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ON THE SENSITIVITY OF A REGRESSION COEFFICIENT TO MONOTONIC TRANSFORMATIONS

by

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Comments Invited

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On The Sensitivity of a Regression Coefficient to Monotonic Transformations*

by

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Abstract

This note presents a procedure which enables the user to check whether a monotonic transformation can change the sign of a regression coefficient. One possible use of this procedure is to examine the robustness of key regression coefficients.

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On The Sensitivity of a Regression Coefficient to Monotonic Transformations

Economic theory, unaided by strong assumptions, does not offer a complete specification of models. In most cases, theory can help determine the variables to be included in a model and the expected sign of the coefficient of certain variables. For example, when dealing with consumer behavior, theory suggests that the substitution effect is negative while the income effect is unsigned. The exact specification of the model is left to the judgement of the investigator. Readers may therefore be curious about the robustness of the model's specification to some of its perturbations.

The sensitivity of estimated regression coefficients to omitted variables has been studied by Leamer (1975) and Visco (1978). Leamer shows that when a variable is dropped from a linear regression there can be no change in sign of any estimate that has a larger t-value than the one dropped from the regression. Visco shows that the converse is not generally true. That is, deleting a variable with a high t value is not a sufficient condition for changing the sign of other regression coefficients. An alternative way of affecting the sign of a regression coefficient is to use monotonic transformations of the variables.

The aim of this note is to suggest a simple procedure which enables the investigator to examine the sensitivity of the sign of a regression coefficient to monotonic transformations of the variables. By reporting the results the investigator can inform the reader about the sensitivity of the regression coefficient to monotonic transformations. The first section provides a description of the method while the second presents an illustration. The proof and the illustration are restricted to the simple regression case. The same method can be applied to a multiple regression, but then one cannot guarantee that it will not change the sign of other coefficients too.

I. The method.

Assume the following model

$$y = \alpha + \beta x + \epsilon$$

The estimator b = cov(Y,X)/Cov(X,X). Let T(y) be a monotonic transformation of y. ($T'(y)>0$.) The sign of the regression coefficient between T(y) and X depends on the sign of cov(T(y),X)). The covariance is a weighted sum of negative and positive elements. By choosing T(y) some elements may be increased (decreased), thereby affecting the sign of the covariance. However, positive and negative elements are scattered, while T(y) affects ranges of the variate. Sorting ranges of the variable according to their contribution to the covariance can be done with the help of the Absolute Concentration Curve (ACC) and the Line of Independence (LOI).

The properties of concentration curves are described in Yitzhaki and Olkin (1988). Here only the relevant properties are presented.

Definition 1. The Absolute Concentration Curve (ACC) of X with respect to Y is the cumulative value of X as a function of the cumulative distribution of Y.

The horizontal Axis is F_Y, which is estimated in the sample by the Rank of Y divided by n, the sample size. The ACC curve presents the cumulative value of X as a function of F_Y.

Definition 2. The Line of Independence (LOI) is defined as $L(F_Y)=\mu_X F_Y$, where μ_X denotes expected value. (If X and Y are independent, the ACC and LOI coincide.)

The ACC and LOI are equal at the two extreme points F=0 and F=1. The area between ACC and LOI is equal $\mathrm{cov}(F_Y(Y),X)$. To see this note that

$$\mathrm{cov}(X,F_Y(Y))= \int \{g(t)-\mu_X\}\ F_Y(t)\ f_Y(t)\ dt \qquad (1)$$

where $g(t) = E\{X|Y=t)$ is the conditional expectation of X with respect to Y. Using integration by parts with

$$v'= (g(t)-\mu_X)\ f_Y(t) \qquad v=\int_{-\infty}^{y}(g(t)-\mu_X)\ f_Y(t)\ dt$$

$$u= F_Y(t) \qquad u'=f_Y(t)$$

and assuming that second moments exist we get:

$$\mathrm{cov}(X,F_Y(Y))= - \int_{-\infty}^{\infty}\int_{-\infty}^{y}(g(t)-\mu_X)\ f_Y(t)dt\ \ f_Y(y)dy$$

and by using the transformation $F=F_Y(Y)$ we get:

$$\mathrm{cov}(X,F_Y(Y))= \int_0^1 [\mu_X\ F-\Theta_X(F)]\ dF \qquad (2)$$

where $\Theta_X(F_Y)=\int_{-\infty}^{y^*} g(t)f_Y(t)dt$ is the absolute concentration curve of X with respect to Y and y^* is implicitly defined as:

$$F_Y(y^*)=\int_{-\infty}^{y^*} f_Y(t)dt.$$

The following theorem presents the main result.

Theorem: If the concentration curve and the LOI intersect, it is possible to find two monotonic transformations, $T_1(y)$ and $T_2(y)$, so that

$cov(T_1(y),X)\ cov(T_2(y),x)<0$. On the other hand, if ACC and LOI do not intersect then $cov(T_1(y),X)\ cov(T_2(y),x)>0$ for all T_1, T_2.

The formal proof is given in Yitzhaki and Olkin (1988). However, it is easy to present an intuitive proof. Using the same procedure which led from eq. (1) to (2), the cavariance between Y and X can be written as:

$$cov(X,Y)= \int_{-\infty}^{\infty} [\mu_X F(y) - \Theta_X(F(y))]\ dy \qquad (3)$$

Note that a monotonic transformation does not affect the expression in square brackets in (3) and (2) as a function of F. Also it does not affect F. However by substituting y in (3) by T(y), dy is substituted by $T'(y)dy$. If $[\mu_X F-\Theta_X(F)]$ has the same sign for all F, it will be impossible to change the sign of the covariance. If, on the other hand, $[\mu_X F-\Theta_X(F)]$ changes signs, then transformations with $T'<1$ when the sign is "wrong" and $T'>1$ when the sign is "right" may affect the sign.

The ACC and LOI are not affected by transformations. The sections of the horizontal line for which ACC is below (above) the LOI tells us the percentage of overall observations which contributes to a positive (negative) covariance. Hence, by presenting the curve the author can inform the reader whether one can change the sign of the regression coefficient, whether it can be easily done, and the percentage of observations which caused the sign.

To check whether transformations on the independent variable can change the sign, all we have to do is to change the role of the variables.

A similar procedure can also be applied in the multiple regression case. However, in this case a transformation may change the sign of other coefficients too [see Yitzhaki and Olkin (1988)].

II. An Illustration.

The example is based on the small synthetic example used by Huber (1981, chapter 7) to introduce his discussion of robust regression. [For further discussion of this example see Atkinson (1985)]. This example demonstrates the non-symmetric nature of the procedure. Only transformations of the dependent variable can change the sign of the regression coefficient.

Consider the following example with 6 observations:

$$X = -4\ ,\ -3\ ,\ -2\ ,\ -1\ ,\ 0\ ,\ 10$$
$$Y = 2.48,\ 0.73,\ -0.43,\ -1.44,\ -1.32,\ 0.0$$

Table 1 presents ACC minus LOI of X as a function of F_Y ($ACC_{XY}(F)-\mu_X F$) and the ACC minus LOI for Y as a function of F_X ($ACC_{YX}(F)-\mu_Y F$). The first one changes signs while the second remains positive. This means that the sign of a regression coefficient of T(y) on X (T'>0.) depends on the choice of the transformation, while the sign of a regression coefficient of Y on any T(x) is always negative. Hence, it is impossible to change the sign of the regression with transformations of X. However, it will be possible to do so with transformations of Y.

Table 1: Absolute Concentration Curves <u>minus</u> Line of Independence

F	0	1/6	2/6	3/6	4/6	5/6	1
$ACC_{XY}-\mu_X F$	0	-.167	-.167	-.50	1.16	.667	0
$ACC_{YX}-\mu_Y F$	0	.412	.532	.46	.22	.001	0

Note that the curve is negative for low values of F_Y. Hence, a transformation which expands low values of Y (T'>1) and shrinks large values of Y (T'<1) may yield a positive coefficient while a regression of Y on X yields a negative sign. Let us try the transformation $z = \ln(y+1.45)$; we get the following regressions:

$$y = .008 - .076x \quad (R^2 = .07)$$
$$\quad (1.57) \quad (.14)$$

$$z = -.68 + .003x \quad (R^2 = .00)$$
$$\quad (2.51) \quad (.22)$$

Table 1 shows that three observations contribute to a positive sign and the other three to a negative one. R^2 of the second regression can be increased by using more complicated transformations. Note, however, that the sign of a regression coefficient can also be changed by simultaneously applying transformations of the independent and the dependent variables. This issue is beyond the scope of this note.

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