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REGIONAL COOPERATION IN THE USE OF IRRIGATION WATER, EFFICIENCY AND GAME THEORY ANALYSIS OF INCOME DISTRIBUTION

by

Dan Yaron and Aharon Ratner

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D. Yaron* and A. Ratner**

* Institute of Agricultural Economics, University of Oxford, on leave from the Hebrew University of Jerusalem.

** The Hebrew University of Jerusalem.

Abstract

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The paper presents an analysis of the economic potential of regional cooperatives in water use in irrigation under conditions characterized by a general trend of increasing salinity. Income maximizing solutions for the region are derived and the related income distribution schemes are solved for with the aid of cooperative game theory algorithms. Distinction is made between transferable and non-transferable income situations. The reasonableness and the acceptability of these schemes is later critically evaluated.

INTRODUCTION

In most parts of the world with irrigated agriculture, the allocation of irrigation water to farms is dictated by water rights (quotas) which have been institutionally determined many years previously. Generally, these rights have not been changed since their determination, nor adjusted to the significant technological changes in agriculture and the farming systems which have occurred. The inevitable result is inefficiency in the interregional and interfarm allocation of water.

The inefficiency of the institutional water allocation system has been exacerbated recently by the increasing use in irrigation of low quality water (e.g. drainage water or brackish water from marginal sources) in regions which suffer from water scarcity (e.g. Western USA, Israel). In those regions where a dual supply is being developed, with differentiation according to water quality (salinity), the institutional system can hardly cope with water allocation problems. With one water-quality level, the allotment of water quotas to farms involves at least two parameters - the annual and the peak-season quantities.[1] With a dual water supply, there are four allocation paremeters to be considered.

Another problem arises when the National Water Authority and a particular region are faced with the option of increasing the salinity of the water supply to the region and compensating the region by increased quantity. If the water supplied to the region has to be of just one salinity level (contrary to the previous situation) and the region's farms have different preferences with respect to the desired quanity-salinity (Q-S) mix, the determination of the "optimal" mix is a difficult problem. This problem is addressed in the present paper.

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1 Sometimes the number of parameters is larger.

The growing complexity of water-allocation issues in the situations described above and in others will, necessarily, increase the inefficiency of the institutional allocation system and emphasise the need for inter-farm and interregional water mobility in correspondence with economic considerations.

One way to increase the efficiency of water allocation among farms within regions is through the establishment of farmers' regional water associations or cooperatives. The cooperatives must be established voluntarily: the historical institutional water rights of its farm members will be retained (changing them seems to be an extremely difficult or even impossible task), but the members of the association will be able to exchange water quotas (1) among themselves and (2) with other entities (e.g. the National Water Authority). Such associations already exist and operate in certain regions with irrigated agriculture.

The objective of this paper is to analyse the economic potential of such cooperative associations from the point of view of both efficiency and equity under conditions characterised by a general trend of the increasing salinity of the water supplied to agriculture.

The next section of the paper presents a model for the determination of the optimal Q-S mix for a regional cooperative association. In Section 3 an application of the model to a quasi-empirical situation (specified later) is demonstrated. In Section 4 several income distribution schemes based on cooperative game-theory approaches are presented and evaluated, with a distinction being made between transferable utility (TU) and non-transferable utility (NTU) situations. Section 5 summarises the paper, offering conclusions and evaluating the potential for cooperation in water use under the conditions referred to.

2. THE DETERMINATION OF THE OPTIMAL WATER QUANTITY - SALINITY MIX FOR A REGIONAL WATER USERS COOPERATIVE

Consider a region with I farms and a given allotment of water ($G\bar{W}$) of given salinity level (R_o). $G\bar{W}$ is the sum of the individual farm quotas (GW_i), $G\bar{W} = \sum_{i=1}^{I} GW_i$.

Assume that the National Water Authority can supply the region with a higher water quantity at the cost of increasing its salinity. For a given $G\bar{W}$, the substitution between water quantity $B\bar{W}$ and its salinity R is subject to a transformation curve determined by the National Water Authority:

(1) $F(B\bar{W}, R|G\bar{W}) = 0$

It is further assumed that the quality of water R must be the same for the whole region and all of its farms. Any decision regarding R and the receipt of a larger quantity of water but of higher salinity must be mutually agreed by all the farms in the region; this provides the essential motivation to encourage the region's farms to cooperate within the framework of a water users' association.

Obviously, $\sum_i BW_i = B\bar{W}$, with BW_i denoting the quantity of water quality R allocated to farm i.

Farms' i income y_i is a function of BW_i and R :

(2) $y_i = y_i\ (BW_i, R) \qquad i=1,2,\ldots,I$

The cooperative's problem is to determine simultaneously (a) the optimal quantity-quality ($B\bar{W}$-R) mix for the regional cooperative and (b) the quantity of water (BW_i) of quality $R \geq R_o$ allotted to each of the individual farms.

This is a classical problem of equilibrium in the supply and use of "public" and "private" goods. In the present context, we refer to water quality R as a "public good" which "all enjoy in common in the sense that each individual's consumption leads to no subtraction from any other individual's consumption of that good" (Samuelson (1964)). This is opposed to a "private good" (in this context, the quantity of water) which can be parcelled among different individuals so that the consumption of that good by one individual <u>does</u> reduce the quantity remaining for the consumption of the others.

The optimality conditions for the relationship between $\bar{BW}$ and R are derived from the following problem:

(3) Maximize $W = \sum_{i=1}^{T} \lambda_i y_i$

subject to

$$F(\bar{BW}\ R \mid \bar{GW}) = 0$$

(4) $\sum_{i=1}^{I} BW_i - \bar{BW} = 0$

with W being the cooperative's welfare function and λ_i - relative weights assigned to the ith farm income ($\lambda_i > 0, \Sigma\lambda_i = 1$).[1] The optimality conditions satisfy:

(5) $$\sum_{i=1}^{I} \frac{\partial y_i}{\partial R} \Big/ \frac{\partial y_i}{\partial BW_i} = \frac{F_R}{F_{\bar{BW}}}$$

with F_r and $F_{\bar{BW}}$ being the partial derivative of F with respect to R and $\bar{BW}$ respectively. Relationship (5) implies that the sum of the individual farms' marginal rate of substitution of water quantity for quality in water use should equal the technical marginal rate of substitution between water quantity for quality in supply. By parametrically varying the λ_i weights the "efficiency frontier" in the I farms' income space can be derived. The efficiency frontier is the locus of Pareto-optimal points with the property that any move from such a point aimed at improving the income of one farm must necessarily reduce the income of some other farm(s).

Ratner (1983) has applied a linear programming model to the empirical estimation of such an income efficiency frontier for a quasi-empirical

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1 The magnitudes of the λ_i values do not affect the optimality conditions. The issue of income distribution is discussed in Section 4.

regional cooperative in Israel (the Negev). In the transition from the theoretical background to the operational model, it was decided for computational convenience to refer to R as an exogenous parametrically varying variable[1], R = 260, 300, 350, 400 ppm Cl with the water quality of $\overline{GW}$ being 220 ppm (current salinity level). For each level of R the following problem was solved:

Maximize f

(6) $$f = \sum_{ij} c_{ij}^R x_{ij}^R$$

subject to

(7) $$\sum_j w_{ij}^R x_{ij}^R - BW_i^R \leq 0 \qquad i=1,2,\ldots,I$$

$$\sum_j d_{ij}^R x_{ij}^R - SW_i^R \leq 0$$

(8) $$\sum_i BW_i^R - \overline{BW}^R \leq 0$$

(9) $$\sum_i SW_i^R - \overline{SW}^R \leq 0$$

(10) $$\frac{1}{\alpha^R} \overline{BW}^R \leq \overline{GW}$$

(11) $$\frac{1}{\alpha^R} \overline{SW}^R \leq \beta\overline{GW}$$

(12) $$\sum_j a_{pij} x_{ij} \leq b_{pi} \qquad i=1,2,\ldots,I \quad p=1,2,\ldots,p$$

(13) $$\sum_j c_{ij}^R x_{ij}^R \geq K_i \qquad i=2,3,\ldots,I$$

where c_{ij} = income per activity unit j on farm i [$];

x_{ij} = level of activity j on farm i ;

w_{ij}, d_{ij} = respectively, total and high season water inputs per activity unit j;

α^R = regional coefficient of substitution between good-quality water (GW) for water of quality R ($\alpha < 1$);

β = maximal share of high-season water used out of the annual total ($0 < \beta < 1$);

a_{pij} and b_{pi} = respectively, input coefficient and availability level of restriction p other than water;

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[1] Feinerman (1980) and Feinerman and Yaron (1983) have incorporated R into a linear programming model as an endogenous decision variable, along with the quantities of water to be applied to the farms' crops. Their approach however, is computationally cumbersome and has not been applied here.

K_i = restriction level on income of the ith farm

$i=2,\ldots,I$.

Assuming that restrictions (8)-(11) are binding, denoting their shadow prices at the optimal solution by $\bar{u}^o$, $\bar{v}^o$, $\bar{w}^o$ and $\bar{z}^o$ respectively, and substituting $\bar{SW}^R = \beta \bar{BW}^R$, we get from the optimality conditions:

(14) $\bar{u}^o = \frac{1}{\alpha^R} \bar{w}^o$ or $\bar{w}^o = \alpha^R \bar{u}^o$

and $\bar{v}^o = \frac{1}{\alpha^R} \bar{z}^o$ or $\bar{z}^o = \alpha^R \bar{v}^o$

(15) $$\frac{d\ \bar{BW}^R}{d\bar{GW}^R} = \frac{\partial f/\partial \bar{GW}}{\partial f/\partial \bar{BW}^R} = \frac{\bar{w}^o + \beta \bar{z}^o}{\bar{w}^o + \beta \bar{v}^o} = \alpha^R$$

Relationship (15) implies that the marginal rate of substitution between water of quality R and water of current quality $\bar{GW}$, in use on all farms, should equal the technical rate of substitution α^R in the regional supply transformation relationship.

When restriction (13) is deleted, the optimal solution of (6) through (12) determines the highest income obtainable for the region for water quality R. By incorporating (13) and parametrically varying the K_i levels, interfarm income efficiency frontier is obtained. Note that restriction (13) does not affect the Pareto-optimality conditions (14) and (15) which hold for any point on the income efficiency frontier. For any given set of K_i levels ($i=2,\ldots,I$) the optimal R can be found parametrically by solving (6) through (13).

3. A QUASI-EMPIRICAL APPLICATION

a) Cooperation between two farms

The above model has been applied to the analysis of a small area in the Negev region of Israel. We first present partial results relating to a regional cooperative with two farms only.[1]) The farms have at their

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[1] The two farms are kibbutzim (plural for a kibbutz) - namely collective farms based on voluntary membership and democratic management.

disposal certain annual and high-season quotas of water from the regional water project, hereafter Mekorot water (with a current salinity of 220 ppm Cl); self-owned wells of saline water (1000-1200 ppm Cl) with annual and high-season pumping quotas determined by the National Water Authority; and certain areas of irrigable land. The major differences between these two adjacent farms are:

(a) The land on Farm 1 is practically unlimited while the cropping area on Farm 2 is limited.

(b) The share of the high-season water supplied by the regional project out of the annual total is 20% and 12% respectively for Farms 1 and 2;

(c) The high-season pumping quota from the self-owned wells is 10% and 30% respectively for the two farms.

The income (value added) derivable from these resources by the two farms with no cooperation (and the status quo water salinity of 220 ppm Cl) is \$948,000 and \$768,000 respectively at January 1982 price level.

The regional cooperation model was applied to the two farms using the following assumptions:

1) The regional supply transformation relationship between water quality R and quantity $B\bar{W}$ is:

R ppm Cl	Compensational $B\bar{W}$ at % of GW
220	0
260	3.0
300	5.5
350	8.0
400	10.0

2) The income of each farm under cooperation must be equal to or higher than its income before cooperation ("individual rationality").

3) The $B\bar{W}$ - R combination for the region is determined as a part of the cooperative arrangement; so are the quantities of water allocated to each farm.

4) The cooperation in water use involves both the Mekorot water and the saline wells' water. Regarding the latter case, the recipient of the other's well-water covers the cost of water pumping and conveying.

5.1) The cooperation involves both water and direct-income transfers ("side payments");

<u>or</u>

5.2) The cooperation is restricted to transfers of water only; the distribution of income among the farms is determined by those transfers. (For example, Farm 1 transfers water to Farm 2 in June, and a reverse transfer takes place, say in July).

Assumptions 1 through 5.1 and 1 through 5.2 will be referred to hereafter as Scenarios 3 and 4 respectively. A few other scenarios were analysed by Ratner (1983); they will not be discussed here.

The model results for Scenarios 3 and 4 are presented in Table 1 and Figure 1. As Table 1 indicates, the additional income generated due to cooperation varies within the range of 5.9-22.5 per cent of the income with no cooperation, depending on the scenario.

The major share of the additional income is due to water and income transfers (19.1% as compared with the overall 22.5% with respect to Scenario 3). Only a minor share of the additional income is attributable to the change in water salinity (from 220 to 350 ppm Cl) and the addition of 5.5% to the water quota. This latter result is not surprising - it suggests that the compensation for the increasing salinity is about right.

The significant rise in the regional income under conditions of Scenario 3 results from more than doubling the total crop area of Farm 1 (the more efficient farm), which becomes possible by transfer of water from Farm 2 to Farm 1. Before income transfers being made the income of Farm 1 rises by 63% in

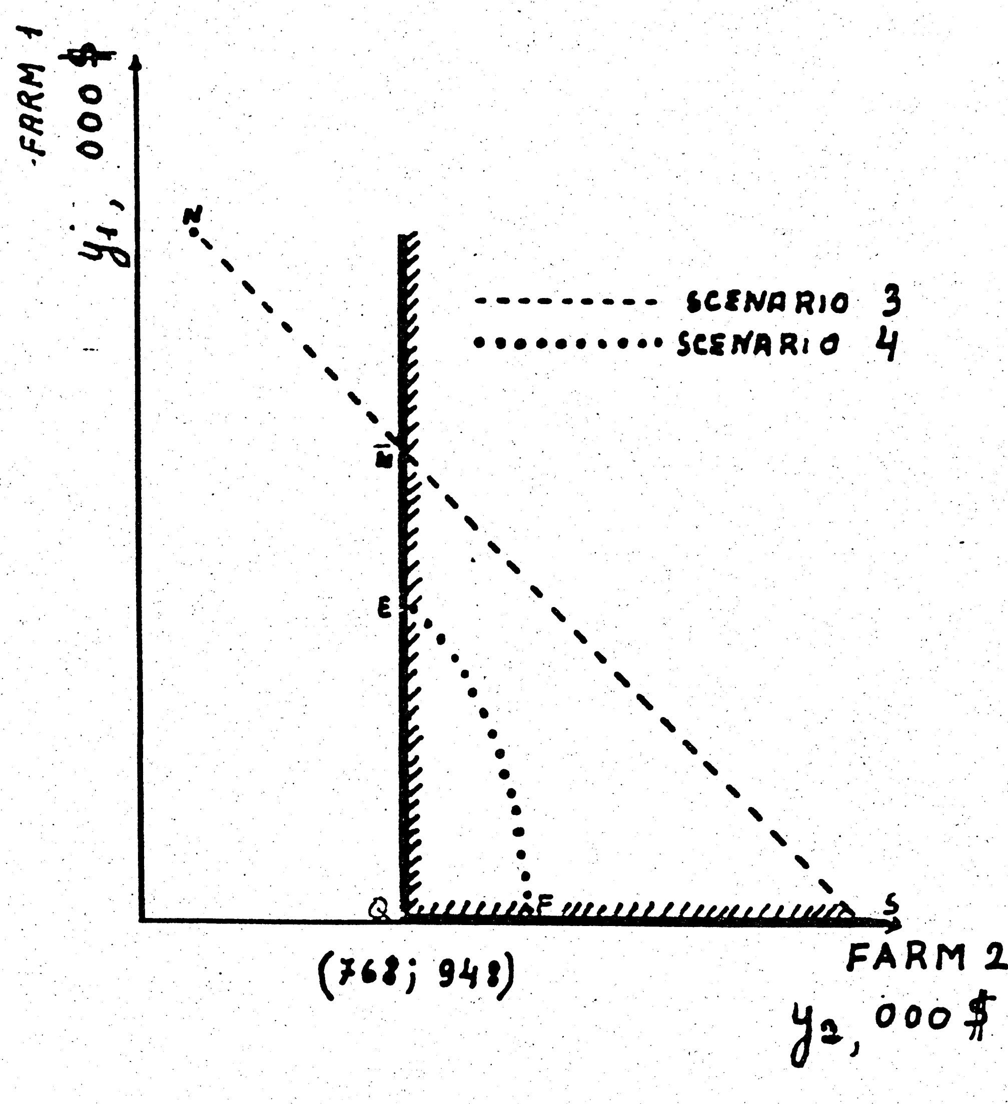

Fig. 1 Income transformation curves between Farms 1 and 2, Scenarios 3 and 4.

comparison with the non-cooperative income and that of Farm 2 falls by 27%. The incomes of the two farms after income transfers depends on the income allocation along the NS transformation curve in Figure 1. More details can be found in Ratner (1983).

Figure 1 presents the transformation curves of income between the two farms. The point Q represents the income derivable with no cooperation; the feasible region for income allocation lies "north-east" of Q. The point N corresponds to the maximal cooperative income under Scenario 3 but it lies outside the feasible region; the feasible part of the income transformation curve is $\bar{N}S$. For Scenario 4 it is EF.

Table 1. Additional Income Generated by the Cooperation of Farms 1 and 2 with Reference to Two Alternative Scenarios

Scenario #		3	4
Elements of cooperation		Water and Income Transfers	Water Transfers Only
Income with no cooperation	000$	1,716	1,716
Additional income[1]	000$	386	102-240
	%[2]	22.5	5.9 -14.5
Of the above	%		
a) due to increased water quota and salinity		3.4	0.8 - 2.9
b) due to water and income transfers[3]		19.1	5.1 - 11.6

1) Additional income generated due to cooperation as percent of the two farms' income with no cooperation.

2) Computed with reference to unrounded numbers.

3) Income transfers - when applicable (Scenario 3). Computed with reference to the additional income generated with no change in water salinity and no compensation in quantity.

b. Cooperation among three farms

The analysis of a two-farms cooperative has the advantage of simplicity in graphical presentation of the results, but its simplicity may obscure some issues (e.g. "partial coalitions", "group rationality"). Therefore in this subsection, we turn to the discussion of a three-farms cooperative.

The unique feature of Farm 3[1], which is now assumed to join the cooperative, is its sensitivity to salinity, due to a large share of salinity-sensitive (perennial) fruit crops in its crop mix (40% of the irrigated area compared with zero and 5% respectively on Farms 1 and 2). An auxiliary analysis showed that if each of the three farms would have been confronted individually with the option of substituting quantity of water for quality according to assumption 1 above, the optimal salinity levels would have been 220 ppm Cl for Farm 3 as compared with 300 and 350 ppm Cl for Farms 1 and 2 respectively. This reflects a conflict of interest which the three farms must resolve in order to cooperate.

As with the case involving two farms, the cooperation model was applied to the three-farms cooperative with respect to Scenarios 3 and 4.

The income obtainable by the three-farms cooperative under conditions of Scenarios 3 and 4 is shown in Table 2.

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[1] Farm 3 is a Moshav - a cooperative village of 80 small family farms. The village cooperative acts as a credit organization and provides production and marketing services. For simplicity the Moshav village is referred to as an aggregate in our expository analysis.

Table 2. Maximal Additional Income due to the Cooperation among Farms 1, 2 and 3

Scenario #		3	4
Additional Income	000$	385	(244)[4]
	%[2]	16.6	10.5
Additional Income due to water transfer only[1]	000$	364	(233)[3]
	%	15.7	10.1
Optimal salinity level	ppm Cl	300	260

1 With no change in water salinity.

2 Computed with reference to rounded figures.

3 Maximal additional income on the three farms' income-transformation surface. The actual additional income depends on income distribution.

The following observations are made with respect to Table 2:

a) The additional income of the three-farms cooperative is somewhat lower than that of the two farms (1 and 2). This is due to the relative sensitivity of Farm 3 to salinity and its low capacity to benefit from the substitution of quantity of water for quality, an option open to the region.

Why then a three-farms cooperative? If Farm 3 is an integral part of the region and its water-supply system cannot be separated from the other two farms, then a three-farms agreement on the salinity level of the water supplied to them, and the cooperation implied by this, are necessary. If this is not the case - and Farm 3's water supply can be separated - the establishment of the three-farms cooperative will not be justified.

b) As in the case of the two farms cooperative, a major share of the incremental income is due to water transfers. Again, this implies that the increased salinity of the water supplied to the region is compensated fairly by the increased quantity.

c) Table 2 presents the maximal additional income values. For Scenario 4 these values are not quite meaningful. The actual additional income depends on its distribution.

4. INCOME DISTRIBUTION

The second part of the study is essentially an exercise in the application of game-theory models to the problem of the allocation of the benefits generated by the cooperative among its members.[1] The attempt to solve this problem with the aid of game-theory models was made in view of the recently abundant and apparently promising literature in this field with respect to variety of issues - e.g. water-resource development (Young et al. 1981), charging for airfield use (Littlechild and Thompson 1974), telephone switchboard use (Billera et al. 1978), while the number of publications reporting less successful applications is apparently small (e.g. Heany and Dickinson 1982).

It should be noted that viewing the "region" and its water supply-demand relationships as a competitive market was considered inappropriate, for the following reasons:

a) The number of farms in the region is small,[2] the farms are not anonymous and partial cooperation agreements ("partial coalitions") are possible,

b) It is assumed that deliberate increasing of water salinity in the region necessitates an agreement by all the farms in the region and by the National Water Authority.

The above assumptions were considered by the authors as reasonably representing the prevailing institutional background for the allocation of the income generated by cooperation among the farms. With these assumptions

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1 Another approach, namely pricing for water transfers according to its shadow cost, was also applied (Ratner (1983)). In the opinion of the authors it yielded unreasonable results, apparently due to the conditions prevailing in Israel - and other countries - where the shadow cost of water is significantly higher than the price paid by the farms. The shadow-pricing approach is not described in this paper.

2 The authors have in mind a viable region with 20-25 kibbutz or other villages, a number considerably larger than in our expository discussion.

as the terms of reference, it was evident that the payments for water transferred and the related income-distribution must be mutually agreed upon.

Payments for water transfers according to its shadow price was considered. However, this approach - a priori apparently very sound economically, was found to be inappropriate under the conditions prevailing in Israel. The reason is that water is rationed according to institutionally determined quotas, its price to farmers is subsidised and lower than its marginal value product. Thus water quotas allocated to farmers may bear a considerable rent component, as in our case study. With water being considered as a national natural resource, owned and administered by the nation, it is claimed that farmers have the right to benefit from the rent derivable from water if and only if they use water for production. If the water had been private property of the farmer this restriction on benefits from water quota rent would have been removed.

Note that reference to the (inefficient) system of quota allocation and water pricing in Israel as given does not imply its approval. On the contrary, suggestions for its revision have been set forth (e.g. Yaron 1971 (Hebrew)); this issue, however, falls beyond the scope of the present paper.

At the following stage several cooperative game-theory models were applied, the aim being an "objective" income allocation scheme, one which is not based on value judgement, but rather on mutually agreed principles or axioms. The essence of cooperative game-theory approaches is that once the participants in a cooperative enterprise agree on certain axioms believed to satisfy selected criteria such as efficiency, fairness and reasonableness, the income (or cost) allocation can have only one outcome (Loehman and Whinston (1971, 1974), Nash (1950), Shapley (1953) and Schmeidler (1969)). The object of the authors in this part of the study was to find out to what extent the schemes based on a priori reasonable axioms lead to reasonable and acceptable results.

Distinction was made between the transferable and non-transferable utility (income) situations in accordance with Scenarios 3 and 4.

a. The Transferable Income Situation - Scenario 3

The following approaches were applied, the core (Owen (1982)), the Nucleolus (Schmeidler (1969)), the Shapley value (Shapley (1953)) and the Nash-Harsanyi solution (Nash (1950), Harsanyi (1959)).

The core of the three farms cooperative is a three-dimensional polyhedron defined by relationships (16) and (17):

(16)
$$946 \leq y_1 < 1,261 \quad (000\$)$$
$$769 \leq y_2 < 1,151 \quad \text{(rounded numbers)}$$
$$605 \leq y_3 < 791$$

(17) $$y_1 + y_2 + y_3 = 2,705$$

where y_i is the total (value added) income of the i-th farm.

[1] A comprehensive discussion of game theory can be found in Owen (1982) or Luce and Raiffa (1957).

While logically and morally easy to accept, the core concept is not conclusive; it provides only bounds on the claims of the members of the cooperative, and is therefore only partially useful. It may serve as the starting point for either (i) negotiations among the members or (ii) continuation of the analysis. A direct progression is the application of the Least Core and the Nucleolus (Schmeidler (1969)). The latter is needed when the Least Core does not lead to a unique solution, as it happened in this problem.

The Nucleolus, the Shapley value and the Nash solutions are shown in Table 3. Scrutiny of Table 3 raises questions with respect to the reasonableness and the acceptability of the schemes examined, due to the following reasons: Farm 1, which is the most efficient, generates the major part (58%) of the total cooperative income and more than 100% of the incremental income generated. At the same time Farm 1's share in the cooperative gains is less or at most equal (Nash solution) to that of Farm 2. The opposite prevails with respect to Farm 2. Farm 3, whose weighted average contribution to the cooperative's gains according to Shapley value is 8% only is allocated 24 and 33.3 percent respectively in the Nucleolus and Nash solutions. Note that Shapley value allocates the highest share of the cooperative gains to Farm 1 as compared to the other schemes, but still its share is smaller than that of Farm 2. Only the Nash solution allocates equal shares to all the three farms.

Table 3 Comparison of Alternative Schemes of Allocation of the Income Generated by Regional Cooperation with Reference to Scenario 3.

Farm			1	2	3	Total
1.	Income with no cooperation	000\$ [1]	941	769	605	2,320
2.		% [2]	41	33	26	100
3.	Income genreated under cooperation	000\$	1,579	568	558	2,705
4.		%	58	21	21	100
			Allocation of cooperation gains			
5.	Nucleolus	000\$	112	179	93	385
6.		%	29	47	24	100
7.	Shapley value	000\$	159	194	32	385
8.		%	41	51	8	100
9.	Nash solution	000\$	128	128	128	385
10.		%	33.3	33.3	33.3	100

1. Rounded numbers.
2. Percentages computed with reference to unrounded numbers.

b. The Non-transferable Income (NTU) Situation - Scenario 4

Except for the Nash-Harsanyi solution finding solutions for income allocation when side payments are not possible is subject to difficulties which are primarily computational, and in some cases, conceptual as well.

The Nash-Harsanyi Solution

We start with the Nash solution, which can be derived with the aid of the following model:

$$(18) \quad \max z = \prod_{i=1}^{3} (y_i^R - y_i^o)$$

subject to

$$(19) \quad y_i^R \leq y_i^o \qquad i = 1, 2, 3$$

and restrictions (7) - (13) with $y_i^R \equiv \sum_j c_{ij} x_{ij}^R$ and y_i^o being the income of the i-th farm with no cooperation. By logarithmic transformation of (18) and maximizing log z (a monotonic function of z) a separable objective function is obtained, and the problem can be solved by a separable programming routine available in most computers. The optimal water quality R is determined by solving the problem parametrically, for the five discrete levels of R chosen (see above). The results are shown in Table 4.

Table 4. Nash-Harsanyi Solution for a Three-Farms Cooperative with reference to Scenario 4

Farm		1	2	3	Total
1. Income with no cooperation,	000$	946	769	605	2,320
2.	%[1]	41	33	26	100
3. Income generated on farm	000$	1,031	811	672	2,514
4.	%[1]	41	32	27	100
5. Additional income	000$	85	42	67	194
6.	%	43	22	35	100

[1] Percentages computed on the basis of unrounded numbers

Scrutiny of Table 4 suggests that the percentage-wise distribution of total income generated on the farms among the three cooperating farms is about the same as before establishing the cooperative. From this point of view, the results seem intuitively acceptable. The aggregate income of the cooperative rises by 8% in comparison with the non-cooperative situation, while the incomes of Farms 1-3 rise by 9, 5 and 11 percent respectively.

The Core Solution

The Core of a game is formally defined as the set of feasible allocations which can be improved upon by no coalition s in S, where S denotes the set of all coalitions (Hildenbrand and Kirman, 1976).

The Core of the three-farms cooperative in the NTU situation was defined by the following relationships:

$$y_2 \geqq g_{21} (y_1)$$

(20) $y_2 \geqq g_{23} (y_3)$

$$y_3 \geqq h_{31} (y_1)$$

(21) $y_1 + y_2 + y_3 = V(1, 2, 3)$

where

$g_{2i} (y_i)$ - is the income transformation function for the partial cooperation (coalition) of Farms 2 and i (i=1,3)

$h_{31} (y_1)$ - is the income transformation function for farms 3 and 1

V (1, 2, 3) - is the aggregate income of the three-farms cooperative (grand coalition). Note that its magnitude depends on the income allocations $y_1 - y_3$.

The functions $g_{2i}(y_i)$ and $h_{31}(y_1)$ were derived for the corresponding two-farms cooperatives, as shown in Section 2 for Farms 1 and 2.

Note that: any point $[g_1(y_1), y_2]$ satisfying $y_2 \geq g_{21}(y_1)$ satisfies also $y_1 \geq g_{21}^{-1} (y_2)$; similar inverse relationships hold for $g_{23}(y_3)$ and $h_{31}(y_1)$; restrictions (20) satisfy the individual rationality requirements of the farms as well.

The Core of a three-players game in the NTU situation is the set of feasible income points. These points lie on a three dimensional surface determined by (a) the intersection of the three two-players cores (restrictions (20) above), and (b) the three players' income transformation surface (restriction 21). A schematical presentation of such an intersection is shown in Figure 2, following Hildenbrand and Kirman (1976).

In our study, the Core of the three-farms cooperative was parametrically solved by maximising y_1^R subject to restrictions (7)-(13) and (20) for various combinations of y_2 and y_3 (arbitrarily chosen). The non-linear

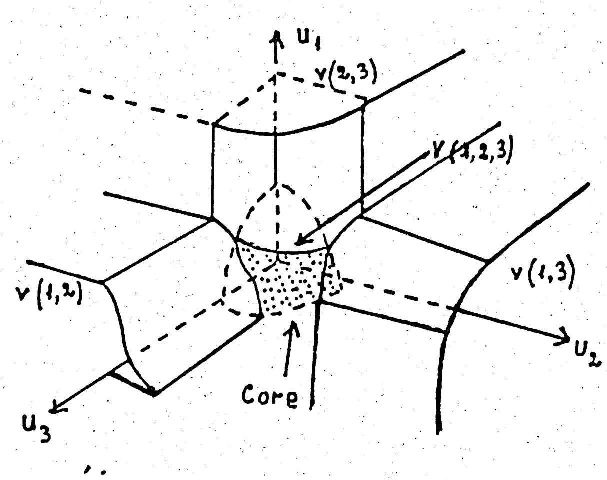

Fig. 2 A schematical presentation of the core of three players game in the NTU situation following Hildenbrand and Kirman (1976)).

functions $g_{21}(y_1)$, $g_{23}(y_3)$ and $h_{31}(y_1)$ were piecewise linearly approximated and a separable programming routine was used. The problem was solved five times for the different preselected values of R. It is noted that the optimal R for the three-farms cooperative may differ from that which is optimal for any two-farms cooperative.

The Core thus computed is presented in Figure 3 in terms of the additional income generated by cooperation. The point (0, 0, 0) represents the income obtained by the three farms with no cooperation and the values along the three axes are expressed in terms of the additional income ("normalised game"). The additional incomes of Farms 2 and 3 are clearly marked in the graph, while the income of the 1st Farm is represented by the height of the tridimensional surface (which is the Core). The shadowed area in the $y_2^* \, y_3^*$ plane represents $y_2^* \, y_3^*$ combinations which do not satisfy the group rationality conditions (restriction (20)).

We make the following observations:

a) The Core is not a convex set. This eliminates the possibility of generating new points in the Core by convex combinations of selected points lying in the Core.

b) The Core in the case studied is quite large.

c) Heavy computational burden is involved in computing the core for the three-farms cooperative. The computations might be prohibitive for cooperatives (coalitions) with a larger number of members.

In view of the above, one may wonder whether the Core is a practical concept in applications such as the one dealt with here.

Considering the difficulties involved in the computation of the Core, no attempt was made to compute the NTU equivalent to the Nucleolus (which was computed for Scenario 3). Furthermore, the nucleolus concept in the NTU case is not clear.

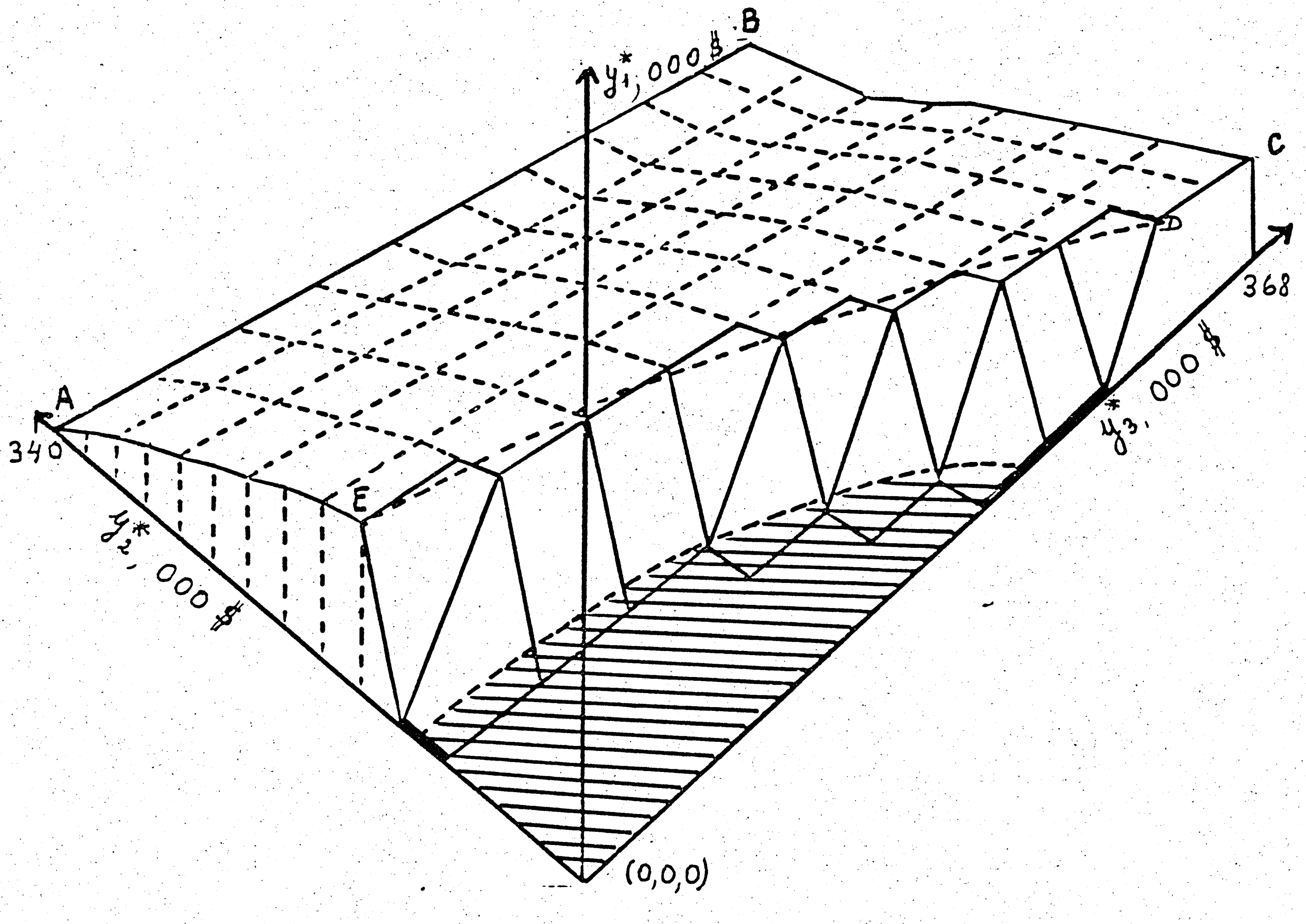

Fig. 3 The core of the three farms cooperative game in the NTU situation.

5. SUMMARY AND CONCLUDING COMMENTS

The paper presents an analysis of the economic potential of regional cooperatives in water-use in irrigation, under conditions characterized by a general trend of increasing water salinity. Specifically, it is assumed that the farms of a region face the option of substitution of larger water quotas for lower quality (higher salinity), and that the salinity levels must be the same for all the region's farms. This is the motivation for cooperation.

The potential gains from cooperation in the case of a quasi-empirical example are in the range of 6 - 22% of the income with no cooperation, depending on the specific conditions referred to.

The major share of the additional income is due to the exchange of water quotas among the cooperating farms with no change in water salinity. This implies that the potential for cooperation in water use should be studied as well, under conditions in which salinity is not an issue.

The potential for additional income due to cooperation is higher when direct income transfers ("side payments") are possible. However, the soundness of such transfers with no *a priori* reference to the price per unit of water may be questioned. On the other hand, if the lump-sum payments transferred will satisfy some notion of a "fair price", the objection might be mild or even non-existent. The question remains open for further study.

Cooperative game-theory models were applied in an attempt to formulate an objective income-distribution scheme for the cooperating farms based on mutually agreed axioms, rather than on value judgement and arbitrary negotiations. A distinction was made between the transferable and the non-transferable income situations. The results of this part of the study are inconclusive. While the analysis with the aid of game-theory models provides a better understanding of the problem, it does not provide an objective, indisputable solution to income distribution with reference to the case studied. Moreover, except for the Nash-Harsanyi solution the computational burden is heavy; in the case of computation of the Core with non-transferable income it may become prohibitive. It seems that further theoretical study and further empirical applications are needed in order to evaluate fully the applicability of game-theory approaches to problems similar to the one discussed in this paper.

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