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Abstract

The interaction of selfish, rational actors in the provision of a public good is analyzed, in an attempt to explain recent experimental findings as well as real-world cases of voluntary collective action. The model, which allows for more sophisticated behavior than that assumed by the conventional Cournot theory, predicts a higher level of collective action that that theory. The impacts of income effects and of the price elasticity of demand for the public good are given special emphasis.

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1. Introduction

The theory of voluntary collective action is only beginning to be explored by economic theorists, despite the accumulation of extensive evidence from experimental studies (e.g. Marwell and Ames, 1979, Schneider and Pommerehne, 1981) that individuals do make voluntary contributions to the provision of public goods, even in large groups. This evidence is supplemented by the everyday occurence of people voting, non-commercial radio stations being funded, and so forth.

The conventional, Cournot theory can explain voluntary collective action only at a suboptimal level.[1] According to this theory, actors take each others' contributions as given and choose their contributions so as to maximize their utility. For normal (i.e., non-luxury) goods, the theory predicts that one actor's contribution will decrease in response to an increase in the contribution of another actor. Income effects dampen this decrease, but normally do not eliminate it altogether.

The Cournot assumption, it is generally recognized, makes sense only in a one-period game, in which actors have no opportunity to learn the response of others actors to their contributions. If such learning could occur, it is commonly thought that contributions would considerably diminish, since a decrease in other actors' contributions makes it more costly for the actor in question to purchase a given increase in the quantity of the public good (Becker, 1974).

But the higher level of rationality involved in actors' anticipating each other's reactions can easily lead in the opposite direction - to a <u>higher</u> level of provision of public goods. An actor who knows that his reaction to increases in the contributions of other actors is correctly anticipated by those actors will make his reaction itself a decision variable. Just as a negative reaction -

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[1]For expositions of this theory, see Buchanan (1967), Olson and Zeckhauser (1966), Chamberlin (1974), McGuire (1974) and Guttman (1976).

as the Cournot model predicts - raises the "effective price" of the public good to other actors and thus decreases their optimal contributions, a positive reaction will lower that effective price and lead to an increase in contributions. The individual actor, if he is truly rational, will take account of this fact and may therefore find it optimal to respond positively - i.e., match increases in other actors' contributions, in order to encourage them to contribute.

This paper formalizes this idea and explores its application to a world of positive income effects. The model developed here is a generalization of the model of "matching behavior" described in Guttman (1978, 1982) for the case of zero income effects. The complexity of the general case with income effects, however, forces us to restrict attention to the case of identical actors. It is shown that income effects tend to make matching behavior less profitable and thus reduce the quantity of the public good. In contrast, the Cournot model implies that the stronger the (positive) income effects, the larger will be the quantity of the public good.

In developing the model, we employ the following simplified framework. In a Nash noncooperative one-period game, actors choose their optimal "matching rates". These matching rates determine the effective prices of the public good to the other actors, because these rates are undertakings to match other actors' "flat" contributions (which are as yet undetermined). There follows another one-period game, in which the "flat" (unconditional) contributions are made in the manner of Cournot, but with the reactions of the other actors (the matching rates) taken into account. The amount of the public good is then

$$x = \sum_i a_i \left(1 + \sum_{j \neq i} b_j\right) , \tag{1}$$

where the a_i are the flat contributions and the b_j are the matching rates of the various actors.

This particular way of modeling the process of collective action was chosen because of its relative simplicity and because of the shortcomings of alternative modes of analysis. The alternatives include (a) a supergame model, in which the "repeat business" idea (in the background of our model) is formalized, (b) a model of sequential commitments, such as that of Thompson and Faith (1981), (c) a model of simultaneous choice of matching rates and flat contributions. The difficulties involved with these alternative approaches will be discussed after developing the model.[2]

2. The Model

a. Asymmetrical Action

In order to develop an intuitive understanding of the model, it is helpful to begin with a set-up slightly different from that which was outlined in the introduction, in which two agents act sequentially rather than simultaneously. In Figure 1, RF_1 and RF_2 depict reaction functions for two actors in a setting in which each can contribute as much as he likes to the provision of a public good. The public good is perfectly divisible in production, and produced at a constant marginal cost (here normalized to equal unity). These reaction functions, which are familiar from the literature in the Cournot model (see, e.g., Olson and Zeckhauser 1966), indicate that each actor's contribution x^i is a decreasing function of the other actor's contribution, but that (assuming a positive income effect and that the public good is not a highly "superior" good) each actor's contribution does not decrease unit-for-unit with an increase in the other actor's contribution. This is because an increase in the other actors' contributions increases actor i's wealth and thus increases his demand for the public good, so that it is not optimal to reduce his contribution unit-for-unit. Becker (1974) has shown that the rate of decrease is

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[2]It should be pointed out that the problem analyzed here is not the "revelation problem" discussed by a large and growing literature. The revelation problem assumes the existence of a taxing agent (government), and seeks to develop mechanisms whereby individuals will honestly reveal their preferences for the public good to that government. Here, no government is assumed, so that the problem is voluntary collective action, not demand revelation.

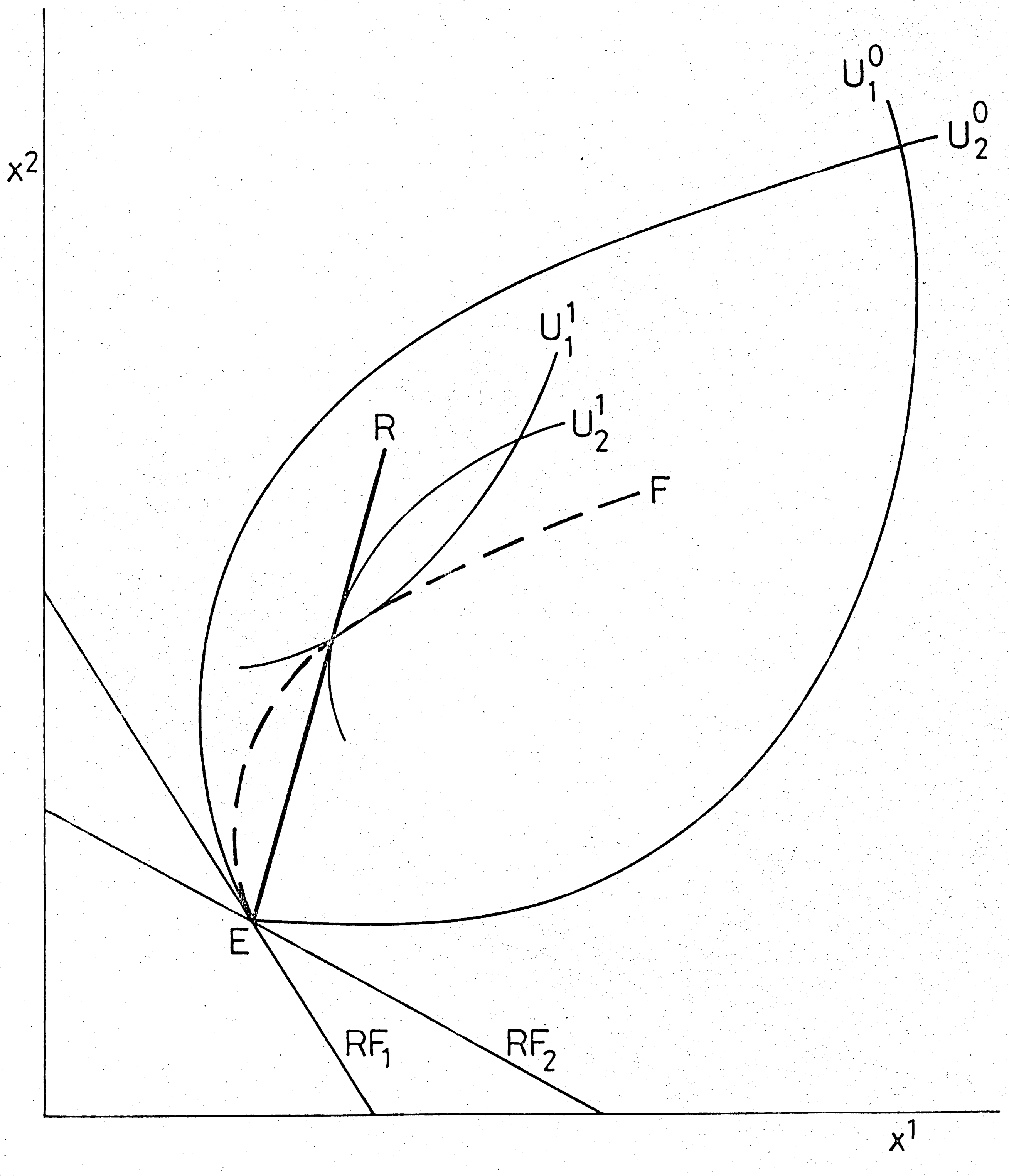

Figure 1 Matching Behavior: Asymmetrical Action

$$\frac{dx^1}{dx^2} = -1 + s_x \eta_x$$

where s_x is the share of the public good in actor 1's total consumption basket and η_x is his income elasticity for the public good. This equation implies that RF_1 and RF_2 will be strictly linear only if $\eta_x = 1$. No generality is lost, however, by assuming this to be the case.

The equilibrium under Cournot behavior is E. At this equilibrium, each actor's contribution is optimal, given the contribution of his fellow actor. Therefore, any further contribution beyond E - given the other actor's contribution - is a "bad" for that actor. Thus U_1^0 and U_2^0 depict indifference curves for the two actors where the two "commodities" are the actors' contributions. The area bounded by the two curves is a Pareto-preferred region relative to E, but is unattainable under the Cournot behavioral assumption.[3]

Now suppose that actor 1 proposes - either explicitly or via his behavior - that if actor 2 increases his contribution by any given amount, he (actor 1) will respond with an increase in his own contribution. Such a proposal can be represented by a "reaction line" such as ER. If actor 2 takes this offer as given,[4] then he will maximize his utility by finding a tangency between ER and his highest attainable indifference curve, U_2^1. Note that this involves actor 2 increasing his contribution substantially - a result of the lowered effective price of the public good that he faces.

The broken curve EF represents an "offer curve" of similar points of tangency to different matching lines (such as ER). This curve is thus the set of attainable equilibrium points facing actor 1 in his choice of an optimal matching rate. Actor 1 chooses the point on EF which is on his highest attainable indifference curve - in this case, U_1^1. Thus actor 1's optimal matching rate is that represented by ER.

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[3] The idea for this diagram is drawn from Markusen (1976).

[4] The questionability of this assumption is discussed below.

While the resulting equilibrium is distinctly Pareto-preferred to the Cournot equilibrium E, it is not Pareto-optimal. A Pareto-preferred region remains, bounded by U_1^1 and U_2^1. Presumably, the same process would now repeat itself. One actor would make a matching offer to his fellow actor, and a further movement towards Pareto-optimality would result. The process would repeat itself again until Pareto-optimality was approached, in the limit.

One objection which one can make against such a model is, why should actor 1 fulfill his matching offer? Having induced actor 2 to increase his contribution, it would seem optimal for him to "renege" and not to match actor 2's contribution. The answer is that we have in mind a situation which is repeated indefinitely over time - i.e., with no known end-point, or with no end-point at all. The literature on repeated games or "supergames" implies that, with "repeat business", it will not be optimal for actor 1 to renege, for the simple reason that he will not be believed the next time he tries to make an offer. Since making a believable matching offer yields permanent, long-run benefits and reneging yields only a one-shot benefit, the former will, in general, be preferable. We accept this argument without formalizing the model as a supergame.[5]

A second difficulty with the above model is more serious. Why should actor 2 take actor 1's matching offer as given? It would seem to be better for him to counter with a matching offer of his own, since the resulting equilibrium, as shown in Figure 1, involves a distribution of the "gains from interaction" favoring the actor making the matching offer. The model of asymmetrical action must acknowledge this difficulty and postulate that the priority of action is predetermined in an earlier game or by chance, or determined randomly, or en-

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[5]Supergame models typically produce an infinite number of equilibria. By restricting the actors' strategy sets and imposing a two step structure, we reduce the equilibria to one outcome.

forced by an outside enforcer. Since none of these options is intuitively attractive, we are led to reformulate the model in a game of symmetrical action. In such a game, the matching offers would be made simultaneously. We can thus make the traditional Cournot-Nash assumption that each actor takes the offer of his fellow as given in determining his own matching offer.

b. Symmetrical Action

We now reformulate the model as a process in which the agents simultaneously choose matching rates. After these rates are chosen, unconditional (flat) contributions are determined; these flat contributions correspond to the behavior of actor 2 in the first stage of the above-described asymmetrical model. Both games take the form of Nash non-cooperative games. The sum of the final contributions is a weighted sum of the flat contributions, the weight for flat contribution a_i being one plus the sum of the matching rates facing actor i, as indicated in equation (1) in the introduction to this paper.

The assumption here is that the matching offers are linear, similar to the matching line ER in Figure 1. This linearity assumption is made in order to keep the analysis manageable. Not only would the analysis be virtually impossible without such a simplification, it seems implausible to assume that real-world actors consider all possible functional forms, since their computational task would then be extremely large.[6] To minimize these computation costs, an actor would choose the function with the minimum number of parameters which would still allow him to exhibit matching behavior - i.e., a linear function.

We begin with a derivation of the effect of one actor's matching rate on the total amount of the public good and on the flat contributions of the other actors.

[6]Tideman has calculated in connection with the standard Prisoner's Dilemma repeated 8 times, that there are 10^{77} possible strategies for each player, and that "a computer that size of the earth, composed of micro-processors one Angstrom (10^{-10} meter) on a side, each capable of evaluating a strategy in a nanosecond (10^{-9} second),...would take about 200 million years" to evaluate all the strategies.

Each actor will make a flat contribution only to the extent that the other actors' flat contributions are insufficient to "purchase" the individually optimal amount of the public good, i.e., the amount that equates the actor's <u>private</u> marginal benefit of the good to its price. The flat contributions a_i are chosen so as to equate the actual amount of the public good to the amount that is individually optimal for actor i, in a manner analogous to the choice by actor 2 of an optimal contribution in the asymmetrical game described above. Denote x_i^* as the individually optimal quantity of the public good for actor i, and r_i as the sum of the matching rates b_j <u>facing actor i</u>. (Note that the subscript i in x_i^* does not indicate that x_i^* is an individual contribution x^i, but rather an individually optimal <u>sum</u> of the contributions x^i.) Then we can rewrite (1), and add the equilibrium condition $x = x_i^*$, to obtain

$$x \equiv \sum_i a_i \; (1+r_i) = x_i^*$$

For identical actors, we can ignore the subscript i and write

$$x = (1+r) \sum_i a_i = x^*$$

Each actor will adjust his flat contribution so that the <u>sum</u> of the a_i will equal an optimal value, i.e.,

$$\sum_i a_i = a^* = x^*/(1+r) \tag{2}$$

The individually optimal quantity of the public good x* is determined quite simply. Let $U(x, y^i)$ be each actor's utility function, where x is the public good and y^i is actor i's private good consumption. Assume, for simplicity, that both are produced at constant marginal cost and that these marginal costs are equal. Then, for optimality

$$\frac{U_x}{U_y} = \frac{1}{1+r}$$

where U_x and U_y are the marginal utilities of x and y. This equation results from the fact that a dollar spent on the public good purchases (1+r) dollars' worth of that good, once the reactions of the other actors are taken into account.

Thus 1/(1+r) becomes the effective price of the public good to each actor.

Figure 2 depicts the effect of an increase in actor 1's matching rate on actor 2's demand for the public good, in a situation where there are only two actors, so that $b_1=r_2$. As b_1 rises, so does actor 2's demand for the public good.

Let the individual actor's uncompensated price elasticity of demand for the public good be ε. Since $1/(1+r)$ is effectively the price of the public good,

$$\varepsilon \equiv - \frac{dx^*}{d(\frac{1}{1+r})} \frac{1}{(1+r)} \frac{1}{x^*} \tag{3}$$

Moreover, actor i takes the other actors' matching rates b_k as given in determining his own matching rate, i.e., $db_k = 0$, and thus we have (subjectively to actor i, at least) $dr_j = \sum_{k \neq j} db_k + db_i = db_i$, so that

$$\frac{d \frac{1}{1+r_j}}{db_i} = - \frac{1}{(1+r_j)^2} . \tag{4}$$

Solving (3) for $dx^*/d[1/(1+r)]$, combining with (4), and assuming that $x=x_j^*$ for all actors j so that r_j equals a common value r for all $j \neq i$, we obtain

$$\frac{dx^*}{db_i} = \frac{\varepsilon x^*}{1+r} . \tag{5}$$

Equation (5) specifies the effect of one actor's matching rate b_i on the individual demands for the public good of all other actors j, assuming these demands to be equal to each other. To obtain the effect of b_i on the sum of the flat contributions a*, we differentiate equation (2) with respect to r, and note, as above, that subjectively to actor i, $dr_j=db_i$. We thus obtain (using (5)),

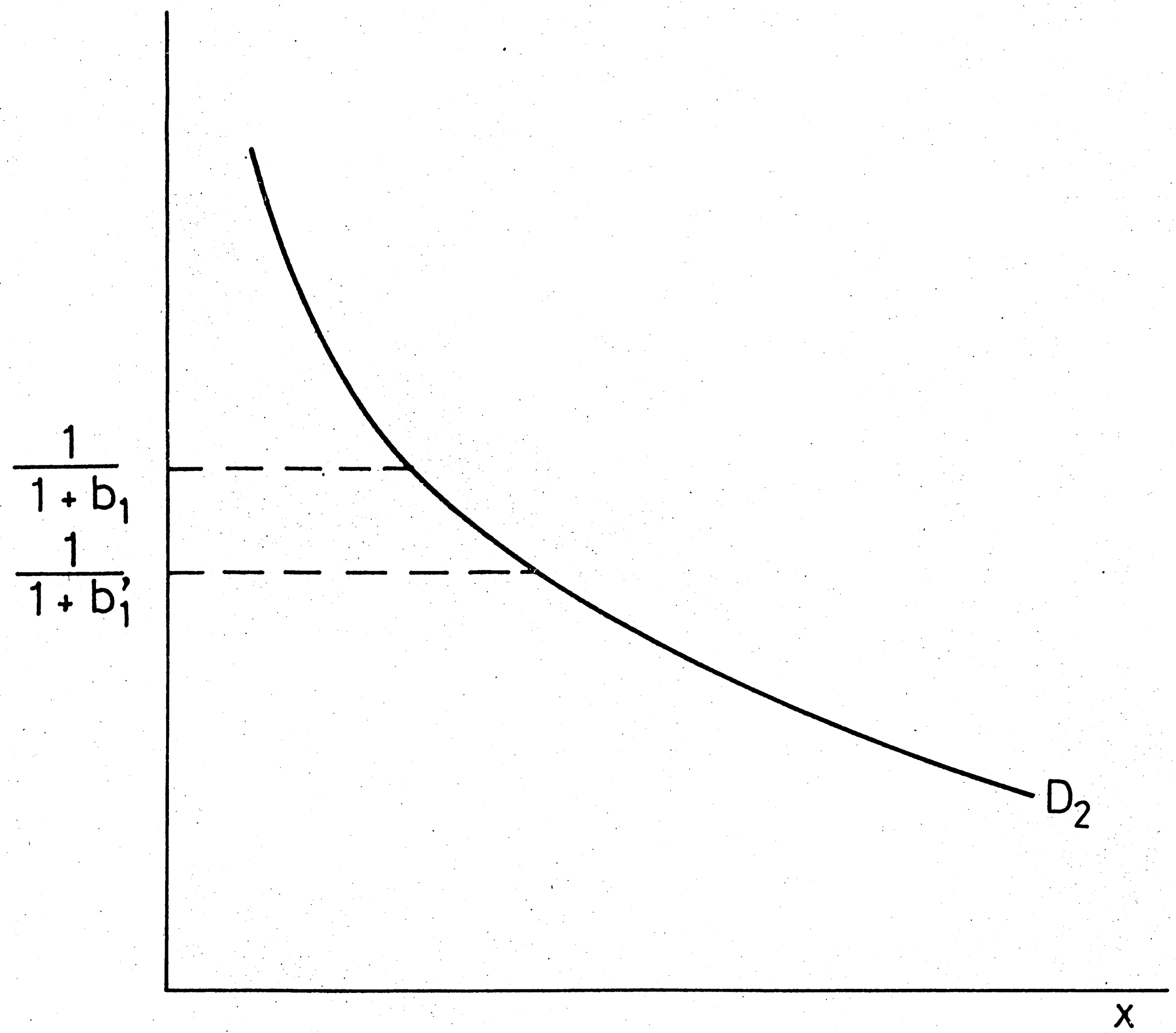

Figure 2 Effect of Increase in Matching Rate on Other Actor's Quantity Demanded

(6) $$\frac{da^*}{db_j} = \frac{a^*(\varepsilon-1)}{1+r}$$ [7]

We now specify the equilibrium sum of the flat contributions, for identical actors, as a function of the number of actors n, among other variables. We wish to take account of the effect of a deviation of b_i from equality with the other (n-1) actors' matching rates, in order to examine the effect of b_i. We thus assume, by symmetry, that the other (n-1) identical actors choose identical matching rates, and have the same optimal x* and a*, different from those of actor i to the extent that b_i differs from the (common) matching rate of the other actors. We thus have a pair of reaction functions in the game determining the flat contributions (the "a-game"):

(7)
$$a_i = a_i^* - \beta_i(n-1)a_j, \text{ and}$$
$$a_j = a_j^* - \beta_j(n-2)a_j - \beta_j a_i, \text{ for the other (n-1) actors,}$$

where $\beta_i = \beta_j$ are reaction coefficients in the a-game, i.e.,

$$\beta_i \equiv -\frac{da_i}{da_j} = 1 - s_x\eta,$$

following Becker (1974), as noted above in the discussion of the Cournot equilibrium.[8] Equations (7) state that each actor sets his flat contribution a_i equal to his individually optimal sum of the flat contributions if the other actors' a_j are zero, and reduces his a_i if the a_j increase.

[7]Differentiating (2) with respect to b_i, letting r_j=r, and $dr_j=db_i$, we obtain

$$\frac{da^*}{db_i} = \left[\frac{dx^*}{db_i}(1+r) - x^*\right]\frac{1}{(1+r)^2}$$

$$= \frac{\varepsilon x^* - x^*}{(1+r)^2} = \frac{x^*(\varepsilon-1)}{(1+r)^2} = \frac{a^*(\varepsilon-1)}{1+r}$$

[8]The β_i will be strictly constant for $\eta \neq 0$, only if $\eta=1$; otherwise s_x will vary. We here treat them as constants. Moreover, to the extent that b_i differs from the (common) matching rate of the other actors, the other actors j will optimally have different β_j with respect to a_j then with respect to a_i. If actors are unable to distinguish between other actors' flat contributions, or if n is large, these differences can be neglected. We ignore them here.

Figure 3 depicts the effect of an increase in b_1 on actor 2's reaction line in the a-game (for 2 actors), and on the resulting equilibrium, which shifts from E_1 to E_2. This shift can be derived analytically for the case of n actors by solving equations (7) simultaneously and multiplying the solution value for a_j by (n-1) to obtain

$$(8) \qquad (n-1)a_j = \frac{a_j^* - \beta_j a_i^*}{\frac{1}{n-1} + \beta_j \frac{n-2}{n-1} - \beta_i \beta_j} .^9$$

By combining (8) with (6), the effect of b_i on $(n-1)a_j$ can be found.

Using (5), (6), and (8), we can derive the equilibrium matching rate, b*. As indicated earlier, the actor's choice of an optimal matching rate is made with a knowledge of how his matching rate affects the equilibrium of the flat contributions in the upcoming a-game. By increasing his matching rate, actor i decreases the effective price of the public good to the other actors and thus increases their demands, resulting in an increase in the quantity of the public good. The rate of increase is given by (5). If the demand for the public good is price-elastic, then the flat contributions of the other actors a_j will also increase. This, in turn, will entail an increase in the matching component of actor i's own contribution, $b_i \Sigma a_j$, and thus a decrease in his consumption of the private good, y. (The increase in the a_j will permit a reduction in actor i's flat contribution a_i, which will offset the increase in $b_i \Sigma a_j$, though, in the region of his optimal matching rate, this offset will be incomplete.)

Formally, we have both the quantity of the public good x and i's private good consumption y^i being a function of b_i, so that, for optimality

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[9] With a large enough number of actors and rapid enough adjustment of the a_i, the equilibrium in the a-game will become unstable (see Frasca, 1980). But, if we assume slow adjustment of the a_j to their optimal values, this problem can be avoided (Fisher, 1961).

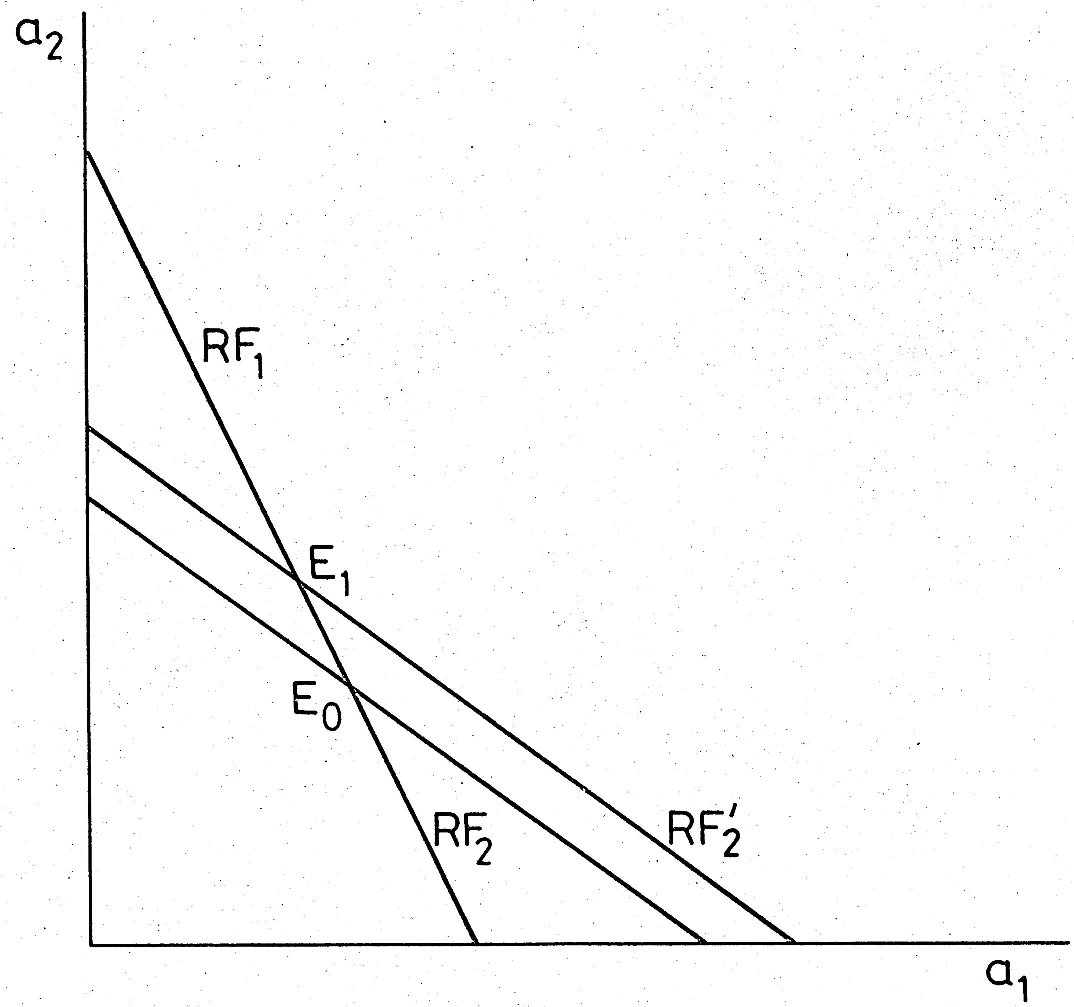

Figure 3 Effect of Increase in Matching Rate on Equilibrium Flat Contributions

(9) $$dU = U_x dx + U_y dy^i = U_x \frac{\partial x}{\partial b_i} db_i + U_y \frac{\partial y^i}{\partial b_i} db_i = 0$$

Moreover, the a-game is expected to be in equilibrium for all values of b_i that actor i chooses. Since all the actors are identical, all will be at an interior solution such that

(10) $$\frac{U_x}{U_y} = \frac{1}{1+r},$$

as indicated above. Thus we can use (10) to simplify (9):

$$dU = [\frac{\partial x}{\partial b_i} + (1+r) \frac{\partial y^i}{\partial b_i}] U_x db_i = 0,$$

or, using (5) and the initial equilibrium condition that x=x*,

(11) $$dU = [\frac{\varepsilon x}{1+r} + (1+r) \frac{\partial y^i}{\partial b_i}] U_x db_i = 0.$$

Since $x^i = a_i + b_i \Sigma a_j$, and since the price of y, like that of x, is unity, the actor's budget constraint is

$$I = x^i + y^i = a_i + b_i \sum_{j \neq i} a_j + y^i,$$

so that

(12) $$\frac{\partial y^i}{\partial b_i} = - \frac{\partial x^i}{\partial b_i} = - [\frac{\partial a_i}{\partial b_i} + \sum_{j \neq i} a_j + b_i \frac{\partial (\Sigma a_j)}{\partial b_i}].$$

Substituting (12) into (11) and dividing through by $U_x db_i (1+r)$, we obtain, for optimal b_i,

(13) $$\frac{\varepsilon x}{(1+r)^2} = \frac{\partial a_i}{\partial b_i} + \sum_{j \neq i} a_j + \frac{b_i \partial \Sigma a_j}{\partial b_i}$$

The left-hand side of (13) is the marginal benefit to actor i of increasing his matching rate b_i, valued in terms of the private good. The right-hand side is his marginal cost (in terms of sacrifice of the private good) of increasing b_i.

The terms on the right-hand side of (13) can be written more explicitly with the help of equations (5), (6) and (8). Regarding the term $\partial a_i/\partial b_i$, we have

$$\frac{\partial a_i}{\partial b_i} = \frac{\partial a_i}{\partial \Sigma a_j} \frac{\partial \Sigma a_j}{\partial a_j^*} \frac{\partial a_j^*}{\partial b_i} .$$

Using the definition of β_i, (6) and (8) - the latter holding for a symmetrical equilibrium only - we obtain

$$(14) \qquad \frac{\partial a_i}{\partial b_i} = -\beta_i\left(\frac{\varepsilon-1}{1+r}\right) a_j^* \frac{1}{\frac{1}{n-1} + \beta_j\frac{n-2}{n-1} - \beta_i\beta_j} .$$

Secondly, the equilibrium value of $\sum_{j\neq i} a_j = (n-1)a_j$ is given by equation (8). Finally, using (6) and (8),

$$(15) \qquad b_i \frac{\partial \Sigma a_j}{\partial b_i} = b_i\left(\frac{\varepsilon-1}{1+r}\right) a_j^* \frac{1}{\frac{1}{n-1} + \beta_j\frac{n-2}{n-1} - \beta_i\beta_j} .$$

Combining (8), (13), (14), and (15), we obtain

$$(16) \qquad \frac{\varepsilon x}{(1+r)^2} = \frac{a_j^* - \beta_j a_i^* + (b_i - \beta_i)\left(\frac{\varepsilon-1}{1+r}\right)a_j^*}{\frac{1}{n-1} + \beta_j\left(\frac{n-2}{n-1}\right) - \beta_i\beta_j}$$

Assuming that, in equilibrium, $a_i^* = a_j^*$, we can use (2) to divide through by $a^* = x^*/(1+r)$ (which equals $x/(1+r)$ in equilibrium) to obtain

$$(17) \qquad \frac{\varepsilon}{1+r} = \frac{1 - \beta_j + \frac{b_i - \beta_i}{1+r}(\varepsilon-1)}{\frac{1}{n-1} + \beta_j\left(\frac{n-2}{n-1}\right) - \beta_i\beta_j}$$

Assuming further that, in equilibrium, $b_i = b_j$, $r=(n-1)b$, and $\beta_i=\beta_j$, we can solve for equilibrium b to obtain

$$(18) \qquad b^*= \frac{\varepsilon(\frac{1}{n-1} + \beta(\frac{n-2}{n-1}) - \beta^2) - 1 + \varepsilon\beta}{(n-1)(1-\beta) + \varepsilon - 1}$$

Note that, as the income effect goes to zero, β approaches 1, and b* also approaches unity. Figure 4 shows the effect of changes in β on the equilibrium matching rate b*, with ε set to equal 1.5. An increase in β represents a decrease in the strength of the income effect - either the income elasticity of demand has decreased or the share of x in I has decreased. Such decreases in the strength of the income effect shift upward the curve relating b* to n. This effect results from the fact that as β increases, so does the size of the effect of b_i on the equilibrium in the a-game, depicted in Figure 3.

An increase in the price elasticity of demand ε increases the incentive to engage in matching behavior, because such behavior works through actor i's reducing the effective price of the public good to the other actors. Figure 5 shows the effect of increases in ε on the optimal matching rates, as ε varies from 1 to 3, with β=.9.

In all cases, the value of b leading to Pareto-optimality is unity. If all the b_i equalled unity, each actor would, by equation (10), equate U_x/U_y to 1/n, since r would equal (n-1) in this case. Thus the sum of the marginal rates of substitution for the whole group would be unity, which is, by our normalization, the ratio of the marginal costs of x and y. Thus, the gap between the matching rates b* and unity gives an index of the degree of suboptimality of provision of the public good. As Figures 4 and 5 indicate, we obtain the usual result of increasing suboptimality with group size, except for the case of a zero income effect, where b* = 1 for all n.

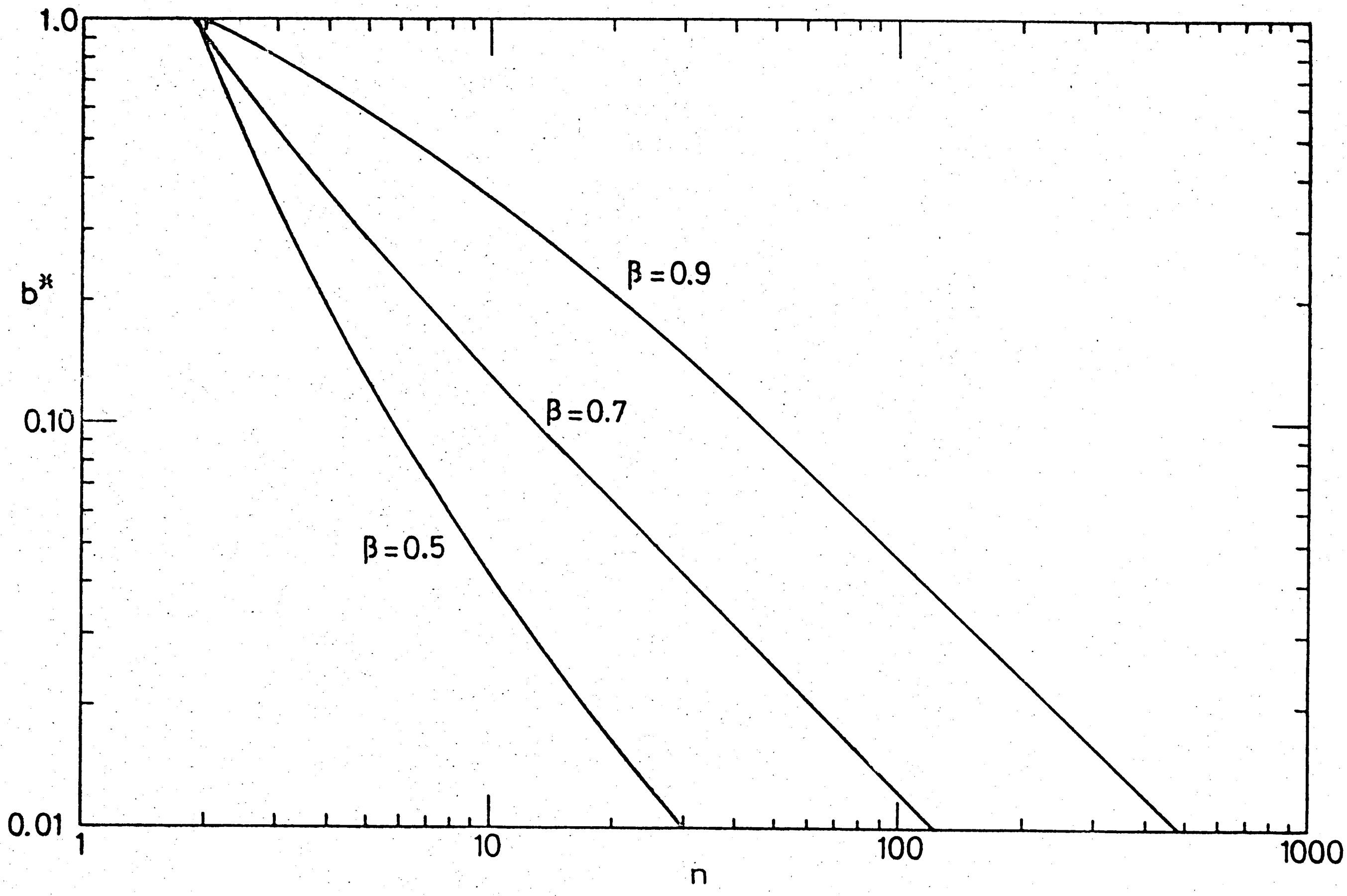

Figure 4 Optimal Matching Rates (b^*) as Function of Number of Actors (n) and a-Game Reaction Coefficient (β); ε=1.5.
(equation 18)

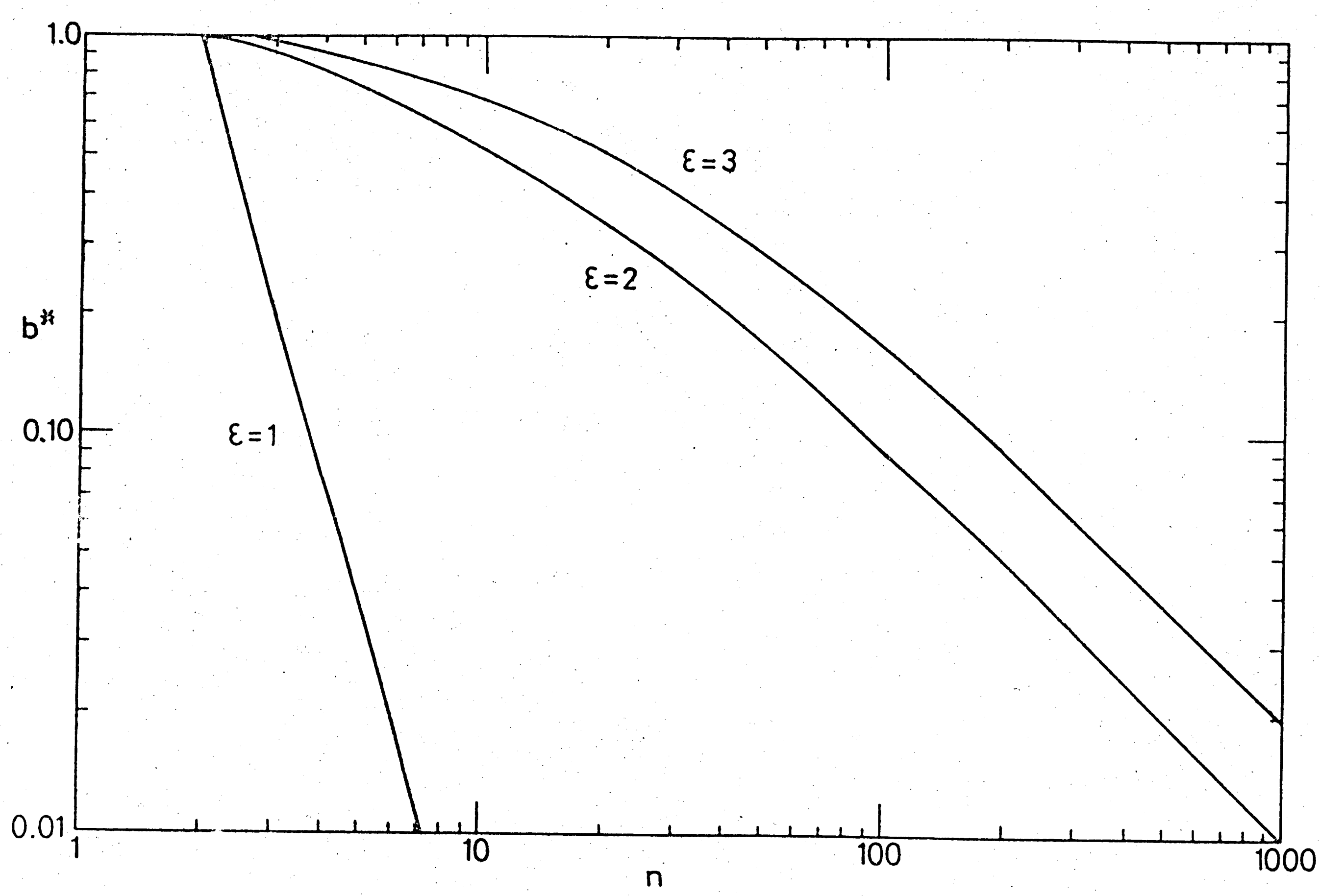

Figure 5 Optimal Matching Rate (b*) as Function of Number of Actors (n) and Price Elasticity of Demand (ε); β=0.9 (equation 18)

It should be borne in mind, however, that even if $\beta=.9$, the income effect is quite strong. Such a value for β implies that the share of x in income times the income elasticity of demand is 0.1. For the case of an income elasticity equal to unity, this implies that 10 percent of income is spent on the public good - seemingly a high percentage for any one good. If we assume that only one percent is spent on the good, then $\beta=.99$; letting $\varepsilon=1.5$ and $n=100$, we obtain $b^*=.33$ - still far short of optimality, but, for this group size, probably high in terms of conventional thinking.

Another interpretation of a value of β less than unity should be mentioned. Even if there is no income effect, an imperfection in the public good will cause β to fall short of unity. An imperfect public good is here defined as a public good whose benefits do not accrue equally to the contributor and to other individuals, but rather the benefits are concentrated, to some degree, in the hands of the contributor. An example would be a research and development program of an individual firm. The firm in question may value of a "unit" of its R&D program at one dollar, while other firms in the industry, who can only incompletely and after a time lag enjoy its benefits, may value the unit at only, say, 50 cents. Thus, the standard Cournot model would predict a reduction in the other firms' R&D programs (neglecting rivalry effects) by only 50 cents, not one dollar, as in the case of a pure public good without wealth effects. Thus, if were to apply the model to this imperfect public good, β would be 0.5. In this case, the Pareto-optimal value of the matching rates would also be less than unity: it would be close to 0.5 in our R&D example, for large groups. But, for large groups, a value of β of 0.5 would lead to matching rates much closer to zero than 0.5, as Figure 4 illustrates.

c. Comparison with Alternative Approaches

In the introduction, it was indicated that there are alternative approaches to modeling the theory of voluntary collective action. These approaches include:

1. The supergame approach. It has become customary to model a problem of the sort analyzed here as a supergame - i.e., a game repeated over many time periods. The difficulty with the supergame approach is that the generality of the typical supergame model - usually considered a virtue - leads to an inability to predict an outcome. Typically, such models produce an infinite number of equilibria, including both Pareto-optimal equilibria and the opposite extreme of a zero level of cooperation. This inability to predict an outcome is complemented by the implausibility of the computation task implied by the large number of strategies open to each actor, as indicated earlier (see footnote 6). If we restrict this strategy space so that the same response to others' actions is made time-period after time-period, given that others' actions do not change, and further restrict the response function to be linear, we obtain the present model as a special-case supergame model. The defense of the linearity assumption is, as indicated earlier, one of mathematical tractability, together with the plausibility of some such restriction of the strategy set.[10]

2. A model of sequential commitments. This approach, which has been developed by Thompson and Faith (1981), is essentially the "asymmetrical action" model described above in subsection (a). Its difficulty has already been indicated: the arbitrariness of any given sequence of action, without an outside enforcer. Hierarchies - established, for example, by wealth differentials - do exist in society, however.[11] The asymmetrical approach therefore can be viewed as complementary to the symmetrical action analyzed here.

3. Simultaneous choice of matching rates and flat contributions. This modeling approach was not taken for the simple reason that it would rule out the motive of choosing positive matching rates: to influence the flat contributions.

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[10]Levine (1981), however, has derived this linearity restriction in the framework of a more general model (of oligopoly).

[11]See Guttman (1980b) for an empirical analysis of "dependency structures" in Indian villages.

The flat contributions, moreover, cannot be chosen without the matching rates being known as data. Therefore the sequential priority assumed here is logically necessary.

3. Concluding Remarks

The model presented here provides an explanation of the phenomenon of voluntary collective action that is consistent with recent experimental evidence as well as real-world phenomena in the economy (oligopoly, cartels) and polity (voting, campaign contributions). The model predicts Pareto-optimal provision of a pure public good only in the absence of income effects. With income effects or imperfections in the public good, there is suboptimality in its provision, which increases with group size.[12]

It should be noted that matching behavior assumes each actor to have a large amount of information - as formulated here, information on other actors' contributions and matching rates, as well as information on their demand functions for the public good. As group size increases and individual contributions decrease, the cost of obtaining such information increases and its private benefit decreases. Thus the feasibility of matching behavior decreases with group size, even aside from the influence of income effects. It is possible to formulate models, however, which overcome this information problem, through the introduction of entrepreneurs who centralize both the information collection and the matching behavior functions.[13]

The Cournot theory of collective action, which postulates less sophisticated behavior than the matching behavior model, can be obtained as a special case of that model, where the matching rates are constrained to equal zero. The "a-game" of the matching behavior model is, in that case, precisely the game described by the Cournot theory. Our results, depicted in Figures 4 and 5, then indicate that, if there are positive income effects or imperfections in the public good, the Cournot theory becomes increasingly valid as group size increases.

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[12] See McMillan (1979a, 1979b) for a similar approach, also with an emphasis on the consumption-production input distinction.

[13] See Guttman (1980a) for an analysis of this kind.

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