is associated with more risk averse choice. This phenomenon, in general known as the *common consequence effect,* directly violates the independence axiom. Choices of a over b and c over d, however, do not imply choices of aa over bb and cc over dd when the compound prospect assumption does not hold. For the cases where the individual's perception of a lottery depends not entirely on the net probabilities, the ranking of the double letter pairs may be different from the ranking of the single letter pairs. Thus, violation of the expected utility property does not imply violation of the independence axiom, although the reverse is true.

It is important to mention that behavior as in Allais paradox appears also under different choice sets. A second example where people's choices have been found to be inconsistent with expected utility maximization is known as the common ratio effect (it is drawn as decision tree at the end of the paper). When faced with these decision problems, most people chose to have the $1M prize with certainty over 0.98 chance of $5M prize in the first case, and 0.98 precent chance of $5M prize over 1 precent chance of $1M prize in the second case. As in Allais paradox, this choice pattern directly violates the expected utility hypothesis.

2.3. The Nature of The Violation of The Expected Utility Hypothesis

The nature and the significance of this empirical evidence are best explained by using a unit triangle. Thus, this section starts with presentation of the expected utility in the simplex.[5] For that purpose, consider the set of all probability triples, (p_1, p_2, p_3)

[5] For more complete presentation of the expected utility in the simplex see Machina 1987.

defined on fixed outcomes, $x_1 < x_2 < x_3$. Since the probabilities sum to one, any lottery can be drawn in the unit triangle in the (p_1,p_3) plane (figure 1). All northwest movements lead to stochastically dominating lotteries since any movement upward shifts some probability weights from x_2 to x_3 and any movement leftward shifts some probability weights from x_1 to x_2. The individual's indifference curves can be found by solving the following equation:

$$\bar{u} = p_1 u(x_1) + (1-p_1-p_3)u(x_2) + p_3 u(x_3).$$

The solution leads to parallel indifference curves with slope of $(u(x_2)-u(x_1))/(u(x_3)-u(x_2))$. Thus, it is enough to know one indifference curve in order to know the whole preferences over the entire triangle. The iso expected profit lines can be drawn in this triangle, as well. They are the solution of the next equation

$$\bar{x} = p_1 x_1 + (1-p_1-p_3)x_2 + p_3 x_3.$$

The solution shows that they are parallel lines with slope of $(x_2-x_1)/(x_3-x_2)$. Movement northeast along those lines keeps the expected profit constant while increasing the variance (it shifts probability mass from the center to the edges). Hence it represents a mean preserving spread or a pure increase in risk in the sense of Rothschild and Stiglitz. The individual's attitude toward risk depends on the slope of the indifference curves relative to the slope of the iso expected profit lines, where steeper indifference curves imply risk aversion and flatter imply risk seeking. Moreover, there is a one relationship between the slope of the indifference curves and Arrow – Pratt coefficient of absolute risk aversion, $-U''/U'$. That is,

$$-\frac{U''}{U'} \equiv A \approx 2\left[\frac{s-1}{s+1}\right]$$

where s is the slope of the indifference curves. This results is derived under the assumption that the outcomes are equally spaced and close to each other.

When one draws the four prospects, a,b,c,d, in the unit triangle (figure 2) they form a parallelogram. Any expected utility maximizer whose indifference curves are steeper than the sides of the parallelogram would prefer a to b and d to c. The opposite choice would occur when the indifference curves are flatter than the sides of the parallelogram. For consistency with the choice as in Allais paradox the indifference curves can not be parallel, they have to have, what Machina (1987) termed, the fanning out property (figure 3). Again, the figure shows that the stochastically dominating pair (a,b) is ranked with more risk aversion than the other pair (c,d).

Indifference curves that fan out are also consistent with a choice pattern as in the common ratio effect. However, in both cases of the common consequence and the common ratio effects, it is sufficient for the indifference curves to fan out only slightly. In other words, the indifference curves can be almost parallel and still consistent with behavior as in Allais paradox, or people can be almost expected utility maximizer and choose as in Allais paradox. Thus, the laboratory experiments show deviation from expected utility maximization but they do not show how deep the deviation is. Furthermore, the laboratory experiments test the expected utility hypothesis for a very special choice set in which the choices form a parallelogram. The probability that people may face such a special choice set in real life is very small. Thus, the failure of the expected utility hypothesis in the laboratory experiment should not imply failure of the expected utility hypothesis to predict real life behavior.

Recently, Conlisk pointed out that the inconsistency with expected utility maximization as in Allais paradox is a systematic one. That is, most of the people choose the pair (a,c) (in Figure 2), rather than choosing (a,c) and (b,d) with equal frequencies, when inconsistency with expected utility maximization is observed. Note that nonsystematic inconsistency with expected utility maximization (equal frequencies of (a,c) and (b,d)) is not much of a concern since similar objects are hard to distinguish, and thus people may get confused when faced with similar choices. Conlisk observed that the systematic inconsistency disappeared when he moved the Allais paradox to the interior of the simplex. Moreover, when the segment d-c (Figure 2) was moved to the upper left corner of the simplex in such a way that point d is in the interior of the simplex and point c is on the line where $P_2=0$, Conlisk observed the same systematic inconsistency as in the original Allais paradox. However, the latter systematic inconsistency implies indifference curves that fan in rather than fan out. Conlisk explains his results by the existence of what Kahneman and Tversky termed the certainty effect. In the original Allais paradox people choose a over b because of the certainty effect. When all the points are in the interior of the simplex there is no certainty effect and systematic inconsistency disappears. In the third version of Allais paradox, the certainty effect ensures the choices of c over d and a over b. Thus, Conlisk's results favor the certainty effect hypothesis of Kahneman and Tversky over Machina's fanning out indifference curves hypothesis.

In view of Conlisk's results, the certainty effect is a main factor leading to the inconsistency with the expected utility hypothesis. Consequently, It is conjectured that when the choice set has only lotteries in which all prizes get positive probability, no inconsistency with the expected utility should be observed. Moreover, when the choice set is consistent of higher dimensional lotteries, a partial certainty effect resulted by assigning some of the prizes zero probability may be insignificance in terms of affecting the decision criterion. Thus, testing the expected utility hypothesis in a real world of higher dimensional lotteries may leads to different results than those obtained by laboratories tests.

3. A Test of The Expected Utility Hypothesis

Checking the empirical validity of the expected utility hypothesis was done not only by lab tests. Lin, Dean and Moore, for example, compared the prediction of profit versus utility maximization in agricultural production. Robison appraised their method, as well as the reliability of a test constructed from responses to hypothetical questions. At a somewhat more theoretical level, Varian (1983) extended Afriat's theorem to the case of expected utility and found necessary and sufficient conditions for the expected utility maximization with respect to portfolio choice. The criterion found in the present paper is, in principle, the same as the one found by Varian. In both cases choices are consistent with the expected utility maximization if there exists a feasible solution to a system of linear inequalities. This is not surprising in view of the fact that the expected utility function is linear in the probabilities. However, both the set over which the maximization takes place as well as the way the solution is derived are different in Varian's work and in the work presented here. Green and Srivastava's work is an extension of Varian's work to the case of a complete market for contingent claims.

In line with the methodology suggested by Friedman (1953), the test presented below tests the predictive power of the expected utility theory rather than testing its axiomatic basis.

3.1. The Three Dimensional Case

The three dimensional case is presented first because it has straightforward graphic interpretation in the triangle diagram. Consider a situation where the decision maker has to choose among lotteries defined on a fixed, ordered, and equally spaced outcome domain. The choice set in the first year, G_1, includes the lotteries g_1^1, g_1^2 and g_1^3, and the choice set in the second year, G_2, includes the lotteries g_2^1, g_2^2 and g_2^3 (figure 4). Suppose that g_1^2 and g_2^2 are the chosen lotteries in the first and second year, respectively. Then, there must be a straight line (an indifference curve) that separates the chosen lottery from all the other lotteries in each of the choice sets. If the utility function has a positive slope, then these indifference curves are restricted to have positive slopes. Furthermore, the slope of the indifference curve in some year, say i, cannot be smaller then the slope of the line g_i^2 - g_i^3, or greater then the slope of the line g_i^1 - g_i^2. Once the range of the slope of the indifference curves is known for the two years, one can detect inconsistency with expected utility maximization by comparing the ranges of these slopes. If the intersection of the slope ranges over the two years is empty then there is no system of parallel indifference curves that indicates choices of g_1^2 and g_2^2, and the choices are inconsistent with expected utility maximization.

As reported by Chalfant and Alston, the power of the test is an inherent problem in nonparametric tests. In the test suggested above, the union of the slope ranges is a simple measure of the power of the test. The smaller the union the stronger the indication that the expected utility hypothesis holds when it is not rejected. For example, if the union is just a point than the slope ranges are identical points. In this case one can conclude that there is a system of parallel indifference curves which is consistent with the choices and there is no system of unparallel indifference curves which is consistent with the choices. Similarly, a simple measure for the significance level of the test would be the distance between the two slope ranges. The longer the distance the less likely the expected utility hypothesis to hold when it is rejected.

Numerical Example

Let p_i^j be the probability triples associated with the lotteries g_i^j of Figure 4. Their values for G_1 and G_2 are: $p_1^1 \equiv (.07, .73, .2)$, $p_1^2 \equiv (.17, .47, .36)$, $p_1^3 \equiv (.33, .2, .47)$; and $p_2^1 \equiv (.33, .6, .07)$, $p_2^2 \equiv (.43, .4, .17)$, $p_2^3 \equiv (.62, .1, .28)$. Visual examination of figure 4. clearly shows that preference of g_1^2 over g_1^1 and g_1^3, and g_2^2 over g_2^1 and g_2^3 is consistent with the expected utility maximization (i.e. there are straight lines with common slope that separate the chosen lotteries from all the other lotteries in their choice set). Algebraicly, the expected utility maximization over G_1 implies

$$.07U(x_1) + .73U(x_2) + .2U(x_3) \leq .17U(x_1) + .47U(x_2) + .36U(x_3)$$

$$.33U(x_1) + .2U(x_2) + .47U(x_3) \leq .17U(x_1) + .47U(x_2) + .36U(x_3).$$

Subtracting the right hand side from both sides and writing in matrix form give

$$\begin{bmatrix} -.1 & .26 & -.16 \\ .16 & -.27 & .11 \end{bmatrix} \begin{bmatrix} U(x_1) \\ U(x_2) \\ U(x_3) \end{bmatrix} \leq 0.$$

Expressing the second column of the probability matrix as minus the sum of the other two columns and dividing through by $U(x_3) - U(x_2)$ yield

$$\begin{bmatrix} -.1 \\ .16 \end{bmatrix} \begin{bmatrix} \frac{U(x_1) - U(x_2)}{U(x_3) - U(x_2)} \end{bmatrix} \leq \begin{bmatrix} .16 \\ -.11 \end{bmatrix},$$

where the solution to the above two linear inequalities is any number in the closed interval, [-1.6, -.69]. Note that the solution to the above system equals the negative of the slope of the indifference curves, s, thus implying that s lies in the close interval [.69, 1.6]. Similarly, the expected utility maximization over G_2 restricts the slope, s, to lie in the closed interval [.57, 1]. The intersection of the two allowable ranges is the set [.69, 1], because it is not an empty set the choices over G_1 and G_2 are consistent with the expected utility maximization. Furthermore, utilizing the relationship between the slope of the indifference curves and the coefficient of absolute risk aversion (given in section 2.3.) shows that the latter, for wealth level x_2, lies in the close interval [-.38, 0]. That is, the individual represented be this arbitrary example is risk seeking. Because an estimate of the coefficient of absolute risk aversion is given for only one level of wealth, no conclusion concerning the effect of the wealth level on the coefficient of absolute risk aversion can be derived.

3.2. The Multidimensional Case

Generalization to the multidimensional case is the next objective. In this case the indifference curves are hyperplanes. Following the previous arguments, the indifference hyperplanes have to be parallel to one another in order to be consistent with the expected utility maximization. On the other hand, the indifference hyperplanes have to be consistent with the choices in any given year. That is, the hyperplane has to separate the chosen lottery from all the other lotteries in the choice set. This restricts the slopes of the hyperplane to some specific ranges. If any of the ranges is disjoint from some other range, then inconsistency is observed. The mathematical derivation of these conditions is given next.

Let G_a be the set of all lotteries of the form (x,p) where x is an increasingly ordered vector of outcomes and p is a vector of probability measure defined on x, both of dimension J. Let G_t be a sub set of G_a consisting of the choice set faced by the individual at time t. It is common to assume that G_t is smaller [6] than G_a because of some resource constraints. G_t is assumed to be convex. It is important to remember that any two lotteries are different only in their probability measure, the outcomes are the same. In other words, different lotteries have different probability distribution functions over common outcome space. Let I be the number of lotteries in G_t and, with out loss of generality, assume that the first lottery is the chosen one. Thus, the results of the expected utility maximization at time t,

$$\max_{(x,p^i)\in G_t} \sum_{j=1}^{J} U(x_j)p_j^i,$$

is (x,p^1). It, then, follows that at any time t: [7]

$$U(x_1)p_1^i + \cdots + U(x_J)p_J^i \le U(x_1)p_1^1 + \cdots + U(x_J)p_J^1 \quad i = 2,...I.$$

Subtracting the r.h.s. from both sides and writing in matrix form yield

$$\begin{bmatrix} p_1^2 - p_1^1 & . & . & p_J^2 - p_J^1 \\ \vdots & \vdots & \vdots & \vdots \\ p_1^I - p_1^1 & . & . & p_J^I - p_J^1 \end{bmatrix} \begin{bmatrix} U(x_1) \\ \vdots \\ U(x_J) \end{bmatrix} \le 0. \qquad (1)$$

[6] Smaller in the sense that $G_t \subset G_a$ and not $G_a \subset G_t$.

[7] Note that the superscript t is omitted when redundant.

Because the elements of each row in the above probability matrix sum to zero, this system of homogeneous linear inequalities can be written as

$$\begin{bmatrix} p_2^2 - p_2^1 & . & . & p_J^2 - p_J^1 \\ . & . & . & . \\ . & . & . & . \\ p_2^I - p_2^1 & . & . & p_J^I - p_J^1 \end{bmatrix} \begin{bmatrix} U(x_2) - U(x_1) \\ . \\ . \\ U(x_J) - U(x_1) \end{bmatrix} \leq 0. \qquad (2)$$

Dividing each linear inequality by $U(x_2) - U(x_1)$ yields equivalent non-homogeneous system of linear inequalities of the following form

$$\begin{bmatrix} p_3^2 - p_3^1 & . & . & p_J^2 - p_J^1 \\ . & . & . & . \\ . & . & . & . \\ p_3^I - p_3^1 & . & . & p_J^I - p_J^1 \end{bmatrix} \begin{bmatrix} (U(x_3) - U(x_1))/(U(x_2) - U(x_1)) \\ . \\ . \\ (U(x_J) - U(x_1))/(U(x_2) - U(x_1)) \end{bmatrix} \leq \begin{bmatrix} p_2^2 - p_2^1 \\ . \\ . \\ p_2^I - p_2^1 \end{bmatrix} \qquad (3)$$

or in short $Ps \leq y$. The solution set[8] for the system in (3) is a convex polytope, it is also not vacuous because of the convexity assumption on G_t. The proposition below summerizes the findings so far.

Proposition

Under the maintained hypothesis of constant preferences over time, an individual can be represented as expected utility maximizer if the intersection set of the solution sets for (3) over t is not a null set.

3.3. Boundaries on The Coefficient of Absolute Risk Aversion

The Arrow-Pratt coefficient of absolute risk aversion (ARA), $-U''/U'$, conveys information on the individual's willingness to undertake a constant gamble at different levels of wealth. Thus, it is plausible to assume decreasing ARA, or equivalently, the wealthier the individual the more likely that uncertain projects will be undertaken. It will now be shown that exists a one to one correspondence between a solution for (3),

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[8] A general method for solving linear inequalities is in Gale.

s, and the ARA coefficients.

Let x be evenly spaced where the distance between two adjoint outcomes is unitary and let the units be small. Then, a good approximation for the ARA coefficient at a wealth level x_i is

$$A_i = -2\frac{(U(x_{i+1}) - U(x_i)) - (U(x_i) - U(x_{i-1}))}{(U(x_{i+1}) - U(x_i)) + (U(x_i) - U(x_{i-1}))} \qquad i=2,\ldots,I-1 \tag{4}$$

where A_i denotes the ARA coefficient at wealth level x_i; and let s_i be the i'th element of the solution vector s. Then, by simple substitution one can verify that the following relations hold

$$A_2 = -2\frac{s_1 - 2}{s_1} \tag{5}$$

$$A_3 = -2\frac{s_2 - 2s_1 + 1}{s_2 - 1}$$

$$A_i = -2\frac{s_{i-1} - 2s_{i-2} + s_{i-3}}{s_{i-1} - s_{i-3}} \qquad i=4,\ldots,I-1.$$

It follows that the transformation in (5) can be used to calculate simultaneous boundaries on the coefficients of ARA, A_i. The available information, as conveyed by the data, implies that the A_i's are equally likely to be at any point inside the boundaries. In other words, the A_i's are uniformly distributed between the boundaries. Consequently, unbiased point estimates for the A_i's are the mass center of the convex polytope formed by the boundaries. Practically, the average of all the vertices of the convex polytope gives the desired estimates. Now, the point estimates can be tested for trend. For example, if the estimated A_i's become smaller as i increases, there is empirical evidence in favor of decreasing ARA.

4. The Empirical Application

The data set contains a cross section - time series sample on 97 farmers during ten years, 1973-82. It includes the following variables: allocation of land, yield, and revenue for five crops: Bell Peppers, Tomatoes, Onions, Melons, Eggplants. The data set also includes the total expenditure on water and other inputs. The cost for each crop can be recovered via a behavioristic approach described by Just *et al* (1990). The profit for each crop can be calculated by subtracting the recovered cost from the revenue figures.

4.1. Presentation of The Farmers' Decisions as Lotteries

Let $\Pi = (\pi_1, \ldots, \pi_n)$ be the profits domain per unit of land for each farmer, where π_1 and π_n are the minimum and maximum profit per unit of land which might occur for some crop, respectively. Let P be an array of probability measures of dimension $n \times n \times n \times \cdots \times n$ (J times, where J is the number of crops). An element of P, $p_{i_1,\ldots,i_J}$, is the probability of having some specific combination of profits, say $(\pi_{i_1}, \ldots, \pi_{i_J})$ where π_{i_j} is the profit per unit of land of crop j. In other words, P is the joint distribution function of profits.

Along with Muth's concept of rational expectations, the farmer's perceived joint distribution of the profits must coincide with the in-fact (real) joint distribution of the profits. An unbiased minimum variance estimate of the real joint distribution function of the profits is the empirical actual frequencies of each payoff. Hence, it is also an unbiased estimator of the in-mind (perceived) joint distribution of profits of the farmer. This approach of modeling profit expectations, however, is quite naive in the sense that constant expectations over time lead to constant decision over time. That is, the land allocation does not change over time. A quick glance at the data shows that this is not the case. Hence, there is a need to incorporate some mechanism which explains changes in the expectations over time which in turn explains the changes in land allocation among the different crops. One possibility is that the joint profit distribution is not a stationary one. In this case the empirical distribution of profits (equal weights to each realization) is no longer the right estimator, in the sense of biasness and efficiency, for the actual profit distribution. Under milder conditions (Just 1977) a declining moving average (i.e. adaptive expectation) will be a consistent and efficient estimator and therefore a rational expectation one. Alternatively, we may assume that the farmer's information set includes only fixed number of observations into the past.

It is important to understand that for empirical purposes there are two methods for estimating the joint profit distribution. The first is by assigning equal weights to past realizations, and the second is or by a declining moving average of past realizations. The classification of the methods as rational or naive expectation, however, depends on the characteristics of the underlying distribution. The adaptive expectation scheme (i.e. declining moving average) would be naive for stationary distribution and rational for nonstationary one. Giving equal weights to past realizations would be naive for nonstationary distribution and rational for stationary one. Because in our sample the stationarity question is not testable, the interpretation of the expectation mechanism as rational or naive is not unambiguous.

Next, it is necessary to transform any specific land allocation among the different crops to an ultimate lottery in terms of profit. This can be done by finding the total profit which is associated with each event. Formally, the payoff associated with some element of P, say $p_{i_1,\ldots,i_J}$, is $\sum_{j=1}^{J} l_j \pi_{i_j}$, where l_j is the land allocated to crop j. [9] This payoff corresponds to some outcome in the total profit domain $(\pi_1, \ldots, \pi_N)$, say π_k; then setting the probability associated with it, p_k, to be equal $p_{i_1,\ldots,i_J}$. Doing so for

[9] Note that the implicit assumption here is that the technology is a constant return to scale one. In this case this assumption is not so strong because the farmers allocate fixed amount of land among the crops. Thus, the managerial constraint, that usually cause the decreasing return to scale phenomenon, is not binding.

all non zero elements of P will result in an ultimate lottery.

4.2. Some Preceding Tests

Now, after every single decision made by the farmer is formulated as a lottery it is possible to answer the following questions[10]:

1. Does the farmer follow the first order stochastic dominance criteria? Equivalently, is his utility function increasing in its argument?

2. Does the farmer follow the second order stochastic dominance criteria? Is his utility function also concave in its argument?

3. Does the farmer follow the third order stochastic dominance? In other words, is he averse to downside risk?[11]

4. What is the farmer's attitude toward pure increase in risk? Is there any available lottery which has higher expected value and equal or smaller variance than the chosen lottery?

Theoretically, the answer to the second and fourth questions must be the same. However the way the answer is constructed is different. In order to answer the above four questions one must define the set of all other lotteries available at some period for some farmer. Any different land allocation gives a different lottery, hence one must find all lotteries that are associated with all *possible* different land allocations. Possible in the sense that the allocations obey the following constraints:

C1. $\sum l_i \leq L$. There is a limited amount of land.

C2. $\sum t_i l_i \leq T$. There is a limited amount of time

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[10] For theoretical consideration of the stochastic dominance see Hadar and Russell.

[11] See Menezes *et al* for the relationship between aversion to downside risk (U''' > 0) and third order stochastic dominance.

C3. $\sum w_i l_i \le W$. There is a limited amount of water.

Once all possible land allocations are known it is possible, as explained in the previous paragraph, to find the set of all possible lotteries. The way to answer the first question is by checking whether there exists a lottery with $\sum_{i=1}^{j} p_i - p_i^* \le 0$, for all j, with strict inequality at least for one j, where p^* indicates actually chosen lottery and p is any other available lottery which was not chosen. The existence of such a lottery shows that a stochastically dominating lottery was not chosen. The questions concerning higher order stochastic dominance can be similarly answered. Once one has the mean and the variance of each available lottery, a short search procedure will give the answer to the fourth question.

To continue the analysis there is a need for some additional assumptions. Comparison of two decisions has to be based on the assumption that everything else is fixed, either along time, namely the individual's preferences do not changes over time, or at one point of time, namely the preferences do not change across the population. There are arguments in favor of each of the above. The assumption of fixed preferences across the individuals might be based on the fact that the available data came from an Israeli Moshav - a cooperative village. This Moshav is quite isolated and stands as an environmental unit by itself. Consequently most of the individuals interactions are within the Moshav and not with the outside environment. Furthermore, public opinion is a very powerful device which drives the individuals to behave according to it. Hence one can say that the preferences of everybody approach steady state preferences. Contingent on the rejection of the assets integration assumption (Markowitz (1952)), the fixed preferences over time assumption is composed of two other assumptions. The assets integration assumption says that the individual cares not only about the final wealth but also about the initial wealth position so that it makes a difference whether some wealth position was arrived at by a loss of money or by a gain of money. The first assumption is that the wealth does not change over time. At first glance this looks like quite a strong assumption. However, under the permanent income hypothesis of Friedman, where the individual considers all future incomes when evaluating his/her wealth, it may look more plausible. The second assumption is that the preferences defined over initial and final wealth (initial wealth and current prospects) are fixed. In other words, tastes are stable over time. Conditional on the fixed wealth hypothesis, the fixed preferences over time assumption can be empirically tested via the revealed preferences approach. In the present work the fixed preferences over time assumption is employeed. This is because the justifications for the assumption of fixed preferences across individuals are rather *local* ones, while the justifications for the assumption of fixed preferences over time are more *global* and likely to exist in general and not only in special cases. Furthermore, the fixed preferences over time is a testable hypothesis in a way which will be outlined below.

Let G_t be the set of all prospects available to a producer at time t, and g_t^* is the chosen prospect at that time. If g_t is any prospect in G_t, then g_t^* is directly revealed preferred to g_t, $(g_t^* R^o g_t)$. A sequence of directly revealed preferred choices, say $(g_{t_1}^* R^o g_{t_2}^*),...,(g_{t_{n-1}}^* R^o g_{t_n}^*)$, implies that $g_{t_1}^*$ is revealed preferred to $g_{t_n}^*$, $(g_{t_1}^* R g_{t_n}^*)$. Data which were generated by a utility maximizing individual have to satisfy both the Weak Axiom of Revealed Preference and the Strong Axiom of Revealed Preference. They are:

(WARP) If $g_{t_1} R^o g_{t_2}$ and $g_{t_1} \neq g_{t_2}$ then it is not the case that $g_{t_2} R^o g_{t_1}$.

(SARP) If $g_{t_1} R g_{t_2}$ and $g_{t_1} \neq g_{t_2}$ then it is not the case that $g_{t_2} R g_{t_1}$.

WARP and SARP are *necessary conditions* for utility maximization, where only SARP is a *sufficient condition* for utility maximization. Generally, violation of WARP implies a taste change or that the constant utility over time hypothesis must be rejected. Violation of SARP implies either that the preferences are not transitive or a taste change.

5. Results and Conclusions

The methodology suggested in the previous sections for testing the expected utility hypothesis was applied to the data set. For the empirical purpose, the actual land allocations made by the farmer over ten years, were taken as the set of all possible land allocations. The test was preformed in two stages. In the first it was checked whether there is a straight hyperplane that separates the chosen lottery from all other lotteries belonging to its choice set. This stage was preformed by checking existence of feasible solution to the system in (3) for the ninth and tenth year separately. In the second stage, it was checked whether the intersection set of the allowable ranges of the slopes of the separating hyperplanes is empty or not. This second stage was preformed by checking the existence of feasible solution for the ninth and tenth year simultaneously. All computations were done by using the Fortran programming language and the Nag library, the latter was used to check existence of feasible solution for a system of linear inequalities.

Prior to preforming the test, all possible land allocations were presented as feasible lotteries. For this purpose, the domain of total profits was divided to eight equal sub domains, and then the empirical density was estimated. Two schemes for forming expectations have been used: i. adaptive expectations, by mean of assigning geometrically declining weights to past realization of profits;[12] ii. rational expectation, by mean of assigning equal weights to past realization of profits. In both the ninth and the tenth year, the past eight years were assumed to be in the information set. As the last step before preforming the test, it was checked whether the chosen lottery is not first order stochastically dominated by any other lottery in its choice set[13].

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[12] The weights are .24, .19, .15, .12, .10, .08, .06, .05 for the most recent to the least recent, respectively.

[13] Note that if the chosen lotteries in the ninth and the tenth year are not stochastically dominated by any other lottery in their choice set then the WARP is fulfilled. Because there are only two period in this experiment, WARP is necessary and sufficient for utility maximization.

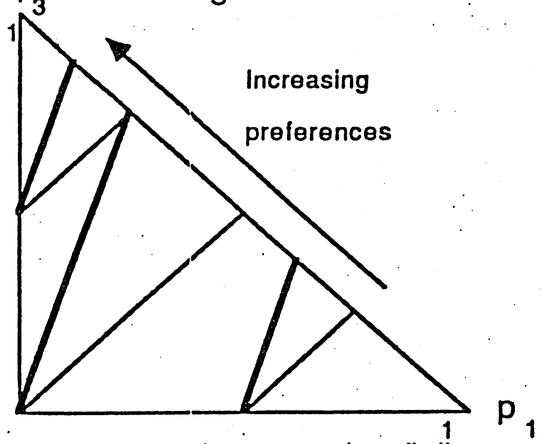
Table I summerizes the results, it gives the failure ratio in the First Order Stochastic Dominance test and the two stages of the test of the expected utility hypothesis for the two expectations schemes. It was found that in 7 out of 194 choices the chosen lottery is first order stochastically dominated by at least one other lottery in its choice set for the case of rational expectations. For the adaptive, expectation only one choice did not meet the first order stochastic dominant criterion. For the rational expectations, four farmers (farmers 8, 9, 14, and 26) failed the first stage of the test, indicating that the indifference curves are not straight hyperplanes. For the adaptive expectations, also four farmers (farmers 9, 14, 26, and 31) failed the first stage of the test. For the rational expectations, farmers 4, 12, 17, 21, 25, 31, 40, 48, 77, and 88 failed the second stage of the test. For the adaptive expectations, farmers 4, 12, 17, 21, 25, 39, 40, 72, 83, and 92 failed the second stage of the test. The same number of farmers failed the first stage of the test, and the same number of farmer failed the second stage of the test. In the first stage 3 out of 4 failed the test for both expectation schemes. In the second stage 6 out of 10 failed the test for both expectation schemes. Thus, it is concluded that the first order stochastic dominance is very sensitive to the expectation scheme, while both stages of the test are not. It should be mentioned that failing the first order stochastic dominance criterion does not imply rejection of the expected utility hypothesis. For both expectation scheme, four out of 194 choices were inconsistent with straight hyperplane indifference curves. Moreover, for both expectation schemes, 10 out of 93 farmers showed inconsistency with parallel hyperplane indifference curves. Hence, It is concluded that the phenomenon of inconsistency with straight hyperplane indifference curves is less likely to appear than the phenomenon of inconsistency with parallel hyperplane indifference curves. However, the extent to which individuals exhibit inconsistency with expected utility maximization in laboratory experiments is significantly larger then the extent to which individuals shows inconsistency with expected utility maximization in reality.

It is suggested that two factors cause the differences between laboratory experiments and reality. The first is that in the former artificial limited situations are presented to individuals and these limited situations are rare in reality. The second is that the laboratory experiments use lotteries of a low dimension (usually 3), while in reality it is expected to face lotteries of a higher dimension. The higher the dimension of the lotteries the less is the increase in the certainty effect when an element of a lottery is set to zero. As shown by Conlisk, the certainty effect is a main factor causing deviation from the expected utility hypothesis. It, then, follows that in a world of higher dimension lotteries (reality) violation of the expected utility is less likely to appear. Thus, the growing concerns about the reliability of expected utility based models for empirical analysis should be dismissed.

Table I

	Adaptive Expectation	Naive/Rational Expectation
First order Stochastic Dominance	1/194	7/194
The First Stage	4/97	4/97
The Second Stage	10/93	10/93

Figure 1.

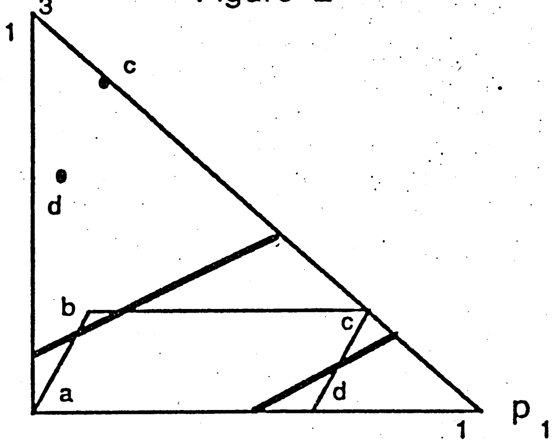


Thin lines are iso-expected profit lines.

Thick Lines are indifference curves

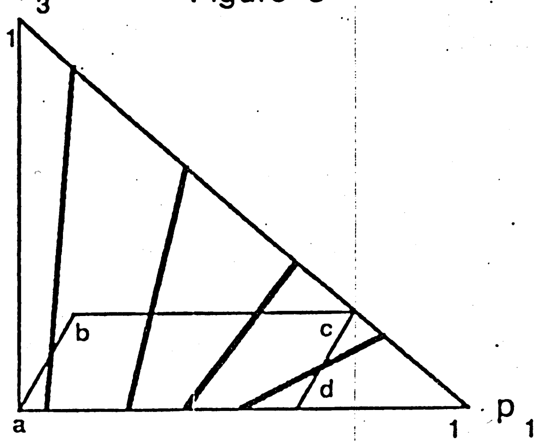
This case represents risk aversion.

Figure 2



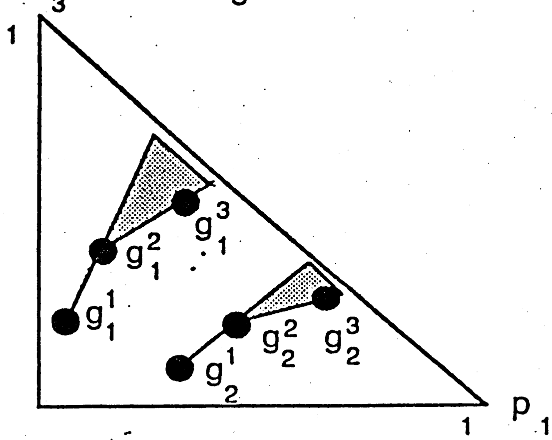
Thick lines are indifference curves.

Figure 3



Indifferece curves that fan out.

Figure 4



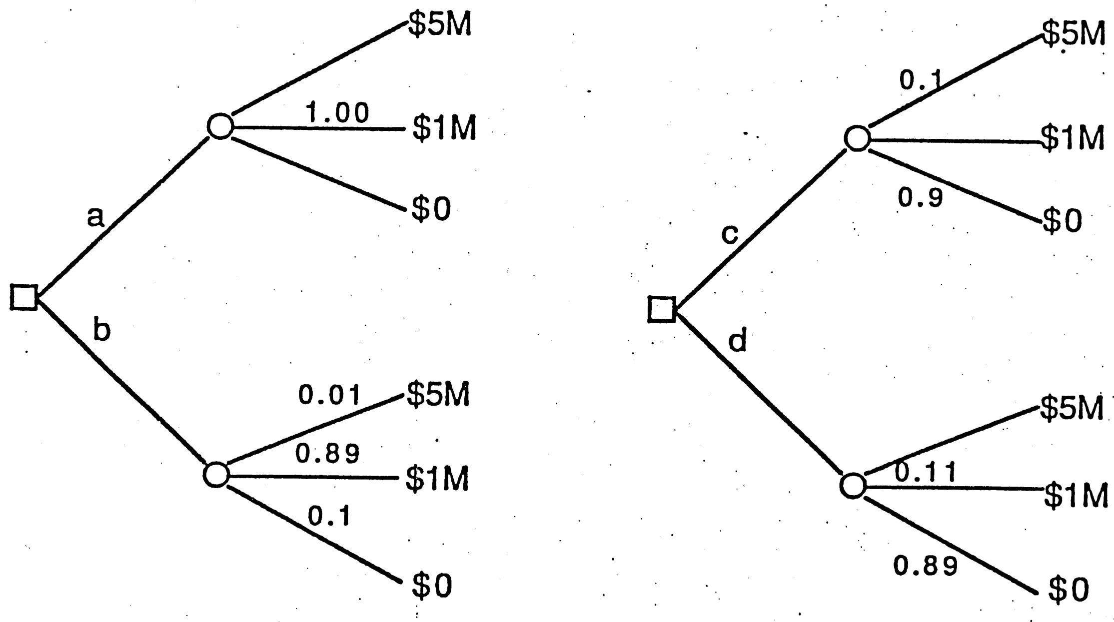
The dark area indicate allowable

range of the indifference curves.

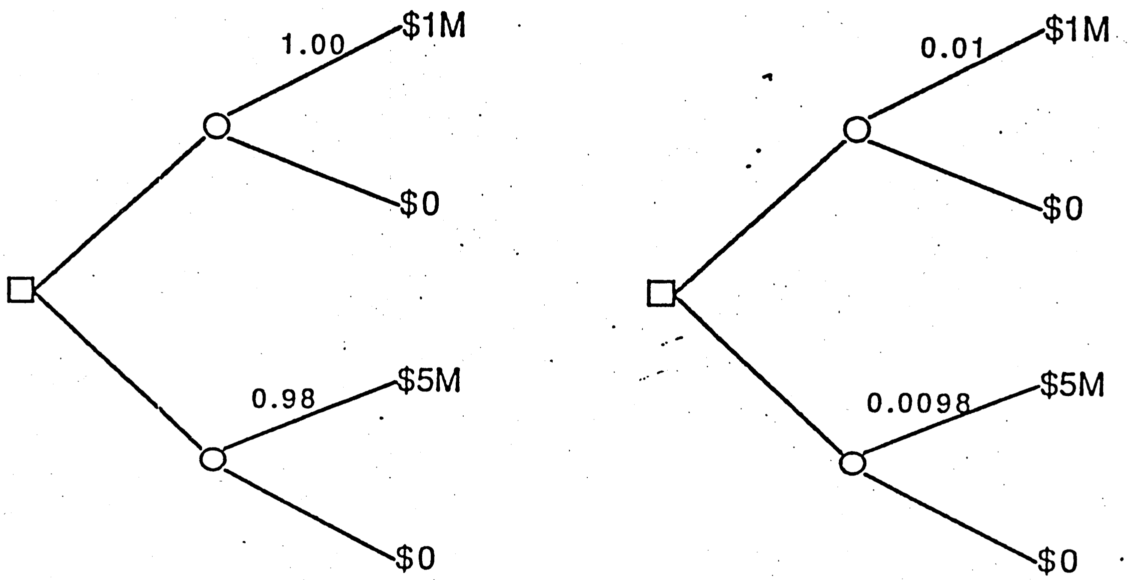
In this case the choice is consistent

with the expected utility maximization.

Allais Paradox



Common Ratio Effect



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