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EVALUATING THE MEAN-GINI APPROACH TO PORTFOLIO SELECTION

by

Haim Shalit and Shlomo Yitzhaki

Rehovoth, Israel, P.O.B. 12

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This paper evaluates the empirical properties of the mean-Gini (MG) and the mean-extended Gini (MEG) efficient sets by comparing their performance to the mean-variance (MV) portfolio selection. The analysis focuses on the similarities and differences existing between the MV, the MG, and the various MEG efficient sets. In addition, the risk parameter for which the MEG efficient set is best supported by the market data is estimated. The analysis is carried out with respect to the Tel-Aviv Stock Exchange to present empirically a new approach to portfolio selection. (PORTFOLIO THEORY; OPTIMAL PORTFOLIO SELECTION; MEAN-GINI ANALYSIS; MEAN-EXTENDED GINI; MEAN VARIANCE PORTFOLIO).

Evaluating the Mean-Gini Approach to Portfolio Selection

1. Introduction

The use of the mean-Gini (MG) and the mean-extended Gini (MEG) in risk analysis was proposed and developed by Yitzhaki (1982, 1983). Recently, the approach was extended to finance theory and portfolio analysis in an article by Shalit and Yitzhaki (1984). Being a statistic used mainly in income inequality, the Gini, as a measure of dispersion, has several qualities that make it a favorable candidate for describing risk. The mean-Gini approach uses, like the mean-variance (MV), two summary statistics to charaterize the distribution of a risky prospect. However, because MV analysis requires the perfect knowledge of all prospects' probability distribution, it might fail to rank portfolios of prospects consistently, according to individual preferences. On the other hand, Mean-Gini analysis also provides necessary conditions for stochastic dominance and thus is appealing to investigators because it prevents them from choosing a portfolio which can be considered inferior.

In the present paper, we evaluate empirically the mean-Gini and mean-extended Gini methods, compare them to mean-variance analysis, and appraise their respective merits. This type of analysis has been performed recently by Bey and Howe (1984) who compared the empirical properties of the MG efficient set to the mean-variance, mean-semivariance, and stochastic dominance efficient sets. Their study tested the performance of prespecified possible portfolios. The present paper departs from Bey and Howe's approach in two aspects: Firstly, we find efficient sets of portoflios for the extended Gini, but more important we compare MG, MEG, and MV efficient portfolios that are obtained from minimizing the portfolio risk for given expected rates of return using an optimization algorithm. It is the first time that such a procedure is used with respect to the mean-Gini and mean-extended Gini efficient sets.

The advantage of the Gini over the variance as a measure of dispersion and risk has been established by Yitzhaki (1982). The Gini's properties valid for finance theory were analyzed and motivated in our previous article. Here, we present only the main features of the analysis. First, the MG method allows for the construction of efficient portfolios that are all included in the set of first and second degree stochastic dominance (FSD and SSD) portoflios, regardless of the probability distribution of the returns.[1] Second, the Gini provides an intuititive measure of investment risk since the statistic is defined as the expected distance between two possible realizations of the prospect outcome. In the context of a portfolio, the interpretation of the Gini is the expected difference between the returns on two dollars of investment randomly drawn from the portfolio. Third, the Gini can be extended into a family of statistics that differ from each other by a single parameter ranging from one to infinity (Yitzhaki 1983). The value of one represents risk as viewed by a risk-neutral investor while the other extreme, infinity, shows risk as perceived by a maximin individual. The extension allows for the construction of mean-extended Gini efficient portfolios that are all <u>included</u> in the second degree stochastic dominance efficient set. Finally, if one considers only the set of probability distributions that intersect at most once (for example, the normal, lognormal, uniform, Gamma, and exponential distributions), the union of all the MEG efficient sets becomes the SSD efficient set.

It is important to stress that, from a theoretical point of view, all the efficient sets obtained by using different parameters of the extended Gini are equivalent. Hence one cannot conclude, without further information, which extended Gini is supported by the data. However, if there exist data on a portfolio chosen by the investors (e.g., the market portfolio) one can ask what efficient set is closest to that portfolio. The extended Gini, whose efficient set is closest to the market portfolio, is the one which is best supported by the data. Its parameter represents the risk aversion index of a representative investor in that market.

The purpose of this paper is to evaluate the empirical properties of MG and the MEG efficient sets by comparing their performance to the MV portfolio selection. In particular, we focus on three issues:

(i) How similar are MV and MG efficient sets?

(ii)How similar are the different MEG efficient sets?

(iii) What can be the risk averse parameter able to characterize the representative investor in the stock market. In other words, for what parameter, the MEG efficient set is best supported by the data.

The analysis is performed with respect to the Tel-Aviv Stock Exchange whose aggregate data is published by the Israeli Central Bureau of Statistics. First, we present the mean-Gini method and briefly motivate its use. In §3, we analyze the data and the caomposition of the MG and MV portfolios. §4, we determine the risk index by comparing the actual market portoflio composition with the various MEG efficient portfolios.

2. Mean-Gini Efficient Portfolios

The two-parameter MG portoflio analysis consists of determining assets combinations that are efficient in mean-Gini or mean-extended Gini space. With that respect the construction of MG efficient portfolios is similar to the method of finding MV efficient portfolios. For a given number of securities, one searches for the mix of prospects that minimizes the portfolio's Gini (or extended Gini) given an expected rate of return. Mathematically, the optimization problem is stated as

$$\min_{x_1,\ldots,x_N} \quad \Gamma(\nu) \tag{1}$$

$$\text{subject to} \sum_{i=1}^{N} x_i\bar{R}_i = \bar{R}_0,$$

$$\text{and} \sum_{i=1}^{N} x_i = 1; \; x_i \geq 0,$$

where $\Gamma(\nu)$ is the extended Gini for a given parameter ν, $\bar{R}_i$ the average return on security i, x_i the share of security i in the portfolio, $\bar{R}_0$ the required average return on the portfolio, and N the number of securities available. The extended Gini of a portoflio is defined as follows:

$$\Gamma(\nu) = -\nu\mathrm{cov}[R_p, (1-F_p)^{\nu-1}], \tag{2}$$

where R_p is the portfolio return, F_p its cumulative distribution and ν

the extended Gini parameter. For $\nu = 2$, Gini's mean difference (the Gini) is obtained by the following formula:[2]

$$\Gamma = 2 \text{ Cov } [R_p, F_p(R_p)]. \quad (3)$$

In other words, the Gini is twice the covariance of the random variable and its cumulative probability distribution. This representation is quite similar to the variance; However, in the case of the Gini, $F(R_p)$ is used instead of the variate itself.

For higher values of ν, Equation (2) is viewed as a "weighted covariance" between the variate and its cumulative distribution; the higher ν becomes, the larger the weights that are attributed to the lower portions of the variate's distribution. In the extreme case where $\nu \to \infty$, the extended Gini reflects the attitude towards risk of an investor who cares only for the lowest realization of the return; in other terms it is the maximin investor. If, on the other hand, ν is close to 1, the weights become equal reflecting the risk attitude of a risk-neutral individual. Some additional light on the role of ν is shed when one analyzes its effect on the nondiversifable risk of a security in a portoflio. Consider a given portfolio $x^0 = (x_1^0, \ldots, x_N^0)$ whose rate of return is given by $R_p = \sum_{i=1}^{N} x_i^0 R_i$, where R_i is the return on prospect i. Following equation (2), the extended Gini of the portfolio is given as follows:

$$\Gamma_0(\nu) = -\nu \sum_{i=1}^{N} x_i^0 \ \mathrm{cov}[R_i, \ (1-F_p)^{\nu-1}]. \tag{4}$$

An investigation of Equation (4) reveals that for high values of ν, the individual performance of prospect i is relatively important when portfolio returns are low. What matters here is the covariance between porspect i and the rank of the portfolio, provided that the actual portfolio returns are low. On the other hand, if ν equals 1, equal weights are given to the whole range of the portfolio distribution.

For a given parameter ν, the efficient set of portoflios is found by solving problem (1) for different values of $\bar{R}_0$. The algorithm used in this procedure is presented in the appendix. Although non-linear programming techniques applying gradient methods, on the one hand, and piecewise linear programming algorithms, on the other hand, can be used in the case of $\nu = 2$, they are inapplicable when ν differs from 2. Once the set of efficient portfolios is determined, they can be related to the second stochastic dominance (SSD) efficient set by the following propositions:[4]

<u>Proposition 1</u> (Yitzhaki 1982): $\bar{R}_i \geq \bar{R}_k$ and $\bar{R}_i - \Gamma_i(\nu) \geq \bar{R}_k - \Gamma_k(\nu)$ are necessary condition for portfolio i to dominate portoflio k according to SSD rule.

Whereas the necessary conditions for SD rules are used for any probability distribution, the sufficient conditions hold for families of cumulative distributions that intersect at most one, e.g., the normal, lognormal, uniform, and Gamma distributions. The sufficient conditions are stated as:

<u>Proposition 2:</u> Let R_i and R_k be two prospects with equal expected return. Assume also that the cumulative distributions $F_i(R)$ and $F_k(R)$ intersect at most once. Then $R_i - \Gamma_i(\nu) > \bar{R}_k - \Gamma_k(\nu)$ for any $\nu > 1$ is a sufficient condition for R_i to dominate R_k according to SD rule.

The mean-extended Gini necessary condition for SD requires that there is no other portfolio in the feasible set such that Proposition 1 holds. In theory, the proposition should be applied to all portfolios. However, in practice we can calculate only a finite number of efficient portoflios. Therefore, for empirical purposes, the MG and MEG portfolios constructed are SD efficient with respect to all portfolios considered. The MEG method, although restrictive in this context, is a two-parameter model able to construct SD efficient portfolios.[5] The union of efficient portfolios for all ν is also SD efficient.

3. Data Analysis and Results

The data base consists of eleven asset classes of stocks and bonds traded on the Israeli Stock Exchange of Tel-Aviv from January 1977 to January 1983. The nominal rates of return on those classes are computed on a monthly basis by the Israeli Central Bureau of Statistics to measure the total return on securities including cash receipts (dividends and interest payments net of taxes), bonuses, splits, and rights to other shares or options. The nominal rates of return were adjusted for monthly inflation by the Consumer Price Index and only real rates of return are used in the present analysis. The asset classes considered represent a break-down of the entire Stock Exchange in Israel including all stocks and bonds traded. The choice for this specific period was dictated by the publication of the indices by the Central Bureau of Statistics that started in December 1976. The short period prevented us from performing some sensitivity analysis on the data, our main purpose being to use the MG method for others to follow.

Throughout the paper, it is assumed that investors' expectations about assets future performance are consistent with past returns. This means that all <u>available</u> information resides in historical performance which will be used by investors to select the porfolios. Although this assumption is restrictive in the sense that it can rule out large classes of assets in an optimization algorithm, the approach was maintained to present a simple recipe for the use of mean-Gini analysis. Hence, the results obtained are far from definitive and are provided here as an example.

Summary statistics presented in Table 1 show that the ranking of prospects according to the Gini is identical to the ranking according to the standard deviation indicating some similarity between the two statistics. The highest mean return is obtained for the group of Real Estate Firms which also shows one of the largest values of risk in terms of standard deviation and Gini. The lowest dispersion according to the two statistics is obtained for the Class of Bonds linked to the Consumer Price Index. The group of Commercial Banks seems to allow for lower risk (in S.D. and Γ) at high mean return implying that this class of securities will participate in most of the required expected return portoflios. We also remark that some classes of assets exhibit negative monthly mean real rates of return (very close to zero). This must not be surprising since one of alternatives of not holding CPI linked Bonds is cash at a real loss equal to the rate of inflation.

In Table 2, we show the mean-variance efficient portfolio sets for selected expected rates of return. The class of Commercial Banks participates in all required expected return positions with 21.1% of the portfolio when the monthly expected return is 0.5% and with 80.8% of the portoflio when the return is 1.94%.[6] The largest expected return is obtained, jointly with an increase in the portoflio's standard deviation, by the groups of Manufacturing and Real Estate Firms. For a lower expected return and lower risk, the Class of CPI Linked Bonds forms the bulk of the portfolio. We note that some assets never participate in the optimal portfolio although they are traded on the Exchange. We suggest three reasons for that anomaly to the theory. First, we use ex-post statistics whereas investors have ex-ante expectations. Second not all investors use MV analysis nor other optimization methods; (Although with MG and MEG as we will see, all assets enter, at one point or the other, the efficient portfolio. Third, the time horizon seems too small for the analysis.

We now compare the mean-variance efficient set with the mean-Gini efficient set shown in Table 3. In general, the MG efficient portfolio is more concentrated in classes of securities with relatively higher return and relatively lower dispersion such as Commercial Banks. The mean-Gini criterion is also a better discriminator than MV since it forces the inclusion of more Real Estate Firms and less Manufacturing Firms securities; the first having a $\Gamma/\bar{R}$ ratio of 2.59 vs 3.18 for the second. The importance of the Manufacturing Firms in the MV efficient set seems to be rooted in its lower correlation coefficient with the Commercial Banks that allows for a better diversification. However in the MG efficient set, this feature erodes. One can now determine the subset of the stochastic dominance efficient set in the MG efficient set. Following Proposition 1 the necessary conditions for stochastic dominance are satisfied for all the efficient portoflios with expected rates of return that are greater than 1.37%.

We now present the mean-extended Gini portfolios with $\nu = 1.3$ as shown in Table 4. Here the subset of stochastic dominance is the set of efficient portfolios with expected rates of return greater than 2.50%. Although Commercial Banks shares dominate the efficient portfolios, there is an increasing participation of the group of Investment Firms, suggesting a higher diversification of individual securities in the portfolio. In general, the efficient MEG set with $\nu = 1.3$ is more diversified than the MG set with $\nu = 2.0$

We now consider the case of $\nu = 6.0$ which is presented in Table 5. Those required return portfolios are likely to be held by more risk-averse individuals. First, note that all the different required return configurations are in the SD efficient set. What is remarkable is the concentration of Commercial Banks and Bonds Linked to CPI for lower expected yield portoflios whereas the holdings of Commercial Banks and Real Estate Firms are predominant for higher rates of return. Indeed, since risk aversion is exhibited by the relatively large weight attributed to the worst realized outcome, investors conforming to that behavior tend to prefer efficient portfolio with prospects that have less of such bad outcomes.

4. Risk Index and Market Portfolio

We now determine the rissk parameter ν for which the efficient portoflio set is closest to the portfolio held by most investors as represented by the actual market position. In the previous section, we have demonstrated the importance of ν in portfolio selection. This risk parameter essentially determines the weights attached to the different sections of the returns distribution. In addition as shown by Shalit and Yitzhaki (1984), different ν can be used to construct different Capital Asset Pricing Models which may be similar or different depending on whether or not the individual securities are normally distributed. However, finding the risk parameter used by the representative investor in his portoflio selection remains an open question we answer by estimating what ν fits best the actual stock market data.

Since portfolio composition changes quite substantially with the risk parameter, the position of the securities in the efficient portfolio is affected especially by their diversified and systematic risk. Thus, the choice of ν is crucial in the identification and characterization of the securities in the portfolio. We propose a methodology for determining the value of ν that provides a set of efficient portfolios closest to the market portfolio. This will enable us to determine from the data which risk parameter represents, on average, investors in the stock market. The market portfolio is given by the actual position of all the classes of securities held by the public and valued at market prices. The weights are obtained by dividing the values of shares of each class by the total value of the stock exchange. At equilibrium prices this position is most desired by all investors since, if it was not, sales and purchases of individual shares will not only affect the relative position but also their value. Define $x^M = (x_1^M,\ldots,x_N^M)$ as the weights of the market portfolio with expected rate of return $\bar{R}_M$. In addition, let $x(\nu) = [x_1(\nu),\ldots,x_N(\nu)]$ be the solution of the optimization problem for different values of ν and a given expected return $\bar{R}_M$. The distance between the two vectors x^M and $x(\nu)$ is defined as

$$d(\nu) = \{\sum_{i=1}^{N} [x_i(\nu) - x_i^M]^2\}^{1/2}. \qquad (3)$$

We propose to use $d(\nu)$ as a measure of goodness of fit and find for what value of ν, that distance is minimized. Since $d(\nu)$ does not necessarily behave monotonically, a minimum for $d(\nu)$ will be found by searching over the entire range of ν. In Table 6, we present the distance $d(\nu)$ and the efficient portfolio composition for several values of ν together with the MV portfolio and the actual position of the market.[7] The value of ν that minimizes the distance $d(\nu)$ is around 2.5. Hence, the solution to the optimization problem closest to the actual market position implies investors who are generally <u>more risk averse</u> than investors using the simple Gini index (ν = 2) as a measure of risk. The same conclusion applies when using the variance as a measure of risk, implying that investors seem to attach a higher weight to possible losses than the weight suggestewd by MV analysis. This finding, although sensitive to the data set, is important because the simple MG allocation is similar to the MV allocation.

In addition, the composition of asset classes in the various MEG efficient sets is quite different. In the case of $\nu = 2$, the class of Commercial Banks account for 72% of the portfolio and CPI Linked Bonds for 17%. However for $\nu = 2.5$, the portfolio is composed of 38% Commercial Banks and 27% CPI Linked Bonds. This solution is much closer to the market position of 34% Commercial Banks and 20.5% CPI Linked Bonds. Thus, the index of $\nu = 2.5$ not only provides us with the smallest distance but also with a better fit of the securities distribution. We should then expect the CAPM calculated on a basis of $\nu = 2.5$ to perform better than the CAPM calculated on any other ν. However, further empirical evidence will be needed to establish whether the estimated risk parameter typifies the average Israeli investor in the stock market. Again, a word of caution is necessary since the results obtained in the present study can be sensitive to the period and the sample chosen.

5. Conclusion

In this paper, we have derived the mean-Gini and mean extended Gini efficient sets of risky prospects and compared the results with those obtained from mean-variance analysis. Contrary to the Bey and Howe (1984) approach, who calculated MG and MV efficient sets for prespecified portoflios, our results were obtained via an optimization algorithm for given expected rates of return. Hence, the approach makes a relevant and different comparison involving <u>optimal</u> portfolios given a set of prosect returns.

The MG and MEG analysis is motivated primarily by the simplicity of computation needed in the optimization procedure. It also has the convenience of the MV analysis and provides the necessary conditions for stochastic dominance. Hence, its importance in portfolio selection whenever the mean variance analysis might fail. This is especially true whenever assets returns are not normally distributed or whenever their distribution is unknown.

With the proposed method, we are now able to evaluate risk for uncertain prospects, construct optimal efficient portfolios from prospective returns, and establish necessary conditions for stochastic dominance. Furthermore, by deriving and comparing the various mean-extended Gini efficient sets, we obtained the risk parameter most likely to be held by investors. For the data set analyzed, the estimated risk parameter revealed that mean-variance efficient portofolios sets, or MG efficient portoflio sets underestimates the risk aversion of most investors in this specific market. In this respect, we can state that for a value of $\nu = 2.5$ the Capital Asset Pricing valuation will perform better than the CAPM on any other ν given the set of returns on the Tel-Aviv Stock Exchange. Only for this value we have obtained a composition of optimal efficient sets that fits the position held by most investors. However, the question whether or not this result hold for other time periods of other groups of investors still remains open.

Appendix

The efficient set is found by solving problem (1). Unfortunately, the derivatives of $\Gamma(\nu)$ with respect to the portfolio weights, x, cannot be derived analytically. Hence, it is simpler to solve Problem (1) as the unconstrained optimization problem:

$$\underset{x_1,\ldots,x_N}{\text{Min}} \quad \Gamma(\nu) + \lambda_1(\sum_{i=1}^{N} x_i\bar{R}_i - \bar{R}_0)^2] + \lambda_2(\sum_{i=1}^{N} x_i - 1)^2 + \lambda_3(\sum_{i=1}^{N} x_i^*)^2$$

$$\text{where } x_i^* = \{ \begin{matrix} x_i \text{ if } x_i > 0 \\ 0 \text{ if } x_i \leq 0. \end{matrix}$$

and the λ_i are penalty values that are found by trial and error. If these values are too low, the solution will not satisfy the constraints. If they are too high the solution will not be optimal since only the constraints will matter without considering to the objective function. The procedure used to carry out the minimization is the numerical optimization algorithm developed by Daks (1972) and its is based on the variable metric method of Fletcher (1970). Anyhow, it is worth mentioning that any algorithm which does not require an analytical derivation of derivatives can be used.

FOOTNOTES

[1] Search algorithms for constructing SD efficient portfolios are nonexistent. The only other method able to yield SSD efficient portfolios seems to be the mean-semivariance approach, see Bey (1979). Furthermore, as Dybvig and Ross (1982) recently showed, the SSD efficient set is not necessarily convex, implying that a search algorithm to derive SD efficient sets will be difficult to construct.

[2] The various representations of the Gini are developed and presented in Dorfman (1979), Kendall and Stuart (1977) Shalit and Yitzhaki (1984).

[3] In the discrete case (with K observations), one uses the rank of portfolio realizations (R_p) and calculate the Gini for a portfolio as

$$\Gamma_p(\nu) = -\frac{\nu}{K} \sum_{i=1}^{K} R_{pi} \cdot (Z_{pi}-\bar{Z})$$

where $R_{pi} = \sum_{j=1}^{N} x_j^o R_{ij}$

and $Z_{pi} = [(K - \text{Rank}(R_{pi}))/K]^{\nu-1}$

$$Z = \frac{1}{K} \sum_{i=1}^{K} Z_{pi}$$

[4] This procedure is familiar for portfolios construction. The alternative method is to select first a set of portfolios and apply the various efficiency criteria to this common set of portfolios (Porter,

Wart, and Ferguson (1975) or Porter (1979). This method was, however, criticized by Frankfurter and Philips (1975).

[5] The algorithm used in the optimization is available from the authors upon request.

[6] It seems that the true risk implicit in these shares was <u>not</u> reflected in their returns during the period of the study since ten months after the study ended, commercial banks shares crashed.

[7] The market shares used in the analysis are those of the last periods observations, assuming that the investors had the same information we possessed.

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Table 1

Means of Monthly Real Rates of Return, Standard Deviations, Gini and Correlation Coefficients of the Securities Traded on the Tel-Aviv Stock Exchange (I-1977 to I-1983)

Index Number	Mean a/	S.D. a/	Γ(2) a/	Correlation Coefficients Between Classes									
				1	2	3	4	5	6	7	8	9	10
1	1.94	6.81	3.48	-									
2	1.95	14.38	8.07	.66									
3	1.23	15.71	8.53	.56	.82								
4	2.84	13.98	7.68	.58	.81	.79							
5	1.65	13.04	7.36	.53	.78	.75	.83						
6	3.29	20.85	10.48	.35	.55	.53	.61	.71					
7	3.52	16.40	9.11	.57	.72	.69	.75	.77	.52				
8	-0.02	2.87	1.60	.10	-.02	-.07	-.07	.13	.06	.01			
9	-0.18	3.99	2.15	.32	.02	-.07	.00	.04	.08	-.07	.29		
10	-0.18	5.48	2.84	.37	.22	.05	.12	.19	.12	.09	.46	.67	
11	1.37	10.49	5.45	.78	.75	.70	.75	.74	.50	.68	.09	.15	.32

Classes of

1. Commercial Banks
2. Mortgage Banks
3. Industrial Financial Institutions
4. Investment Firms
5. Trade and Services
6. Manufacturing
7. Real Estate Firms
8. Bonds linked to CPI
9. Bonds traded in foreign currency
10. Bonds linked to foreign currency
11. Bonds convertible into shares

a/ Values in percentages.

Table 2

The Mean Variance Efficient Set

Portfolio Return	Portfolio S.D.	Class Number										
		1	2	3	4	5	6	7	8	9	10	11
0.50	2.83	21.2	–	–	4.1	–	–	–	66.6	8.1	–	–
1.00	3.87	43.1	–	–	2.3	–	1.6	1.7	51.4	–	–	–
1.23	4.47	52.6	–	–	1.1	–	2.5	3.0	40.7	–	–	–
1.37	4.90	58.3	–	–	0.4	–	3.1	3.8	34.4	–	–	–
1.65	5.66	69.4	–	–	–	–	4.0	5.0	21.6	–	–	–
1.94	6.63	80.8	–	–	–	–	4.9	6.1	8.3	–	–	–
2.50	8.89	63.2	–	–	–	–	9.1	27.6	–	–	–	–
2.84	11.05	41.2	–	–	–	–	12.4	46.4	–	–	–	–
3.00	12.16	30.9	–	–	–	–	13.9	55.2	–	–	–	–
3.29	14.28	12.1	–	–	–	–	16.7	71.1	–	–	–	–
3.52	16.40	–	–	–	–	–	–	100.0	–	–	–	–

Classes of
1. Commercial Banks
2. Mortgage Banks
3. Industrial Financial Institutions
4. Investment Firms
5. Trade and Services
6. Manufacturing
7. Real Estate Firms
8. Bonds linked to CPI
9. Bonds traded in foreign currency
10. Bonds linked to foreign currency
11. Bonds convertible into shares

Table 3

The Mean-Gini Efficient Set ($\nu = 2$)

Portfolio Return	Portfolio's Gini	Class Number										
		1	2	3	4	5	6	7	8	9	10	11
0.50	2.0	2.2	1.5	1.2	6.5	8.2	0.7	2.6	77.1	–	–	–
1.00	2.1	53.1	–	–	–	–	–	–	34.8	11.8	0.2	–
1.23	2.4	63.1	–	–	0.2	–	–	0.5	33.9	2.3	–	–
1.37*	2.6	67.1	–	–	0.6	0.3	0.1	30.4	–	–	–	–
1.65	3.0	77.4	–	–	–	–	1.6	2.8	17.4	–	0.7	–
1.94	3.5	99.4	–	–	–	0.3	0.1	–	–	0.1	–	–
2.50	4.8	63.3	–	–	–	–	6.3	30.3	–	–	–	–
2.84	6.1	41.8	–	–	–	–	8.4	49.7	–	–	–	–
3.00	6.8	32.1	–	–	0.8	–	1.0	66.1	–	–	–	–
3.29	8.0	12.4	–	–	0.3	–	11.0	76.3	–	–	–	–
3.52	9.1	–	–	–	–	–	0.3	99.7	–	–	–	–

* Stochastic Dominant Efficient Set for $\overline{R}_p \geq 1.37$

Table 4

The Mean-Extended-Gini Efficient Set ($\nu = 1.3$)

Portfolio's Return	Portfolio's Γ (1.3)	Class Number										
		1	2	3	4	5	6	7	8	9	10	11
-0.02	0.6	0.1	-	0.9	0.3	0.5	0.2	0.3	68.6	21.3	7.5	0.3
1.23	1.0	62.5	0.1	-	0.4	-	0.3	0.2	36.2	0.2	-	-
1.37	1.1	69.7	0.6	-	0.4	0.3	0.4	0.7	0.8	26.6	0.4	-
1.65	1.3	85.7	-	-	-	-	-	0.2	0.6	13.3	0.1	-
1.94	1.4	99.7	-	-	0.2	-	-	-	-	-	-	-
2.50*	1.9	61.1	-	-	2.3	-	9.2	26.8	-	-	-	0.7
2.84	2.4	35.7	-	-	13.3	-	11.0	39.9	-	-	-	-
3.00	2.7	30.6	-	-	0.2	-	13.6	55.6	-	-	-	-
3.29	3.2	8.4	-	-	4.2	-	23.1	63.8	-	-	-	
3.52	3.7	-	-	-	-	-	-	100.0	-	-	-	-

* Stochastic Dominant Efficient Set for $\overline{R}_p \geq 2.50$.

Table 5

The Mean-Extended-Gini Efficient Set ($\nu = 6$)

Portfolio's Return	Portfolio's Γ (6.)	Class Number										
		1	2	3	4	5	6	7	8	9	10	11
0.50*	3.8	29.8	-	-	-	-	-	-	32.7	37.5	-	-
1.00	4.9	51.4	-	-	-	0.1	-	0.4	48.0	-	-	-
1.23	5.7	56.0	-	-	-	-	-	4.5	39.4	0.1	-	-
1.37	6.3	66.1	-	-	3.7	-	-	-	24.5	5.8	-	-
1.64	7.2	78.5	-	-	-	-	-	3.7	17.8	-	-	-
1.94	8.3	90.5	-	-	-	-	-	5.3	4.2	-	-	-
2.50	11.6	64.2	-	-	-	-	0.2	35.6	-	-	-	-
2.84	14.3	43.1	-	-	-	-	-	56.9	-	-	-	-
3.00	15.7	32.6	-	-	-	-	-	67.4	-	-	-	-
3.29	18.4	14.1	-	-	-	-	-	85.9	-	-	-	-
3.52	20.5	-	-	-	-	-	0.5	99.5	-	-	-	-

* Stochastic Dominant Efficient Set for $\overline{R}_p \geq 0.50$.

Table 6

Market Shares, Efficient Portfolios and Relative Distance

Extended Gini Parameter ν	Distance Measure $d(\nu)$	Portfolio Return $\bar{R}$	Class Number										
			1	2	3	4	5	6	7	8	9	10	11
Market		1.726	33.9	3.0	0.3	12.4	13.9	3.9	8.4	20.5	2.1	0.5	1.2
1.2	42.4	1.726	72.3	-	-	3.1	-	3.3	3.7	17.2	0.2	-	-
1.5	48.6	1.724	77.5	-	-	3.1	-	0.8	3.5	9.2	5.9	-	-
1.8	42.6	1.718	24.3	-	4.0	6.3	0.1	0.6	30.0	2.7	27.9	4.0	-
2.0	41.8	1.720	71.6	-	-	2.9	-	3.3	4.2	17.2	0.4	0.2	-
2.2	42.0	1.719	71.7	-	-	2.2	-	-	7.7	18.2	0.1	-	-
2.4	24.5	1.716	37.7	-	0.5	22.0	-	-	11.0	9.8	13.3	5.3	-
2.5	19.0	1.713	38.1	-	-	13.5	-	-	17.3	27.0	4.1	-	-
2.6	27.2	1.715	48.9	2.0	-	0.1	0.4	1.5	19.2	27.8	-	-	-
3.0	36.8	1.700	19.0	-	-	6.7	-	0.9	32.8	20.2	19.0	1.1	-
4.0	49.8	1.710	71.9	-	-	-	-	-	9.9	1.4	18.0	-	-
6.0	44.6	1.700	74.8	-	-	6.9	-	-	1.6	16.7	-	-	-
8.0	51.1	1.68	79.8	-	-	2.0	-	2.4	0.2	9.8	4.3	1.5	-
M.V	43.8	1.726	73.0	-	-	-	-	4.1	5.1	17.8	-	-	-

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