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המרכז למחקר בכלכלה חקלאית

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THE CENTER FOR AGRICULTURAL ECONOMIC RESEARCH

Working Paper No. 8303

BREAK-EVEN ANALYSIS UNDER INFLATION

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by

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Rehovoth, Israel, P.O.B. 12

רחובות, ת.ד. 12

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Break-even analysis, often called cost-volume-profit relationships, is considered as an important tool in many decisions and planning activities. It involves the interrelationship between per unit price and variable costs, fixed costs and volume of activity, and is aimed to check the sensitivity of sales and profit stemming from changes in the relevant parameters (product prices, input costs and interest rate).[1]

Conventional break-even analysis, however, does not take the time element into consideration.[2] Thus the cost of time (interest rate) and changes in price level do not affect the break-even point. These deficiencies do not affect the results significantly as long as the interest and inflation rates are low. In time of inflation, the nominal interest rate and the price level are functions of time. Therefore, the time element should be taken into consideration in break-even analysis, especially when imperfections in the money and goods markets exist. This paper extends the break-even analysis to include the time element.

Once the time element is taken into consideration, a company may look for the minimum level of production in a given period (quarter, year) that covers the fixed costs, or the minimum time required for covering these costs. Accordingly, this paper covers the two cases (Model I and Model II, below).

I. Notations

T = The whole period under analysis (say 12 months).

t = Time in periods.

F = Fixed costs for the whole period T, at time $t = 0$.

$Q(t)$ = The total number of units produced up to time t.

Q^* = The volume at which break-even occurs: $Q^*_I = Q(T)$; $Q^*_{II} = Q(t^*)$.

t^* = The number of periods at which break-even occurs, under model II.

D = Average rate of production per period = capacity.

D^* = Required average rate of production per period needed to break-even in period T (Model I).

S = Selling price per unit at time $t = 0$.

V = Variable costs per unit at time $t = 0$.

M = S-V = Contribution margin per unit at time $t = 0$.

p = Average inflation rate per period (say, per month).

a = Rate of increase in sales price, per period (say, per month).

b = Rate of increase in variable costs per period.

$S(t) = S\,e^{at}$ sales price at time t.

$v(t) = V\,e^{bt}$ variable costs per unit at time t.

r' = Pre-inflation rate of interest, per period (say per month).

r = Nominal rate of interest per period, corresponding to inflation rate p.

II. Break-Even Relationships

The basic relationship in conventional break-even analysis is

(1) $$SQ^* - VQ^* = F$$

This yields

(2) $$Q^* = \frac{F}{S-V} = \frac{F}{M}$$

and the average break-even rate of production per period is

(3) $$D^* = \frac{Q^*}{T}$$

Relation (1) is applicable when prices and costs are stable, and the rate of interest is zero. When these parameters (prices, costs, interest) are functions of time, the following two models are considered.

Model I: This model is set to find the quantity Q_I^* produced in a given time T, at which break-even occurs.[3] Stated differently, it is required to find the minimum break-even average production (D^*) in one period, to cover the fixed costs, such that:

$$Q_I^* = D^* \cdot T \tag{4}$$

Model II: This model is set to find the quantity Q_{II}^* in a period t^* in T at which break-even occurs.[4] In this case, we assume a fixed average production per period (D). It is required to find the shortest time; that is, the break-even period t^* in T, to cover the annual fixed costs, such that

$$Q_{II}^* = D \cdot t^* \tag{5}$$

In the case of stable prices and zero interest rate, the two models are equivalent, that is, $Q_I^* = Q_{II}^*$.

In order to simplify the computations and to arrive at operative formulas, the analysis in this paper is based on the following three assumptions: (1) constant returns to scale in the relevant range of production, (2) the company cannot affect the prices in the market, and (3) the interest rate and the rates of price level changes are equal among periods.

In the following analysis the fixed costs are stated in terms of the price level at the beginning of the period. In other words, the firm has to calculate the present value of the fixed costs stream, using the proper interest rate.

III. Model I, Stable Prices

When the interest rate is positive, each unit sold and input purchased earns compound interest from date t to end of period of analysis (T).

The break-even relationship is then

(6) $$\int_o^T S\, e^{r'(T-t)}\, dQ - \int_o^T V\, e^{r'(T-t)} dQ = F\, e^{r'T}$$

where $dQ = D^*dt$, as $Q(t) = D^*t$.

Integrating relation (6) and collecting terms, we have

(7) $$Q_I^* = \frac{F}{M} \quad \frac{T}{a_{\overline{T}|r'}}$$

and

(8) $$D^* = \frac{F}{M} \quad \frac{1}{a_{\overline{T}|r'}}$$

where

$$a_{\overline{T}|r'} = \frac{1-e^{-r'T}}{r'} \quad \text{in the continuous case.}$$

$$= \frac{1-(1+r')^{-T}}{r'} \quad \text{in the discrete case.}$$

= the present value of a (T) period annuity, earning

r' per period.

Note that the term $\frac{T}{a_{\overline{T}|r'}}$ is always greater than one, so that when interest compounding is taken into consideration, the break-even volume is larger than that arrived at by conventional analysis $(\frac{F}{M})$. The order of magnitude of this effect can be seen from the factors of $\frac{T}{a_{\overline{T}|r'}}$ for various levels of time and monthly rates of interest, for the descrete case:

Interest rate:	0.5 %	1.0 %	2.0 %
3 months	1.010	1.020	1.040
6 months	1.018	1.035	1.071
12 months	1.033	1.066	1.135

IV. Model I, Inflationary Conditions

Under inflationary conditions, prices and costs rise continuously, so that these terms are functions of time. The rate of interest also rises to compensate the inflation effect. Because of the decline in the money value, it is assumed that the fixed costs should retain their pre-inflation value, that is, their value should rise according to the Fisher rule.[5] Thus, the corresponding compounding rate for the fixed costs is

$r' + p$ in the continuous case

$(1+r')(1+p)-1$ in the discrete case.

Under the above assumptions, relation (6) becomes:

$$(9) \qquad \int_o^T S\ e^{at+r(T-t)}\ dQ - \int_o^T V\ e^{bt+r(T-t)}\ dQ = F\ e^{(p+r')T}$$

Integrating relation (9) and collecting terms, we have

$$(10) \qquad Q^*_I = \frac{T\ F\ e^{(p+r')T}}{e^{rT}[S\ a_{\overline{T|}(r-a)} - V\ a_{\overline{T|}(r-b)}]}$$

In the discrete case, instead of using the rates (r-a) and (r-b), use $\frac{1+r}{1+a} - 1$ and $\frac{1+r}{1+b} - 1$, respectively. In the case that (r-a) or (r-b) are negative, note that $a_{\overline{T|}-r} = S_{\overline{T|}r}$[6]

where $S_{\overline{T|}r} = \dfrac{e^{rT}-1}{r}$ in the continuous case.

$= \dfrac{(1+r)^T-1}{r}$ in the discrete case.

= the future value of annuity.

In a special case, when the nominal interest rate follows the Fisher rule and the rates of increase in prices and costs are the same as the inflation rate, that is,

$$r = r' + p$$

$$a = b = p$$

then relation (10) reduces to

$$Q_I^* = \frac{F}{M} \cdot \frac{T}{a_{\overline{T}|r'}} \tag{11}$$

which is the same as relation (7), developed under stable prices.

V. Model II

The relations for Model II are the same as those in Model I, except that the integration ranges are from 0 to t^*, $Q = Dt^*$ and $dQ = D\,dt$. In this case, it is required to determine the break-even period t^* .

For example, relation (6) becomes:

$$D\left[\int_0^{t^*} S\, e^{r'(t^*-t)} dt - \int_0^{t^*} V\, e^{r'(t^*-t)} dt\right] = F\, e^{r't^*} \tag{12}$$

which reduces to

$$D = \frac{F}{M} \frac{1}{a_{\overline{t^*}|r'}} \tag{13}$$

and

$$a_{\overline{t^*}|r'} = \frac{F}{M\ D} \tag{14}$$

Given F, M, D and r' , we search the tables of the factors for present value of annuities for the corresponding t^* that satisfies relation (14).

As a first approximation to t^*, we can use

$$\hat{t} = \frac{Q_I^*}{D} = \frac{D^*}{D} \cdot T, \quad \text{thus } \hat{t} \text{ is proportional to } D^*. \tag{15}$$

Note that D^* is a decreasing convex function of t, as shown in the figure. Thus the approximate solution $\hat{t}$ is usually overstated. The fact that $\hat{t}$

is overstated leads to the conclusion that $Q^*_{II} \leq Q^*_{I}$.

[figure here]

VI. Example

Consider the following data: the fixed costs, F, in present value terms, are \$120; S=\$7, V=\$5, D=7 units, T=12 months, and r' = 0.01 per month in discrete terms. The break-even point under conventional analysis (relation 2) is 60 units. Taking into consideration the element of interest costs, the break-even point (Model I, relation 7) is 64 units, which is larger than the former break-even point by 6.7%.

The break-even time needed to cover the fixed costs, according to model II is found to be 9 months. This is found by using relation (14) as follows:

$$\frac{F}{MD} = \frac{120}{2x7} = 8.57 = a_{\overline{t^*|}0.01}$$

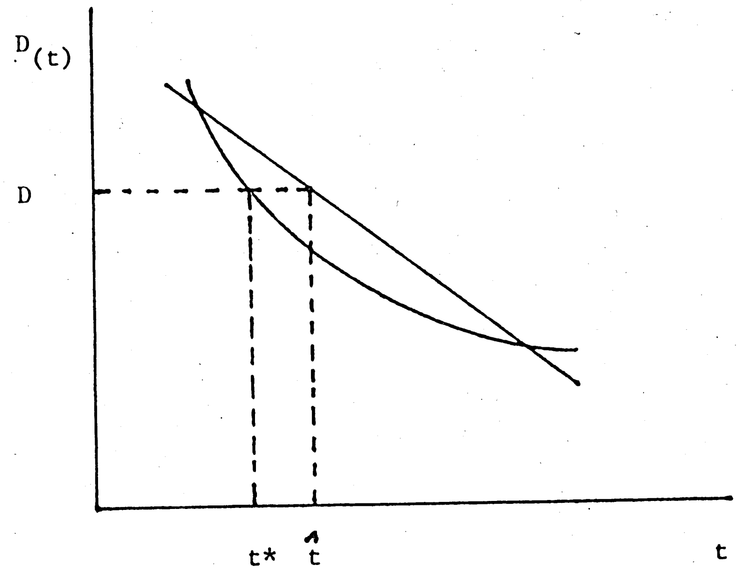
Searching the table of present values of annuities corresponding to 1% interest rate we find the entry of 8.57 under 9 periods. The approximated time, $\hat{t}$ (relation 15) is 64 ÷ 7 = 9.14 months, which is higher than the true value, as inferred from the figure. The corresponding break-even point in volume terms (Model II, relation 5) is 63 units (9 months time 7 units capacity) which is lower than that arrived at under Model I.

Consider now inflationary conditions, where the following monthly discrete rates are anticipated: p = 0.02, a = 0.018, b = 0.021, and r = 0.03. The break-even point, in volume terms (Model I, relation 10) is found to be 68 units.

VII Conclusions

This paper shows that the conventional break-even analysis is valid in time of inflation as long as inflation is neutral; that is, when all the parameters in the discussed models increase at the same rate. This is because discounting uses the real rate of interest which is relatively low and discounting does not change the magnitude of the break-even result. However, facts of life indicate that the inflation effect in most cases is not neutral, affecting differently product prices, variable costs, fixed costs and the nominal interest rate. In this case the use of the two models is of special importance to reach correct conclusions regarding the annual break-even volume (Model I) and the minimum time to cover fixed costs (Model II). The illustrated examples indicate that the break-even point will be higher during inflation. However, changing the parameters - for instance, lower price level changes of fixed assets (and/or variable costs) relative to the change in the nominal interest rate (and/or the price of the product) - may reverse the above conclusion.

Under nonneutral inflation there is a place to use both models, each answering a different question. The general conclusion regarding the break-even point is that the required volume under Model II is smaller than that under Model I.



$D_{(t)}$
D
t*
$\hat{t}$
t

Footnotes

1. Break-even analysis (BE) is discussed in many texts in the field of cost and managerial accounting, finance, marketing etc. For a comprehensive discussion see, e.g. Garrison (1979), Horngren (1982) and Kaplan (1982); for BE under uncertainty, see, e.g., Kaplan (1982) and Beirman and Dyckman (1976); for BE and sensitivity analysis see, e.g. Horngren (1982) and Shashua and Goldschmidt (1983); for BE and target pricing see, e.g., Kotler (1978); for BE and dividend determination see, e.g., Shillinglaw (1977).

2. All of the texts mentioned in note 1 carry out break-even analysis of a period ignoring discounting. However, in a multi-period analysis discounting is taken into account between periods disregarding discounting within the period (e.g., Brealy and Myers, 1981, and Kaplan, 1982).

3. The texts mentioned in note 1 analyze the break-even according to this model.

4. A similar model was used by Dhavale and Wilson (1980).

5. Ignoring the inflation effect (or the discounting factor) on fixed costs, causes $Fe^{r,T}$ in Relation (6) to become F, and Relation (7) will take the form of $\frac{F}{M} \cdot \frac{T}{S_{\overline{T}|r'}}$

where $S_{\overline{T}|r'} = \frac{e^{rt} - 1}{r}$

Noting that $\frac{T}{S_{\overline{T}|r'}} < 1$

the break-even volume under discounting (or under neutral inflation) is smaller than the conventional one. This solution contradicts common sense. Let us mention that Dhavale and Wilson (1980) disregarded discounting the fixed costs.

6. In the discrete case

$$a_{\overline{T}|-r} = (1+J)s_{\overline{T}|J}$$

where $J = \frac{r}{1-r}$ or $1+J = \frac{1}{1-r}$

for example,

$$a_{\overline{T}|-0.05} = \frac{1}{0.95}\, s_{\overline{T}|\frac{0.05}{0.95}}$$

where $J = \frac{r}{1-r} = \frac{0.05}{1-0.05} = \frac{0.05}{0.95}$

and $1 + J = \frac{1}{1-r} = \frac{1}{1-0.05} = \frac{1}{0.95}$

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